Personal Equation in Astronomy*

By RAYNOR L. DUNCOMBE

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SECTION III

Experimental Psychology and Personal Equation

Experimental psychology, a relatively new science, developed in the latter half of the nineteenth century mainly from a confluence of the streams of modern philosophy and of experimental physiology. The contribution of philosophy stems principally from the British school of philosophical psychology as represented by John Locke, James Mill, and John Stuart Mill. This philosophy was based on empiricism; that is, it regarded individual experience as the basis for the genesis of the mind. Diametrically opposite was the German school of Leibnitz, Kant, and Fichte, which propounded intuitionism. Physiology gave the new science a background of study of the brain, the nervous system, of sensation, and most important, the experimental method.

The three men who stand out as the founders of experimental psychology in the nineteenth century are Fechner, Helmholtz, and Wundt. Prior to their work, psychology was regarded as being in the realm of philosophy and also of physiology; with them it became a science in its own right. Briefly, Fechner perfected the Weber-Fechner law on the relation of stimulus and sensation, Helmholtz experimented with sense-physiology and measured the rate of nervous impulse, while Wundt proposed the first complete system of psychology and established at Leipzig a laboratory to verify experimentally the tenets of his system. To this new science, the oldest also made a contribution, for astronomy gladly gave to the new experimental psychology the problem of personal equation, with extensive data, a method, and a few ideas.

The history of psychology since the time of Wundt deals with a series of different schools of thought, each with its own system in vogue for a period of time. Titchener, a pupil of Wundt, founded the American structuralist school, against which the functionalist group, led by Dewey and Angell, arose in protest at the beginning of the present century. About 1910 the German Gestalt school and American behaviorism appeared on the scene to supplant earlier systems. The "purposive" tenets of McDougall and Freudian theory also appeared, but in the last two decades open-minded psychologists have tended to accept the good points of all these schools, rather than succumb completely to any one dogma.

When the new experimental psychology undertook to analyze, about

*Note: The references in this paper will be given in chronological order at the end of the final installment.

1860, the bothersome astronomical question of personal equation, no distinction was made between the Bradley method and the key-and-chronograph method of observing; both were classed merely as reaction experiments. As a result, much of the early work at Wundt's laboratory in Leipzig centered around the measurement of simple reaction times.

The visual-auditory complication reaction, as Wundt later came to call the Bradley method, could not be explained by the mere measurement of simple reaction times, however, and it was not until 1885 that von Tschisch, a pupil of Wundt's, discovered the importance of attention in this experiment. Wundt described attention as being composed of a *Blinkfeld* and a *Blinkpunkt,* an attention field and an attention point or focus. Von Tschisch found in the Bradley method that the predisposition of the focus of attention could change the apparent order of stimuli by changing the order of perception. It is remarkable that Bessel, in 1822, very nearly stated the importance of attentional predisposition when he noted that one observer might go "over from seeing to hearing, and the other from hearing to seeing."

The method Wundt used for studying the complication seemed practical and direct. It consisted of breaking down th plex reaction into its component simple reactions, measuring the simple reaction times, and then reconstructing the whole complication reaction as the sum of its parts. This additive method set an example for the chronometry of compound reactions which remained in vogue through the first decade of the present century. More recently the additive method of analyzing compound reactions has been discounted, especially by Gestalt psychology, which believes that the "whole" of any act equals more than the sum of its component parts.

The complication experiment has gradually become of less use in psychology because the original astronomical combination of a moving visual stimulus plus a complicating remittant sound has been retained. The eye movements thus introduced tend to invalidate the results. More recently the experiment of comparing a distinct sound with a discrete touch has been made, and has shown that the observer perceives first the stimulus for which he is predisposed. While a complete analysis of the mental processes involved in the complication reaction is still lacking, the effect of attentional predisposition, plus the measurable reaction times, seem to be sufficient to explain the relative personal equations found with the Bradley method.

The simple reaction, as Wundt termed the key-and-chronograph method, has lent itself more readily to psychological experimentation and analysis. The simple reaction was measured first in Wundt's laboratory by recording on a time-registering device the time taken by an observer to react to a signal. These tests established the fact that if the observer was given a ready signal about two seconds before the stimulus, his reaction time was more rapid and more consistent than without the signal. It was therefore decided, that, in order to secure definitive reaction times which could be reproduced by similar experiments in other laboratories, the warning signal must always be given shortly before the stimulus. This is known as simple reaction with preparatory set, while without the warning signal there is no preparatory set. The key-and-chronograph method of observing transits is really a series of simple reactions with preparatory set, so Wundt's method of testing is analogous to this type of astronomical observation.

It was not until 1888 that Lange showed that there are two types of simple reaction with preparatory set, muscular and sensorial, dependent upon the observer's focus of attention. William James,[32] who was first to interpret the "new" German psychology in America, noted that the reaction time was found to differ in the same person according to the direction of his expectant attention. That is, if the subject thought as little as possible of the movement which he was to make, and concentrated upon the signal to be received, the reaction time was longer; if, however, he concentrated upon the muscular response, it was shorter. Lange found his own "muscular" reaction time to average $0^s.123$, while his "sensorial" reaction time averaged about $0^s.230$.

The difference between the "sensory" and "muscular" types of reaction, dependent upon the focus of attention, as discovered by Lange, has been fully verified and the "muscular" response found to be about $0^s.1$ more rapid. Of this difference Pillsbury[93] says, ". . . the muscles of the arm that is to react are more tense when attention is upon movement than when it is upon response. The tension is more diffuse, it spreads through more muscles in the sensory type of adaptation. Physiologically this means a readiness to react with any muscle rather than with the right hand alone, as in the motor form of reaction." The simple reaction times with preparatory set vary with the individual, *e.g.*, for sound from $0^s.120$ to $0^s.180$, for touch from $0^s.117$ to $0^s.183$, and for sight $0^s.150$ to $0^s.225$.

As stated previously, the key-and-chronograph method of observing meridian transits is essentially a series of simple reactions with preparatory set. Method A, however, seems to be the "sensory" type reaction, while method B is apparently the "muscular" type. In method B, where the observer waits until the wire bisects the star before starting to react, the attention is mainly on movement. It may be expressed by $T + t_m$, where T equals the true transit time and t_m equals the "muscular" reaction time. In method A, (bird on wing), the observer's attention is directed toward the reception of the stimulus, since he must decide how far in advance of the actual transit to start his reaction. It can be expressed by $T - \Delta t + t_s$ where T equals the true transit time, Δt equals the distance in time between the star and the wire when the reaction was commenced, and t_s equals the "sensory" reaction time.

Thus, the reaction time with method B equals t_m while with method

A it equals t_s. The total reaction time is greater for A than for B, but since Δt affects T negatively, compensating for t_s, the apparent personal equation, as shown by star transits, is smaller in Method A than in B. That method A has the larger probable error, may also be illustrated by these formulas. While t_s is greater than t_m, each value seems to be fairly consistent in the same observer. The Δt, however, depends on the judgment of the observer, and is therefore variable. It is this judgment term, absent in method B, which increases the probable error of method A. The variation of the personal equation with declination as found with method A also is due to the inability of the observer to adapt his judgment of Δt to varied speeds of the stars.

To astronomers at the beginning of the present century, the various forms of personal error revealed by experiments seemed to be inherent ills of the key-and-chronograph method of observation. Despite Wundt's discovery that the reaction to sound was shorter than the reaction to touch or to taste, some astronomers went back to the Bradley method in an attempt to cut down personal equation by combinations of stimuli to different senses. John Locke of Cincinnati mentioned, in describing his new chronograph and transmitting clock, that with the Bradley method, if the observer so desired, he could feel the clock "beat," or put the transmitting wire into his mouth and taste it. Several intrepid experimenters actually wired themselves into the clock circuit, but no decrease in personal equation was found.

Other astronomers, however, turned their attention toward the new Repsold hand-driven, traveling-wire micrometer, which was just emerging from the developmental stage. Experiments with this new type micrometer had given promising results in the diminution of personal equation, and astronomers hoped that here, at last, was the foil of personality in transit circle work.

SECTION IV

The Traveling-Wire Micrometer

Before tracing the history of the Repsold or traveling-wire micrometer, it should be noted that there is no distinct boundary between the previous period of key-and-chronograph observation and the present period. The traveling-wire micrometer was successfully adopted at Munich and at Potsdam as early as 1890, but on the other hand, the key-and-chronograph remained in use with the nine-inch transit circle at Washington until 1936. The mean date of the adoption of the Repsold micrometer at Cape, Greenwich, and by the six-inch transit circle at Washington is 1909, but no more definite date can be set for the transition in the mode of observing.

The search for some method of following the motion of a star with the transit instrument started early in the nineteenth century. One idea was to move the entire instrument so that the star remained bisected by a stationary wire in the reticle. Emmanuel Liais suggested this in a memoir to the Paris Academy of Sciences in 1858, and Repsold later made the same suggestion in 1887. The plan was considered impractical, however, since the mechanism required was too massive.

Another plan was to keep the instrument stationary, and to follow the star transit by means of movable wires in the reticle. The earliest definite references found to something of this nature are to a plan by Wheatstone. At a Royal Astronomical Society meeting in 1864, Warren De La Rue[15] mentioned that Wheatstone's idea was to have the micrometer wires follow the star's motion and make automatic contacts for the chronograph record.

In 1859 Liais[11] gave a detailed plan for a hand-driven, moving-wire micrometer. Contacts on the head of the micrometer screw operated two chronograph pens, one recording hundredths, and the other whole revolutions. Two more chronograph pens recorded seconds and minutes from the standard clock, while a fifth pen, operated by an observing key, indicated which part of the chronograph record the observer wished to use.

An idea for a traveling-wire micrometer driven by a clock-work mechanism which was mounted right on the side of the micrometer was suggested several years later by C. Braun.[67] In 1863 he revised his plan, and proposed in a letter to Airy that the clock-work mechanism be mounted on a pier beside the transit instrument, and that the micrometer be driven by means of rods through one pivot and down through the cube to the micrometer. In this same year, M. Redier[16] at the Paris Observatory devised a micrometer with the eyepiece and one movable wire driven by clock-work. There is no description of the manner in which the chronograph contacts were made, however, or of how the speed of the wire was controlled.

In 1889 Repsold[30] gave a plan which reverted in principle to Liais' idea for a traveling-wire micrometer. Repsold's scheme for an "impersonal" micrometer was to introduce motion to the wire by hand drive. A rod, having knurled wheels at its extremities, extended through the base of the micrometer box. This rod was geared to the screw moving the eyepiece slide, which in turn was connected by a gear train to the screw operating the movable wire. A contact arm attached to the micrometer box and bearing against a break circuit drum on the micrometer screw, automatically made the chronograph record as the wire moved across the field. In using this type of micrometer, the observer, by means of the knurled wheels, turned the driving rod at a speed to conform with the star's motion, thus keeping the movable wire on the star as it crossed the field. Micrometers of this type were tried by Seeliger at Munich and the Geodetic Institute at Potsdam with success.

Gill, in his *"History and Description of the Royal Observatory, Cape of Good Hope,"* gives an interesting discussion of the Repsold micrometer. Gill wanted an impersonal micrometer for the new transit circle being planned at Cape Observatory in 1890. He seriously considered a method of making a photographic star trail, but, in experimenting with a three-element photo-visual lens, found that the central lens did not react to temperature changes as quickly as the two outer elements. This unequal temperature effect caused an undesirable change in the focal length of the lens, so the scheme was abandoned.

Gill then turned his attention to the traveling-wire micrometer. In 1896 he tried a hand-driven Repsold impersonal micrometer at Repsold's shop in Munich, but felt an undue sense of strain in following the star transits, which he later attributed to lack of practice with this method. He expressed the opinion that it could be improved by some way of driving the wire mechanically. As previously noted, Redier had used a clock drive for a traveling-wire micrometer at Paris in 1863. In 1901 Karl Struve[47] at Königsberg added clock-work driving mechanism to a Repsold micrometer, and in 1902 Cohn[49] discussed some results with this method. In this instance the clock-work and cone to regulate the speed of the wire were mounted right on the micrometer. The wire was set to run slightly slower than the star and the additional impetus was added by the hand drive. Thus, the observer still had to drive by hand to record an accurate transit.

Gill did not like this idea of mounting the clock-work directly on the micrometer, and Repsold stated that "the addition of a clock-work drive was neither necessary nor desirable"[64] in the use of his impersonal micrometer. Repsold advocated, however, that, if astronomers insisted on using some sort of motor drive, C. Braun's scheme of mounting the motor independently of the micrometer and transmitting the driving motion by rods through the pivot and cube of the transit instrument be used. Gill finally settled on an electric motor drive for the new Cape transit circle although Repsold refused to adapt the new micrometer for motor drive until Gill agreed to try the hand drive method as well as the motor.

The micrometer was put into use on the new transit circle in 1905, and the motor drive evolved by Gill was a decided improvement over the previous clock-work drives. It consisted of a constant speed electric motor, a Grubb sun-planet differential gear, and a driving cone mounted beside one pier of the instrument. Rods through the pivot and cube transmitted the motion to the micrometer. Guiding keys, held by the observer, operated the differential gear. The driving mechanism in use with the transit circle at Cape today appears to be essentially the same as Gill's design.

At Greenwich a hand-driven traveling-wire micrometer was adapted to the old transit circle in 1915.[88] More recently a new seven-inch transit circle, designed for motor drive, and closely modeled after the Cape instrument, has been put into use.[101] Since its development by Repsold, the traveling-wire micrometer has also come into use at Washington, Paris, Bergedorf,[78] Trieste,[91] Brussels,[92] and several other principal observatories. At Washington it was first used in 1907 on the six-inch transit circle.

In 1929 C. B. Watts[82] of the U. S. Naval Observatory developed a type of motor drive distinctly different from Gill's design. Essentially it consists of a small, light-weight, two-phase synchronous motor mounted directly on the micrometer box, and connected with the gear train driving the micrometer screw. The speed of the moving wire can be varied to observe stars of different declinations by changing the frequency of the power input. Slight corrections for guiding on a star are applied by introducing small changes in the initial frequency of the input current. This type of motor drive has been in use with the six-inch transit circle at Washington since 1933, and with the nine-inch transit circle since 1936.

M. B. Snyder[51] seems to have been the first to seriously consider mounting an electric motor drive directly on the micrometer. In 1902 he gave the following outline of the three types of motor drive available at that time: "I. A small electric motor may be placed on or near the head of the transit instrument, with its axis parallel to that of the instrument. The varying rate of motion required for change in declination may then be secured by regulating the field of the motor and, if necessary, also that of a small dynamo supplying the current. The main difficulties of this plan are the wide range of speed regulation and the interference due to inertia at starting. II. Equatorial speed that is absolutely constant but slightly regulable may be given the motor, similarly placed, and the differing rate of motion proper to each declination determined by mechanical gearing, consisting principally of two friction disks placed at right angles to each other, or by some other mechanical equivalent . . . III. It may in some instances be desirable to place the electric motor on a separate support near the base of the instrument, and then by means of a light steel shaft entering the axis of the transit finally communicate the required motion to the micrometer screw . . ." Snyder constructed a motor drive of the second type mentioned above which was used on a small transit instrument at Philadelphia in 1902. Interest in the project must have waned, however, as there is no further mention made of it. The type of motor drive suggested in plan I did not become feasible until the later development of a suitable synchronous motor.

The inherent ills of the key-and-chronograph method of observing transits had been the variation of personal equation with the declination, magnitude, and direction of motion of the star. With the advent of the traveling-wire micrometer astronomers were interested to see if these personality factors had been diminished or eliminated. The first encouraging word came from Gill and Hough[56] at Cape. They conducted tests for variation of personal equation with star magnitude, using the old transit circle with the key-and-chronograph, and the new transit circle with the hand-driven traveling-wire micrometer. The magnitude personality was very evident in the results of the old instrument, but no trace of this error could be found in the observations with the impersonal micrometer. Again, J. C. Hammond[71] found no evidence of a magnitude equation between fifth and eighth magnitude in observations with a hand-driven impersonal micrometer on the six-inch transit circle at Washington. R. M. Stewart[61] also found no measurable magnitude equation evident in observations with an impersonal micrometer.

The variation of personal equation with declination in key-and-chronograph observations as caused by the use of observing method A has been previously discussed. This variation entered because the observer attempted to compensate for his reaction time in pressing the key by anticipating the moment the star would cross the micrometer wire. With the traveling-wire micrometer, however, the star is kept continuously bisected as it crosses the micrometer field. The observation then becomes merely a comparison of the position of the object and of the micrometer wire traveling at approximately the same speed. This eliminates the necessity of anticipating the bisection of a moving object with a stationary wire, and thus the variation of personal equation with declination due to this form of anticipation disappears.

An observation with a traveling-wire micrometer may be described as a series of complex reactions with preparatory set. The preparatory set in this case is of the sensorial type; that is, a general readiness on the part of the observer to react in one of two ways. The observer, in guiding, must discriminate which way the star has drifted, and must correct the traveling wire accordingly. In this type of observation, where the observer must discriminate between two stimuli and their respective responses, the reaction times are more variable and on the whole longer than in the so-called simple reaction. With the key-and-chronograph method the observer's reaction time entered the right ascension determination with full weight. In impersonal micrometer observations, however, the reaction time of the observer affects the right ascension determination very slightly, if at all, since the micrometer wires and the star are moving at approximately the same speed. Thus, while in observations with the impersonal micrometer the personal reaction time may be actually greater than in key-and-chronograph observations, its influence on the observed right ascensions is markedly less.

In conjunction with the impersonal micrometer, two methods of registering the position of the movable wire have developed. The earlier type consists of a contact drum connected to the micrometer screw which sends signals to a chronograph at known positions of the traveling wire.[64] More recently, Watts[115] has developed the photographic register, which photographs the micrometer head at four-second intervals as determined by the standard clock. The cardinal difference between the two methods is the time needed for a complete transit observation and the resultant fatigue of the observer. With the contact drum the time required for a complete observation depends on the speed at which the traveling wire is moving, and thus varies with the declination. An equatorial observation may take only thirty seconds, while a near-pole transit will require four or five minutes. With the photographic register, six photographs of the micrometer head, at four-second intervals compose the transit observation; thus it takes no longer to observe a circumpolar star than an equatorial star.

Aside from the time-saving advantage in observing stars of high declination, the photographic register may also offer a definite increase of accuracy in these observations. As mentioned previously, an observation with the impersonal micrometer is a series of reactions with preparatory set. This preparatory set, however, cannot be sustained over very long periods, since muscular and neural fatigue develop, inducing the observer to adopt a less attentional attitude. Thus in observations of near-pole stars with the contact-drum method, the observer is extremely liable to become fatigued and lose his attitude of preparatory set; while with the photographic-head method the observer is able to retain his state of preparatory set while observing near-pole stars as completely as with equatorial stars.

It is common practice in using contact-drum registration to give the observer an audible signal after each drum contact, so that during observations of near-pole stars the observer can relax between signals and thus avoid extensive fatigue. After each succeeding period of relaxation during the observation, however, it does not seem probable that the degree of preparatory set attained by the observer would be as complete as that reached by an observer making a continuous and complete observation in a short period. Another method to avoid fatigue in near-pole observations with contact-drum registration is used at both Cape[66] and Greenwich. The micrometer wire is set to move at equatorial speed and allowed to travel back and forth across the star image; with each coincidence of star and wire the observer operates a punch key. An objection to this method might be, however, that the azimuth stars are not observed in the same manner as the other stars of the program, with a resultant variance in personal equation.

With the increased accuracy of observation afforded by the impersonal micrometer, a new personal error came to light; namely, leading error. In using the Repsold micrometer the observer keeps the movable wire on the star by turning the hand-driven micrometer screw. Thus the observer always has a tactual reminder that the wire and the star are in motion. Also, the fixed wires of the micrometer field drifting past the movable wire and star give a visual impression of relative motion. These reminders that the wire and star are in motion destroy the illusion that the star is being bisected at rest. Thus the observer tends to drive the wire a little ahead, or slightly behind, the center of the star. Leading error should not be confused with bisection error, since the former is present only when the observer is conscious of the motion of the star and wire, and does not exist in measures made at rest. Also the leading error is not eliminated by the reversing prism, as is bisection error, for although the prism reverses the apparent direction of motion of the star, the micrometer screw continues to turn in the same direction. R. M. Stewart[61] found evidence of this leading error with the Repsold micrometer, and M. N. Stoyko[84] has given a mathematical expression of its effect on observed transits. Since the known personal errors evident in the key-and-chronograph method are theoretically eliminated in the Repsold micrometer, the personal equations found with this type of micrometer should be composed mainly of the leading error. This is on the assumption that the bisection error is negated through use of the reversing prism.

The personal equations found with the hand-driven traveling-wire micrometer of the six-inch transit circle in 1930 for five observers were $+0^s.012$, $+0^s.004$, $0^s.000$, $+0^s.015$, and $+0^s.010$.[96] The values are noticeably smaller and much more accordant than the personal equations detected in previous methods of observing.

With the perfection of motor-drive methods for the Repsold micrometer, the leading error has been greatly reduced. The observer no longer has the tactual reminder that the star and wire are in motion, and thus the illusion of bisection at rest is more nearly complete. The motor drive has also reduced the accidental errors of the observation. Thus the personal equation found with the motor-driven Repsold micrometers today are much smaller, more accordant, and more nearly constant for each observer than were those noted with the key-and-chronograph method. For example, the absolute personal equations determined in the current program of the nine-inch transit circle run from $+0^s.001$ to $-0^s.004$, $\pm 0^s.002$, while the relative personal equations determined recently on the six-inch transit circle vary from $-0^s.004$ to $+0^s.005$, $\pm 0^s.002$. The greater range in the latter example may be attributed to the fact that these relative personal equations include the errors in determining the instrumental constants in addition to the errors of observation.

It seems highly probable that the small leading error, evinced by the above personal equations as still evident in the motor-driven Repsold micrometer method, is due mainly to the fixed wires in the micrometer field. These wires apparently moving past the star and movable wire give a visual impression of relative motion which hampers the bisection-at-rest illusion. At present the leading error, and the personal equations, are smaller than the other errors of the transit observations. In the future, when the accuracy of transit observations has been increased to the point where the leading error becomes appreciable, the fixed wires in the micrometer field might be removed, thus completing the bisection-at-rest illusion. The main function of the fixed vertical wires is to indicate to the observer the position of the traveling wire in the field. This same function might be performed by audible signals, which would not give the visual impression of relative motion that the fixed wires do. This change should eliminate all indication that the star and traveling wire are in motion, except perhaps for a slight appreciation of muscular sensation as the observer moves his head to accommodate for the motion of the eyepiece slide. The leading error seems to vary approximately as sec δ. Near the pole, however, at that declination where the observer is no longer conscious of diurnal motion, the leading error ceases to exist. This declination, dependent upon the individual's discrimination threshold, probably varies to some extent among observers.

The bisection error, which was first noticed in key-and-chronograph observations, continued to be present in the traveling-wire method. D. B. Nugent[62] found bisection errors of $0^s.0016$ and $0^s.0260$ in two observers but discovered no correlation between bisection error and the magnitude or declination of the star observed. Other astronomers have also established the presence of bisection error and in the last decade a number of observatories have adopted the reversing prism to eliminate this error. A series of transits observed by one man, without reversing prism, will be affected systematically by the amount of his bisection error. Even if he makes his own determination of the instrumental constants by which these observations are to be reduced, a partial effect of the bisection error will still be present. In bringing together the work of several observers, done without reversing prism, the residual of their combined bisection errors will appear as a systematic error in the resultant star positions, for in combining the results of several observers there is no assurance that their bisection errors will cancel out. According to R. S. Woodworth,[75] in an experiment on judging the center of an interval, the mean estimate of ten subjects in the horizontal coördinate was 0.509, and in the vertical coördinate 0.486.

The psychological reason behind the bisection error is not clear. The nativistic and empiristic schools are at odds as to the manner of development of our space perception. But whether inborn or learned by experience, both schools agree that there is a definite variance in judgment among individuals in estimating the mid-point of an interval. This improper judgment, which becomes habit through usage, seems more correct to each individual than does the true center of the interval as determined by an impersonal machine. One solution to bisection error might be remedial training of judgment for each observer, but a much simpler expedient is the use of the reversing prism.

The reversing prism, however, is not a remedy for all instances of bisection error. To be fully effective in use with the Repsold micrometer having double traveling wires, the observed object must appear symmetrical when centered between the movable wires. The stars satisfy this requirement suitably, but some of the planets, and more notably the sun and moon, do not, since their apparent diameters greatly exceed the angular separation of the movable wires. In these cases the bisections are made at the visible limbs of the object, and it seems to be in these instances that the reversing prism fails to eliminate bisection error in its entirety.

Although there has been extremely little experimentation on this point, it seems highly probable that in centering an arc between the movable wires an observer's bisection error for a right-hand arc, *i.e.*, extremities toward the right, will differ from that for a left-hand arc. This statement applies also to limb declination observations. There is no implication that this difference is constant; its amount undoubtedly varies among individuals. In observations of the sun where limbs I and II are observed both direct and reversed, the residual of the bisection errors for right-hand and left-hand arcs would not affect the observed position of the sun but would appear as a change in the observed diameter of the sun. If a reversing prism is not used in the observation, the residual bisection error would have the effect of shifting the observed position of the sun. In the case of the moon and of the planets where often only one limb is visible, the residual bisection error of observations made with a reversing prism would enter directly the observed position of the object.

This seems to be the case with observations of Mercury made during the latter part of the nineteenth century; and the introduction of more accurate observing methods in the present century, rather than eliminating this effect, has tended to delineate it more clearly. As noted by H. R. Morgan,[107] personal errors of observation tend to average out in a synodic period. Thus the residual bisection error might average out of the observed position of the object in a synodic interval, but it would still affect the observed diameter of the object. If this residual bisection error exists, then in a synodic period it should be revealed as a difference between the observed diameter and the true diameter of the object; and this difference seems to be confirmed in meridian transit observations of Mercury.[110] These observations of Mercury are complicated somewhat by the fact that the center of light is usually observed instead of the limb. Since the visible figure of the planet appears unsymmetrical, except when very near superior conjunction, it seems reasonable to assume that this residual bisection error is present here as well as in limb observations. Another indication of residual bisection error seems present in observations of the moon with the six-inch transit circle at Washington. In combining, by observer, the observations in each synodic period, the resultant semi-diameters have been found to differ systematically. The use of a single movable wire to eliminate residual bisection error in limb observations might be suggested. The inaccuracies introduced in attempting to keep an opaque micrometer wire tangent to a luminous limb might prove greater than the amount of the residual bisection error, but would probably be accidental, rather than systematic.

There are two other forms of bisection error not previously mentioned. One form is that resulting from unsymmetrical star coma. This unequal flaring of light, especially noticeable on brighter stars, is a result of the optical properties of the objective. The presence of coma often induces the observer to bisect the center of illumination, rather than the star itself. This bisection error cancels out of the final position of the star, however, if the instrument, and hence the objective, are reversed at frequent intervals. Again, in observations near the horizon, the star images are elongated in the declination coördinate due to atmospheric dispersion. In effect the images become tiny spectra of the star light. Due to personal differences of color sensitivity, individual observers may bisect the elongated image at different points. Reversal of the instrument does not lessen this effect.

The use of the reversing prism in an observational program is important in determining an accurate relative system of stars. Using modern transit circle methods, the probable error of a single observation as determined at Washington, is $\pm 0^s.011$ in right ascension. Therefore, with approximately thirty observations of each star, it is possible to establish a relative system of stars accurate (with regard to accidental errors) to $0^s.002$ in right ascension. These stars, however, cannot be put on a system of fundamental coördinates with this same degree of accuracy, since the equinox can be determined with only about one-fifth of the accuracy with which the relative star positions can be established.

The astronomers of today are reaping a grim harvest, in fundamental and definitive catalogues, of the personal errors sown in the observational catalogues of the nineteenth century. The variation of right ascension determinations with the declination has been previously noted as a defect of method A in key-and-chronograph observations. This error, which is evident in many of the older observational catalogues, has been reflected in the definitive systems based on these catalogues. In another instance, the corrections for variation of right ascension with star magnitude applied to these old observational catalogues in forming a definitive catalogue have been incorrect, with the result that in the final definitive system the bright stars are rotated with respect to the faint ones.

The personal errors contained in the old observational catalogues also may affect the derived proper motions. In determining accurate proper motions it is desirable to have a relatively long period of time between the end catalogues. H. R. Morgan[107] has pointed out that it is impossible to get very reliable proper motions in ten or twenty years, since the errors of the end positions are too large. Thus in deriving our present proper motions the primary end catalogues were mainly those of the nineteenth century. The best determined of these proper motions have been used to form the basis of certain statistical relations, which have been applied to the great mass of stars. Therefore, if the errors