DIFFUSE RADIATION IN THE GALAXY

L. G. HENYEY AND J. L. GREENSTEIN[1]

ABSTRACT

Observations have been obtained to verify the existence of diffuse interstellar radiation. A Fabry photometer, attached to the 40-inch refractor at the Yerkes Observatory, was used to measure the brightness of regions over a wide range of galactic latitude. The intensities in the photographic region of the spectrum were calibrated by means of the Polar Sequence stars. The mean of four such runs across the Milky Way, on circles of nearly constant longitude, $l = 40°$, shows a maximum of brightness of 80 stars of the tenth magnitude per square degree for the diffuse extra-terrestrial radiation. The mean of three runs near $l = 140°$ shows a maximum of 35 in the same units.

It is shown that the observed intensity of diffuse light may be explained as scattered stellar radiation if the phase function governing the scattering of starlight by the interstellar matter is strongly forward-throwing. The concentration of the diffuse light toward the galactic circle is also in agreement with this property of the phase function. The observations also indicate that the scattering efficiency, or albedo, of the particles is greater than 0.3.

INTRODUCTION

The light of the night sky has been partly resolved into its various components by van Rhijn,[2] Dufay,[3] Wang Shih-Ky,[4] and others. After subtracting the contributions to the total radiation which arise from the permanent aurora, the zodiacal light, and starlight, a residual illumination was found which was ascribed to stellar radiation diffused by interstellar material. In view of the great optical thickness of a dark nebula, the scattering power of such an object should be high; and a dark nebula, illuminated by stars, should be brighter than the surrounding regions of the Milky Way if they are relatively transparent. Observations by Struve and Elvey[5] revealed only small contrast between the dark nebulae and normal regions of the Milky Way. From this observation they concluded that either the scattering efficiency (albedo) of the particles was very low or there existed considerable diffuse radiation in the Milky Way. The latter hypothesis was shown to be correct by Elvey and Roach,[6] who measured photoelectrically the surface brightness of the night sky and determined the variation of the intensity of light of galactic origin with galactic latitude and longitude. After subtracting the component arising from starlight, a considerable residual intensity was found, the mean value of which was 57 stars of the tenth magnitude per square degree in the galactic plane.

The measures by Elvey and Roach are subject to some uncertainty because of the large area of the sky included in each measurement. The measured intensity must be corrected for the light of all stars within a field of 9 square degrees. The Fabry photometer, constructed by Struve and Elvey[5] for the 40-inch refractor at the Yerkes Observatory, can be used for a more detailed study. The photometer records the surface brightness of the sky, or of a star, in an image 0.5 mm in diameter, with five minutes' exposure on Eastman 40 plates. A complete set of observations covering 60° across the Milky Way, and including exposures on standard stars, requires four hours for completion. Observations can be made only on perfect photometric nights free from any trace of

[1] National Research Fellow, at the time of the beginning of this investigation.

[2] *Pub. Astr. Lab. Groningen*, **31**, 1921.

[3] *Etude de la lumière du fond du ciel nocturne*, Paris, 1934.

[4] *Pub. Obs. Lyon*, **1**, No. 19, 1936.

[5] *Ap. J.*, **83**, 162, 1936.

[6] *Ap. J.*, **85**, 213, 1937.

polar aurora. We present here the results of the detailed investigation of two regions in the Milky Way—one in Cygnus near $l = 40^{\circ}$, the other in the region of the dark nebulae in Taurus and Auriga, near $l = 140^{\circ}$.

OBSERVATIONAL TECHNIQUE

The field of the sky included in each exposure is defined by a circular diaphragm in the focal plane of the 40-inch, with a diameter of 126″. The observer chooses a field, near a predetermined galactic latitude and longitude, such that all stars brighter than the limit of visibility with the 40-inch refractor are excluded from the field of the diaphragm. This limit is fainter than visual magnitude 15, and it is probable that all stars are excluded brighter than photographic magnitude 16. For this reason the correction for unresolved starlight is less serious than it was in the observations of Elvey and Roach and is probably free from large systematic errors depending on galactic latitude. The observing program for each night was so designed that points at a nearly constant galactic longitude, with galactic latitudes ranging from $+30^{\circ}$ to -30°, could be observed with a minimum change in zenith distance. The observations were taken near the height of the pole. For purposes of calibration, exposures were made on stars of the North Polar Sequence, usually on numbers 14, 15, 16, and 17, with the same exposure time but with a diaphragm 18″ in diameter. The images produced by the stars are of the same quality and size as those of the sky; the smaller diaphragm serves merely to reduce the effect of the sky background on the stellar images. Exposures with the larger diaphragm were made on the sky at the North Pole, to serve as a check on the constancy of the polar aurora and on the transparency. The surface brightness of the night sky in the Milky Way is only 0.3–0.4 mag. brighter than near the galactic pole. The North Polar Sequence stars are too widely separated in magnitude to define the characteristic curve of the plate adequately, and a tube photometer was used to supply the plate calibration. The zero point of the photometer curve was set by the densities of the stellar images. The densities were measured on a Ross photometer. It was found that the total mean error of a determination of surface brightness was of the order of ± 0.03 mag. The image density of the sky exposures was close to that at the center of the straight-line portion of the characteristic curve. No systematic deviation of the North Polar Sequence stars from the characteristic curve could be detected.

The measured surface brightnesses include direct and scattered auroral light, zodiacal light, and the light of faint stars. The auroral and zodiacal light both vary from night to night, and their absolute amounts are not accurately known. The procedure of reduction comprises two steps. The intensity of the zodiacal light at the ecliptic latitude and longitude may be derived, approximately, from the isophotes published by Elvey and Roach.[6] The variation of the auroral light with zenith distance can be theoretically predicted if the absolute intensity is known at any one point. Unfortunately, it was found that the auroral light usually varies with azimuth at Yerkes, being stronger at the North Pole than at equal altitude in other azimuths. A variation of the permanent auroral emission with the geomagnetic latitude may be present. It seems best to assume that the diffuse galactic radiation is small, or zero, at points 30° distant from the galactic circle. We use the observed values of the residual surface brightness at these points, after subtracting the zodiacal light and starlight, to fix the absolute amount of the auroral light for the mean azimuth of the observations. This method of reduction also corrects for any errors in the adopted values of the zodiacal light, as well as for the extra-terrestrial radiation scattered in the earth's atmosphere. The theoretical variation of auroral light with zenith distance permits us to subtract out the auroral light at each point, and we finally obtain the residual galactic light. We depend, essentially, on the rather sudden increase in the galactic light near the Milky Way plane.

The observations must also be corrected for the light of the stars fainter than the sixteenth magnitude. Since the star counts are not available for the actual fields observed, the corrections were based on the star counts of Seares and Joyner[7] and of van Rhijn,[8] interpolated for the galactic latitude and longitude of the observations. The extrapolation of the tables for fainter stars introduces, in general, only small uncertainty into the correction for the unresolved stars. To determine more precisely the total starlight, the mean star counts were corrected by using the detailed counts to the eighteenth magnitude by Miller[9] in Cygnus and to the fifteenth magnitude by McCuskey[10] in Tau-Aur. The star correction is uncertain near the galactic plane in Cygnus, where the actual illumination contributed by stars of each successively fainter magnitude group is only slowly decreasing at the limit of the available star counts.

TABLE 1

MEAN INTENSITY OF STARLIGHT IN TENTH-MAGNITUDE STARS PER SQUARE DEGREE

b	Seares and Joyner	Van Rhijn	b	Seares and Joyner	Van Rhijn
0°	121	201	40°	34	33
5	109	139	50	28	28
10	88	93	70	22	22
20	57	61	90	20	20
30	42	44			

While investigating the problem of the correction of the observed light for the contribution of the faint stars, a serious difficulty was encountered which does not seem to have been noticed previously. Since our knowledge of the frequency of faint stars is based on the Mount Wilson counts in the Selected Areas, we may expect the distribution tables of Seares and Joyner and of van Rhijn to yield accordant results. The methods used in these two investigations in extrapolating the counts to the galactic circle and in reducing the data for the southern Milky Way seem, however, to have been quite different. The nature of this discrepancy may be seen from an examination of Table 1, which gives the integrated starlight, I_0, from 1 square degree, as derived from the distribution tables of Seares and Joyner[7] (Table XIV) and van Rhijn[8] (Table 6). If the counted number of stars between apparent magnitude $m+\frac{1}{2}$ and $m-\frac{1}{2}$, at galactic latitude b, is $A_{m,b}$, then

$$I_0(b)=\int_{-\infty}^{\infty} A_{m,b}\, 10^{-0.4(m-10)}\, dm\,.$$

Counts in the catalogues of the bright stars were made for stars brighter than magnitude 5, and an extrapolation was made for the fainter stars which contribute only a small fraction of the light.

It is necessary to distinguish between these two results, and it was found that the counts of Miller and of McCuskey agreed more closely with the distribution tables of van Rhijn (Table 10) than with those of Seares and Joyner. The final adopted values of the starlight, based on van Rhijn's tables, are displayed in Figure 1. Little uncertainty

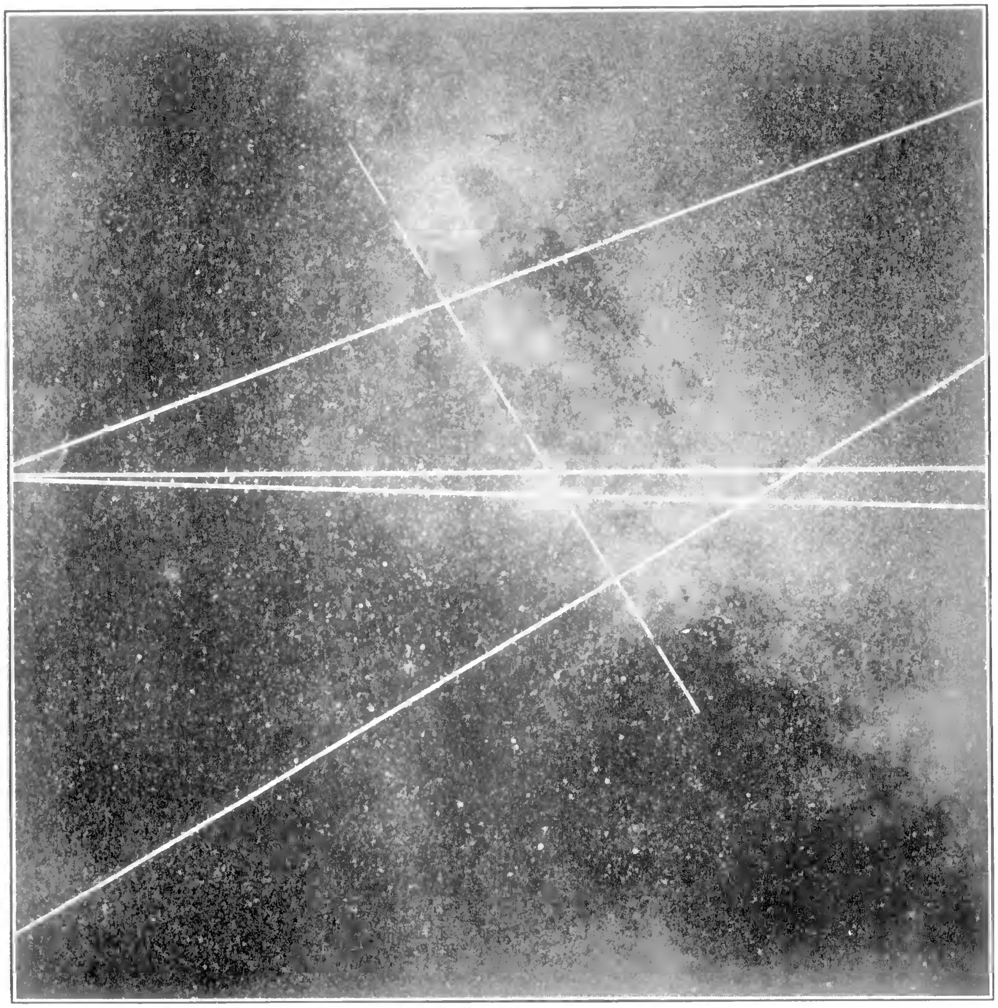
[7] *Ap. J.*, **67**, 24, 1928.

[8] *Pub. Astr. Lab. Groningen*, **43**, 1929.

[9] *Harvard Ann.*, **105**, No. 15, 1937; *Contr. Perkins Obs.*, No. 13, 1939.

[10] *Ap. J.*, **88**, 209, 1938.

PLATE VI



PHOTOGRAPH FROM THE ROSS AND CALVERT *Atlas of the Milky Way* OF THE REGION IN CYGNUS IN WHICH THE DIFFUSE RADIATION WAS MEASURED

The solid lines indicate the tracks along which observations were made. The broken line indicates the galactic equator.

exists in the correction for unresolved stars in the field of the photometer; in any case, we have used the larger values.

The surface brightness of the Cygnus region, one of the brightest found by Elvey and Roach, has been investigated in four complete runs across the Milky Way, from $b = -30°$ to $b = +20°$. The regions covered by the observations are shown in Plate VI. The individual observations are shown in Figure 2, *b*, where we have plotted the corrected intensities of the diffuse light against galactic latitude. Some uncertainty exists in the northern half of the Milky Way because of the large correction for starlight. The radiation in the southern half is determined more exactly because of the small star correction required in the regions of great obscuration by the dark nebulae of the rift. The diffuse light is roughly the same in the rift and in the star clouds, in agreement with the measures of Struve and Elvey.[5]

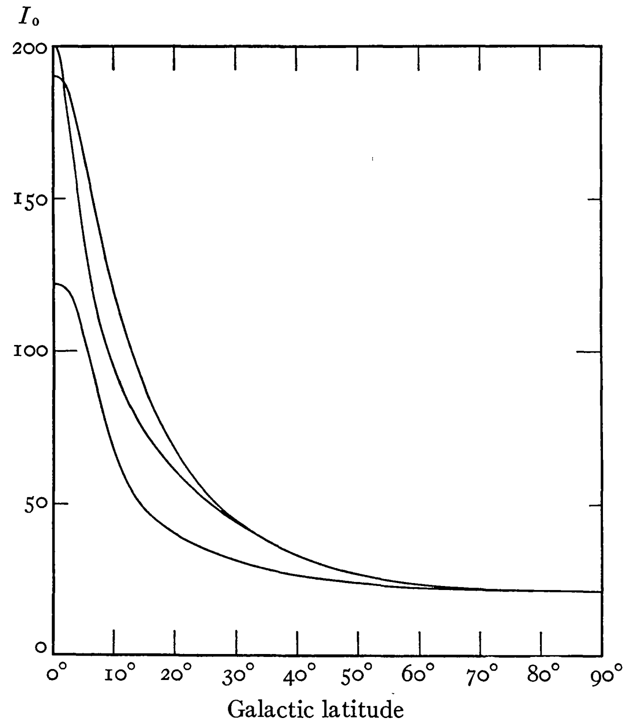


FIG. 1.—The adopted values of I_0, the surface brightness of the starlight, are given. The curve reaching the highest value is based on the mean star counts over all longitudes. The next curve is for the Cygnus region, $l = 40°$; the bottom curve is for the Tau-Aur region, $l = 140°$. The intensities per square degree are expressed in terms of the photographic brightness of one tenth-magnitude star.

In spite of the lack of knowledge of the diffuse intensity at high galactic latitudes it is probable that the galactic light is more sharply concentrated to the Milky Way plane than is the mean starlight. From the mean star counts around the sky the increase of starlight in the range $b = 15°$ to $b = 0°$ is found to be 60 per cent of its value at $b = 0°$. For the diffuse light our method of reduction makes this increase 100 per cent. To reduce it to the same ratio of increase as the starlight, however, we would have to assume that the diffuse light is 90 (in units of tenth-magnitude stars per square degree) at $b = 15°$. So high a value seems improbable on the basis of the photoelectric observations.

We must also consider the possible effects of the extended regions with bright emission-line spectra, discovered by Struve and Elvey.[11] The photographic emulsion we have used has low sensitivity at $H\beta$, and the absorption of the optical system excludes the lines $H\zeta$ and λ 3727. The line emissions at $H\beta$, $H\gamma$, $H\delta$, and $H\epsilon$ may affect our measured surface brightnesses. If we use the Balmer decrement obtained in our study of diffuse nebulae,[12] we can compute a maximum total brightness of the pertinent lines, in terms of $H\alpha$. Struve and Elvey have estimated the order of magnitude of the emission in $H\alpha$, and after some correction we obtain the photographic surface brightness of the extended nebulae as about 3 tenth-magnitude stars per square degree. The uncertainty of the surface brightness of $H\alpha$ is considerable, but it is safe to assume that the intensity is less than 15 and that only a small fraction of the diffuse radiation in the Milky Way is in the form of line emission. This is reasonable on the basis of the theory of nebular

[11] *Ap. J.*, **88**, 364, 1938.

[12] Greenstein and Henyey, *Ap. J.*, **89**, 653, 1939.

excitation, since the mean temperature of the stimulating radiation in the Milky Way is low, compared to that in normal diffuse emission nebulae. We may expect the spectrum of the nebula associated with the Galaxy as a whole to be dominantly of the reflection type.

The results of three sets of observations of the Milky Way near $l = 140°$ are shown in Figure 2, *a*. As a result of preliminary trials it was found necessary to begin far south of the Milky Way, near $b = -40°$, to obtain measures free from diffuse radiation. The very extended dark nebulosities in Tau-Aur stretch far south, and Hubble[13] has found regions of deficiency in the counted number of extragalactic nebulae at $-40°$ in this longitude. The effect of these absorbing regions is visible in the asymmetry of the intensity of the diffuse light. At northern galactic latitudes it falls sharply to the adjusted

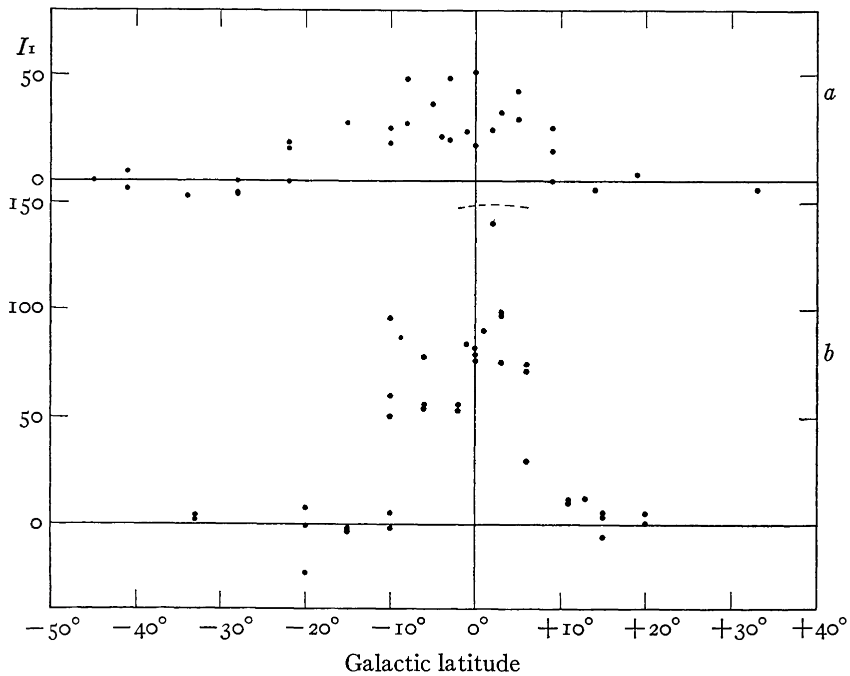


FIG. 2.—The individual measured values of I_1, the surface brightness of the diffuse radiation, are plotted as a function of galactic latitude. Fig. 2, *a*, at top, refers to the region in Tau-Aur near $l = 140°$; Fig. 2, *b*, refers to the region in Cygnus, near $l = 40°$.

zero near $b = +15°$. The intensity is high as far south as $-20°$, however. The maximum light in the Milky Way is less than in Cygnus and reaches a value of less than 50. The concentration of the diffuse light would be no greater than that of starlight if the intensity of the diffuse light were 20 at $b = 20°$; this possibility cannot be excluded by our data.

In the course of the surveys exposures were obtained at various points of special interest. The dark nebula near o Persei (*Barnard Atlas of Selected Regions of the Milky Way*, Chart 3, *B4*) has an intensity of about 15 brighter than the normal fields around it. The faintly luminous nebulosity preceding o Persei is brighter than its surroundings, by 55. The Pleiades are very bright, and we have observed a surface brightness of 2.5 mag. per square degree in the nebula near Merope. NGC 1977, which has a faint emission spectrum as well as a reflection spectrum, reaches 1.8 mag. per square degree. The center of NGC 1976 is too bright for accurate measurement with our present technique and is probably brighter than 0 mag. per square degree.

[13] *Ap. J.*, **79**, 8, 1934.

The results of our detailed investigations may be compared with those of Elvey and Roach. For the Cygnus dark region they obtain 110; for the bright region, 78. Both these figures are based on zero intensity at 40°. Since we have set our zero of intensity at $b = 15°$ in Cygnus, we should transform their values to 96 and 58, respectively, on our scale. Our observed values are 80 and 100; the difference may depend on the actual regions observed and on the star corrections they have used. In the Tau-Aur region they find the maximum diffuse radiation in the galactic plane to be 50, compared to our value of 35. Their observations do not reveal the considerable intensity at southern galactic latitudes, because of the overcorrection for starlight involved in the use of the mean star counts in the star-poor regions of the dark nebulae. Nevertheless, the order of magnitude of the diffuse light found by two completely independent methods is essentially the same, and the presence of diffuse radiation throughout most of the Milky Way may be considered established. The mean value of the galactic light in the Milky Way plane is given by Elvey and Roach as 57. This is almost identical with the mean intensity, over all galactic latitudes and longitudes, of starlight as derived from the star counts, i.e., 61 from the van Rhijn tables.

THEORETICAL DISCUSSION

In comparing the observational results with theoretical predictions, it is convenient to divide the problem into two parts. We consider first the intensity of diffuse light along the galactic circle and then the concentration toward the galactic circle.

The diffuse light has its origin at each point along the line of sight in the scattering of starlight and diffuse light. The contribution of an element of optical thickness, $d\tau$, is

$$d\tau \int I \Phi d\omega ,$$

where the integral[14] represents the amount of radiation scattered into the line of sight. The total intensity of starlight and diffuse light is I, and the scattering is governed by the phase function, Φ. The intensity of light reaching us is reduced by the factor $e^{-\tau}$, where τ is the optical thickness between the point at which the scattering takes place and the observer. The observed intensity of diffuse light, I_1, is the integral of the contributions up to some limiting optical thickness, τ_0, or

$$I_1 = \int_0^{\tau_0} e^{-\tau} d\tau \int I \Phi d\omega = \overline{\int I \Phi d\omega} (1 - e^{-\tau_0}) \tag{1}$$

if we take the appropriate mean value of the term $\int I \Phi d\omega$ along the line of sight. In computing I_1, we restrict our attention, for the time being, to its value along the galactic circle.

Our region of the Galaxy may be regarded, to a first approximation, as a slab of emitting and scattering matter, stratified along parallel planes. The observed diffuse light originates within comparatively short distances, of the order of 1500 parsecs, from the sun. Within such distances we have no reason to expect violent fluctuations of the term $\int I \Phi d\omega$. This integral depends on the incident intensity, which is a function of the density of absorbing and emitting matter averaged over large regions of space. Its value varies more slowly than does the star density. We shall compute the value of the observable intensity by using the value of $\int I \Phi d\omega$ near the sun.

Little information concerning the nature of the phase function for interstellar scatter-

[14] Henyey, *Ap. J.*, **85**, 107, 1937.

ing is now available. We have carried out our computations, using a phase function of the form

$$\Phi(\alpha) = \frac{\gamma(1 - g^2)}{4\pi} \frac{1}{(1 + g^2 - 2g \cos \alpha)^{3/2}} . \tag{2}$$

The phase angle is α, defined as the deviation of the ray from the forward direction; γ is the spherical albedo; the parameter g measures the asymmetry of the phase function, according to the expression

$$\gamma g = \int \Phi(\alpha) \cos \alpha \, d\omega . \tag{3}$$

For $g = 0$ we have an isotropic distribution of the scattered radiation; for $g = +1$ all the radiation is thrown forward. A representation of the cases $g = +\frac{1}{3}$ and $g = +\frac{2}{3}$, with $\gamma = 1$, is given in Figure 3. Such forward-throwing functions resemble those com-

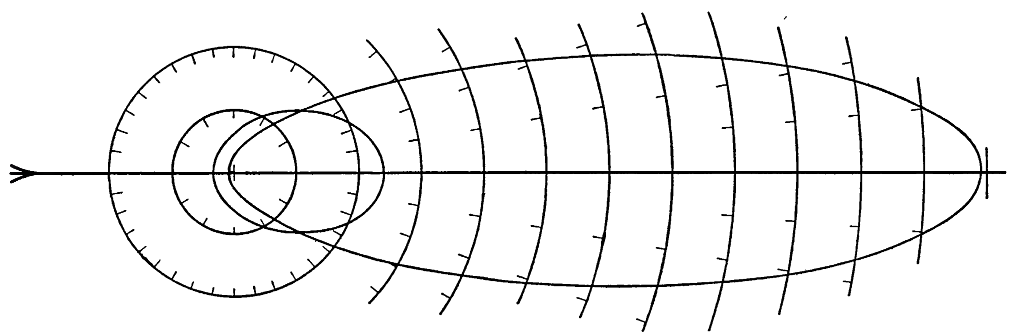


FIG. 3.—Polar diagram of the phase function of equation (2), for $\gamma = 1$. The more elongated curve is for $g = +\frac{2}{3}$; the other, for $g = +\frac{1}{3}$. The radiation is incident on the particle from the left, as shown by the arrow.

puted on the basis of the Mie theory for particles whose radius is near a wave length. If the sign of g is negative, we have backward-throwing phase functions of the same form as those in Figure 3. Suitable combinations of forward and backward phase functions of the form of (2) can also be used.

In order to compute the quantity $\int I\Phi d\omega$, we must know I as a function of position over the sky. The principal variation of I is in galactic latitude, with a smaller variation in longitude. Let us carry out the computation of $\int I\Phi d\omega$ as though the longitude variation is zero and the variation with latitude is (a) the mean over the sky and (b) the actual variation at the particular longitude which we are considering. The true value is evidently intermediate between these two. For this purpose we have used the mean star counts of van Rhijn[8] and have computed the light of the stars, $I_0(b)$. The results have been given in Figure 1 for the mean over all longitudes and for $l = 40°$ and $l = 140°$. We have already discussed the difficulty arising from the discrepancy between the distribution tables of Seares and Joyner[7] and of van Rhijn.[8] To $I_0(b)$ must be added $I_1(b)$, the diffuse light, which can be obtained from our Figure 2, a and b, for Cygnus and Tau-Aur. For the average over all longitudes the values observed by Elvey and Roach[6] were used. As may be seen from the curves in Figure 1, the values of $I_0(b)$ for Cygnus are close to those for the average over all longitudes; in view of the uncertainty of the diffuse light arising from the absence of data for the southern hemisphere the mean light was taken equal to that in Cygnus.

The results of the integration, which was carried out numerically, are shown in Figure 4. In the figure we have plotted the ratio, Γ, of the observed maximum intensity of diffuse light to the value predicted for $\gamma = 1$, $\tau_0 = \infty$, and various values of g. The ratio, Γ, measures the effectiveness of the interstellar matter in scattering light; according to equation (1), it is

$$\Gamma = \gamma(1 - e^{-\tau_0}) = \frac{\text{Observed light}}{\text{Predicted light}} . \tag{4}$$

The results for Cygnus are given by the top curve, where case (*a*) and (*b*) coincide. The two other full-line curves were obtained for Tau-Aur; the upper of these curves is for

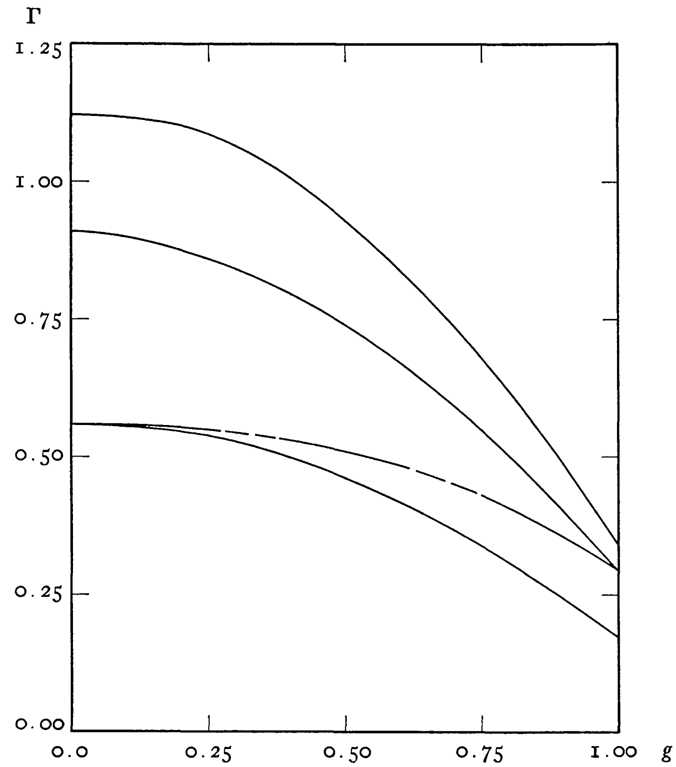


FIG. 4.—The ratio, Γ, of the observed diffuse intensity to the predicted value for albedo unity is plotted as a function of g. The uppermost curve refers to the results for Cygnus; the three lower curves, to the results for Tau-Aur.

case (*b*), the bottom curve for case (*a*). The broken-line curve indicates the probable trend of the actual value of Γ for Tau-Aur.

The first point of interest is the systematic difference between Cygnus and Tau-Aur. The ratio of the value of Γ for Tau-Aur to that for Cygnus lies between 0.50 and 0.85. This difference may arise either from a real variation of the optical properties of the interstellar matter from point to point in the Galaxy or from a much smaller total optical thickness, τ_0, in Tau-Aur. That there is a real difference in the total amount of material seems quite reasonable. Let us accept this possibility and assume that the optical thickness in the direction of Cygnus is effectively infinite. For Tau-Aur

$$1 - e^{-\tau_0} = \frac{\Gamma_{\text{Tau-Aur}}}{\Gamma_{\text{Cygnus}}} ,$$

and we have the condition $0.50 < 1 - e^{-\tau_0} < 0.85$. Then,

$$0.7 < \tau_0 < 1.9 .$$

The upper limit corresponds to $g = +1$. We require, therefore, considering the uncertainty of the observations, that τ_0 be less than 2.5.

Since counts of extragalactic nebulae have originally indicated large absorption in the Milky Way, the low total absorption required in Tau-Aur must be examined. An absorption of 2.7 mag. ($\tau_0 = 2.5$) corresponds to a decrease in the counted number of nebulae, to a given limiting magnitude, to one-thirtieth the number at the galactic pole. On the plates used by Hubble this would leave about 3 nebulae per plate. Such a number could possibly escape detection. A lower absorption, say $\tau_0 = 1.5$ mag., corresponds to 12 nebulae per plate—probably sufficient to be mentioned, in spite of the high star density in the Milky Way. These values of τ_0 may gain some support from the colors of distant B stars in this region of the Milky Way. Stebbins, Huffer, and Whitford[15] find no color excesses as large as 0.25 mag. between $l = 120°$ and $l = 210°$, up to distances of 1500 parsecs. We may summarize by the statement that the observed weakness of the diffuse light in Taurus can be explained by the condition

$$1.5 < \tau_0 < 2.5$$

and an extremely forward-throwing phase function. The value of g required, $> +0.9$, cannot be taken as a definite lower limit, because of observational uncertainties. The observations indicate that there is about an even chance that $g > 0.9$; it is almost certain that $g > 0.65$.

Examining the curve in Figure 4 for the Cygnus region, we find that for $g < 0.4$ the possible values of Γ exceed unity. By the nature of Γ this is impossible. We thus have another strong indication that the phase function is moderately or strongly forward-throwing. Since the total absorption, τ_0, in Cygnus is effectively infinite, the quantity Γ equals the spherical albedo, γ. Then, since $g > 0.65$, it follows that

$$0.3 < \gamma < 0.8 ,$$

with some preference for the lower limit. These limits depend on the observations and on our assumptions concerning $\int I \Phi d\omega$; they are somewhat uncertain. In any case, there exists a strong indication that γ is not very small. It should be remembered that, if the interstellar material consisted of small metallic particles, γ would be much smaller,[16] because of the consumptive absorption within the particles.

We have considered the intensity of light on the galactic circle. Our observations can be used to define another observational quantity, the concentration of the diffuse light toward the galactic plane. To discuss this problem we must introduce some model for the Galaxy of stars and interstellar matter. We will regard the Galaxy as a slab of emitting and scattering material stratified along parallel planes and extending to infinity in all directions along the galactic circle. The most serious deviations from such a model in the actual Galaxy arise from the existence of local clouds of absorbing material, or dark nebulae. Provided they are concentrated toward the galactic plane, they have little effect on the intensity of the diffuse light. For example, the galactic light in Cygnus (Fig. 2, *b*) hardly varies, from the brilliant star cloud to the dark nebulae in the rift.

[15] *Ap. J.*, **90**, 209, 1939.

[16] Henyey and Greenstein, *Ap. J.*, **88**, 601, Table 2, 1938.

The illumination of a slab such as we are considering is governed by the following equation of transfer:

$$\cos\theta \frac{dI}{d\tau} = I - \int I\Phi d\omega - \alpha . \tag{5}$$

The meaning of some of the symbols is given by Figure 5. The differential equation refers to the intensity, I, traveling in the direction of a ray defined by θ at point τ. The optical thickness is τ measured from the central plane; and τ_1 is the half-thickness of the slab. To make the problem specific, we have assumed that the sun is actually in the central plane, at the point $\odot$, in Figure 5. The scattering term is $\int I\Phi d\omega$.[14] The quantity α is the ratio of the stellar emission per unit volume into unit solid angle to the absorption coefficient per unit distance. The intensity I is the total intensity, in-

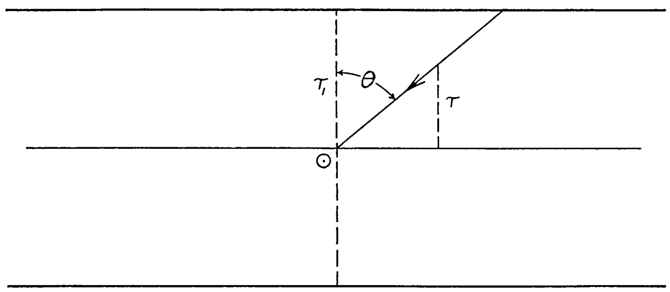


FIG. 5.—Sketch illustrating the meaning of the symbols used in the discussion of the plane-parallel model.

cluding both starlight and diffuse light. The intensity of the starlight, I_0, is the solution of the equation

$$\cos\theta \frac{dI_0}{d\tau} = I_0 - \alpha . \tag{6}$$

The diffuse intensity, I_1, is

$$I_1 = I - I_0 . \tag{7}$$

The solution of equation (5) for I is, for $0 < \theta < \pi/2$,

$$I = \sec\theta\, e^{\tau \sec\theta} \int_{\tau}^{\tau_1} e^{-\tau' \sec\theta} \left(\int I\Phi d\omega + \alpha \right) d\tau' .$$

Likewise, for I_0,

$$I_0 = \sec\theta\, e^{\tau \sec\theta} \int_{\tau}^{\tau_1} e^{-\tau' \sec\theta}\, \alpha d\tau' . \tag{8}$$

Hence, the diffuse light I_1 is

$$I_1 = \sec\theta\, e^{\tau \sec\theta} \int_{\tau}^{\tau_1} e^{-\tau' \sec\theta} \left(\int I\Phi d\omega \right) d\tau' . \tag{9}$$

It may be seen from this discussion that the illumination depends on the values of α at various points in the slab. Bottlinger[17] considered the problem of the representation of the observed intensity of starlight in terms of a model for the Galaxy. He found that the light, averaged over all longitudes, could be represented by a plane-parallel model with, in our terminology, a constant value of α. In case α is constant we have from equation (8), for $\tau = 0$,

$$I_0 = \alpha(1 - e^{-\tau_1 \sec \theta}) . \tag{10}$$

We have examined the representation by equation (10) of the star counts by van Rhijn, averaged over all longitudes and over the northern and southern hemispheres. We find that a good fit is given by taking $\alpha = 195$ and $\tau_1 = 0.12$. Table 2 gives a comparison of the observed and the predicted starlight. It may be seen that the observations are represented within an error of 10 per cent. We choose, therefore, to carry through our analysis regarding α as constant.

TABLE 2

OBSERVED AND PREDICTED STARLIGHT

b	Van Rhijn Observed	I_0 Predicted	b	Van Rhijn Observed	I_0 Predicted
90°	20	22	20°	61	58
70	22	23	10	93	96
50	28	28	5	139	146
40	33	33	0	201	195
30	44	41			

To compute I_1, we must have information concerning $\int I\Phi d\omega$ at all points along the ray. A solution of a restricted case of the present problem of the diffusion of radiation in the Galaxy has been made by Wang Shih-Ky,[4] who considered the problem of a uniform phase function and unit albedo. A more general solution may be obtained by considering the well-known symbols, J, F, and K.[18] Then we have

$$\left.\begin{aligned} \frac{1}{4}\frac{dF}{d\tau} &= (1-\gamma)J - \alpha , \\ \frac{dK}{d\tau} &= \tfrac{1}{4}(1-\gamma g)F , \end{aligned}\right\} \tag{11}$$

where γ is the spherical albedo and g the asymmetry of the phase function, given by equation (3). Using the approximation $K = \frac{1}{3}J$ and assuming that γ and g are constants, we find that equations (11) lead to the equation

$$\frac{d^2J}{d\tau^2} = p^2 J - 3\alpha(1-\gamma g) , \tag{12}$$

where

$$p^2 = 3(1-\gamma)(1-\gamma g) .$$

[17] *Zs. f. Ap.*, **4**, 370, 1932.

[18] Cf. Milne, *Handb. d. Ap.*, **3**, 121.

The solution of (12) for constant a is

$$J = \frac{a}{1-\gamma} - A \cosh p\tau ,$$

where A is a constant of integration. (No term in sinh $p\tau$ appears, because the model is symmetrical about the central plane, $\tau = 0$.) To evaluate the constant of integration, we use the usual boundary condition, $|F| = 2J$, at the surface $\tau = \tau_1$. The application of this condition leads to

$$J = \frac{a}{1-\gamma}\left[1 - \frac{\cosh p\tau}{\cosh p\tau_1 + \frac{2}{3}\frac{p}{1-\gamma g}\sinh p\tau_1}\right] . \tag{13}$$

We must here introduce some form of the phase function to evaluate the scattering term, $\int I\Phi d\omega$. The optical properties of our model for the Galaxy should depend primarily upon those properties of the phase function which we specify by γ and g. We have seen that in the differential equations (11) and (12), at least as far as J and F are concerned, this is certainly true, within the range of validity of the approximation, $K = \frac{1}{3}J$. There is, consequently, little loss of generality in supposing that the phase function is of the form

$$\Phi = \frac{\gamma(1-g)}{4\pi} + \gamma g \Psi , \tag{14}$$

where Ψ is a perfectly forward-throwing phase function of unit albedo. It can be verified that γ and g in this relationship conform to their former definitions. It follows that

$$\int I\Phi d\omega = \gamma(1-g)J + \gamma g I .$$

Introducing this expression into equation (5), we have

$$\cos\theta \frac{dI}{d\tau} = (1-\gamma g)I - \gamma(1-g)J - a .$$

The value of I at $\tau = 0$ may be computed as before:

$$I = \sec\theta \int_0^{\tau_1} e^{\tau'(1-\gamma g)\sec\theta}(\gamma(1-g)J + a)d\tau' .$$

Substituting for J from (13) and carrying out the integration, we find for I:

$$I = \frac{a}{1-\gamma}\left[1 - e^{-\tau_1(1-\gamma g)\sec\theta} - \frac{\gamma(1-g)}{(1-\gamma g)} \times \frac{1 - e^{-\tau_1(1-\gamma g)\sec\theta}\left(\cosh p\tau_1 + \frac{p}{1-\gamma g}\cos\theta \sinh p\tau_1\right)}{\left(1 - \frac{3(1-\gamma)}{1-\gamma g}\cos^2\theta\right)\left(\cosh p\tau_1 + \frac{2p}{3(1-\gamma g)}\sinh p\tau_1\right)}\right] .$$

Subtracting the intensity of starlight as given by equation (10), we have, finally, the diffuse light, I_1, as

$$I_1 = \frac{a}{1-\gamma}\left[\gamma + (1-\gamma)e^{-\tau_1 \sec\theta} - e^{-\tau_1(1-\gamma g)\sec\theta} - \frac{\gamma(1-g)}{(1-\gamma g)} \times \frac{1 - e^{-\tau_1(1-\gamma g)\sec\theta}\left(\cosh p\tau_1 + \frac{p}{1-\gamma g}\cos\theta \sinh p\tau_1\right)}{\left(1 - \frac{3(1-\gamma)}{1-\gamma g}\cos^2\theta\right)\left(\cosh p\tau_1 + \frac{2p}{3(1-\gamma g)}\sinh p\tau_1\right)}\right]. \tag{15}$$

When $\gamma = 1$, the right-hand side in equation (15) becomes indeterminate, but the correct form may be obtained by passing to the limit.

TABLE 3

RELATIVE CONCENTRATION, σ, DIFFUSE LIGHT *minus* STARLIGHT

g	γ = 0	1/3	2/3	1
	τ_1=0.1			
0	0.00	0.00	0.00	0.00
1/2	.18	.19	.21	.22
1	0.22	0.23	0.24	0.25
	τ_1=0.2			
0	0.00	0.00	0.01	0.01
1/2	.24	.27	.30	.33
1	0.33	0.37	0.40	0.44

In view of the uncertainties inherent in the observational problem, it seems undesirable to make a detailed comparison between the observational and the theoretical results. We can, however, make a comparison between the observed and the predicted concentration of light toward the galactic circle. For this purpose we introduce a parameter, μ, to measure the concentration.

$$\mu = \frac{I(b = 0°) - I(b = 20°)}{I(b = 0°)}\,. \tag{16}$$

The relative concentration, diffuse light *minus* starlight, is

$$\sigma = \mu_1 - \mu_0\,.$$

In the Cygnus region the observed value of μ_0 is 0.64 $\pm$ 0.07. This probable error has been estimated by allowing for the errors of sampling and for the errors due to the faint stars. According to equation (10), this corresponds to $\tau_1 = 0.15 \pm 0.04$. The value of μ_1 as derived in the Cygnus region becomes exactly unity. The concentration can never exceed unity, and the error must be all one-sided. The situation can be represented by saying that $\mu_1 = 0.95 \pm 0.06$. From this set of values we find the relative concentration, σ, to be $\sigma = 0.31 \pm 0.09$.

Equations (10) and (15) have been used to compute σ for a number of values of γ, g, and τ_1. Table 3 contains the results of these computations. It may be seen that for $g = 0$ there is very little relative concentration.

TABLE 4

RELATION BETWEEN γ AND g FOR THE OBSERVED CONCENTRATION

γ	0	1/3	2/3	1
g	>1	1.0	0.9	0.8

Let us adopt a value of $\tau_1 = 0.15$; then we find in Table 4 for each value of γ the value of g which is required to yield the observed relative concentration in Cygnus. Here, again, we have a positive indication of the strongly forward-throwing character of the phase function. The observational errors again limit our conclusions, but we may state that the observed concentration supports our former statement that g is very probably greater than 0.65. Table 4 also confirms our conclusion that γ is very probably greater than 0.3.

It is a pleasure to acknowledge our indebtedness to Dr. C. T. Elvey for his discussion of the possibility of investigating the diffuse galactic light by means of the Fabry photometer.

YERKES OBSERVATORY
March 1940