

THE ROTATION OF THE SPIRAL NEBULA MESSIER 33*

N. U. MAYALL AND L. H. ALLER[†]

ABSTRACT

The investigation is based on radial velocity observations of 25 bright-line condensations well distributed over the spiral (Pl. I), and upon several long exposures that record the spectrum of the unresolved nebulosity of the nuclear region. As preliminary steps in the derivation of the relation between rotational velocity and nuclear distance, the *velocity of the system* is found to be -167 ± 5 km/sec from three different groups of the data; also, the *inclination to the line of sight* of the principal plane of the spiral is found to be 33° from long-exposure direct photographs on $6\frac{1}{2} \times 8\frac{1}{2}$ -inch plates taken with the Crossley reflector. By using these results, together with the assumption of simple circular motion in the plane, rotational velocities of low precision (the linear dispersion of the spectra is 304 Å/mm at λ 3950) are first obtained for the individual condensations (Fig. 1) and then combined to give a first approximation to the *rotational-velocity-curve* (Fig. 2). The latter may be briefly described as follows: the *main body* of the spiral, some 18' in radius, appears to rotate almost like a solid body (rotational velocity increases fairly uniformly with distance), while the *outer parts*, represented by a zone having least and greatest radii of 18' and 30', respectively, appear to rotate like a *planetary system* (rotational velocity decreases with distance). The transition between these two types of motion occurs in the general vicinity of 16' (1000 parsecs) from the center, at which distance the rotational velocity attains a *maximum* of approximately 120 km/sec. The observations define fairly well the character of the motion out to the extreme limits of the spiral (30', or nearly 2000 parsecs) shown on the photographs. Comparison of the results for these most distant regions of M 33 with similar ones for the solar neighborhood of the Galaxy strengthens the current hypothesis that *the sun is located at a considerable distance from the main body of our own stellar system*.

I. INTRODUCTION

The investigation of the rotational motions of the spiral nebulae is of value in at least two respects. In the first place, a knowledge of such motions is fundamental in the study of the dynamics of stellar systems. In the second, a comparative study of the rotational motions of spirals and of the Galaxy may contribute something toward the solution of certain problems of galactic rotation, such as, for example, the determination of the distribution and total mass of the Galaxy, the orbital velocity of the sun, and the position of the solar neighborhood relative to the main body of our stellar system.

The best means available at present for the measurement of nebular rotational velocities is the spectroscope. Unfortunately, its use is attended with many difficulties, owing to the extreme faintness of spirals and their component parts; and the measurements accordingly lack precision. Nevertheless, the application of the spectroscopic method gives a general idea of the character of the motions, especially when, as in the present instance, the velocity measures extend to the outermost parts of a spiral.

A good deal of the earlier work in this field is represented by the pioneer observations of V. M. Slipher² and of F. G. Pease,³ together with the more recent work of M. L. Humason⁴ and of H. W. Babcock.⁵ In most of these investigations the rotational velocities were measured only in the immediate vicinity of the nucleus; the measures stopped at distances far short of the full size of the nebula. A notable exception in this respect is Babcock's study of the Andromeda nebula, in which the rotational velocities are traced to the farthest extensions recognizable on long-exposure photographs.

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[†] Society of Fellows, Harvard University.

² *Lowell Obs. Bull.*, **2**, 65, 1914; *Pop. Astr.*, **23**, 21, 1915; **25**, 36, 1917; **29**, 272, 1921.

³ *Proc. Nat. Acad. Sci.*, **2**, 517, 1916; **4**, 21, 1918; *Pub., A.S.P.*, **28**, 191, 1916.

⁴ *Ann. Rept. of the Director of Mt. W. Obs.*, 1936-1937, p. 31; 1938-39, p. 27.

⁵ *Lick Obs. Bull.*, **19**, 41, 1939 (No. 498).

In the belief that velocity measures which describe the rotational motion of a spiral as a whole are likely to be of more value in general studies of the dynamics of stellar systems, the present investigation has been undertaken as a natural sequel to that of Babcock for M 31. In order to separate the observations from their interpretation, this paper is limited to a presentation of the observational material. The theoretical discussion of the data is made the subject of a paper by Wyse and Mayall, entitled "Distribution of Mass in the Spiral Nebulae Messier 31 and Messier 33" (see p. 24 of this issue).

The particular spiral selected for study is M 33, often termed the "spiral nebula in Triangulum."⁶ It was chosen for two reasons. First, it is the nearest late-type spiral, and its distance, 220,000 parsecs, is accurately known from Hubble's observations of Cepheid variables.⁷ Consequently, the constituent parts are sufficiently bright, and the angular size is sufficiently large, to allow of spectroscopic investigation with a telescope of moderate light-gathering power and of medium focal length. Second, no late-type spiral has hitherto been studied for rotation by the spectroscopic method, despite the significant fact that late-type spirals occur more frequently than others.

M 33 is typically "Sc" in Hubble's system of classification.⁸ Its principal characteristic, besides irregular spiral structure, is a high degree of resolution into stars, groups or clusters of stars, and numerous nebulous condensations. The three brightest of the latter—NGC 604, observed by Slipher⁹ and by Pease,¹⁰ and NGC 588 and 595, observed by Hubble¹¹—have long been known to possess bright-line spectra. In each spectrum the intensity of $H\beta$ is much greater than that of N_2 . On the basis of this criterion, the validity of which has been established by a comparative study of the galactic nebulae,¹² Hubble has inferred that these three condensations in M 33 are similar to the diffuse galactic nebulae.

The occurrence in spirals of condensations that exhibit a bright-line spectrum is a phenomenon of much importance in at least two respects. In the first place, it points to a relationship between nebular contents and nebular type. The relationship, suggested by recent work on the spectra and colors of nebulae, is in the sense that emission patches are common in the Sc or late-type spirals and are seen rarely, if at all, in the Sa or early-type spirals. In the second place, the presence of bright-line objects in spirals is of the greatest value for spectroscopic investigations of internal motions, that is to say, for the measurement of rotational velocities. Owing to the apparent faintness of most condensations (few are brighter than the twelfth photographic magnitude), it is necessary to employ spectrographs with low dispersion, of the order of 250–500 Å/mm. Under these conditions an emission spectrum can be measured more accurately for radial velocity than can an absorption spectrum. This circumstance suggested the possibility that M 33 might be a suitable object for rotational study, provided that a sufficiently large number of emission patches could be identified, and that the principal plane of the spiral was not too greatly inclined from the line of sight. These two conditions are, accordingly, considered first of all.

II. INCLINATION OF THE PRINCIPAL PLANE OF M 33

A casual glance at published photographs of M 33 is likely to convey the impression that the principal plane of the spiral is nearly at right angles to the line of sight. How-

⁶ The position of the nucleus, NGC 598, for 1900 is $\alpha = 1^h 28^m 2$, $\delta = +30^\circ 9'$.

⁷ *Ap. J.*, **63**, 83, 1926; *Mt. W. Contr.*, No. 310; also *The Realm of the Nebulae*, p. 143, Yale University Press, 1936.

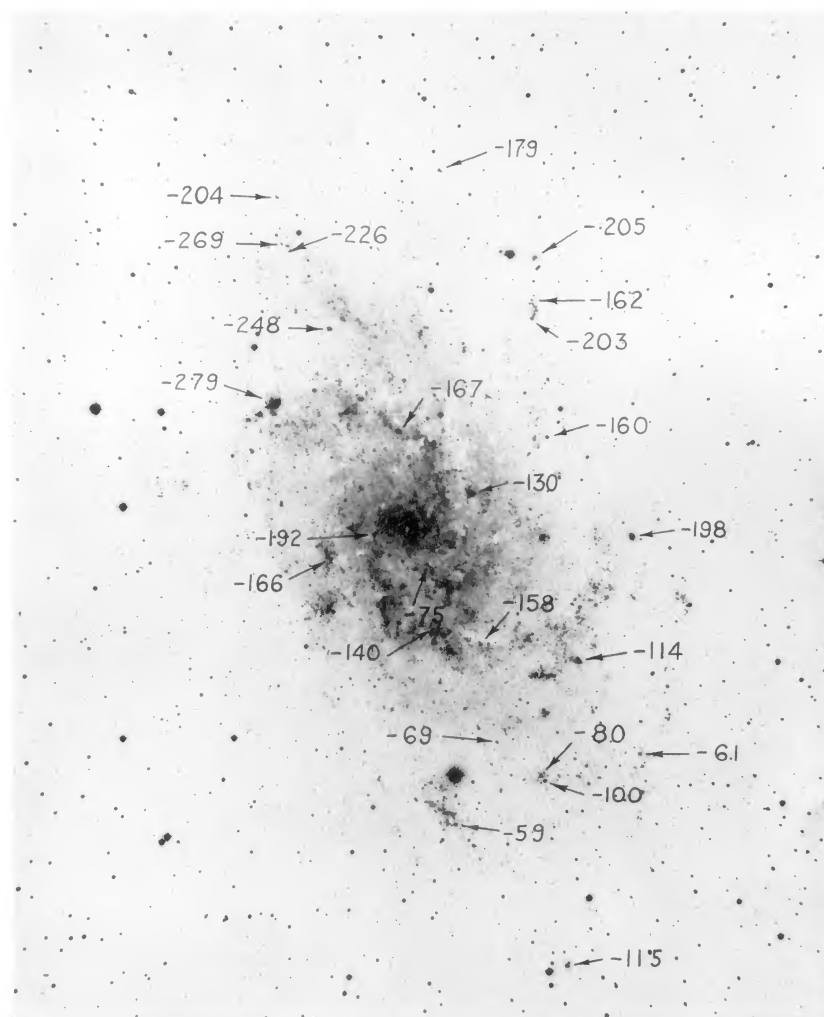
⁸ *Ap. J.*, **64**, 321, 1926; *Mt. W. Contr.*, No. 324.

⁹ *Pop. Astr.*, **23**, 23, 1915; *Proc. Amer. Phil. Soc.*, **56**, 406, 1917.

¹⁰ *Pub. A.S.P.*, **27**, 239, 1915.

¹¹ *Ap. J.*, **63**, 236, 1926; *Mt. W. Contr.*, No. 310.

¹² *Ap. J.*, **66**, 217, 397, 1922; *Mt. W. Contr.*, Nos. 241 and 250.

PLATE I*b*

OBSERVED RADIAL VELOCITIES OF 25 EMISSION CONDENSATIONS IN M 33

ever, as a result of some experimental direct photography with the 36-inch Crossley reflector, in which a plate larger than usual was used,¹³ it became evident that the spiral extends over an approximately elliptical area whose major and minor axes are 62' and 34', respectively, with the major axis lying in position angle 20°. These dimensions are rather well defined on 3-hour exposures made on the fastest available plates, and they represent the limits to which the faintest stars (about the twentieth photographic magnitude) in the spiral can be traced with reasonable confidence.¹⁴ If it is assumed that the outline of the spiral in its own plane is circular, it follows from the above dimensions that M 33 is tipped only 33° from the line of sight.¹⁵ Accordingly, rotational velocities in the principal plane are decreased, in projection, by a factor of $1 - \cos 33^\circ$, or only 16 per cent, and hence their measurement by the spectroscopic method is not precluded by an unfavorable inclination of the spiral.

III. CONDENSATIONS OBSERVED IN M 33

The direct photographs of M 33 referred to in the preceding section were closely examined for condensations that seemed likely to be emission patches. The survey immediately showed that the spiral contains a considerable number of such objects, and that some restrictive rule would have to be employed in selecting those to be observed. The most natural thing to do was to observe them in the order of their brightness. As the work progressed, it became necessary to make exceptions to this rule, in order to extend the observations to the limits of the system, and to obtain a satisfactory distribution of objects along the major and minor axes.

Plate Ia shows the outline of the spiral, with arrows marking 31 condensations whose spectra are available for discussion. Of these objects, 25 exhibit one or more bright lines in their spectra, while 6 show only a continuous spectrum.¹⁶ There are undoubtedly many more emission knots in M 33. In several cases very faint condensations, indistinguishable from stars, happened to fall on the slit when it was placed on a brighter patch, and their spectra are characterized by a single bright line, $\lambda 3727$ [O II]. Since there are innumerable such condensations shown on the photographs, and since most of those observed spectroscopically show bright lines, it seems safe to conclude that emission lines are of very general occurrence in this system. This is not altogether unexpected, in view of a recent report from this observatory on the spectra of some 80 extragalactic nebulae.¹⁷

All the spectroscopic observations were made with a specially designed nebular spectrograph¹⁸ mounted at the primary focus of the Crossley reflector. The exposures range from 1 to 18 hours, the most frequent exposure being one full night in length, although toward the end of the program, when the brighter objects had been observed, the average exposure was of two nights' duration. In a number of instances the maximum slit-

¹³ J. H. Moore, N. U. Mayall, and J. F. Chappell, *Astronomical Photographs Taken at the Lick Observatory*.

¹⁴ Through the courtesy of Dr. Hubble one of the best negatives of M 33 in the Mount Wilson Observatory files was made available for comparison with the Crossley plates. The negative is an 8×10 Seed 30 plate, exposed 6½ hours on August 4–5, 1921, by J. C. Duncan, with the 100-inch reflector stopped to 84 inches. Essentially the same limiting outline for the spiral was obtained from this 100-inch plate as from the Crossley plates.

¹⁵ This angle agrees closely with that obtained from microphotometer tracings by Miss Patterson (*Harvard Bull.*, No. 914, p. 9, December, 1940), who found the inclination to the plane of the sky to be 60°. She also located the major axis in position angle 20°.

¹⁶ All six of these continuous spectra are so narrow as to show few or no absorption lines. On this account, the spectral types are difficult, if not impossible, to estimate. In most cases the distribution of intensity in the spectra suggests an early-type classification. Further discussion of these continuous spectra does not appear warranted.

¹⁷ *Lick Obs. Bull.*, 19, 33, 1939 (No. 497).

¹⁸ *Pub. A.S.P.*, 48, 14, 1936.

length, 6'.2 was used, and in this way it was often possible to record simultaneously the spectra of several objects.

Plate II illustrates the general appearance of several of the spectra. The most conspicuous line—the one farthest to the left in all of the spectra—is $\lambda 3727$ of [O II]. The group of three lines to the far right on some of the plates is composed of (left to right) $H\beta$, N_2 , and N_1 . Lines lying between are Balmer lines, the only exception being, in a

TABLE 1
IDENTIFICATION AND POSITIONS OF CONDENSATIONS IN M 33

Object (1)	x (2)	y (3)	θ (4)	sec θ (5)	R (6)	Spectrum (7)
Anon. 12.....	+29'.0	— 5'.7	19°.0	1.06	30'.8	Continuous
9a.....	+22.4	+ 1.4	6.7	1.01	22.6	
15.....	+20.6	— 8.9	38.2	1.27	26.2	Only $\lambda 3727$
9b.....	+19.4	+ 2.0	10.5	1.02	19.7	
9c.....	+19.0	+ 2.1	11.3	1.02	19.4	Only $\lambda 3727$
3.....	+13.4	+ 1.0	7.7	1.01	13.5	
IC 132.....	+13.4	—12.9	60.5	2.03	27.1	
Anon. 20.....	+11.0	—11.9	63.5	2.22	24.3	Only $\lambda 3727$
IC 133.....	+10.0	—11.6	64.8	2.35	23.4	
NGC 604.....	+10.0	+ 5.8	46.7	1.46	14.6	
Anon. 5.....	+ 7.2	+ 1.8	24.6	1.10	7.9	Continuous
IC 142.....	+ 5.8	— 1.1	19.1	1.06	6.1	
131.....	+ 2.4	— 9.9	82.5		18.3	
NGC 595.....	+ 0.5	— 4.1	86.2		7.5	
Anon. 11.....	0.0	+ 2.6	90.0		4.7	
2.....	— 0.4	+ 5.8	87.8		10.6	
4.....	— 0.4	+ 8.6	88.6		15.7	Continuous
18.....	— 2.9	0.0	0.0	1.00	2.9	Only $\lambda 3727$
NGC 592.....	— 3.6	— 7.6	75.5		14.4	Continuous*
588.....	— 5.5	—12.8	76.8		24.0	
Anon. 13.....	— 6.5	— 1.3	20.3	1.07	6.9	Continuous
16.....	— 6.8	+ 0.6	9.2	1.01	6.9	
IC 137.....	— 8.6	+ 0.5	6.0	1.01	8.6	Continuous
Anon. 14.....	— 8.8	— 1.5	17.1	1.05	9.2	
7.....	—11.8	— 7.0	47.3	1.47	17.4	
IC 135.....	—15.0	— 0.4	2.7	1.00	15.0	
Anon. 8a.....	—18.0	— 2.4	13.7	1.03	18.5	
8b.....	—18.5	— 2.5	14.0	1.03	19.1	
6.....	—18.8	— 8.7	40.3	1.31	24.6	Only $\lambda 3727^\dagger$
19.....	—19.3	+ 3.8	19.7	1.06	20.5	
1.....	—29.8	0.0	0.0	1.00	29.8	

* Hubble has reported (*Ap. J.*, **63**, 260, n. 2, 1926) no trace of diffuse nebulosity around this object. The spectrum shows no bright lines, not even a faint $\lambda 3727$.

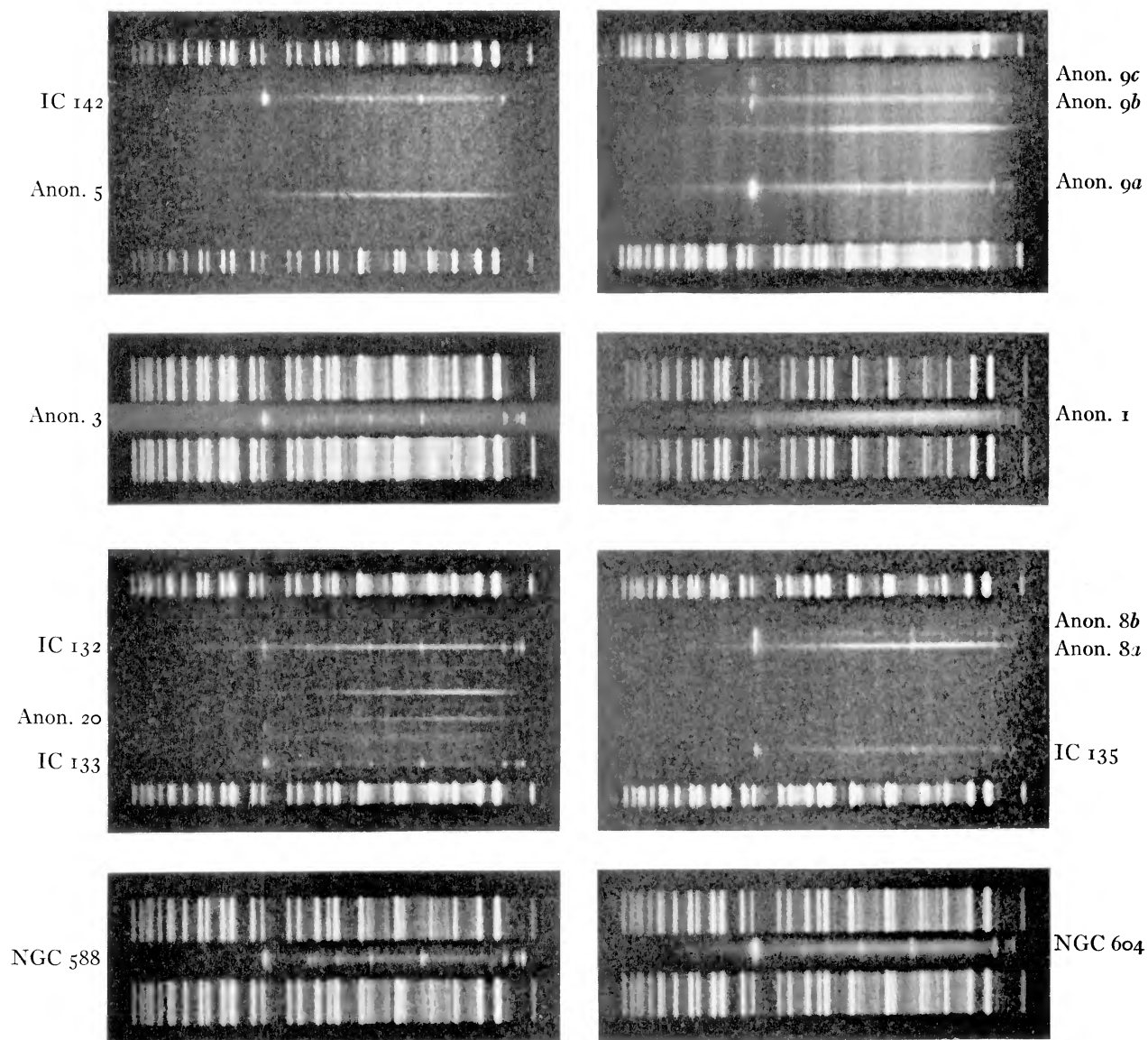
† This appears to be a foreground star superposed on a small and faint patch of nebulosity.

few cases, a line at $\lambda 3869$, which is the violetward component of the [Ne III] pair. The redward component, $\lambda 3968$, is blended with $H\epsilon$.

Table 1 gives data relating to the positions and identifications of the objects marked on Plate I. Column 1 of the table contains, in so far as possible, the catalogue numbers of the condensations observed;¹⁹ in cases for which no such identification is available, arbitrary numbers are assigned. Columns 2 and 3 list the rectangular co-ordinates, x and

¹⁹ Two of the identifications made by means of Dreyer's *First Index Catalogue* (*Memoirs of the Royal Astronomical Society* [London], **51**, 185, 1895), namely, IC 135 and 137, are uncertain because of the inadequacy of the summary description and the inaccuracy of position. The objects listed as IC 134, 136, 139, and 140 could not be found. NGC 603 appears to be a small group of three faint stars.

PLATE II



TYPICAL SPECTRA OF EMISSION CONDENSATIONS IN M 33

The scale of enlargement is $11\times$ the original negatives, on which the linear dispersion at $\lambda 3950$ is $304 \text{ \AA/mm}$

y ; the reference frame is centered on the nucleus, with axes corresponding to those of the apparent elliptical outline of the spiral. The major axis is taken to lie in position angle 20° ; positive x is in the same direction north-following the nucleus, while positive y is in position angle 110° south-following the nucleus. The next three columns give certain data which, calculated from the rectangular co-ordinates and the angle of inclination, and hence referred to the plane of the spiral,²⁰ are used later for the derivation of the relation between rotational velocity and nuclear distance. Column 4 lists for each object the angle, in the plane of the spiral, between the major axis and the radius vector from the nucleus to the object. This angle, hereafter referred to as the azimuth angle θ , is found from the obvious relation $\tan \theta = (y \csc i)/x$, where i is the angle of inclination with respect to the line of sight (33°), as indicated in a preceding paragraph. Column 5 is self-explanatory, while column 6 contains for each object the nuclear distance, R , obtained from the simple formula $R = \sqrt{x^2 + (y \csc i)^2}$. Lastly, column 7 notes those condensations which show either a continuous spectrum or only one bright line: $\lambda 3727$.

IV. RADIAL VELOCITIES OF THE EMISSION-LINE CONDENSATIONS

As soon as the first few spectra had been measured for radial velocity, it became clear that the rotational velocities in M 33 are rather small, being, on the average, only a few times the probable error of a velocity. Hence, in order to improve the accuracy of the measures, a considerable number of duplicate plates has been obtained, and, furthermore, almost every plate has been measured twice by each observer. All this material is collected in Tables 2a and 2b, which are arranged in a manner designed to facilitate the discussion of the accuracy of the data in Section V.

Column 1 of Table 2a requires no comment, but for Table 2b a few remarks are necessary. In order to ascertain whether there are any anomalies in the rotational velocity in the immediate vicinity of the nucleus, such as those found by Babcock²¹ for the Andromeda nebula, the long slit was placed on the minor axis with the nucleus central, and then on the major axis, first, with the nucleus at one end of the slit, second, with it central, and third, with it at the other end. In all cases the image of the nebula was held steady on the slit, and, as the exposures were purposely made long enough to record the spectrum of the faint, unresolved nebulosity, the spectrum of the nucleus itself was, naturally, "burned out." To obtain the spectrum of the nucleus alone, two plates of shorter exposure, each of 6 hours, were obtained; and, since the nucleus is semistellar, it was allowed to drift along the slit by an amount (about $0.5'$) large enough to give a satisfactorily wide spectrum. These spectra of the nucleus and its immediate surroundings are separately discussed in Section VI (b).

Columns 2 and 3 of Tables 2a and 2b are self-explanatory, although, to complete the record, it may be stated that three different kinds of fast emulsions were used. These were: Imperial "1200," Imperial "Eclipse Soft" H & D 850, and Agfa "Spectral Blau Ultra Rapid." It may be of interest to note the rather lengthy average exposure: 9.1 hours. Columns 4 and 5, labeled "Set I," and columns 7 and 8, labeled "Set II," contain the first and second measured velocities, V_1 and V_2 , for each plate; they have been corrected only for the orbital motion of the earth. Set I represents measures by Mayall; Set II, by Aller. The measures are given in detail to indicate the uncertainties inherent in small-scale spectra; differences between the first and the second measures of each plate are given in column 6 for Set I and in column 9 for Set II. These differences are used, in Section V (a), to calculate the probable error of measurement for each observer. Col-

²⁰ This calculation contains the implicit assumption that all the objects lie in the plane of the spiral, and it may be justified by analogy with the galactic diffuse nebulae, which are known to lie close to the galactic plane.

²¹ *Op. cit.*, p. 48.

TABLE 2a
RADIAL VELOCITIES OF BRIGHT-LINE CONDENSATIONS IN M 33

OBJECT IN M 33 (1)	DATE (PST) (2)	EXP. TIME (3)	SET I			SET II			MEAN V (10)	WT. w (11)
			V ₁ (4)	V ₂ (5)	Δ (6)	V ₁ (7)	V ₂ (8)	Δ (9)		
Anon. 9a....	1939 Sept. 8, 9	15 ^h 45 ^m	-184	-176	8	-206	-194	12	-186	1.0
9a....	39 Oct. 8, 9	18 20	220	235	15	234	179	55	221	1.0
15....	39 Nov. 7, 9, 10	16 15	187	210	23	133	171	38	184	0.3
15....	40 Oct. 26, 27, 29	16 00	157	186	29				172	0.3
9b....	39 Sept. 8, 9	15 45	310	300	10	370	328	42	318	1.0
9b....	39 Oct. 8, 9	18 20	198	204	6	244	282	38	220	1.0
9c....	39 Sept. 8, 9	15 45	222	222	0	328	241	87	241	0.3
9c....	39 Oct. 8, 9	18 20	185	166	19	311	272	39	211	0.3
3....	38 Aug. 27	5 00	284	253	31	281	271	10	270	1.0
3....	39 Oct. 15	5 30	238	215	23	234	211	23	225	1.0
IC 132.....	38 Aug. 30	7 00	269	271	2	290	285	5	275	1.0
132.....	39 Oct. 19, 20	9 43	150	138	12	75	67	8	122	1.0
132.....	40 Sept. 7, 8	10 00	217	204	13	218	246	28	217	1.0
Anon. 20....	39 Oct. 19, 20	9 43	142	127	15	104	114	10	126	0.3
20....	40 Sept. 7, 8	10 00	195	206	11	192	202	10	199	0.3
IC 133.....	39 Oct. 19, 20	9 43	102	93	9	166	162	4	118	1.0
133.....	40 Sept. 7, 8	10 00	302	266	36	282	313	31	288	1.0
NGC 604....	38 Aug. 24	4 24	281	281	0	241	284	43	275	1.0
604....	39 Jan. 21	1 00	305	290	15	332	264	68	298	1.0
604....	39 Sept. 4	0 15	248	243	5	221	228	7	239	1.0
604....	39 Sept. 7	1 00	303	302	1	308	309	1	304	1.0
IC 142.....	39 Oct. 14	9 00	161	168	7	170	180	10	167	1.0
131.....	38 Aug. 28	4 50	179	214	35	190	197	7	195	1.0
131.....	39 Oct. 18	5 40	140	134	6	99	98	1	125	1.0
NGC 595....	38 Sept. 1	5 35	169	181	12	198	179	19	179	1.0
595....	39 Sept. 15	3 50	87	82	5	99	59	49	81	1.0
Anon. 11....	38 Sept. 22, 23	15 45	204	162	42	207	217	10	192	1.0
2....	38 Sept. 2, 3	7 50	158	175	17	136	162	26	161	1.0
2....	39 Oct. 16	5 30	174	155	19	206	175	31	172	1.0
18....	39 Oct. 11, 12	15 00	117	66	51	42	27	15	75	0.3
NGC 588....	38 Aug. 31	5 45	178	197	19	210	178	32	190	1.0
588....	39 Aug. 21	4 50	212	194	18	211	221	10	207	1.0
Anon. 16....	39 Sept. 17	6 55	125	158	33	134	137	3	140	1.0
14....	39 Nov. 6	10 00	128	176	48	163	181	18	158	1.0
7....	38 Oct. 20	8 40	85	84	1	100	140	40	95	1.0
7....	40 Sept. 9, 10	7 25	127	156	29				142	0.7
IC 135.....	39 Nov. 4, 5	12 30	56	59	3	106	85	21	69	1.0
Anon. 8a....	38 Nov. 20	9 00	89	58	31	50	86	36	72	1.0
8a....	39 Nov. 4, 5	12 30	98	115	17	38}*	53}*	15	88	1.0
8b....	39 Nov. 4, 5	12 30	87	113	26				100	0.7
6....	38 Oct. 19	9 00	55	65	10	26	103	77	61	0.3
19....	40 Oct. 30, 31	13 00	56	61	5				58	0.7
19....	40 Nov. 23, 24	11 30	63	56	7				60	0.7
1....	38 Nov. 17, 18	13 00	109	140	31	130	157	27	130	1.0
1....	39 Sept. 15, 16	16 46	-96	-90	6	-130	-103	27	-100	1.0

* 8a and 8b measured as one object.

umn 10 contains the mean velocity, V , for each plate, deduced as described in Section V (a). Lastly, column 11 gives the weight, w , for each mean plate velocity, V . These weights are assigned on the following basis: a plate for which there are two measures by each observer, $w = 1.0$; one for which the measures are missing in Set II,²² $w = 0.7$; and one for which only $\lambda 3727$ was measured, $w = 0.3$. The latter assignment depends upon the circumstance that, when more than one line is measured per plate, $\lambda 3727$, because of its favorable position and good intensity in the spectra, contributes, on the average, nearly one-third of the total weight from all the lines.

At this point it is appropriate to describe the manner in which the velocities V_1 and V_2 were obtained. The number of bright lines measured per plate ranged from one ($\lambda 3727$)

TABLE 2b
RADIAL VELOCITIES OF THE NUCLEUS (NGC 598) AND NUCLEAR REGION OF M 33

OBJECT IN M 33 (1)	DATE (PST) (2)	EXP. TIME (3)	SET I			SET II			MEAN V (10)	WT. w (11)
			V_1 (4)	V_2 (5)	Δ (6)	V_1 (7)	V_2 (8)	Δ (9)		
6'.2 nf to nucleus...	1939 Oct. 10, 11	14 ^h 00 ^m	-190	-189	1	-89	-156	67	-170	1.0
3'.1 nf to 3'.1 sp....	40 Oct. 3, 4	18 00	183	180	3				182	0.7
Nucleus drifted.....	38 Aug. 23	6 00	166	161	5	164	152	12	162	1.0
Nucleus drifted.....	38 Aug. 25	6 00	185	164	21	200	195	5	181	1.0
3'.1 np to 3'.1 sf*...	38 Sept. 22, 23	15 45	186	141	45	127	81	46	146	1.0
Nucleus to 6'.2 sp...	39 Oct. 11, 12	15 00	-182	-162	20	-149	-161	12	-167	1.0

* Slit along minor axis; in all other cases, along the major axis.

to ten ($\lambda 3727$, $H\eta$, 3869, $H\zeta$, 3968+ $H\epsilon$, $H\delta$, $H\gamma$, $H\beta$, N_2 , and N_1). The scatter in the velocities from individual lines on a single plate is large, sometimes amounting to as much as 250 km/sec. The range of intensity between the lines is also considerable: from an emission so faint as to be hardly distinguishable, to a radiation so strong as to appear "burned out." The linear dispersion ranges from 240 Å/mm for $\lambda 3727$ to 688 Å/mm for N_1 . In order to take account of all these factors in deriving a velocity for a plate, the shifts for each line have been given weights, which, in the judgment of each observer, seemed most likely to represent the reliability of the measures. Hence the individual velocities, V_1 and V_2 , are, in a sense, weighted means.

V. ACCURACY OF THE RADIAL VELOCITIES

This question may be examined by comparing (a) the repeated measures of a single plate, which will give an estimate for the error of measurement, and (b) the measures of duplicate plates, which will lead to an estimate for the error of a single plate of unit weight.

a) Since there are available two sets of measures, it is possible to derive an error of measurement for each set, to calculate their relative weights, and to combine the two sets. Before this is done, however, it is desirable to know whether any large systematic difference exists between the two sets. From the values of V_1 and V_2 for Set I and Set II given in Tables 2a and 2b it is found that the systematic difference, in the sense Set I minus Set II, amounts to +1.6 and +4.5 km/sec, for the first and second measures in each set, respectively. Since the mean difference, +3.0 km/sec, is so small, it may be concluded that no systematic correction need be applied to one set in order to reduce it to

²² The lack of a few measures in Set II is due to the fact that Aller left the Lick Observatory before all the plates were obtained.

the other. The probable error of measurement for each set may be obtained from the formula

$$\pm 0.6745 \sqrt{\frac{\Sigma v^2}{N - n}} = \pm 0.6745 \sqrt{\frac{\Sigma \Delta^2}{2n}},$$

where $v = \pm \frac{1}{2}\Delta$, $\Sigma v^2 = 2\Sigma(\Delta/2)^2$, n is the number of plates, and $N = 2n$ is the number of observations. If ϵ_I and ϵ_{II} are these errors for Set I and Set II, respectively, then the values of Δ given in columns 6 and 9 of Tables 2a and 2b lead to the following probable errors:

$$\epsilon_I = \pm 10.2 \text{ km/sec}, \quad \epsilon_{II} = \pm 15.9 \text{ km/sec}.$$

Since the relative weights are inversely proportional to the squares of the probable errors, it is evident that

$$w_I = 0.7, \quad w_{II} = 0.3.$$

By using these values of the relative weights, the two sets of measures may be combined and a single velocity deduced for each plate. If V_I and V_{II} are the means of V_I and V_{II} for Set I and Set II, respectively, then the combined velocity, V , may be calculated from the relation

$$V = 0.7V_I + 0.3V_{II}.$$

The values of V obtained in this manner are listed in column 10 of Tables 2a and 2b. The error of measurement associated with V , if termed ϵ_m , is given by the formula

$$\epsilon_m^2 = \left(\frac{\partial V}{\partial V_I}\right)^2 \epsilon_I^2 + \left(\frac{\partial V}{\partial V_{II}}\right)^2 \epsilon_{II}^2 = (0.7)^2 (10.2)^2 + (0.3)^2 (15.9)^2,$$

or

$$\epsilon_m = \pm 8.6 \text{ km/sec}.$$

b) An examination of the data in column 10 shows that there are 16 objects with duplicate plates, one (IC 132) with 3 plates, and one (NGC 604) with 4 plates. Thus there is available a total of 39 plates of 18 objects for a determination of the probable error of a plate of unit weight. Let this quantity be denoted by ϵ_{p+m} to indicate the fact that it is composed of two parts—one, ϵ_p , inherent in the plate, the other, ϵ_m , due to measurement. Then, to take account of the different weights of the plates, it is evident that

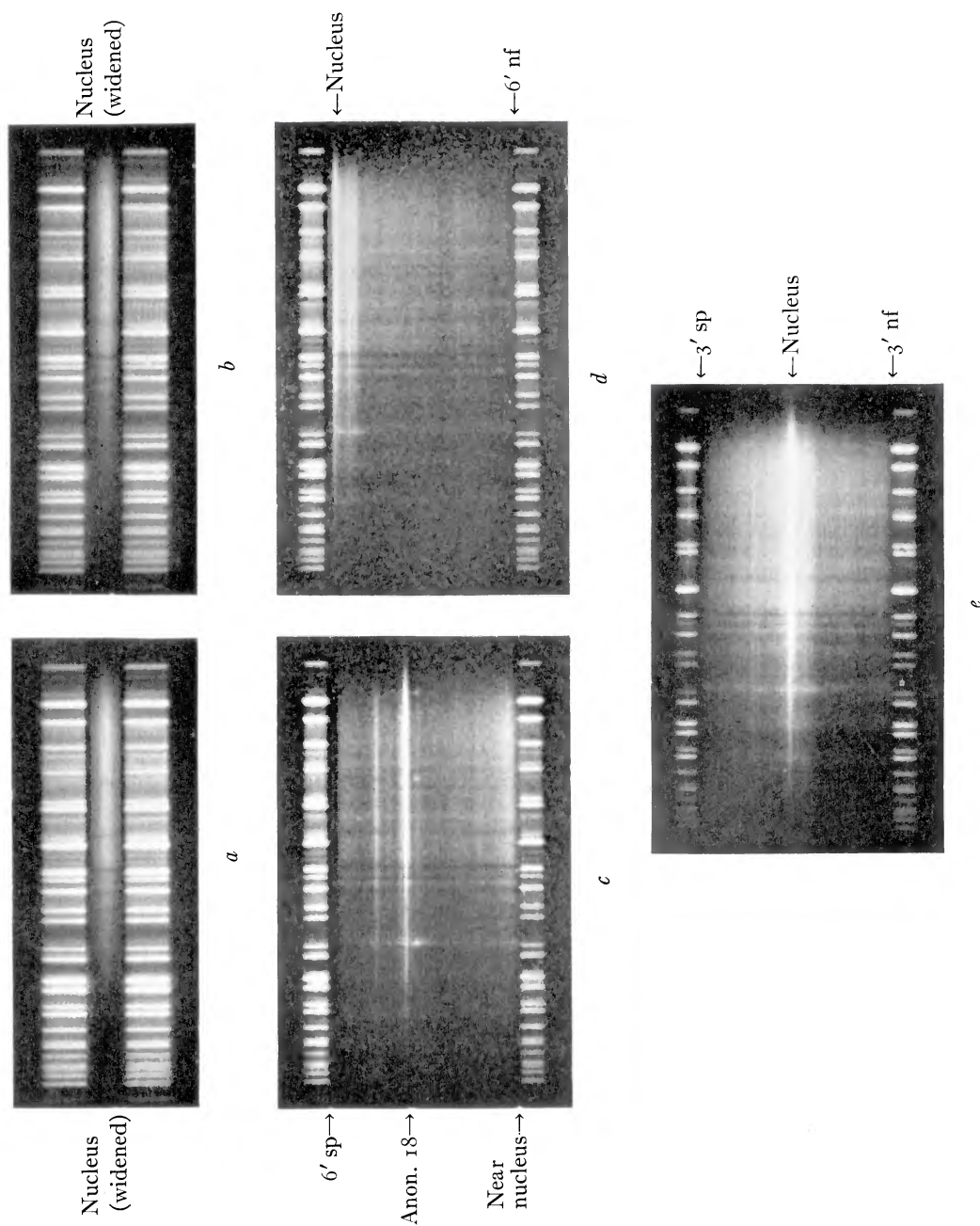
$$\begin{aligned} \epsilon_{p+m} &= \pm 0.6745 \sqrt{\frac{\Sigma wv^2}{N - n}} = \pm 0.6745 \sqrt{\frac{47734}{39 - 18}}, \\ &= \pm 32.1 \text{ km/sec}. \end{aligned}$$

This is a rather large probable error for a single plate; hence, the advantage of taking the mean of two plates, thereby reducing the error to $\pm 32.1/\sqrt{2} = \pm 22.7$ km/sec, is readily apparent. Since a probable error of 23 km/sec corresponds to a displacement, at $\lambda 3950$, of precisely 1μ , this result indicates the microscopic size of the velocity shifts measured in the spectra.

Since the probable error of a plate of unit weight is found to be much larger than the error of measurement, it is clear that the principal source of uncertainty in a derived radial velocity resides in the plate itself. The actual value of this plate error for the present data is given by the relation

$$\epsilon_p^2 = \epsilon_{p+m}^2 - \epsilon_m^2 = (32.1)^2 - (8.6)^2,$$

PLATE III



SPECTRA OF THE NUCLEUS AND NUCLEAR REGION OF M 33

a and *b* are two 6-hour exposures of the nucleus, which was drifted along the slit about 0'.5. *c*, *d*, and *e* are exposures of 15, 14, and 18 hours, respectively, with the slit held steady on the major axis (p.a. = 20°). Enlargement, 11X.

OR

$$\epsilon_p = \pm 30.9 \text{ km/sec.}$$

The result, that the plate error is nearly four times the error of measurement, emphasizes the importance of obtaining several plates per object rather than making repeated measures of a single plate. Unfortunately, when a plate has to be exposed as long as one or two nights, the meeting of this condition consumes a great deal of time. The result is that the measurement of rotational motions in spirals, under present circumstances, proceeds very slowly.

VI. SYSTEMIC RADIAL VELOCITY OF M 33

The first step in determining the rotational velocities of the emission knots observed in M 33 is to obtain a value for the velocity of the spiral as a whole; this quantity will be referred to hereafter as the "systemic" velocity. In its evaluation it will be expedient to discuss separately the spectra and velocities found for (a) the nucleus, (b) the nuclear region, and (c) those condensations on or near the minor axis.

a) *Nucleus*.—The two plates of this object, reproduced in Plate III (a and b), yield a mean velocity of -172 km/sec and a spectral type of A7. Although the spectrum is primarily an absorption spectrum, it is probable that $\lambda 3727$ is present, for measures of a very faint emission feature give a velocity in substantial agreement with that found from the absorption lines.²³ These values for the nuclear velocity and spectral type do not agree very well with earlier Mount Wilson determinations, which are: -70 km/sec and F0 by Adams and Pease,²⁴ -330 km/sec and F0 by Humason,²⁵ and -320 km/sec and F5 quoted by Hubble.²⁶ However, Mr. M. L. Humason, of the Mount Wilson Observatory, kindly informs us, in a private communication, that these early measures may be discarded, since he has recently obtained plates of the nuclear spectrum, with higher dispersion, that are much superior in quality to the earlier ones. His unpublished results for the mean of two plates are: -189 km/sec and A7, which agree quite satisfactorily with the -172 km/sec and A7 mentioned above.

On the basis of all the available material, it would appear that the most probable value of the nuclear velocity lies in the neighborhood of -180 km/sec; but since it is not possible to examine the question of a systematic difference between the two sets of data, the figure -172 km/sec will be used in this paper in order to preserve the homogeneity of the material.

The evaluation of the nuclear velocity has been considered in detail, for the reason that it is important to investigate the degree of accordance that may exist between it and the systemic velocity. On purely dynamical grounds it is difficult to imagine how the two velocities could be appreciably different. In the next two parts of this section, (b) and (c), evidence is adduced which indicates rather definitely that the nuclear and systemic velocities probably are equal to within the errors of observation.

²³ Seyfert (*Ap. J.*, **91**, 538, 1940; *McDonald Contr.*, No. 20) has suggested that the difference of 0.25 mag. between his determination of the color (red index) of the nucleus and that by Stebbins and Whitford (*Ap. J.*, **86**, 255, 1937; *Mt. W. Contr.*, No. 577) "is probably accounted for by the presence of emission lines in the spectrum of the nucleus of M 33," as reported by the present writers (*Pub. A.S.P.*, **51**, 112, 1939). It is to be noted that only one bright line, $\lambda 3727$, was reported, and this in the nuclear region, not in the nucleus itself, so that it is perhaps questionable whether Seyfert's explanation is altogether adequate.

²⁴ *Pub. A.S.P.*, **28**, 34, 1916.

²⁵ *Pub. Lick Obs.*, **18**, 218, 1932 (J. H. Moore's *General Catalogue of Radial Velocities*). Owing to a typographical error, the estimate of type is connected with Slipher's determination of the velocity of NGC 604 from bright lines.

²⁶ *The Realm of the Nebulae*, p. 141. These figures represent a slight revision of the measures given in Moore's *General Catalogue*.

b) Nuclear region.—The spectral characteristics of the unresolved nebulosity out to a distance of some 6' each side of the nucleus, along the major axis (p.a. = 20°), are also shown in Plate III (*c*, *d*, and *e*). As mentioned in Section IV, the region was surveyed by taking three exposures with the nucleus held steady on the south-preceding end, in the middle, and on the north-following end of the slit. All three spectra, as may be seen in the reproduction, appear very similar, and the estimates of spectral type from the three plates fall in the narrow range F9 to G0. A plate taken with the slit placed on the minor axis, nucleus central, has previously been reproduced,²⁷ and from it the spectral type was estimated as G0.²⁸ Therefore, the present data, which have the desirable property of homogeneity, indicate a significant difference in the character of the absorption-line spectrum of the nucleus and its near-by environs, amounting to almost one and a half spectral classes.

Another spectral characteristic of the nuclear region meriting comment is the bright line λ 3727 of [O II]. On all four plates of the nuclear region, this emission may be seen in good strength out to approximately 1' from the nucleus. At that distance its intensity suddenly decreases so much that thereafter it is difficult, except on one plate (that labeled [*c*] in Pl. III), to follow the line across the remaining width of the spectrum. Since there is a more or less corresponding variation in the strength of the continuous spectrum, it is possible that the bright line in question actually is present at distances greater than 1' or 1½' from the nucleus, but that the circumstances of observation were such as to fail to reveal it except on one plate. The presence of λ 3727 in considerable strength, in a small region of unresolved nebulosity immediately surrounding the nucleus, makes it probable that the faint emission feature measured in the spectrum of the nucleus itself is due to unavoidable superposition of both sources of light simultaneously on the slit. So far as can be determined from the data, there is no appreciable variation in the spectral type of the absorption lines within the observed range of 6' each side of the nucleus.

The radial velocity measures of the three major-axis plates and the one minor-axis plate of the nuclear region yield quite consistent results, as may be seen from the values of *V* given in column 10 of Table 2*b*. The range in velocity is from -146 to -182 km/sec. The tabulated values, however, represent the means for each plate of a number of measures made at 1' or 2' intervals from the nucleus. The reason for making these additional measures was to find out whether or not there are small rotational velocities in the nuclear region of M 33, similar to those found by Babcock in M 31. Although, with one exception (slit along the major axis, nucleus central), each spectrum was measured twice by each observer, there is no definite evidence of any velocity variation from 1' to 6' from the nucleus. The scatter in the measures is so considerable that it is not possible to detect the slight inclination of the lines to be expected from the mean rotational velocities, shown below in Figure 2, where, according to the smooth curve, the maximum rotational velocity difference from the nucleus to a distance of 6' would amount to only 20 km/sec in the plane of the spiral. In view of these measures, it seems justifiable to average all the velocity measures for each plate of the nuclear region. When this is done, there results a weighted mean value for the systemic velocity, from the four plates described, of -166 km/sec. This figure may be compared with that found above for the nucleus itself, namely, -172 km/sec. Hence, the present observations furnish support for the assumption that there is little, if any, relative line-of-sight motion between the nucleus of M 33 and the unresolved nebulosity in which it is imbedded.

c) Condensations on or near the minor axis.—Obviously, radial velocities for objects so situated in the spiral that their rotational motions are nearly across the line of sight may also be used to estimate the systemic velocity. It must be assumed in this case, of course, that the motions along the radius vector from the nucleus to the object are so small as to be negligible. To what degree this assumption is valid for the objects ob-

²⁷ *Pub. A.S.P.*, 51, 112, 1939, Pl. XV.

²⁸ *Lick Obs. Bull.*, 19, 36, 1939 (No. 497).

served in M 33 cannot be estimated from the approximate data at hand; but, in the light of current theories of stellar dynamics, it seems a permissible one to make, especially in a first approximation. Accordingly, a third check on the systemic velocity may be obtained by calculating the weighted mean velocity for those condensations in Table 1 for which no value of $\sec \theta$ is given, namely, IC 131, NGC 595, Anon. 11, Anon. 2, and NGC 588 (Anon. 4 and NGC 592 are omitted because their spectra are continuous). The result is a mean velocity of -167 km/sec.

Thus, three different groups of data yield values for the systemic velocity of M 33 as follows: nucleus, -172 ; nuclear region, -166 ; and minor-axis condensations, -167 . All three ways of estimating the same quantity agree so closely that it appears justifiable to form the weighted mean of all the material. Table 3 summarizes the foregoing results and leads to the final value of the systemic velocity, V_s , and its probable error, ϵ_s .

TABLE 3
SYSTEMIC RADIAL VELOCITY OF M 33

Object or Region	Mean Observed Velocity*	Total Assigned Weight	Region
NGC 598.....	-172 km/sec	2.0	Nucleus
6'.2 nf to nucleus.....	-170	1.0	Nuclear region
3'.1 nf to 3'.1 sp.....	-182	0.7	
3'.1 np to 3'.1 sf.....	-146	1.0	
Nucleus to 6'.2 sp.....	-167	1.0	
IC 131.....	-160	2.0	Condensations on or near minor axis
NGC 595.....	-130	2.0	
Anon. 11.....	-192	1.0	
2.....	-166	2.0	
NGC 588.....	-198	2.0	

* The velocity is a mean of two plates when the weight is 2.0.

Systemic velocity, $-167 \pm 5 = V_s \pm \epsilon_s$

VII. ROTATIONAL VELOCITIES OF 20 BRIGHT-LINE CONDENSATIONS IN M 33

At this point it is necessary to introduce a fundamental assumption, which seems to be unavoidable. This assumption is that all the bright-line objects observed for radial velocity move in circular orbits around the nucleus and in the principal plane of the spiral. The justification for this hypothesis of simple circular motion comes mainly from theoretical investigations of dynamics of stellar systems and from studies of the rotational motions of stars in the Galaxy.²⁹ Moreover, unless the departures from circular motion are fairly large in M 33, they will be masked by the rather large errors of observation; and, in a first crude attempt to find out something about the general character of the motion, it appears permissible to use the simplest possible method to derive the rotational velocities. Under the assumption mentioned, the rotational velocity V_R is found from the obvious relation

$$V_R = (V_o - V_s) \sec \theta \sec i.$$

In this equation, θ is the azimuth angle and i the inclination angle—quantities previously defined and evaluated. V_o is the observed velocity for each object, obtained by forming the weighted mean velocity from the several plates, according to the appropriate values of the plate velocity, V , and its weight, w , as listed in columns 10 and 11 of Table 2a.

²⁹ Cf., in this connection, J. H. Oort, *Ap. J.*, **91**, 277, 1940; *M.N.*, **99**, 374, 1939.

Furthermore, if ϵ_0 is the probable error of V_0 , and ϵ_s that of V_s , then the probable error of a rotational velocity, ϵ_R , is easily seen to be given by the formula

$$\epsilon_R^2 = (\epsilon_0^2 + \epsilon_s^2) \sec^2 \theta \sec^2 i,$$

in which the effect of an error in the inclination is neglected.

Application of the above two equations to the data already described leads to the quantities in Table 4. The objects in column 1 are arranged according to their nuclear

TABLE 4
ROTATIONAL VELOCITIES OF 20 BRIGHT-LINE CONDENSATIONS IN M 33

Object (1)	<i>R</i> (2)	<i>V</i> ₀ (3)	ϵ ₀ (4)	<i>w</i> ₀ (5)	<i>V</i> _{<i>R</i>} (6)	ϵ _{<i>R</i>} (7)	<i>w</i> _{<i>R</i>} (8)
		km/sec	km/sec		km/sec	km/sec	
Anon. 1.....	29.8 sp	-115	±23	2.0	+62	±28	2.0
6.....	24.6 sp	61	60	0.3	+165	94	0.2
19.....	20.5 sp	59	28	1.4	+136	36	1.2
8 <i>b</i>	19.1 sp	100	39	0.7	+82	48	0.7
8 <i>a</i>	18.5 sp	80	23	2.0	+107	29	1.9
7.....	17.4 sp	114	25	1.7	+93	45	0.8
IC 135.....	15.0 sp	69	33	1.0	+117	40	1.0
Anon. 14.....	9.2 sp	158	33	1.0	+11	42	0.9
16.....	6.9 sp	140	33	1.0	+32	40	1.0
18.....	2.9 sp	75	60	0.3	+110	72	0.3
IC 142.....	6.1 nf	167	33	1.0	0	42	0.9
Anon. 3.....	13.5 nf	248	23	2.0	-98	28	2.0
NGC 604.....	14.6 nf	279	16	4.0	-195	29	1.9
Anon. 9 <i>c</i>	19.4 nf	226	42	0.6	-71	51	0.6
9 <i>b</i>	19.7 nf	269	23	2.0	-124	29	2.0
9 <i>a</i>	22.6 nf	204	23	2.0	-44	28	2.0
IC 133.....	23.4 nf	203	23	2.0	-101	66	0.4
Anon. 20.....	24.3 nf	162	42	0.6	+13	112	0.1
15.....	26.2 nf	179	42	0.6	-18	64	0.4
IC 132.....	27.1 nf	-205	±19	3.0	-92	±47	0.7

distances, *R*, in column 2; the upper half of the table contains those south-preceding the nucleus, the lower half, those north-following. Columns 3, 4, and 5 contain, respectively, the observed velocity, *V*₀, its probable error, ϵ₀, and its weight, *w*₀. The values given for ϵ₀ are found from the relation

$$\epsilon_0 = \frac{\pm 32.1}{\sqrt{w_0}},$$

in which, as already stated, 32.1 is the probable error of a plate of unit weight, while *w*₀ is, of course, the sum of the weights of all plates of the same object. In order to indicate, as directly as possible, the principal observational result, before the rotational velocities are deduced, the mean observed velocities for the 25 emission condensations listed in Tables 3 and 4 are shown in Plate 1*b*. Columns 6, 7, and 8 list, respectively, the rotational velocity, *V*_{*R*}; its probable error, ϵ_{*R*}; and its weight, *w*_{*R*}. The values of *V*_{*R*} and ϵ_{*R*} were obtained as indicated in the preceding paragraph, while the weights *w*_{*R*} were calculated from the relation *w*_{*R*} = 1600/ϵ_{*R*}², in which the factor of proportionality obviously is the square of the probable error for a rotational velocity having unit weight, i.e., ϵ_{*R*} = 40 km/sec for *w*_{*R*} = 1.0.

To indicate the run of rotational velocity with distance from the nucleus, the values of V_R in Table 4 are plotted against the nuclear distance, R , in Figure 1. Also, to show the uncertainties inherent in the data, the probable errors ϵ_R are shown as vertical lines extending equal distances above and below the points corresponding to the appropriate values of V_R . The figure plainly shows two features of the rotational motion that are worthy of mention. First, the apparent direction of rotation seems well established, for all objects south-preceding the nucleus indicate velocities of recession with respect to the nucleus, while all objects but one (Anon. 20) north-following the nucleus show relative velocities of approach. The actual direction of rotation—in particular, that with respect to the curvature of the spiral arms—is, in this spiral, quite indeterminate. Second, it is

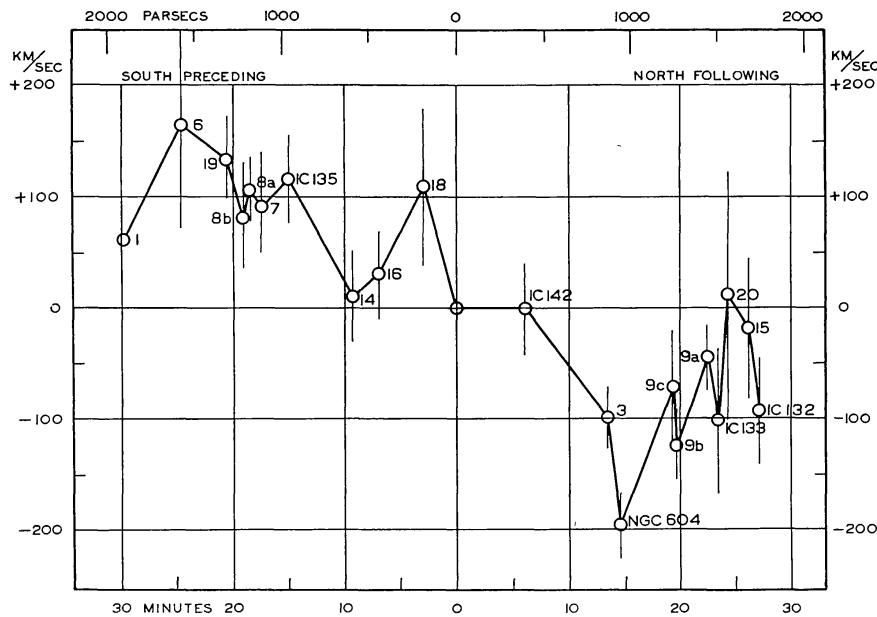


FIG. 1.—Rotational velocities of 20 emission condensations in M 33

clear that the errors of the rotational velocities are not sufficiently small to allow of any but the most general conclusions regarding the nature of the rotation. It is manifestly impossible to decide, for example, whether or not the relation between rotational velocity and nuclear distance is symmetrical about the nucleus. To a first approximation, it appears to be so, and this property of the motion will therefore be assumed in an attempt to define, as far as practicable, the character of the relation between V_R and R .

With so large a scatter in the rotational velocities, it is desirable to try to minimize the effect of the errors of observation on the relation between V_R and R . This may be attempted in one of several ways—for example, by taking the weighted means of several values of V_R corresponding to several consecutive values of R . In other words, overlapping or running weighted means for three objects at a time may be formed, and this procedure was tried for the data in Table 3. However, the resulting “rotational curve” displayed curious irregularities close to, and distant from, the nucleus that were directly traceable to several of the objects (Anon. 18, 6, and 20) whose velocities are those most inaccurately determined. Furthermore, this way of smoothing the data disclosed that one object, NGC 604, tended to distort the curve in the region of maximum rotational velocity. The disturbance was in the sense that inclusion of this one object raised the curve by an amount equal to 50 km/sec over a distance of nearly 10' in R . Finally, the practice of taking running means, while perhaps permissible in this case where large errors and gentle

slopes are involved, introduces systematic errors in the shape of a derived curve. For these reasons, the curve resulting from smoothing the velocities in the manner stated was discarded. Nevertheless, the process proved useful in pointing out NGC 604 as an object having an anomalous velocity. Since NGC 604 is by far the largest and brightest condensation in M 33, it is, perhaps, reasonable to regard it more as a satellite system than a component part of the main spiral. If this were the case, its velocity may be appreciably different in direction and amount from that to be expected on the basis of simple circular motion about the nucleus of M 33.³⁰

Another way in which the general run of the rotational velocities with nuclear distance may be more definitely established is to take weighted means of V_R for different

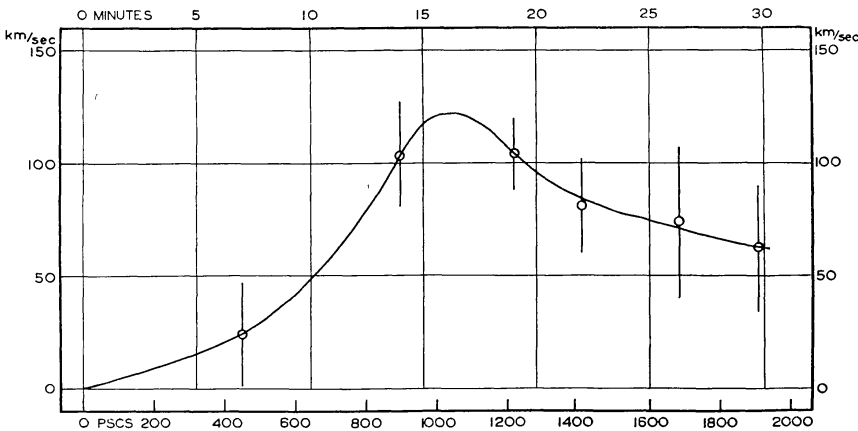


FIG. 2.—Mean rotational velocities in M 33

intervals of R , and several trials of this method have led to a form of the relation between V_R and R that appears to be the most satisfactory one deducible from observations of such low precision. The data, as finally combined into group means, or normal places, are tabulated herewith and are shown in Figure 2.

MEAN ROTATIONAL VELOCITIES IN M 33

Bright-Line Objects in M 33	Interval in R	$\bar{R}$	$\bar{V}_R$ Km/Sec	ϵ	Wt.
IC 142; Anon. 14, 16, 18.....	0'-12'	7'.0	24	± 23	3.1
IC 135; Anon. 3.....	12-16	14.0	104	23	3.0
Anon. 7, 8a, 8b, 9b, 9c.....	16-20	18.9	104	16	6.0
IC 133; Anon. 9a, 19.....	20-24	22.0	81	21	3.6
IC 132; Anon. 6, 15, 20.....	24-28	26.3	74	34	1.4
Anon. 1.....	28-32	29.8	62	28	2.0

The grouping shown in the accompanying table, which omits NGC 604 for the reason previously stated, is for intervals between multiples of 4' in R , except for the region from the nucleus to $R = 12'$. Since in this interval there are only four objects, one of which (Anon. 18) has a large probable error, it has appeared best to take the weighted mean of them all. In any case, the manner in which the group means is taken is arbitrary, and the particular method adopted here represents an effort to form normal places from

³⁰ An analogous situation may exist between the Andromeda nebula (M 31) and its close companion, NGC 221 (M 32). Cf. Hubble, *The Realm of the Nebulae*, p. 141.

those objects whose low-weight velocities display a large but rather symmetrical scatter about the mean (Anon. 6 and 20), as well as to guide the final mean relation through individual velocities having relatively high weight (Anon. 1).

The principal result of this investigation is graphically exhibited in Figure 2, in which the group means tabulated above are plotted together with their probable errors, ϵ . The points are connected with a smooth curve, which is drawn with the realization that it indicates, in only the most general way, the relation between rotational velocity and nuclear distance. In judging the value of the result, it must not be forgotten that departures from the fundamental assumption of simple circular motion will, of course, change the positions of the mean points on the velocity axis and will also increase the dispersion about each mean. It is difficult, if not impossible, to estimate the magnitude of these noncircular components; but, by analogy with the known peculiar motions of the galactic diffuse nebulae, it may be presumed that they are, in general, fairly small, of the order of 10 km/sec.³¹ In any event, it may be supposed that the peculiar motions of the bright-line objects in M 33 are not so large as that indicated for NGC 604, that is to say, of the order of 50 km/sec.

VIII. DISCUSSION OF RESULTS AND COMPARISON OF M 33 WITH OTHER SYSTEMS

Although, as stated at the beginning of this paper, the theoretical discussion of the data is made the subject of another article, there are, nevertheless, a few points in connection with the observations that may appropriately be considered at this time.

The rotation of M 33 may conveniently be described by supposing the spiral to be divided into two parts, corresponding to two regions in which the character of the motion is significantly different. These two regions are: (1) the "main body" of the spiral, some 18' in radius; (2) the "outer parts," represented by a zone having least and greatest radii of 18' and 30', respectively. Briefly stated, the rotation of the main body appears to be of the "solid-body" type, since the more or less uniform increase of the rotational velocity with distance from the center indicates an approach to constant angular velocity, while for the outer parts the rotation appears to be of the "planetary" type, since the rotational velocities decrease with distance from the center.

While the results given here for M 33 may possess some interest and value considered only by themselves, possibly their significance would become plainer if they could be compared with similar results for other spirals. However, an attempt to make such a comparison at the present time meets with two difficulties. In the first place, M 33 is an average spiral in respect to its more important characteristics, such as luminosity and diameter, while the other systems for which rotations are available, are, with one probable exception, giants. Whether the internal motions in giant and average spirals are comparable in respect to their rotations is, on dynamical grounds, open to question, for the larger and more massive systems would be expected to rotate more slowly than average systems. In the second place, the rotation measures that have heretofore been published for other spirals, with the exception of the Andromeda nebula, have not yet been published in sufficient detail; nor do they extend far enough throughout the spirals to allow of a fair comparison with those reported here for M 33. In order to bring out more plainly these difficulties and to indicate in which directions further rotational measures appear to be especially desirable, the following paragraphs and tables summarize the principal results for the few nebulae whose rotations have been investigated in some detail.

There are currently available detailed results for five systems: the Andromeda nebula (M 31), Messier 33, and NGC 4594, 4111, and 3115. The following table gives the more

³¹ This figure is based on the result found by Campbell and Moore (*Pub. Lick Obs.*, 13, 182, 1918) that "the average radial velocity of the 5 extended irregular nebulae with reference to the stellar system is 11 km/sec." The five nebulae are: Orion, η Carinae, Trifid, M 8, and the Horseshoe.

important physical characteristics of these systems, as collected from the various sources noted below.

Nebula (1)	Type (2)	Distance (Parsecs) (3)	Luminosity (Pg. Mag.) (4)	Diameter (Parsecs) (5)
M 33.....	Sc	2.2×10^5	-14.9	4,000
M 31.....	Sb	2.1×10^5	17.5	12,000
4594.....	Sa	1.9×10^6	18.5	3,900
4111.....	Sa	1.6×10^6	14.5	1,600
3115.....	E7	1.5×10^6	-16.2	1,700

The distance, luminosity, and diameter given in columns 3, 4, and 5, respectively, for M 31 and for M 33, are taken from Hubble's book, *The Realm of the Nebulae*, page 126. The distances listed for 4594 and 4111 are calculated from their observed velocities,³² corrected for galactic rotation,³³ using the velocity-distance relation.³⁴ The distance quoted for 3115 is an estimate by Oort,³⁵ who gives reasons for believing that the distance obtained from the velocity-distance relation, 8.6×10^5 parsecs, probably is too small, although he qualifies his estimate with the remark that "the true distance may well differ from it by a factor as large as 2 in either direction." The absolute magnitudes for the same three objects are based on the tabulated distances and on Shapley and Ames's apparent total magnitudes,³⁶ corrected for zero point by -0.08 mag., as suggested by Stebbins and Whitford's photoelectric work on nebular magnitudes;³⁷ they are also corrected by -0.25 (csc $b - 1$) for galactic absorption, as suggested by Hubble's counts of extragalactic nebulae.³⁸ The red-shift correction to the apparent magnitudes may be neglected for these near-by systems.

Thus, in respect to their luminosity and size, it appears that M 31, NGC 4594, and NGC 3115 are giants, while M 33 and NGC 4111 are average systems. If the observed rotational velocities are now used to calculate the periods of rotation,³⁹ and the latter, in turn, are used to estimate the masses,⁴⁰ then it likewise seems to be true that there exists a similar division of this group of nebulae in respect to their masses. The results of such a calculation are given in the table herewith, which also includes the current estimates of the mass of the Galaxy and of the period of rotation for the solar neighborhood. In order not to make misleading comparisons and to represent the observations properly, it seems advisable to list the periods for three regions: the nuclear region, the

³² These are: for 4594, $+1180$ km/sec, Pease (*Proc. Nat. Acad. Sci.*, **2**, 517, 1916; *Mt. W. Comm.*, No. 32); for 4111, $+810$ km/sec, Humason (*Ann. Rept. of the Director of Mt. W. Obs.*, 1936-1937, p. 31).

³³ The solar galactic rotation term is taken as $+275 \cos(l - 55^\circ) \cos b$, where l and b are galactic coordinates.

³⁴ The velocity-distance factor employed here is 525 km/sec/ 10^6 parsecs (Hubble, *Ap. J.*, **84**, 292, 1936; *Mt. W. Contr.*, No. 549).

³⁵ *Op. cit.*, p. 295.

³⁶ *Harvard Ann.*, **88**, No. 2, 1932.

³⁷ *Ap. J.*, **86**, 264, 1937; *Mt. W. Contr.*, No. 577.

³⁸ *Ap. J.*, **84**, 273, 1936; *Mt. W. Contr.*, No. 549.

³⁹ If V_R is the rotational velocity in km/sec, R the nuclear distance in seconds of arc, and d the distance to the spiral in parsecs, then P , the period of rotation in years, is given by the formula $P = 29.7 \times d \times R/V_R$.

⁴⁰ The mass in units of the sun, $M_\odot$, may be found from the relation

$$M_\odot = 1.13 \times 10^{-3} \times d \times R \times V_R^2 = 3.81 \times 10^{-5} \times P \times V_R^3.$$

ROTATION OF MESSIER 33

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main body, and the outer parts; this is done in columns 3, 4, and 5 upon the basis of the sources noted below.

NEBULA (1)	TYPE (2)	PERIOD IN YEARS			MASS IN SUNS (6)
		Nuclear Region (3)	Main Body (4)	Outer Parts (5)	
Galaxy.....	(Sc)			2.2×10^8	1 to 2×10^{11}
M 33.....	Sc		5.9×10^7	0.6 to 2.0×10^8	1.7×10^9
M 31.....	Sb	1.1×10^7	9.2×10^7		9.5×10^{10}
4594.....	Sa		2.0×10^7		2.8×10^{10}
4111.....	Sa	1.4×10^6			
3115.....	E7		4.5×10^6		1.5×10^{10}

The periods given in columns 4 and 5 for M 33 are derived from the mean observed rotational velocities shown in Figure 2 of this paper; for M 31 the periods are those deduced by Babcock;⁴¹ for 4594 it is computed from Pease's rotation measures;⁴² for 4111 and 3115 they are obtained from Humason's results.⁴³ The masses given in column 6 for M 33 and M 31 are those deduced in the following paper, while for 4594 and 3115 they have been calculated, as indicated in the preceding paragraph, from the periods. For the Galaxy, M 31, M 33, 4594, and, in lesser degree, for 3115, the masses tabulated may suggest the correct order of magnitude of the total mass, since the rotational measures in these systems extend to distances fairly large compared with the size of the systems. In the case of 4111, however, Humason has published his results in detail only for the central lens; for the rest of the nebula, out to at least $50''$, he finds that "the rotation decreases with increasing distance in a manner which suggests the operation of an inverse quadratic law." Until the measures for these outer parts are published in detail, it does not appear worth while to try to estimate the total mass for 4111.

The foregoing table shows rather definitely that the Galaxy, M 31, 4594, and probably 3115 are giant systems in respect to their masses. Furthermore, M 33, with the lowest mass but with other characteristics that are average, presumably is also of average mass. On the same grounds, 4111 may be assumed to be of average mass. Hence, in this group of six nebulae there probably are four giant and two average stellar systems. One of the latter is M 33, and, therefore, its rotational characteristics are strictly comparable only with the other, NGC 4111, for which, at present, detailed rotation measures are available only for the nuclear region. This circumstance suggests that further comparison of these two average spirals—at least for the present—is unprofitable.

A comparison of the rotation of the main body of M 33 with the rotations of the corresponding parts of M 31, 4594, and 3115 probably means little, for the reason already stated, namely, that M 33 is an average spiral, while the other three are giants. There seems to be no serious objection, however, to comparing the rotational properties of the principal parts of these giants; and, if this is done, there is a suggestion that the period of rotation increases with advancing nebular type, for the above table yields the sequence of periods in years, 4.5×10^6 , 2.0×10^7 , and 9.2×10^7 , corresponding to E7, Sa, and Sb, respectively. The correlation has already been noted by Hubble, who has described, as follows, a similar result based upon Humason's rotation measures: "From E7 to

⁴¹ *Op. cit.*, p. 48.

⁴² *Proc. Nat. Acad. Sci.*, **2**, 520, 1916.

⁴³ *Ann. Rept. of the Director of Mt. W. Obs.*, 1936-1937, p. 31.

the end of the sequence, the periods of rotation, *as indicated by the table*, seem to increase steadily and rapidly—from about 3 million years to 200 million years.”⁴⁴ The table referred to presented results for the six systems NGC 3115 (E7), 2685 (So), 4111 (Sa), 4594 (Sa), M 31 (Sb), and the Galaxy (Sc). The number of objects in this group is reduced to four, however, if the correlation of the periods of rotation is made for only the main bodies of these spirals; as mentioned above, the available data lead to periods for 4111 and for the Galaxy that refer to the nuclear and outer regions, respectively. The detailed measures for 2685 have not yet been published.

It is an interesting question whether the period of rotation for the principal part of the Galaxy, presumably a giant Sc, is still longer than any of the above periods, and also whether the periods for the earlier-type elliptical nebulae increase from E7 to E0, as might be expected purely from their decreasing ellipticity. If further observations should demonstrate that a minimum rotation period, or a maximum spin, occurs in the vicinity of E7 to Sa—a stage in the sequence termed “So” by Hubble—then the rotation measures would yield a datum of much significance for the study of the dynamics of nebulae, for it is just at this stage in the system of classification that spiral structure becomes recognizable.

As for the rotation of the outer part of M 33, it appears that this, likewise, stands by itself, to an even higher degree than for the other parts of the spiral. Rotation measures for the corresponding region in other spirals are not yet available, for the very good reason that such measures are extremely difficult to obtain. Even in the case of the Andromeda nebula, in which the measures extend to 100' from the center, it is doubtful whether the outermost parts have been reached, for the rotational velocity is observed to be still increasing. The photoelectric measures of surface brightness in the vicinity of this object made by Stebbins and Whitford⁴⁵ and the distribution of the globular clusters discovered by Hubble⁴⁶ are additional evidence that M 31 extends considerably beyond the point where the rotation measures stop.⁴⁷

There is one point in connection with the rotation of the outer parts of M 33 that may well be emphasized, because of its bearing on the estimation of the sun's relative position in the Galaxy. In the solar neighborhood it is now definitely established that the rotational velocity decreases with the distance from the center and that the rate of decrease, according to an authoritative estimate,⁴⁸ is 0.005 km/sec/parsec. In the outer parts of M 33, at a nuclear distance of 1600 parsecs, the rotational velocity also decreases, approximately 0.030 km/sec/parsec. Since the sun is of the order of 8000 parsecs from the galactic center, the comparison of these figures strongly suggests the hypothesis that the solar neighborhood lies far out from the main body of the Galaxy. This idea is, of course, by no means new; for example, Lindblad has expressed it, on the basis of more general dynamical considerations.⁴⁹ However, the present observations may be regarded as furnishing a bit of direct observational evidence in support of the hypothesis mentioned.

The comparison of the outer parts of M 33 and of the Galaxy given in the preceding paragraph suggests that it may also be of interest to calculate Oort's rotational constant A for several points in the outer regions in M 33. For this purpose it is convenient, in

⁴⁴ Reported in an unpublished paper read at the dedication of the McDonald Observatory, May 7, 1939.

⁴⁵ *Proc. Nat. Acad. Sci.*, **20**, 93, 1934; *Mt. W. Comm.*, No. 113.

⁴⁶ *Ap. J.*, **76**, 44, 1932; *Mt. W. Contr.*, No. 452.

⁴⁷ After this was written, R. C. Williams and W. A. Hiltner (*Michigan Univ. Obs. Pub.*, **8**, 103, 1941) published a paper on the “Dimensions and Shape of the Andromeda Nebula.” By using a new type of highly sensitive microphotometer of their own design, which traces directly the isophotal contours from photographs, they found that “the apparent diameter of the Andromeda nebula along the major axis must be at least 400', or 6°.7.” The linear diameter, therefore, is not less than 25,000 parsecs, or 80,000 light-years.

⁴⁸ Oort, *M.N.*, **99**, 375, 1939.

⁴⁹ *Festschrift Tillägnad Östen Bergstrand*, p. 30, 1938.

order to use the data presented in Figure 2, to transform the usual expression for A , which involves the central force K , into a form which includes only the rotational velocity V_R and the distance R . From the relation, $V_R^2 = KR$ it follows that

$$A = \frac{1}{2} \left(\frac{V_R}{R} - \frac{dV_R}{dR} \right).$$

From the smooth curve drawn through the mean points plotted in Figure 2, it is easy to determine graphically values of dV_R/dR , which lead to the following approximate values of A :

R	20	25	30 minutes of arc
R	1280	1600	1920 parsecs
V_R	98	74	62 km/sec
dV_R/dR	-0.091	-0.051	-0.029 km/sec/parsec
A	+0.084	+0.048	+0.030 km/sec/parsec

According to Oort,⁵⁰ a provisional value of $A = +0.018$ km/sec/parsec may be adopted for the region surrounding the sun in the Galaxy. Hence, even at a nuclear distance of 1900 parsecs in M 33, a region has not yet been reached wherein the rotational motion is comparable with that existing in the solar neighborhood. The rather marked variation of A with nuclear distance in the outer parts of M 33 is also a point of difference worth noting; and, if the variation indicated above is maintained beyond where the observations end, a rough extrapolation suggests that A would decrease to $+0.018$ km/sec/parsec at a nuclear distance of approximately 2200 parsecs. This distance is nearly twice the radius of the main body of M 33, and, if an analogy with the Galaxy may again be drawn, then the diameter of the main part of the Galaxy may be inferred to be of the order of 8000 parsecs. Since this inference is justifiable only to the degree that the unknown distributions of mass in the two systems are similar, its speculative character is obvious.

The foregoing review of the rotational motions observed in spirals appears to shed more light on what is needed in the way of further observations than it does on the nature and significance of the rotational motions measured up to now. From the point of view of correlating these motions with the progressive structural changes that are so well arranged by means of the system of classification, it would seem that rotational measures are highly desirable for, first, a representative collection of nebulae of all types.⁵¹ Second, in order to insure a valid comparison between the different types of objects, it would be advantageous if the nebulae selected for observation had nearly the same physical properties, such as luminosity, size, and mass. Third, in order to make an adequate comparison of the homologous parts of nebulae of different types, it is important that the rotation measures extend from the nucleus to the outermost parts. Since the meeting of any one of these conditions presents an observational problem of the first magnitude, it is probable that considerable time will be required before a well-rounded body of factual knowledge is acquired in the field of nebular rotations.

LICK OBSERVATORY
MOUNT HAMILTON, CALIFORNIA
March 1941

⁵⁰ *M.N.*, **99**, 374, 1939.

⁵¹ It is a curious fact that no rotation measures for the barred spirals have yet been published.