

A PHOTOELECTRIC LIGHT-CURVE OF EROS

F. E. ROACH and LAURENCE G. STODDARD

ABSTRACT

Photoelectric observations of Eros made on February 4-5, 1938, when the earth was in or near the asteroid's equatorial plane, are presented. A new analysis of the form and size of Eros, following Krug and Schrutka-Rechtenstamm, leads to a three-axis ellipsoid with axes of 35.0, 15.6, and 7.2 km.

In a recent discussion of the photometric data of Eros, Watson¹ predicted that the earth would be in a plane 90° from Eros' pole of rotation on about February 5, 1938. Independent of any hypothesis of the geometrical figure of the asteroid and the law of reflection, the range of light variation should be a maximum at this time, the value predicted by Watson being 1.5 mag.

Eros' period of rotation is about 5^h16^m . In this interval there occur two maxima and two minima. Our photoelectric observations, starting 1938, Feb. $5^d4^h24^m$ G.C.T., cover just one-half of the rotation period, and include a light maximum at 5^h07^m and a minimum at 6^h21^m . The observed range is 1.38 mag. Since the color index is a constant equal to about $+0.80$ mag. throughout the rotation period², this range is the same as would have been obtained visually.

THE PHOTOMETER

The photoelectric cell of our photometer is a Kunz quartz cell having a sensitive surface of potassium hydride. The photoelectric current is amplified by a Pliotron FP-54 tube in a balanced circuit similar to that developed by DuBridge and Brown.³ Characteristics of the photometer and amplifier are assembled in Table 1. The cell characteristics are for an anode potential of 300 volts, although it is possible to use somewhat higher voltages with correspondingly greater sensitivity.

The instrument was made, under the senior author's supervision, by two graduate students of the University of Arizona, Messrs.

¹ *Harvard Circ.*, No. 419, 1937.

² *Ibid.*, p. 12.

³ *Rev. Scientific Instruments*, 4, 537, 1933.

Amos H. Hoff and Willard G. Schmidt. It may be used at any one of the three foci of the 36-inch reflector. The observations here reported were made at the Newtonian focus.

TABLE 1

PHOTOMETER AND AMPLIFIER CHARACTERISTICS

Galvanometer.....	Leeds and Northrup, Type H. S.
Current sensitivity.....	2.1×10^{-10} amps/mm at 1 meter
Amplifier:	
Grid leak resistance.....	5.65×10^{10} ohms
Current amplification.....	7.9×10^5
Resultant current sensitivity..	2.66×10^{-16} amps/mm
Photoelectric cell:	
Anode potential.....	300 volts
Stellar magnitude correspond- ing to 1 mm deflection.....	14.4
Total light of 14.4 mag. as col- lected by 36-inch reflector..	9.65×10^{-12} lumens
Light sensitivity of cell.....	27.5 microamperes/lumen

THE OBSERVATIONS

The observational data are recorded in Table 2. At the start there was in the western sky a five-day-old moon, which set at 5^h50^m G.C.T. Nine sky background readings were made, and the recorded readings in Table 2 have been corrected for this. The comparison star was HD 34020 (BD+21°800). Between the fifth and sixth observations the anode voltage was increased. The maximum difference in the secants of the zenith distances of the asteroid and the comparison star was 0.01. This would lead to maximum differential extinction of about 0.002 mag. In the reductions differential extinction has not been applied. Alternate settings were made on the comparison star and on the asteroid, with four galvanometer readings per setting. The times in Table 2 refer to the observations of Eros. The comparison star readings of the third column have been taken from settings both before and after the Eros readings, each corrected for the rapidly changing sky background. The results are plotted in Figure 1.

TABLE 2

THE OBSERVATIONS

TIME (G.C.T.) FEB. 5, 1938	GALVANOMETER DEFLECTION (MM.)		Δ mag	mpg OF EROS (mpg HD 34020 = 9.70)
	Eros	HD 34020		
4 ^h 24 ^m	18.6	50.1	1.08	10.78
4 39.....	23.5	49.2	0.80	10.50
4 59.....	30.3	49.05	0.52	10.22
5 19.....	30.6	48.7	0.50	10.20
5 31.....	25.3	47.8	0.69	10.39
5 51.....	16.0	49.65	1.23	10.93
6 07.....	10.4	44.55	1.58	11.28
6 20.....	8.4	45.75	1.84	11.54
6 31.....	9.2	45.2	1.73	11.43
6 41.....	12.9	43.2	1.31	11.01
6 50.....	13.1	39.15	1.19	10.89

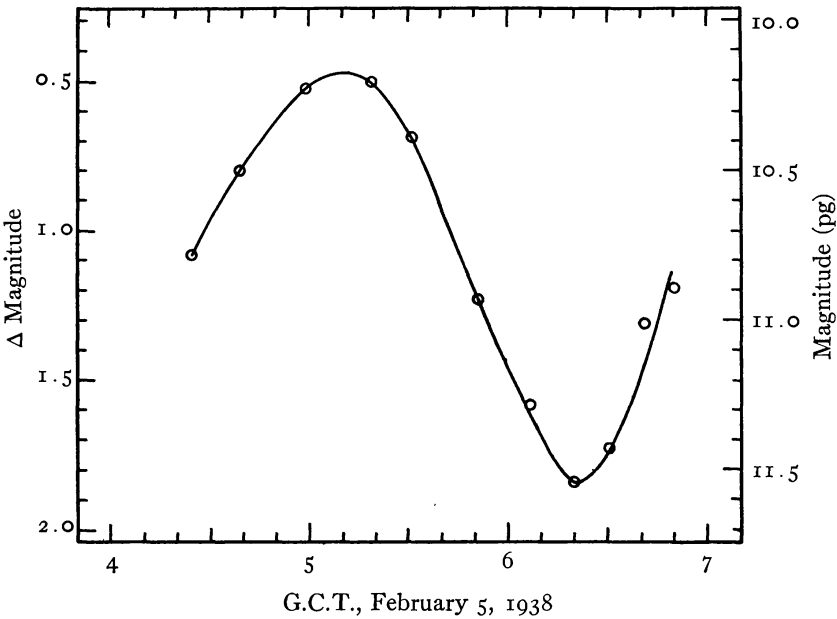


FIG. 1

ANALYSIS OF OBSERVATIONS

In a recent paper⁴ Krug and Schrutka-Rechtenstamm have given an analysis of the form and size of Eros based upon the following assumptions: (a) the object is a three-axis ellipsoid at full phase rotating about its shortest axis; (b) it reflects according to Lambert's law. They obtained equations giving the brightness in terms of the ellipsoid's semiaxes, a , b , and c , where $a > b > c$. We reproduce here their equation (1):

$$\left. \begin{aligned} Q &= Q_A \cos^2 A + Q_B \cos^2 B + Q_C \cos^2 C, \\ &= Q_C \cos^2 C + \sin^2 C [Q_A \cos^2 (\omega t + \phi) + Q_B \sin^2 (\omega t + \phi)], \end{aligned} \right\} \quad (1)$$

where Q is the observed brightness; Q_A , Q_B , and Q_C are the brightnesses as seen along the a , b , and c axes, respectively; A , B , and C are the angles between the line of sight and the a , b , and c axes, respectively; ω is the angular velocity of rotation; t is the time; and ϕ is a phase angle. At maximum and minimum light this becomes

$$\left. \begin{aligned} Q_{\max} &= Q_C \cos^2 C + Q_B \sin^2 C, \\ Q_{\min} &= Q_C \cos^2 C + Q_A \sin^2 C. \end{aligned} \right\} \quad (2)$$

If $C = 90^\circ$, i.e., if the observer is in the equatorial plane of the ellipsoid,

$$\left. \begin{aligned} Q &= Q_A \cos^2 (\omega t + \phi) + Q_B \sin^2 (\omega t + \phi), \\ Q_{\max} &= Q_B, \\ Q_{\min} &= Q_A. \end{aligned} \right\} \quad (3)$$

Equations (1), (2), and (3) apply to any geometrical figure at full phase reflecting according to Lambert's law. A least-squares solution of equation (3) for our data gives $Q_A = 1$, $Q_B = 3.60$, $\omega = 0.0213$ radians per minute, and $\phi = 0$ if $t = 0$ at $3^{\text{h}}52^{\text{m}}$ G.C.T. This solution is plotted in Figure 2, together with the observations.

Equations (2) in the paper by Krug and Schrutka-Rechtenstamm give the dependence of Q_A , Q_B , and Q_C on the semiaxes of a three-

⁴ *Zs. f. Ap.*, **13**, Part I, 1, 1936.

axis ellipsoid. We wish to give an analysis based upon the ratios Q_B/Q_A and Q_C/Q_B , which follow at once from these equations, viz.,

$$\frac{Q_B}{Q_A} = \frac{a^2}{b(b^2 - c^2)} \left[\frac{\sqrt{a^2 - c^2}(b^2 - c^2) - (a^2 - c^2)bE + b(a^2 - b^2)F}{-b\sqrt{a^2 - c^2} + b^2E + (a^2 - b^2)F} \right], \quad (4)$$

$$\frac{Q_C}{Q_B} = \frac{b(a^2 - b^2)}{c^2} \left[\frac{b^2E - c^2F}{(b^2 - c^2)\sqrt{a^2 - c^2} - b(a^2 - c^2)E + b(a^2 - b^2)F} \right], \quad (5)$$

where

$$E = \int_0^\phi \sqrt{1 - k^2 \sin^2 z} \, dz,$$

$$F = \int_0^\phi \frac{dz}{\sqrt{1 - k^2 \sin^2 z}},$$

$$k = \frac{a\sqrt{b^2 - c^2}}{b\sqrt{a^2 - c^2}},$$

$$\phi = \arcsin \frac{c}{a}.$$

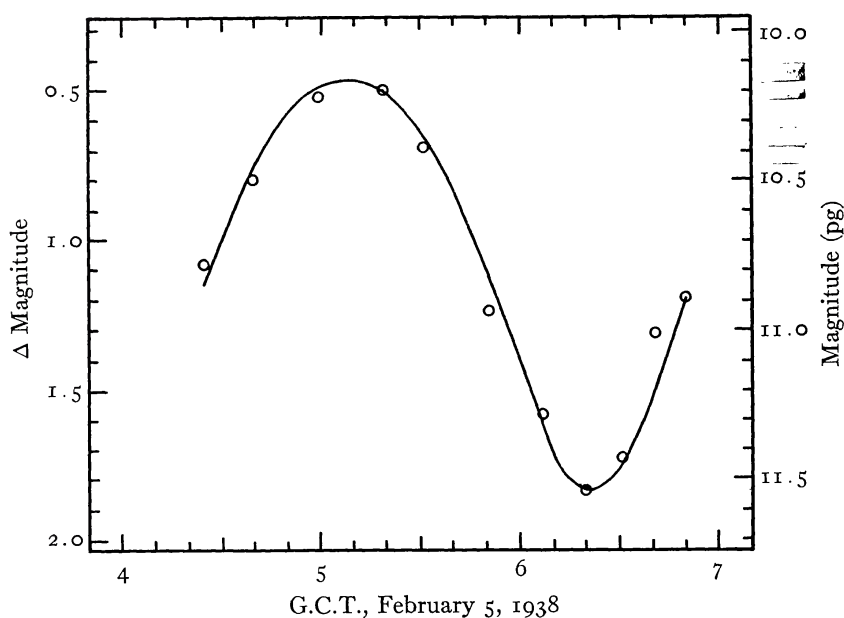


FIG. 2.—Plot of least-squares solution of equation (3)

The functions E and F with k and ϕ as arguments may be found in *Formules et Tables Numériques* by L. Potin.

Krug and Schrutka-Rechtenstamm have utilized forty-four observations of the magnitudes and light variations of Eros and have made a least-squares solution for (1) the position of the north pole of rotation; (2) the values of Q_A , Q_B , and Q_C ; and (3) the corresponding values of a , b , and c . A summary of their results follows: Position of the north pole of rotation: $a = 22^{\text{h}}11^{\text{m}}6$, $\delta = +47^{\circ}7$; $Q_A = 0.827$; $Q_B = 2.168$; $Q_C = 2.830$; $2a = 34.6$ km; $2b = 19.3$ km; $2c = 16.3$ km. From these results we find $Q_B/Q_A = 2.622$, corresponding to a maximum light range of 1.046 mag.; $Q_C/Q_B = 1.305$; if $a = 1$, $b = 0.558$ and $c = 0.471$.

A criticism of these results which was pointed out by Krug and Schrutka-Rechtenstamm is that the maximum amplitude of light variation permitted by the model is definitely lower than is observed as the earth approaches the equatorial plane, i.e., as C approaches 90° . For example, at the time of the observations reported in this paper the value of C was 84° . With this value of C their solution gives for the range of light variation 1.025 mag., as compared with the observed 1.38 mag. of this paper.

We have made a new analysis of the data of Krug and Schrutka-Rechtenstamm on the following basis: (1) In their table of observations we have omitted all observations in which no variation was observed; (2) we assume $Q_B/Q_A = 4.00$, corresponding to $\Delta m = 1.50$ mag., and solve for Q_C/Q_B ; (3) we assume their pole of rotation; (4) this analysis gives no weight to the *actual* magnitude of any observation but only to the range of light variation. The latter is doubtless known with greater accuracy than the former. The analysis of Krug and Schrutka-Rechtenstamm gave weight to both the actual magnitude and the range. A least-squares solution gives $Q_C/Q_B = 3.25$.

A simultaneous solution of equations (4) and (5), assuming $a = 1.00$, yields $b = 0.447$ and $c = 0.207$. If $2a$ is taken as 35 km⁵ this gives $2b = 15.6$ km and $2c = 7.2$ km. Our solution gives for $C = 84^{\circ}$ a range of 1.40 mag., as compared with the 1.38 mag. observed.

⁵ Cf. Watson, *op. cit.*, p. 9.

Two difficulties have always appeared in previous efforts to analyze the observations of Eros. First, computations of the phase coefficient have shown a wide variation from one opposition to another. Second, the absolute magnitudes derived at various oppositions have shown a decided lack of agreement. These two constants are not independent, as the value of the absolute magnitude in any given case depends on the assumed phase coefficient. Our model

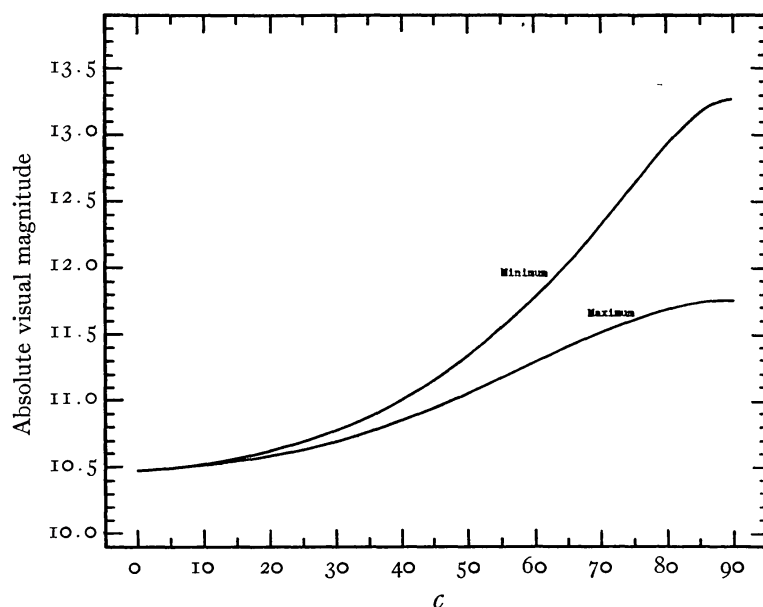


FIG. 3.—Predicted absolute visual magnitude of Eros. The abscissa is the angle between the line of sight and the axis of rotation.

gives a mechanism for the possible clearing-up of these enigmas, as does the model of Krug and Schrutka-Rechtenstamm in a lesser degree. In the first place, there will be no single value of the absolute magnitude, but rather a variation from maximum brightness, if the observer is in the direction of the pole of rotation, to a minimum value when $C = 90^\circ$. Similarly, the phase coefficient will depend upon the rate at which C changes during a series of observations.

In Figure 3 we show a plot of the absolute visual magnitudes at maximum and minimum, on the basis of our solution. This curve depends upon the assumptions that the photographic magnitude

of HD 34020 is 9.70 mag., that the color index of Eros is $+0.80$ mag., and that the phase coefficient is 0.022 mag. per degree of phase angle. The variation of the predicted range with the angle C is shown in Figure 4.

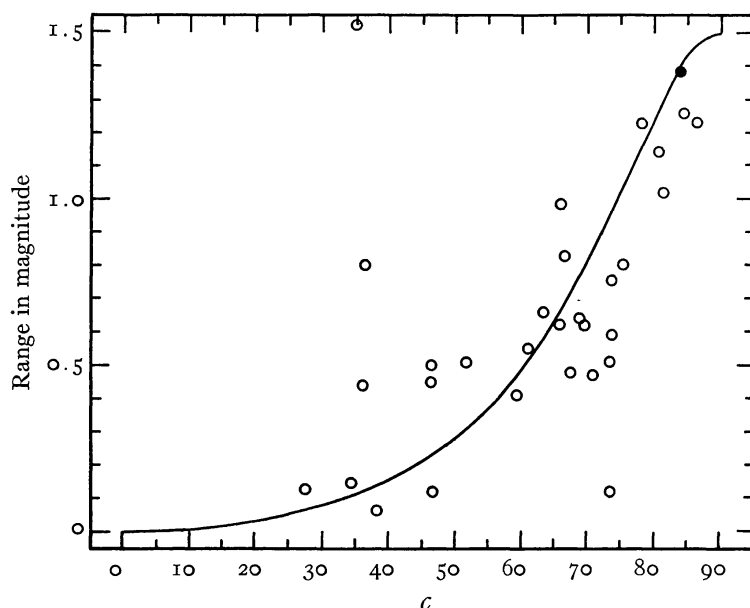


FIG. 4.—Predicted variation of light range with the angle C . The open circles represent the data of Krug and Schrutka-Rechtenstamm. The solid circle represents the data of this paper.

CONCLUSION

It should be remarked that this analysis assumes Lambert's law of reflection, and that Eros is at full phase. The second of these assumptions is definitely not true for our observations, since the phase angle was $46^{\circ}44'$. The fair agreement of our observations and equation (3) may, therefore, be fortuitous.

STEWART OBSERVATORY
UNIVERSITY OF ARIZONA
TUCSON, ARIZONA
May 23, 1938