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THE ZERO POINT OF THE PERIOD-LUMINOSITY CURVE*

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ABSTRACT

From studies of radial velocities the group motion of 67 RR Lyrae variables has been found to be 119 ± 15 km/sec, and that of 157 Cepheids, 28.1 ± 1.4 km/sec. Their respective mean peculiar motions are 72 ± 5 and 14.0 ± 0.6 km/sec; and their rotational coefficients, 37 ± 20 and 27.4 ± 1.4, about a center in galactic longitude 326°.

Mean parallactic and peculiar motions in seconds of arc have been derived from the proper motions of 141 of these stars. Comparisons of these with the values in kilometers per second derived from the radial velocities give mean parallaxes of the proper-motion stars. Assuming Joy's value of the rotational constant, $A = 20.9$ km/sec per kiloparsec, the rotational coefficients give measures of the mean distances of the radial-velocity stars. The two sets of parallaxes, in combination with the corresponding sets of mean photographic magnitudes, determine two sets of absolute magnitudes which are in fair accord. The weighted means of the two determinations indicate corrections to the photographic period-luminosity curve amounting to ±0.00 ± 0.2 for the short-period variables, and −0.14 ± 0.2 for those of longer period, if we correct for galactic absorption of 0.85 mag. (photographic) per kiloparsec. When the data are further subdivided into period groups, the computed absolute magnitudes are in good agreement with those derived from the curve.

The conclusion is that the radial-velocity and proper-motion data now available confirm both the zero point and shape of the photographic period-luminosity curve.

Because the period-luminosity curve is a most effective yardstick for the measurement of cosmic distances, its zero point, the fundamental basis of the scale of distance, has been the subject of several investigations since Shapley[1] based his original determination upon the parallactic motions of but eleven of the brighter Cepheids. Studies by Gerasimovič,[2] Lundmark,[3] Oort,[4] Bok and Boyd,[5] and the writer,[6] among others, have indicated corrections to the zero point covering a range of approximately 1.5 mag. (absolute). This value is best determined through the relations between the mean parallactic and peculiar motions derived from proper motions and from radial velocities. The difficulty has been, and to a lesser extent still is, that we have had neither radial velocities nor good proper motions in sufficient numbers to warrant a great deal of confidence

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[1] *Mt. W. Contr.*, No. 151; *Ap. J.*, **48**, 89, 1918.

[2] *A.J.*, **41**, 17, 1931.

[3] *Lund Medd.*, Ser. 2, **6** (No. 60), 1931.

[4] *B.A.N.*, **4**, 92, 1927.

[5] *Harvard Bull.*, No. 893, 1933.

[6] *A.J.*, **35**, 35, 1923.

in the result. The best determinations of mean parallactic and peculiar motions in kilometers per second, for instance, were based upon radial velocities of but 26 RR Lyrae variables and 37 Cepheids.[7] The strongest determinations in seconds of arc depend upon proper motions of but 47 and 51 variables, respectively.

Justification for further study at the present time of the mean absolute magnitudes of these variables is based upon three considerations: (1) the greatly increased number of radial velocities now available through observations at the Mount Wilson Observatory; (2) an appreciable increase in the quality and quantity of the proper motions of the Cepheids; and (3) knowledge of the effect of galactic absorption upon both the photometric data and the rotational constant. Furthermore, pending the completion of the programs for determining photographically the proper motions of these stars now under way at the Mount Wilson and McCormick observatories, there appears little prospect of an effective increase in the amount of data for some time to come.

I. DATA

Radial velocities.—Recent publications by Joy[8, 9] have given us the radial velocities of 67 RR Lyrae variables and 155 Cepheids. To the latter have been added the velocity of α UMi from Moore's *General Catalogue of Radial Velocities*[10] and an unpublished Mount Wilson value for V 383 Cyg. All have been used with equal weight.

Proper motions.—The faintness of the RR Lyrae variables in general precludes the determination of the proper motions of any considerable number of them from meridian observations of position. We are forced to rely almost exclusively upon the photographic determinations of Luyten, Mrs. Bok, and Miss Boyd at Harvard. To these we have added three based on meridian observations and one weak determination by Kapteyn and van Rhijn.[11] Several have been modified by combining two or three independent determinations with weights proportional to the quoted probable errors. The 55 proper motions of these stars are listed in Table 1.

[7] *Mt. W. Contr.*, No. 293; *Ap. J.*, **61**, 371, 1925.

[8] *Pub. A.S.P.*, **50**, 303, 1938.

[9] *Mt. W. Contr.*, No. 578; *Ap. J.*, **86**, 363, 1937.

[10] *Lick Obs. Pub.*, **18**, 10, 1932.

[11] *B.A.N.*, **1**, 40, 1922.

TABLE 1

PROPER MOTIONS OF RR LYRAE VARIABLES

Star	α_{1900}	δ_{1900}	m_{pg}	Per.	μ_α		μ_δ		Auth.*
SW And. . .	0^h18^m0	$+28°51'$	9.3	0.44	$+0''.003\pm$	5	$-0''.016\pm$	5	H2, W
RR Cet. . . .	1 27.0	+ 0 50	8.6	.55	+ .014	4	− .064	4	H2, K
U Tri. . . .	1 49.7	+33 17	11.6	.45	+ .019	5	− .022	5	H2, K
TU Per. . . .	3 1.8	+52 48	11.7	.61	+ .018	8	− .042	8	H2
X Ari. . . .	3 3.1	+10 4	9.4	.65	+ .049	7	− .078	7	H2
SS Tau. . . .	3 31.4	+ 5 2	12.2	.37	+ .026	8	+ .032	8	H2
RX Eri. . . .	4 45.2	−15 54	9.2	.59	+ .011	6	+ .001	6	H2, W
U Lep. . . .	4 52.0	−21 22	9.5	.58	+ .047	7	− .049	7	H2
RZ Cam. . .	6 23.6	+67 6	12.0	.48	+ .019	12	− .008	12	H1
TZ Aur. . . .	7 4.6	+40 56	11.6	.39	+ .015	20	− .001	20	H1
RR Gem. . .	7 15.2	+31 4	10.2	.40	− .007	12	+ .025	12	H1
AI Vel. . . .	8 11.3	−44 20	7.0	.11	+ .036	9	+ .054	9	B
RW Cnc. . . .	9 13.2	+29 29	11.7	.55	+ .014	9	− .051	9	H2
X LMi. . .	10 0.1	+39 51	12.0	.68	+ .031	7	− .014	7	H2
RR Leo. . . .	10 2.1	+24 29	9.4	.45	+ .007	10	− .004	10	H1
V LMi. . .	10 19.8	+29 8	10.7	.54	− .001	14	+ .016	14	H1
RX Leo. . . .	11 18.7	+27 10	12.2	.65	+ .026	7	− .006	7	H2
SS Leo. . . .	11 28.8	+ 0 31	11.1	.63	− .008	18	− .006	18	H1
SU Dra. . . .	11 32.2	+67 53	9.2	.66	− .034	5	− .074	5	H2, W
UU Vir. . . .	12 3.6	+ 0 6	10.3	.48	+ .001	20	− .005	20	H1
SW Dra. . . .	12 13.1	+70 4	10.0	.57	− .020	7	− .006	7	H2
RR CVn. . .	12 24.2	+35 12	11.0	.56	− .004	8	+ .008	8	H2
SV Hya. . .	12 26.2	−25 37	10.9	.48	− .005	8	+ .014	8	H2
S Com. . .	12 27.8	+27 35	10.9	.59	− .021	12	+ .047	12	H1
U Com. . .	12 35.1	+28 3	11.8	.29	− .043	8	− .016	8	H2
Z CVn. . .	12 45.1	+44 19	10.2	.65	+ .006	11	− .020	11	H1
SX UMa. .	13 22.3	+56 47	9.8	.31	− .038	8	+ .002	8	H2
RV UMa. .	13 29.4	+54 30	9.8	.47	+ .017	8	− .018	8	H2
RU CVn. . .	13 55.1	+32 7	11.1	.57	− .025	5	.000	5	H2, K
W CVn. . .	14 2.2	+38 18	10.3	.55	− .039	4	− .012	4	H2, K, W
ST Vir. . . .	14 22.5	− 0 27	10.8	.41	+ .005	4	− .020	4	H1, K
SW Boo. . . .	14 23.4	+36 30	10.8	.51	− .057	8	+ .009	8	H2
RS Boo. . . .	14 29.3	+32 12	10.3	.38	+ .002	4	+ .005	4	H2, K
SZ Boo. . . .	14 37.9	+28 38	11.6	.52	.000	7	− .007	7	H2
ST Boo. . . .	15 26.8	+36 8	10.8	.62	− .013	7	+ .014	7	H2
RW CrB. . .	15 35.2	+29 56	10.2	.73	− .014	7	− .008	6	W
RV CrB. . .	16 15.5	+29 57	11.5	.33	− .009	7	− .004	7	H2
VX Her. . . .	16 26.2	+18 36	10.4	.46	+ .017	9	− .023	9	H2
RW Dra. . . .	16 33.7	+58 3	10.4	.44	− .006	7	− .002	7	H2
VZ Her. . . .	17 8.8	+36 6	11.5	0.44	−0.026	7	−0.032	7	H2

* Authorities: B, Boss; H1, Harvard, Luyten; H2, Harvard, Bok and Boyd; K, Kapteyn and van Rhijn; W, Wilson.

TABLE 1—*Continued*

Star	α_{1900}	δ_{1900}	m_{pg}	Per.	μ_α	μ_δ	Auth.*
ST Oph...	$17^h28^m.8$	$-1° 1'$	11.6	0.45	$-0''.005 \pm 4$	$+0''.001 \pm 4$	H2, K
TW Her....	17 50.7	+30 26	11.5	.40	+ .003 7	− .029 7	H2
S Ara....	17 51.5	−49 25	10.0	.45	− .015 13	− .010 13	H1
Y Lyr....	18 34.2	+43 52	11.8	.50	− .006 8	− .001 8	H2
RZ Lyr....	18 39.9	+32 42	10.4	.51	+ .004 8	+ .033 8	H2
RR Lyr....	19 22.3	+42 36	7.6	.57	− .122 1	− .193 1	W
XZ Cyg....	19 30.4	+56 10	9.7	.47	+ .081 7	− .027 6	H2, W
XX Cyg....	20 1.3	+58 40	11.8	.13	− .001 8	− .015 8	H2
AA Aql....	20 33.1	− 3 14	12.0	.36	− .007 7	− .002 7	H2
UY Cyg....	20 52.3	+30 3	10.0	.56	− .001 7	+ .016 7	H2
RV Cap....	20 55.9	−15 37	10.0	.45	+ .035 6	− .118 6	H2
SW Aqr....	21 10.2	− 0 20	10.4	.46	− .054 4	− .045 4	H2, K
SX Aqr....	21 31.1	+ 2 47	11.8	.54	− .005 4	− .030 4	H2, K
VV Peg....	22 8.3	+17 46	11.2	.49	+ .001 8	+ .031 8	H2
RZ Cep....	22 35.7	+64 20	9.5	0.31	+0.082 14	+0.179 14	K

The improvement in the proper-motion situation, in so far as the Cepheids are concerned, is not so much in the number now available as in their accuracy from the standpoint of both systematic and accidental error. A good many of these stars appear in the *Boss General Catalogue*.[12] All variables brighter than 8.0 visual magnitude at maximum were placed on the observational program leading up to the formation of this catalogue, and special efforts were made to determine their positions. Likewise, special programs at other observatories—notably Lund, Bergedorf, and Lyon—have emphasized determinations of positions of variable stars. Only those lists which were included in the more general catalogues of position, with sufficient observations of fundamental stars to tie them to the standard system, were used in the *General Catalogue*. The completion of this catalogue, however, afforded the opportunity to derive from the variables themselves approximate values of systematic corrections which would reduce the observations in the various special lists to the *G.C.* system. This reduction has been made by the writer; the positions and proper motions of the *G.C.* have been revised in all cases in which the added material warranted a revision; and some four hundred new proper motions have been determined. Among

[12] *Carnegie Inst. of Washington Pub.*, No. 468, 1938.

these are a number of Cepheids. We have also added a few determinations by Gerasimovič[2] where it was apparent that he had used observations of position as yet unpublished, or at least not listed in current references to published positions. The proper motions of 86 of these stars are listed in Table 2.

The wide range in the accuracy of the proper motions forces the adoption of a system of weights. If the data were abundant, it would be advisable to reject all proper motions with probable errors exceeding 0.″010 and to use the balance without weights. The scarcity of data, however, compels the use of the weaker proper motions. For the RR Lyrae variables we have used all the Harvard data with quoted probable errors up to 0.″020. In view of the minuteness of the average motions of the Cepheids, approximating, as they do, the size of the probable errors involved, we have rejected all proper motions of these stars with probable errors exceeding 0.″015. The following system of weights was then adopted.

P.E.	Weight
0.″001–0.″005	1.5
.006– .010	1.0
.011– .012	0.7
.013–0.014	0.5
0.015–	0.3

II. THE APEX AND THE GALACTIC ROTATION

The relations which lead to the determination of the solar motion are based upon the assumption of at least approximate uniformity of distribution of the reference stars over the sky. Unfortunately, neither the RR Lyrae variables nor the Cepheids whose motions are known are so distributed, nor are their respective distributions at all similar. The former, though fairly uniformly distributed in galactic latitude, are strongly concentrated in longitudes 0°–180°, 70 per cent of them lying on one side of the direction toward the solar apex referred to the stars in general. The latter, on the other hand, show a good distribution in galactic longitude but are overwhelmingly concentrated within a few degrees of the galactic plane, only 6 of the 157 having galactic latitudes in excess of 20°. Nor are the motions of the two classes of stars at all similar. The radial velocities of the RR Lyrae variables range from 0 to 390 km/sec with a high median

TABLE 2

PROPER MOTIONS OF δ CEPHEI VARIABLES

Star	α_{1900}	δ_{1900}	m_{pg}	Per.	μ_α	μ_δ	Auth.*
TU Cas...	0h20m.9	+50°44′	8.3	2.14	−0″.020 ± 5	−0″.009 ± 6	W
UZ Cas...	1 6.4	+60 41	11.9	4.26	− .007 7	.000 7	H
α UMi..	1 22.6	+88 46	3.3	3.97	+ .058 1	− .004 1	B
VX Per....	2 0.8	+57 58	10.0	10.90	− .035 15	− .040 13	G
SU Cas...	2 43.0	+68 28	6.8	1.95	+ .009 2	− .007 2	B
RW Cam..	3 46.2	+58 22	9.4	16.41	− .029 12	− .014 10	W
RX Cam..	3 56.7	+58 23	8.7	7.92	− .021 10	− .009 9	W
SZ Tau...	4 31.4	+18 20	7.0	3.15	− .014 8	− .007 8	B
SV Per....	4 42.8	+42 7	9.3	11.13	− .007 10	− .005 10	G, K, W
RX Aur...	4 54.5	+39 49	8.3	11.62	− .004 7	− .007 6	W
Y Aur...	5 21.5	+42 21	9.7	3.86	+ .011 12	− .010 12	W
β Dor...	5 32.8	−62 33	5.0	9.84	− .007 3	+ .004 3	B
ST Tau...	5 39.4	+13 32	8.7	4.03	+ .001 9	+ .002 8	W
SV Mon..	6 16.1	+ 6 31	9.4	15.23	− .039 12	− .027 11	W
RS Ori....	6 16.5	+14 44	9.2	7.57	− .004 14	− .021 11	W
T Mon..	6 19.8	+ 7 8	7.3	27.01	+ .012 7	.000 5	W
RT Aur...	6 22.1	+30 33	6.0	3.73	+ .005 2	− .016 2	B
W Gem..	6 29.2	+15 24	7.4	7.91	+ .013 6	− .007 7	W
ζ Gem..	6 58.2	+20 43	4.5	10.15	− .004 1	− .003 1	B
X Pup...	7 28.5	−20 42	9.4	25.96	+ .014 12	+ .001 12	W
AH Vel....	8 9.8	−46 24	6.4	4.23	− .010 5	+ .004 5	B
V Car...	8 26.7	−59 47	8.8	6.70	− .004 8	− .016 6	B
RZ Vel....	8 33.6	−43 46	7.6	20.40	− .021 8	.000 6	B
SW Vel....	8 40.4	−47 3	8.7	23.51	− .042 11	− .009 8	B
SX Vel....	8 41.5	−45 58	9.0	9.55	− .011 11	− .006 8	B
V Vel....	9 19.2	−55 32	7.6	4.37	− .009 13	− .025 12	W
l Car...	9 42.5	−62 3	4.8	35.52	− .015 3	+ .007 3	B
UX Car...	10 25.4	−57 6	9.8	3.68	− .021 15	− .027 14	W
Y Car...	10 29.4	−57 59	8.7	3.64	− .023 14	+ .012 12	G
VY Car...	10 40.6	−57 2	8.6	18.98	+ .045 14	− .045 13	W
U Car...	10 53.7	−59 12	8.6	38.72	+ .011 8	+ .006 6	B
ER Car...	11 5.4	−58 18	8.0	7.72	+ .002 8	− .024 6	B
S Mus...	12 7.4	−69 36	6.9	9.66	− .010 8	− .021 7	B
T Cru...	12 15.9	−61 44	8.2	6.73	− .005 8	.000 6	B
R Cru...	12 18.1	−61 4	8.3	5.83	+ .012 9	− .023 7	B
AG Cen...	12 35.7	−59 14	8.8	3.84	+ .018 14	− .010 13	W
R Mus...	12 36.0	−68 52	7.7	7.51	+ .006 8	− .014 6	B
S Cru...	12 48.4	−57 53	7.9	4.69	− .013 9	− .014 7	B
W Vir....	13 20.9	− 2 52	10.4	17.27	+ .009 12	+ .005 9	B
XX Cen...	13 33.8	−57 6	7.6	10.96	−0.012 9	−0.031 8	B

* Authorities: B, Boss; G, Gerasimovič; H, Harvard; W, Wilson.

TABLE 2—*Continued*

Star	α_{1900}	δ_{1900}	m_{pg}	Per.	μ_α	μ_δ	Auth.*
V381 Cen...	$13^h44^m.0$	$-57° 5'$	8.3	5.08	$-0''.033 \pm 12$	$0''.000 \pm 10$	B
V Cen...	14 25.4	−56 27	8.1	5.49	− .024 9	− .021 7	B
R TrA...	15 10.8	−66 8	7.1	3.39	− .004 7	− .027 6	B
S TrA...	15 52.2	−63 30	6.9	6.32	− .005 9	− .011 7	B
U TrA...	15 58.4	−62 38	8.8	2.57	+ .026 15	− .024 13	B
S Nor...	16 10.6	−57 37	7.1	9.75	+ .002 6	− .008 5	B
RV Sco...	16 51.8	−33 27	7.6	6.06	− .005 15	− .036 14	B, G
BF Oph...	16 59.9	−26 27	8.2	4.07	− .001 6	+ .003 5	B
X Sgr....	17 41.3	−27 48	5.1	7.01	− .003 2	− .014 2	B
RY Sco...	17 44.3	−33 40	8.9	20.31	+ .024 9	+ .021 9	B
Y Oph...	17 47.3	− 6 7	7.3	17.12	+ .003 5	− .015 5	W
W Sgr....	17 58.6	−29 35	5.2	7.59	+ .010 3	− .006 3	B
AP Sgr....	18 7.0	−23 9	7.7	5.06	− .008 7	− .010 6	B
WZ Sgr....	18 11.1	−19 6	9.0	21.85	+ .013 8	− .008 6	W
Y Sgr....	18 15.5	−18 54	6.2	5.77	+ .010 5	− .013 5	B
XX Sgr....	18 19.0	−16 51	9.5	6.42	− .013 12	− .002 11	W
U Sgr....	18 26.0	−19 12	7.9	6.74	− .009 4	− .005 4	W
RU Sct...	18 36.7	− 4 13	10.6	19.70	− .028 12	+ .021 12	W
SS Sct....	18 38.3	− 7 50	8.4	3.67	− .012 7	+ .023 6	W
V350 Sgr....	18 39.3	−20 45	8.0	5.15	+ .004 11	− .035 12	W
YZ Sgr....	18 43.7	−16 50	8.0	9.55	− .001 5	− .005 4	W
BB Sgr....	18 45.1	−20 24	7.6	6.64	+ .008 5	− .033 6	W
κ Pav...	18 46.6	−67 21	5.1	9.11	− .006 4	+ .009 3	B
FF Aql...	18 53.8	+17 14	6.0	4.47	− .009 3	− .011 4	B
SZ Aql...	18 59.6	+ 1 9	10.4	17.14	− .014 9	+ .026 9	W
TT Aql...	19 3.2	+ 1 9	8.3	13.75	− .015 4	− .012 4	W
U Aql...	19 24.0	− 7 15	7.3	7.02	+ .031 6	− .013 5	W
U Vul...	19 32.3	+20 7	8.2	7.99	− .007 4	− .028 4	W
SU Cyg...	19 40.8	+29 1	7.3	3.85	+ .004 8	− .009 6	W
SV Vul...	19 46.8	+27 12	8.8	45.13	− .012 6	− .004 5	W
η Aql...	19 47.4	+ 0 45	5.0	7.18	+ .007 1	− .008 1	B
S Sge...	19 51.5	+16 22	6.3	8.38	− .001 2	− .007 2	B
X Vul...	19 53.3	+26 17	9.2	6.32	− .009 8	+ .012 7	W
CD Cyg...	20 0.6	+33 50	9.8	17.07	− .007 9	+ .022 10	G
SZ Cyg...	20 29.6	+46 16	10.3	15.11	− .013 13	− .001 12	W
X Cyg...	20 39.5	+35 14	7.4	16.39	− .017 4	+ .001 3	W
T Vul...	20 47.2	+27 52	6.4	4.44	− .003 3	− .003 2	W
VX Cyg...	20 53.6	+39 48	10.4	20.12	+ .002 14	+ .025 15	W
DT Cyg...	21 2.3	+30 47	5.9	2.50	− .005 3	− .003 3	B
VZ Cyg...	21 47.7	+42 41	9.5	4.86	− .017 13	+ .012 12	W
Y Lac...	22 5.2	+50 33	8.6	4.32	+ .014 11	+ .002 15	G, W
δ Cep...	22 25.4	+57 54	4.2	5.37	+ .012 1	+ .002 1	B
Z Lac...	22 36.9	+56 9	9.1	10.89	− .018 14	− .010 12	G, W
RR Lac...	22 37.5	+55 55	9.3	6.42	− .001 12	− .025 11	W
V Lac...	22 44.6	+55 48	9.3	4.98	− .019 11	+ .017 10	W
X Lac...	22 45.0	+55 54	9.1	5.44	+0.017 14	−0.024 12	G

value; those of the Cepheids range from 0 to 93 km/sec. But, as Joy has shown,[13] the greater part of the latter range is due to rotation effects. The mean proper motions of the former are likewise known to be large relative to the latter. We are thus forced to treat the two sets of stars separately, hereafter designating the RR Lyrae variables as "group I" and the Cepheids as "group II." Since the two groups have widely differing distributions and motions, and since the radial velocities and proper motions are affected differently by the anomalous distributions, it would hardly be reasonable to expect that the four sets of data would give even approximately the same values for the direction of the solar apex. Before we adopt a value for the apex, however, it is desirable that we find out what the data do indicate.

Radial velocities.—The radial velocities have been analyzed by means of the well-known relation

$$V = K + X \cos b \cos l + Y \cos b \sin l + Z \sin b + du + ev \, ,$$

in which K denotes the K term; X, Y, and Z, the components of the solar motion in galactic co-ordinates; and l and b, the galactic longitude and latitude. Also

$$u = \bar{r}A \cos 2l_0 \, , \qquad d = \sin 2l \cos^2 b \, ,$$

$$v = \bar{r}A \sin 2l_0 \, , \qquad e = - \cos 2l \cos^2 b \, ,$$

where A is the constant of galactic rotation, $\bar{r}$ the mean distance in kiloparsecs, and l_0 the longitude of the center of rotation. Finally we denote the mean of the peculiar motions taken without regard to sign by θ. The results of these analyses are given in Table 3.

In view of the large probable errors and the peculiar distribution of the stars involved, the results from group I are not disconcerting. The determinations of K and the rotational terms are seriously affected by the distribution and in turn produce large uncertainties in the derived position of the apex.

In group II the geometrical problem is somewhat different, for there we have good distribution in longitude but very high galactic concentration. As a result we should get a good determination of

[13] *Mt. W. Contr.*, No. 607; *Ap. J.*, **89**, 1939; in press.

galactic rotation, a fair determination of the longitude of the apex, but a poor determination of its latitude. We actually get from the rotational terms a direction of the galactic center in excellent agreement with the best modern results and a value of $\bar{r}A$ in good agreement with Joy's results[13] based upon the assumed apex ($A_0 = 271°$, $D_0 \infty +28°$). The longitude of the apex, however, comes out about 15° greater than the conventional position, while the latitude, affected by four times the probable error attached to the longitude, comes very close to the standard position.

TABLE 3

RESULTS FROM RADIAL-VELOCITY SOLUTIONS

	Group I ($P<1$ Day)	Group II ($P>1$ Day)
K	$+10\pm 8$ km/sec	-1.7 ± 0.9 km/sec
X	-86 15 km/sec	-20.7 1.2 km/sec
Y	-118 19 km/sec	-17.7 1.6 km/sec
Z	-34 14 km/sec	-10.0 6.0 km/sec
u	$+67$ 22 km/sec	$+10.4$ 1.4 km/sec
v	$+2\pm 21$ km/sec	-23.4 ± 1.5 km/sec
V_0	150 ± 16 km/sec	29.0 ± 2.9 km/sec
l_a	54° 6°	40°.5 3°.0
b_a	$+13°$ 6°	$+20°.2$ 12°.2
$\bar{r}A$	67 22 km/sec	25.6 1.5 km/sec
l_0	1° 18°	326°.9 2°.8
θ	72 ± 5 km/sec	14.0 ± 1.0 km/sec
No.	67	157

The most surprising result of this solution is, however, that through the quadrupling of the data and the consideration of rotation effects the value of the deduced solar speed is much greater than that resulting from previous investigations. From 37 radial velocities Strömberg[7] derived in 1923 a solar speed of but 11.5 km/sec. After the discovery of galactic rotation Oort[4] showed that its effect would increase Strömberg's value to more nearly the conventional speed of 20 km/sec. The value here derived, in conjunction with the still higher value referred to the RR Lyrae variables, would suggest that, as we go farther out into space, the group motions of the stars, and consequently the reflected solar motions, tend to increase.

Proper motions.—The corrections to the Newcomb precessions derived by Wilson and Raymond[14] were applied to the proper motions in Table 2. In order to attempt the determination of possible rotational effects, all the proper motions were transformed to proper motions relative to the galaxy in the form $\mu_l \cos b$ and μ_b. These quantities were then analyzed by means of the relations

$$\mu_l \cos b = X \sin l - Y \cos l + Q \cos b + C \cos 2l \cos b + S \sin 2l \cos b ,$$

$$\mu_b = X \cos l \sin b + Y \sin l \sin b - Z \cos b - \frac{C}{2} (\sin 2l \sin 2b) + \frac{S}{2} (\cos 2l \sin 2b) ,$$

where X, Y, Z, l, and b have the same significance as in the radial-velocity equations and Q, S, and C are the rotational constants.

$$Q = \frac{B}{4.74} , \quad C = \frac{A}{4.74} \cos 2l_0 , \quad S = \frac{A}{4.74} \sin 2l_0 .$$

The results of these analyses are given in Table 4.

TABLE 4

RESULTS FROM PROPER-MOTION SOLUTIONS

	Group I (Unit, 0".01)	Group II (Unit, 0".01)
X	+0.79 ±0.31	+0.22 ±0.15
Y	+1.33 .30	+ .43 .16
Z	−0.26 .35	+ .56 .11
Q	+0.62 .42	− .06 .12
C	−1.21 .50	− .01 .16
S	−1.11 ±0.48	0.00 ±0.17
$q=h/\rho$	1.57 ±0.32	0.74 ±0.14
l_a	59° 12°	63° 17°
b_a	− 10° 12°	+ 49° 11°
$P=A/4.74$	1.64 0.49	0.01 ±0.17
l_0	291°± 17°	....
No.	55	86
Solar motion only:		
q	2.10 ±0.32	0.75 ±0.14
l_a	53° 12°	63° 17°
l_b	+ 10°± 12°	+ 48°± 11°

[14] *A.J.*, **47**, 57, 1938.

In confirmation of the results of Gerasimovič's investigation we find here no evidence of rotation effects in the proper motions of group II. The proper motions of group I, on the other hand, give indications of a very large but also extremely uncertain rotation about a center 70° distant from that given by their radial velocities. The mean of the two discordant values of the longitude of the rotational center is 326°, a purely fortuitous agreement with that derived from the radial velocities of group II.

The results for the position of the apex given in Tables 3 and 4 show neither accord between the two types of variables nor accord between the results based upon radial velocities and proper motions within each type. We believe that this lack of agreement is primarily due to distribution, the anomalies of which are not only different in the two types but operate differently upon the two sets of data. It is evident, therefore, that in order to reduce the distances to be derived by comparison of the data to a common basis, we must adopt a position of the solar apex. The only consistency in the positions derived above is that all the solutions give an unexpectedly large longitude and hence a high declination for the apex. This is partly due to distribution, but there may be other contributing factors. For example, there is evidence from analyses of both proper motions and radial velocities of stars of all types which indicates an increase in longitude, and hence in declination, of the solar apex with decrease in brightness. Wilson and Raymond[15] have recently shown, also, that the stars of small proper motions and presumably greater mean distance give relatively high apical declinations. So, since the Cepheids themselves give a very uncertain declination of over +50° and the conventional declination, +30°, appears to be somewhat low for stars similar in magnitude and proper motion to the Cepheids, we have adopted as co-ordinates of our standard apex, $A_0 = 270°$, $D_0 = +36°$. A_0 has its conventional value, and D_0 is taken from the determination of Wilson and Raymond referred to 26,978 stars with proper motions $< 10''$ per century. We adopt for the direction of the galactic center the value given by the Cepheids themselves, $l_0 = 326°$.

[15] *Ibid.*, p. 65.

III. THE MEAN PARALLACTIC AND PECULIAR MOTIONS

Radial velocities.—The directions of the apex and the rotational center having been fixed, least-squares solutions were made to determine new values of K, V_0, $\bar{r}A$, and θ by means of the relation

$$V = K + V_0 \cos \lambda + \bar{r}A \sin 2(l - 326^\circ) \cos^2 b \,,$$

the only new symbol introduced being λ, the distance of the star from the adopted apex. The results are given in Table 5.

TABLE 5

RESULTS OF RADIAL-VELOCITY ANALYSES

(Apex and Galactic Center Assumed)

	Group I	Group II
	km/sec	km/sec
K	-3 ± 8	-3.0 ± 1.0
V_0	119 15	28.1 1.4
$\bar{r}A$	+37 20	+27.4 1.4
θ	72 ± 5	14.0 ± 0.6

In group II the adoption of a standard apex has produced rather insignificant changes in the unknowns. The changes produced in group I are much larger, but so also are the probable errors. In this case the use of the five largest radial velocities (>240 km/sec) in the determination of $\bar{r}A$ was questionable. The value derived was +45 km/sec. We preferred to give them no more than half-weight in the determination of this quantity, and the resulting value is that given in the table.

The most significant result of these solutions is the lack of any change in the values of the mean peculiar motions θ, in spite of the adoption of an apex somewhat different from that indicated by the data. Although the apex is practically indeterminate, we feel that the mean peculiar motions are fairly well determined. It should be stated that the mean peculiar motion of group II was determined in two ways. Use of the mean value of $\bar{r}A$ in Table 5 gives $\theta = 14.2$. Using Joy's value of A (20.9 km/sec/kpc) out to about 2 kpc, as determined from the period-luminosity curve, and beyond that point a flat value of 40 km/sec (also from Joy's work), we get $\theta = 13.8$. The mean is given in Table 5.

Proper motions.—The components of the proper motions in the direction of the solar apex, v, and at right angles to it, τ, were computed. For group I they were reduced to the mean photographic magnitude, 10.5. In group II no such reduction is necessary or desirable; the proper motions of the brighter stars are small, and reduction to a mean magnitude would produce large unbalanced errors in

TABLE 6

PROPER-MOTION DATA

(Unit, 0″.01)

	Group I	Group II
$\overline{\log P}$	−0.32	+0.89
$\overline{m}_{pg}$	10.50	7.96
No.	55	86
μ_{med}	2.75	1.60
$\overline{\mu}$	3.62	1.79
Wt.	56.2	85.0
$\overline{\tau}$	−0.08±0.28	−0.03±0.08
$\overline{v}$	+1.39±0.26	+0.59±0.10
τ_{med}	1.40	0.85
τ_{av}	2.27±0.30	1.04±0.09
$\overline{p}$	0.70	0.67
p_0	0.82	0.78
q	1.92±0.34	0.79±0.13

the resulting motions of the fainter stars. Mean, median, and average values of the data in the two groups are given in Table 6. Here $q = \Sigma v \sin \lambda / \Sigma \sin^2 \lambda$ is the mean parallactic motion in seconds of arc per century, τ_{med} and τ_{av} are the median and mean values of τ taken without regard to sign, and $\overline{p}$ is the mean of the quoted probable errors of the determinations of proper motion. The values of τ_{med} and τ_{av} are not the true values of the peculiar motions. In using proper motions with relatively large probable errors we artificially increase the mean or median τ as computed from individually properly weighted τ's taken without regard to sign, an error which is largely compensated for in the mean parallactic motion derived from the algebraic mean.

Kapteyn[16] has shown that the observed and true means of proper

[16] *Groningen Pub.*, No. 30, p. 52, 1920.

motions, taken without regard to sign, are quite approximately related in the form

$$\mu_0^2 = \bar{\mu}^2 - \kappa\bar{\rho}^2 \, ,$$

where μ_0 is the true mean; $\bar{\mu}$, the observed mean; $\bar{\rho}$, the probable error of unit weight; and κ, a factor determined empirically. The values of τ may be treated in the same manner. Thus we have

$$\tau_0^2 = \tau_{\mathrm{av}}^2 - \kappa\bar{\rho}^2 \, .$$

This is equivalent to the well-known relation

$$\tau_0^2 = \tau_{\mathrm{av}}^2 - \eta^2 \, ,$$

where η denotes average error in place of probable error. If the data are sufficient, the equation in the form given by Kapteyn has the distinct advantage of taking account empirically of underestimation of probable error.

TABLE 7

DEPENDENCE OF τ_{av} ON PROBABLE ERROR

(Unit, 0".01)

Group I				Group II			
$\bar{\rho}$	τ_{av}	τ_{med}	No.	$\bar{\rho}$	τ_{av}	τ_{med}	No.
0.41	2.18	1.70	12	0.21	0.59	0.50	13
0.69	1.89	1.20	17	0.44	0.86	0.80	9
0.83	2.54	1.65	16	0.65	1.01	0.70	13
1.20	2.78	1.70	5	0.84	1.02	0.80	19
1.72	3.10	1.40	5	1.05	1.37	1.30	6
				1.22	1.68	1.40	13
				1.44	2.15	1.90	12

The correlations between τ_{av}, τ_{med}, and $\bar{\rho}$, in which $\bar{\rho}$ is derived from the quoted probable errors, are shown in Table 7. Least-squares solutions of these data for κ and τ_0 give:

	Group I	Group II	
κ_1	1.9	1.9	Average
κ_2	0	1.2	Median
τ_{01}	2.06	0.40	Average
τ_{02}	1.40	0.40	Median

The large values of κ_1 in both groups indicate that the probable errors have been underestimated by about 17 per cent. For both groups the true values, ρ_0, are 1.17 times the values of $\bar{\rho}$ quoted in Table 6, or 0.82 and 0.78; the true values of the average errors, η, thus become 0.97 and 0.92, respectively. The difference between the values of the true τ_{01} and τ_{02} in group I is largely due to four apparently abnormal reduced proper motions: SS Tau ($12^{m}.2$), $\mu_r = 9''.2$; RW Cnc ($11^{m}.7$), $9''.5$; RV Cap ($10^{m}.0$), $9''.8$; and RZ Cep ($9^{m}.5$), $12''.4$. The radial velocities are not abnormal (+5, −85, −80, and 0 km/sec, respectively). The omission of these four stars would reduce τ_{av} to $1''.97$, τ_{01} to $1''.71$, and τ_{02} to $1''.35$. The median value, τ_{02}, probably represents better than τ_{01} the mean peculiar motion of group I.

IV. THE MEAN PARALLAXES

From the parallactic and peculiar motions derived above we get the mean parallaxes of the two classes of stars through the relations

$$\pi_q = \frac{kq}{V_0} \quad \text{and} \quad \pi_\tau = \frac{k\tau_0}{\theta}\,,$$

in which $k = 0.04737$, since q and τ_0 were derived in seconds of arc per century. They are, in units of $0''.001$:

	Group I	Group II	
π_q	0.73 ±.13 0.76 ±.13	1.27 ±.24 1.40 ±.22	From Table 4* From Table 6
π_τ	1.36 ±.20 0.92 ±.20	1.35 ±.30 1.35 ±.30	From τ_{01} From τ_{02}

* Mean of two values of q used.

The agreement between the values of π_q and π_τ in group II is eminently satisfactory. The accord in group I is not so good. The large value of π_τ is due partly to the four stars with abnormal proper motions mentioned above. If we leave these out, π_τ from the average τ_{01} reduces to 1.13 and that from the median τ_{02} to 0.89; and there still remains a considerable difference between π_q and π_τ for this group. It is desirable, therefore, to check the mean parallaxes derived above by other methods.

In *Mt. W. Contr.* No. 558,[17] Strömberg summarizes various methods of using proper-motion and radial-velocity data to secure values of mean parallaxes. Let us apply his formula (13),

$$\bar{\pi} = \frac{\Sigma(A\mu_1 + B\mu_2)}{p\Sigma(A^2 + B^2)} = \frac{k\Sigma\mu \sin\lambda}{pV_0\Sigma \sin^2\lambda},$$

in which μ_1 and μ_2 are the proper motions in arc in right ascension and declination, respectively, and A and B are functions of V_0 and the position of the apex. If the group motion projected on an axis toward the standard apex is numerically equal to the standard velocity, p is equal to unity and the formula is equivalent to $\pi_q = kq/V_0$, the formula used above. Inasmuch as we are using the velocity given by the groups under discussion and an apex which apparently represents the data as well as any, presumably we might assume $p = 1$ without introducing any appreciable error. We have, however, computed values of p as a check and find that the assumption is permissible. The first part of formula (13) presents the distinct advantage of deriving a value of $\bar{\pi}$ directly from the proper motions without computing the τ- and v-components. From it we find

$$\pi_q = 0.77 \text{ (I)}, \qquad 1.38 \text{ (II)} \qquad \text{(unit, } 0\overset{''}{.}001) .$$

Since we have already computed the v-components of the proper motions, we may also use formula (15) of the same article,

$$\bar{\pi} = \frac{\Sigma v}{p\Sigma\sqrt{A^2 + B^2}} = \frac{k\Sigma v}{pV_0\Sigma \sin\lambda} .$$

From the first part of this formula we find

$$\pi_q = 0.80 \text{ (I)}, \qquad 1.43 \text{ (II)} \qquad \text{(unit, } 0\overset{''}{.}001) .$$

The stars of group II being strongly concentrated near the galactic plane, we get still another determination of π_q from the projection

[17] *Ap. J.*, **84**, 555, 1936.

of the proper motions on the galactic plane. Using equations (34) of Strömberg's paper in the form

$$\bar{\pi} = \frac{\Sigma A_l \mu_l}{p \Sigma A_l^2},$$

we find

$$\pi_q = 1.11 \text{ (II)} \qquad \text{(unit, } 0\overset{''}{.}001\text{)} .$$

The values of π_q derived by different methods are, therefore, in satisfactory agreement.

For the peculiar motions we have the relations derived from Strömberg's formulae (11),

$$\bar{\epsilon} = \overline{\eta + \frac{\pi\theta}{k}} = \tfrac{1}{2}[\overline{\mu_1 - pA\pi} + \overline{\mu_2 - pB\pi}] ,$$

all means being taken without regard to sign. To solve this equation, we must know an approximate value of $\bar{\pi}$. This we have from the values of π_q. $\bar{\epsilon}$ is then determined from the last term in the equation, and we have

$$\bar{\epsilon}_{\text{av}} = 2\overset{''}{.}30 \text{ (I)} , \qquad 1\overset{''}{.}06 \text{ (II)} ,$$

$$\bar{\epsilon}_{\text{med}} = 1\overset{''}{.}90 \text{ (I)} , \qquad 0\overset{''}{.}90 \text{ (II)} .$$

Values of π_τ are then determined by the relation (16) of Strömberg,

$$\bar{\pi} = \frac{k}{\theta}\sqrt{\bar{\epsilon}^2 - \bar{\eta}^2} ,$$

$\bar{\eta}$ being the average error previously determined. The treatment of the median values of $\bar{\epsilon}$ in order to secure true values is not so simple. The assumption that the same relation holds between median values and probable errors as between average values and average errors will, however, give comparable results. It should be pointed out that, even if the parallax assumed in determining $\bar{\epsilon}$ is essentially correct, $\bar{\epsilon}_{\text{av}}$ will in general be larger than τ_{av}, and the parallax derived from it will be too large. This is due to two factors. First, $\bar{\epsilon}$ is the average of the peculiar motions in right ascension and declination and thus is equivalent to the combination of the average τ with the average residual resulting from correction for parallactic motion. Second, the statistical effect of the use of a constant value π for the solution of equations determining the values of residual v or of $\bar{\epsilon}$ is such as to produce values of these quantities that are too large, and hence mean parallaxes based upon them that are also too large. In dealing with stars with large dispersion in distance, therefore, this method should be used with caution. In taking the means to determine π_τ we have given it half-weight. The values of π_τ derived through $\bar{\epsilon}$ are

$\pi_\tau =$	1.38 (I),	1.76 (II)	average	
	1.10		4 rejections	(unit, 0\".001)
	1.13	1.72	median	

From the well-known relation

$$\pi_\tau = \frac{k}{\theta}\sqrt{\tau_{\text{av}}^2 - \overline{\eta}^2}\ ,$$

which is not subject to the foregoing criticism, we find

$\pi_\tau =$	1.35 (I),	1.62 (II)	average	
	1.13		4 rejections	(unit, 0\".001)
	0.75	1.12	median	

It is clear from these results that the differences between the mean parallaxes derived from the parallactic and the peculiar motions, as well as the range in the latter, arise from weakness in the proper-motion data rather than from the methods of treatment. Moreover, we are dealing with very small parallaxes, and the results are probably carried beyond a point justified by the data. The straight means of each set of determinations certainly give a fair representation of mean parallax. The various determinations are summarized in Table 8.

In determining $\overline{\pi}_m$ from the values of π_q and π_τ for group I a proper weighting is important, in view of the apparently systematic difference between them. From the probable errors of q and τ derived from the solutions the relative weights would be $\pi_q : \pi_\tau ::$ 2.4 : 1 (I) and 1.7 : 1 (II). From Russell's criterion[18] they would be 1.2 : 1 and 1.4 : 1, respectively. It has been shown, however, that the effects of the weaker determinations of proper motion enter systematically into the determinations of π_τ, whereas they tend to eliminate each other in the determination of π_q. Although we have attempted to take account of this correlation in various ways, the

TABLE 8

MEAN PARALLAXES

(Unit, 0".001)

Method	π_q Group I	π_q Group II	Method	π_τ Group I	π_τ Group II
Table 4	0.73	1.27	Kapteyn	1.13	1.35
Table 6	.76	1.40	$\bar{\tau}$-comp.	1.05	1.37
(13)	.77	1.38	ϵ-comp.	1.20	1.74
(15)	0.80	1.43	(wt., $\frac{1}{2}$)		
(34)	....	1.11			
Mean	0.76	1.30	Mean	1.11	1.44

dependence of the results for π_τ upon assumptions as to errors and their effects is such that we certainly cannot place as much reliance upon π_τ as upon π_q. We therefore have adopted the relative weights $\pi_q : \pi_\tau :: 3 : 1$ (I) and 2 : 1 (II). Combining with these weights, we find as the mean parallaxes of the RR Lyrae variables and the Cepheids, the proper motions of which are given in Tables 1 and 2,

$$\bar{\pi} = 0''.00085 \text{ (I)}, \qquad 0''.00135 \text{ (II)} .$$

V. THE ABSOLUTE PHOTOGRAPHIC MAGNITUDES

The mean parallaxes and mean apparent photographic magnitudes being known (see Table 6), the mean absolute photographic magnitudes are determined by the relation

$$\overline{M}_c = \overline{m} + 5 + 5 \log \bar{\pi} .$$

[18] *Ibid.*, 54, 140, 1921.

Hence,

$$\overline{M}_c = +0.15\ (\mathrm{I}),\ -1.39\ (\mathrm{II})\ .$$

These, however, are not true values; they must be corrected first for dispersion in absolute magnitude. Also, since the δ Cephei stars are strongly concentrated near the galactic plane, their mean apparent magnitude is presumably estimated too faintly, owing to galactic absorption, and must be corrected for this effect. Strömberg has shown[17] that if $\overline{M}$ denotes the true and $\overline{M}_c$ the computed mean absolute magnitude and if σ is the dispersion,

$$\overline{M} = \overline{M}_c - 5 \log C\ ,$$

where C is defined by the relation

$$\sigma^2 = 21.71 \log C\ .$$

Hence,

$$\overline{M} = \overline{M}_c - 0.23\sigma^2\ .$$

Since the proper motions of group I were reduced to the same apparent magnitude and among them there is very little dispersion in period and therefore in absolute magnitude, it may be assumed that for this group, $\sigma = 0$, approximately. From comparisons with the period-luminosity curve we find for group II, $\sigma = 0.54$. With these values we have

$$\overline{M} = \overline{M}_c\ (\mathrm{I}), \qquad \overline{M}_c - 0.07\ (\mathrm{II})\ .$$

For the additional correction to the mean absolute magnitude of group II, we use an absorption of 0.85 mag. (photographic) per kiloparsec, which Joy has found to satisfy the rotation factors for these stars. At the distance derived for these stars this correction amounts to -0.63. The true absolute magnitudes are therefore

$$\overline{M} = +0.15\ (\mathrm{I}), \qquad -2.09\ (\mathrm{II})\ .$$

With the values of log P in Table 6, we find from the period-luminosity curve

$$M_{\mathrm{curve}} = -0.03\ (\mathrm{I}), \qquad -1.70\ (\mathrm{II})\ .$$

The corrections to the curve resulting from this analysis are therefore

$$\Delta M = \overline{M} - M_{\text{curve}} = +0.18 \pm 0.23 \text{ (I)}, \qquad -0.38 \pm 0.22 \text{ (II)} .$$

The mean errors are estimated by means of Schlesinger's formula[19]

$$\text{m.e.} = \frac{R}{2n}\sqrt{\frac{(n+1)(n+2)}{3(n-1)}} ,$$

where R is the range in the absolute magnitudes derived from the various mean parallaxes given in Section IV and n the number of determinations.

Approximate values of the mean absolute magnitudes may be derived from the radial velocities without recourse to the proper motions through consideration of the rotation factors $\bar{r}A$. The degree of approximation which can be attained depends not only upon the accuracy with which $\bar{r}A$ and A may be determined but also upon the relation of $\bar{\pi}$ to $\bar{r}$. For A we adopt Joy's determination, 20.9 km/sec per kiloparsec.[12] Since from our radial velocity solutions

$$\bar{r}A = 37 \pm 20 \text{ (I)}, \qquad 27.4 \pm 1.4 \text{ (II)} ,$$

it follows that

$$r = 1.77 \pm 0.96 \text{ kpc (I)}, \qquad 1.31 \pm 0.07 \text{ kpc (II)} .$$

It is known that

$$\bar{\pi} \cdot \bar{r} = e^{(0.46\sigma)^2} ,$$

where σ is again the dispersion in absolute magnitude. By means of this relation we find

$$\bar{\pi} = 0''.00057 \text{ (I)}, \qquad 0''.00088 \text{ (II)} .$$

The mean photographic magnitudes of the stars with measured radial velocities are

$$\bar{m}_{\text{pg}} = 10.84 \text{ (I)} , \qquad 9.85 \text{ (II)} .$$

[19] *A.J.*, **46**, 161, 1937.

Therefore,

$$\overline{M}_c = -0.38 \text{ (I)}, \qquad -0.43 \text{ (II)} .$$

Correcting for dispersion in absolute magnitude and, in group II, for galactic absorption, we have

$$\overline{M} = -0.38 \text{ (I)} \qquad -1.47 \text{ (II)} .$$

Since for these stars

$$\overline{\log P} = -0.32 \text{ (I)}, \qquad +0.97 \text{ (II)} ,$$

we find

$$M_{\text{curve}} = -0.03 \text{ (I)}, \qquad -1.84 \text{ (II)} .$$

Hence the corrections to the period-luminosity curve derived from the rotational coefficients are

$$\Delta M = -0.35 \text{ (I)}, \qquad +0.37 \text{ (II)} .$$

Though subject to considerable uncertainty, these results are quite comparable, quantitatively, with those derived in the previous analysis. If we combine the two sets of results, giving half-weight to the latter, we have

$$\overline{\Delta M} = 0.00 \text{ (I)}, \qquad -0.14 \text{ (II)} ,$$

with a probable error of about 0.2 in each case. It would appear, therefore, that the zero point of the period-luminosity curve, as given in Shapley's *Star Clusters*, is essentially correct. Certainly there is no evidence in the radial-velocity and proper-motion data now available that would justify a change.

VI. THE SHAPE OF THE CURVE

The shape of the period-luminosity curve is based upon the magnitudes and periods of variables in the Magellanic Clouds, globular clusters, and extragalactic nebulae. The fact that the corrections to the zero point derived in the preceding section from two groups of stars with quite different periods are essentially equal is in itself some confirmation of the general shape. The criticism by Schilt[20] that there is a break in the continuity of the curve at a period of 10 days and that the Cepheids of longer period are absolutely fainter than those having periods from 2 to 10 days was perhaps satisfactorily answered by Gerasimovič.[2] There can be little doubt that some, at least, of the anomalies which produced the criticism were due to the scarcity and poor quality of the data at that time available. It seemed desirable, therefore, to see what evidence of continuity might come out of the data recently accumulated. Group II has therefore been broken down into three subgroups with log $P < 0.75$, $= 0.75-1.00$, and > 1.00, the maximum value being 1.65. It would have been desirable to make two groups of those stars for which log $P > 1.00$, but the proper-motion data are not good enough to permit it. The proper-motion data are exhibited in Table 9, following the general form of Table 6.

TABLE 9

PROPER-MOTION DATA

(Unit, 0".01)

	LOG P		
	<0.75	$0.75-1.00$	>1.00
$\overline{\log P}$	0.60	0.88	1.26
m_{pg}	7.75	7.50	8.67
No.	32	27	27
μ_{med}	1.70	1.37	1.80
μ_{av}	1.83	1.53	2.05
Wt.	30.2	29.7	25.1
$\bar{\tau}$	$-0.01\pm.13$	$-0.13\pm.14$	$+0.06\pm.18$
$\bar{v}$	$+0.93\pm.16$	$+0.59\pm.12$	$+0.14\pm.20$
τ_{av}	$0.95\pm.13$	$0.97\pm.13$	$1.24\pm.19$
$\bar{p}$	0.66	0.61	0.79
η	0.78	0.72	0.93
q	$1.23\pm.21$	$0.76\pm.16$	$0.31\pm.26$

While the decrease in v and q with increasing $\overline{\log P}$ is evidence of increase in absolute magnitude with period, the trend in τ_{av} would appear to be in the opposite direction. Unfortunately, we cannot assume that the correlation between τ and ρ is the same in each sub-

[20] *Ap. J.*, **64**, 149, 1926; *A.J.*, **38**, 197, 1928.

division as in the group as a whole. Investigation shows that it is not, κ being small in the first two groups and large in the last; but the data in the subgroups are not sufficient to give determinate values of κ. We cannot, therefore, use Kapteyn's formula. We can, however, determine η from the quoted probable errors and apply the formulae correlating $\bar{\pi}$ and $\bar{\epsilon}$ and $\bar{\pi}$ and τ_{av}, used in Section IV. We must assume that the group motion of each of the subgroups is the same as that of the whole group. Fortunately, this can be approximately verified through the mean peculiar motions. We find θ = 14.4, 14.0, and 14.0 km/sec. Their group motions along the direction of the apex must therefore be approximately the same.

TABLE 10

π_q AND π_τ FROM SUBGROUPS

π_q			π_τ		
q	(13)	Mean	τ_{av}	$\bar{\epsilon}$	Mean
0″.0021	0″.0022	0″.0021	0″.0017	0″.0029	0″.0021
.0013	.0013	.0013	.0022	.0024	.0023
0.0005	0.0003	0.0004	0.0028	0.0009	0.0022

From the values of q and V_0 and from the application of Strömberg's formula (13) we get two sets of values of π_q. Likewise, from the τ_{av} values corrected for the average errors η and from the values of $\bar{\epsilon}$, also corrected for average error, we get two sets of values of π_τ. These are shown in Table 10. The values of π_q and the values of π_τ derived from the $\bar{\epsilon}$'s show the same trend. In view of the equality in the peculiar motions θ, the opposite trend in π_τ, as determined from the τ-components, must be attributed to the weakness of the data. We take the straight means of the two sets of values to determine π_q, and give the value of π_τ based on $\bar{\epsilon}$ half-weight in the determination of $\bar{\pi}_\tau$, as in Section IV. Combining these mean values with the relative weights, 2:1, assigned in the same section, we have the values of $\bar{\pi}$ given in the first column of Table 11. With these and the apparent magnitudes in Table 9, we compute $\overline{M}_c$, which, corrected for dispersion and absorption as in Section V, gives us $\overline{M}$. Comparison with the period-luminosity curve shows the differences in the last column of the table. The projection on the period-luminosity curve of the various values of $\overline{M}$ derived in this and the preceding sections is shown in Figure 1. There is in them no evidence of discontinuity in the curve.

TABLE 11

MEAN ABSOLUTE MAGNITUDES FROM SUBGROUPS

$\overline{\pi}$	$\overline{M}_c$	$\overline{M}$	M_{curve}	ΔM
0".00210..........	−0.64	−1.05	−1.22	+0.17
.00163..........	1.44	1.96	1.66	− .30
0.00100..........	−1.33	−2.18	−2.24	+0.06

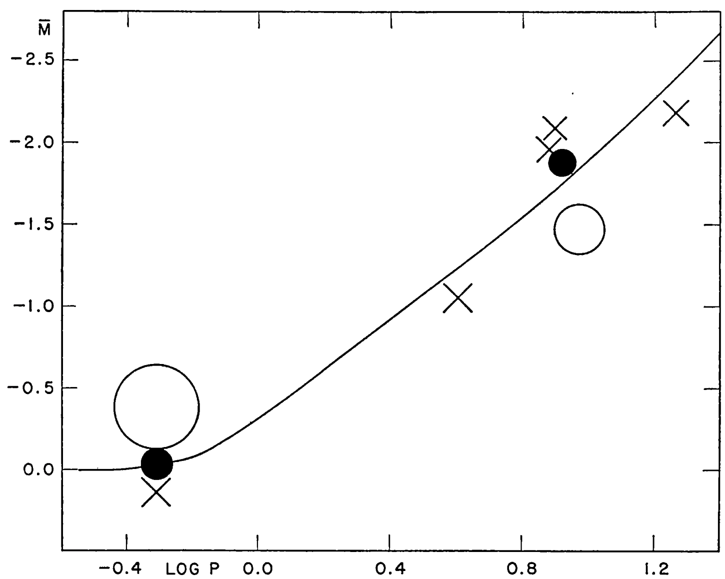

FIG. 1.—The curve represents the photographic period-luminosity relation (Shapley, *Star Clusters*, p. 135, 1930); crosses denote $\overline{M}$ derived from parallactic and peculiar motions; open circles, $\overline{M}$ derived from rotation factors; and filled circles, the concluded mean values.

Since reduced total proper motion should be a direct index of absolute magnitude, we should find evidence of a correlation between period and proper motion. The earlier proper motions gave very little evidence of such a dependence and tended to cast some doubt about the correlation of period and luminosity among the galactic Cepheids. Most investigators have felt that this was largely due to the weakness of the proper-motion data. The mean parallactic motion of these stars is only 0".008, and the peculiar motions are known to be small. It is obvious, therefore, that in the means of small numbers of proper motions with large probable errors, small changes, such as those to be expected from the period-luminosity relation, may be completely hidden, whereas better data might reveal them. Of the 86 stars listed in Table 2, 52 have proper motions with probable errors $\leq$0".008. Their mean magnitude is 6.5. Their proper motions, reduced to this magnitude, show a small but definite decrease with period, as shown in columns 1 and 2 of Table 12.

TABLE 12

THE PERIOD-PROPER-MOTION DEPENDENCE FROM THE BEST PROPER MOTIONS

$\overline{\log P}$	$\bar{\mu}$	$\bar{\pi}$	$\overline{M}_{6.5}$	$\overline{M}_{8.0}$	M_{curve}	ΔM	ΔM_1
0.56..........	0".0192	0".00168	−2.85	−1.38	−1.16	−0.22	−0.02
0.84..........	.0157	.00138	3.41	1.91	1.60	.31	−0.11
1.23..........	0.0134	0.00118	−3.86	−2.36	−2.32	−0.04	+0.16
0.86..........	0.0162	0.00142	−3.34	−1.84	−1.64	−0.20	0.00

We may go farther and compare this dependence with the period-luminosity curve. From all the stars in Table 2 we have found that for $\bar{m} = 8.0$ and $\overline{\log P} = 0.89$, $\mu_r = 0''.0154$ and $\bar{\pi} = 0''.00135$. If we assume that the ratio $\bar{\pi}/\mu_r$, here found, holds in the groups of Table 12, we get the values of $\bar{\pi}$ in column 3 and the values of $\overline{M}_{6.5}$ and $\overline{M}_{8.0}$, corrected for absorption, in the succeeding columns. The small values of ΔM in the last two columns show definitely that the observed decrease in μ_r derived from the best-determined proper motions follows the period-luminosity curve.

The conclusion is reached, therefore, that the radial-velocity and proper-motion data now available confirm both the zero point and the shape of the photographic period-luminosity correlation for the Cepheid variables.

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