into factors depending respectively on the photospheric compressibility and the internal compressibility. The photospheric compressibility can be approximately represented by a polytropic index $s=3$. The resulting luminosity formula appears to represent the main features of the observed mass-luminosity correlation and gives the right order of magnitude for the luminosity. It is shown what inferences are possible about a stellar interior from observable data. The pressure in a stellar photosphere, for stars of similar internal structure, should vary as the fourth power of the effective temperature and inversely as the square of the mass, independent of photospheric opacity. Sundry applications of this result are made.

University Observatory, Oxford:
1937 *April.*

VISUAL OBSERVATIONS OF INTENSITY OF DARK MARKINGS ON THE SUN, AND OF PROMINENCES.

R. v. d. R. Woolley and H. W. Newton.

(Communicated by the Astronomer Royal)

1. In a previous communication to the Society * we have described visual observations of the contour of the line $H\alpha$ with a spectrohelioscope, using a special auxiliary photometer. While we recognise that the spectrohelioscope is not an ideal instrument for the determination of line contours, it is obvious that it offers special advantages for the determination of the spectral contours of transient phenomena, namely, prominences on the limb and bright and dark markings on the disk, and we have felt justified in continuing measurements of this kind with the spectrohelioscope.

A line contour determined with any spectrograph requires correction (*a*) for the finite resolving power of the instrument and (*b*) for what may be called heterochromatic light, unless a monochromator has been used. The presence of light whose wave-length differs from that of the line by an amount large compared with the resolution of the instrument is a well-known phenomenon. (A discussion of the subject will be found in *The Observatory*, **58**, 5, 1935.) The effects of finite resolving power may be computed if the effective resolving power of the spectrograph is known. To eliminate heterochromatic light it is desirable to use a monochromator. Failing this, it is desirable to determine the amount of heterochromatic light experimentally. In our case it has not been possible to erect a monochromator or to make a direct determination of the heterochromatic light, and we have instead determined this amount by comparison of our contour for $H\alpha$ (corrected for finite resolving power) with Thackeray's contour † which was determined with a higher resolution than ours and with a monochromator.

2. *The Correction for Finite Resolving Power.*—Consider first an imaginary spectrograph of infinite resolving power. All the light of wave-length λ

* *M.N.*, **96**, 5, 1935. † *M.N.*, **95**, 293, 1935.

falls on a line in the focal plane which has some co-ordinate x, so that the intensity distribution $I(x)$ is an accurate representation of the intensity distribution $I(\lambda)$ in the spectrum under measurement. In a real instrument of finite resolving power, the line x receives a certain amount of light from wave-lengths contiguous to the wave-length which properly corresponds to x (and loses some of its own light), so that the measured intensity $I_1(x)$ will be given by

$$I_1(x) = \int I(x) f(\xi - x) d\xi, \tag{1}$$

where the function $f(\xi - x)$ depends on the resolving performance of the spectrograph. Its general form is that it has a maximum at $\xi - x = 0$ and that it vanishes when $|\xi - x|$ attains values larger than the practical resolution of the instrument in the x co-ordinate.

In the special case where the lack of resolution is *entirely* due to the use of a wide slit (of width $2h$) we have

$$\begin{aligned} f(\xi - x) &= 1 \qquad |\xi - x| < h \\ f(\xi - x) &= 0 \qquad |\xi - x| \geqslant h, \end{aligned}$$

and the intensity received by the point x takes the simple form

$$I_1(x) = \frac{1}{2h} \int_{x-h}^{x+h} I(x) dx. \tag{2}$$

Since the slits used in the spectrohelioscope were about 0·1 mm. wide, corresponding to 0·5 A., while the resolving power of the grating is 43,000, the lack of resolution may in this case be considered as being due to the wide slits alone. We proceed on that assumption, using equation (2).

In the spectrohelioscope the spectrum is viewed through a second slit. The effect of this second slit as regards resolution is similar to the first, and the intensity $I_2(x)$ which is transmitted by the second slit is given by

$$I_2(x) = \frac{1}{2h} \int_{x-h}^{x+h} I_1(x) dx. \tag{3}$$

Now let us consider equation (2). If $I(x)$ is a constant or is a linear function of x, we shall have $I_1(x) = I(x)$; it is only when the second differences of $I(x)$ do not vanish that $I_1(x) \neq I(x)$. The wide slit is sampling a set of values of a thing (the intensity of the spectrum), and if the thing is linear the mean value is the value in the middle of the range sampled. Only second (and fourth and sixth, etc.) differences upset this. It is only a matter of algebra to develop expressions for $I_1(x)$ and $I_2(x)$ when $I(x)$ has any finite set of differences, but we have found in practice that the fourth differences have no practical significance in our contours (or Thackeray's), and we have accordingly only worked with the second difference. This gives an extremely simple arithmetical procedure for correcting a contour for finite resolving power, as will appear.

If the fourth and higher differences of $I(x)$ may be neglected, then

$$I_1(x) = I(x) + \frac{1}{3!} h^2 \, . \, I''(x) \qquad I_2(x) = I_x + \frac{2}{3!} h^2 I''(x).$$

Conversely, neglecting the fourth and higher differences of $I_2(x)$,

$$I(x) = I_2(x) - \frac{2}{3!}h^2 I_2''(x). \tag{4}$$

It will be noticed that the correction for finite resolving power does not alter the equivalent breadth of the line, since

$$\int_{-\infty}^{\infty} I(x)dx = \int_{-\infty}^{\infty} I_2(x)dx.$$

Equation (4) gives a very convenient way of constructing $I(x)$, the true contour, from $I_2(x)$, the observed contour.

Plot the observed contour $I_2(x)$ and read off a number of values of the intensity for equally spaced values of the abscissa x, choosing this spacing to be a fraction such as one-fifth or one-sixth of the slit width $2h$. Then the second differences of the ordinates read from the graph will give approximate values for $I_2''(x)$. The contour $I_2(x)$ can be corrected to the true contour $I(x)$ by using values of $I_2''(x)$ obtained in this way (and smoothed where necessary) the units in which h is expressed in equation (4) being the unit of spacing of the abscissa. Table I shows an example: the central portion of $H\alpha$ as observed by Newton and Barton. Here $h = 2{\cdot}5$.

TABLE I

Correction of Contour of Normal $H\alpha$ for Finite Resolving Power

Distance in Angstroms from Centre	$I_2(x)$	1st Difference	2nd Difference	2nd Difference Smoothed	$I(x)$
+1·0	67½				
		3			
0·9	64½		0	−0·5	65½
		3			
0·8	61½		−1	−0·5	62½
		4			
0·7	57½		−½	−0·9	59½
		4½			
0·6	53		−1	−0·9	55
		5½			
0·5	47½		−2	−0·5	48
		7½			
0·4	40		0	0·0	40
		7½			
0·3	32½		+2½	+0·7	31
		5			
0·2	27½		+2½	+1·5	24½
		2½			
0·1	25		+½	+1·8	21
		2			
0·0	23		+2½	+1·7	19
		−½			
−0·1	23½		+1½	+1·6	20
		−2			
−0·2	25½				

The unit of $I(x)$ is such that the continuous background outside the line = 100.

The smoothed values are the means of consecutive fives. Half-integers are shown in order to illustrate the correction carefully, but no confidence is to be placed in them from the point of view of probable error.

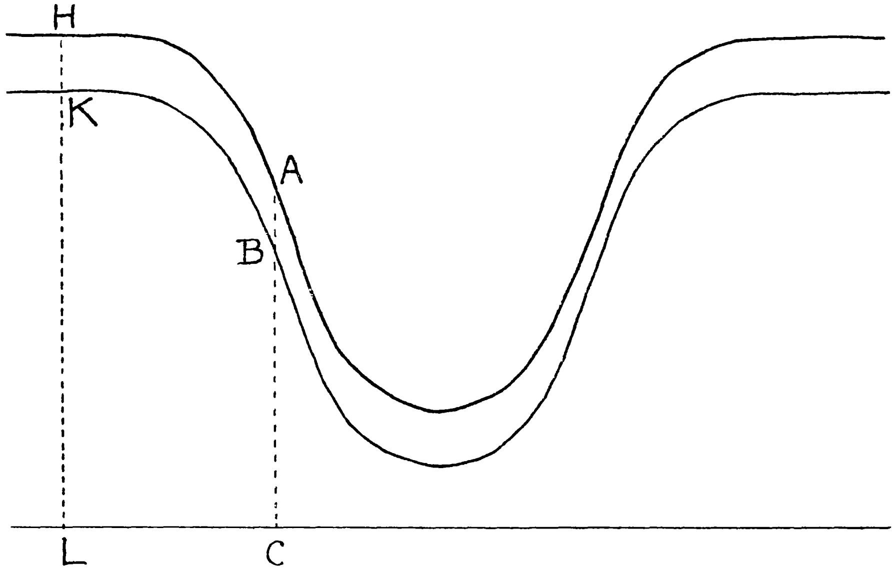

FIG. 1.

3. *Heterochromatic Light.*—Suppose that at any point which is clear of absorption lines the spectrograph overlays the spectrum with the fraction s of heterochromatic light. In fig. 1 the upper curve represents the actual spectrum given by the spectrograph, and the lower curve represents the spectrum which would be given if a monochromator was placed in front of the spectrograph. Then $HK = s \cdot KL$. Consider a point inside an absorption line. The intensity given by the actual spectrograph is $r' = AC/HL$, and the true intensity r is BC/KL. Then

$$r = r' - s(1 - r').$$

As will be seen from Table I, the value of r' at the centre of the line is 0·19. If we assume that the true central intensity of $H\alpha$ on the undisturbed disk is 0·15 (following Thackeray), then $s = 0{\cdot}05$. This is a very reasonable result, as another grating examined at Greenwich by one of us and Mr. Davidson gave precisely this value of s. In neither case is the result supposed to be exact—the 5 per cent. might very well be 6 or 4, and the two gratings need not give the same result.

4. Measurements of the contour of $H\alpha$ on the undisturbed disk have been described in our previous paper. The contour of $H\alpha$ in dark markings has now been determined in the following manner. A calibrated wedge, incorporating a marginal strip of clear glass of the same thickness as that of the wedge itself, is inserted in the field of view between the eye-piece and the rotating prisms of the spectrohelioscope. The dark $H\alpha$ marking is seen through the clear glass and the neighbouring background, in the same wave-length (in the middle of $H\alpha$ if the marking has no radial velocity) is seen through the graduated wedge. The wedge is then adjusted until

a match in intensity is made at the fiducial point in the field between the marking and the background, and a scale reading is taken. The line-shifter is then turned so that the wave-length is increased by successive steps of, say, 0·10 A., and wedge readings are repeated in the same manner at each change of wave-length, until at about 0·65 A. from the centre of the

TABLE II

Contour of Hα in Undisturbed Disk and in Dark Markings

Distance in Angstroms from Centre	Disk		Dark Marking	
	Uncorrected	Corrected	Uncorrected	Corrected
+4·0	95	95	95	95
3·0	87	87	87	87
2·0	78	78	78	78
1·5	76	75	76	75
1·0	68	67	68	67
0·9	65	64	65	64
0·8	61	61	61	61
0·7	58	58	58	58
0·6	53	53	50	50
0·5	48	46	38	36
0·4	40	37	28	23
0·3	32	27	20	14
0·2	27	20	14	5
+0·1	25	17	12	3
0·0	23	15	11	2
−0·1	24	16	12	3
0·2	26	19	14	6
0·3	30	25	18	10
0·4	36	32	24	18
0·5	42	39	32	29
0·6	48	46	44	42
0·7	53	52	53	53
0·8	57	57	57	59
0·9	61	60	61	61
1·0	64	64	64	64
1·5	76	74	76	74
2·0	(78)	(78)	(78)	(78)
3·0	86	86	86	86
−4·0	92	92	92	92

Hα line the dark marking merges into the adjacent background (the wings of the *Hα* line). The wedge readings when reduced give the ratio of intensity of background to dark marking; the observed intensity of the background at distances 0·10, 0·20 . . . 4·0 A. from the centre of *Hα* is known from the previously determined contour of *Hα* observed with the same instrument by means of a lamp photometer (*M.N.*, **96**, 8, 1935). Hence the contour of the dark marking can be inferred with respect to the continuous spectrum (= 100) adjacent to the *Hα* line.

This differential method has the advantage that the comparison between flocculus and background is made directly without the intermediary of a lamp. Use can therefore be made of short spells of fairly clear skies which

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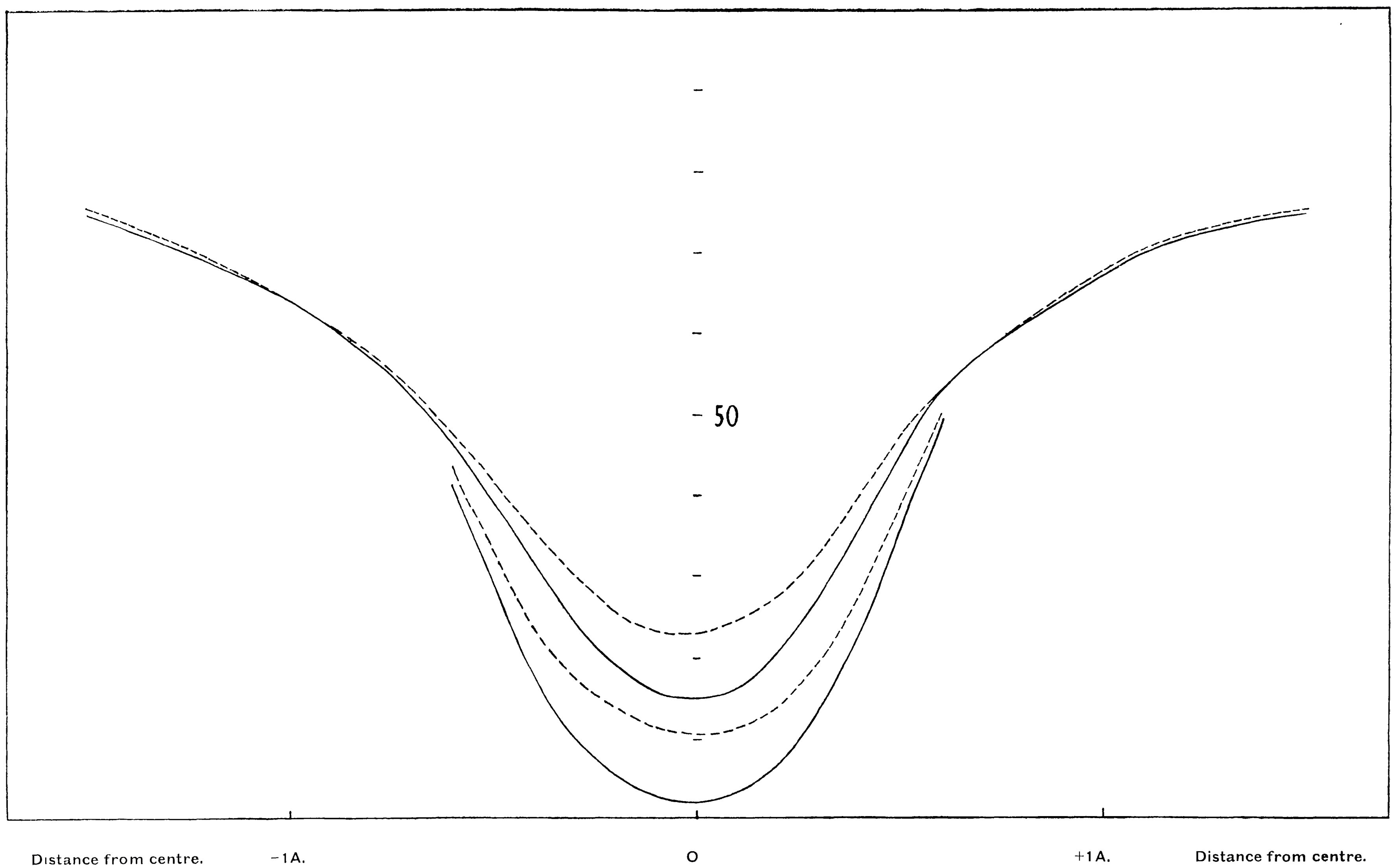

FIG. 2.—*Contour of normal Fraunhofer line (upper pair) and dark marking (lower pair) shown as observed (- - - -) and corrected (——). It will be noticed that the equivalent breadth is actually increased by the correction for heterochromatic light. The contour for the dark marking is not continued beyond ·6 A. on either side of the centre, as the differences from the normal contour are too small to show clearly in the diagram.*

are not quite steady enough in transparency or of sufficient duration to enable the lamp photometer to be used.

The disadvantage of this method is that the reduction of the contour of the dark marking depends on the previously determined contour of the normal $H\alpha$.

Table II and fig. 2 show the contour of $H\alpha$ as given in our previous paper, and also a mean contour for dark markings, constructed from observations of five different markings. The contours are given both uncorrected

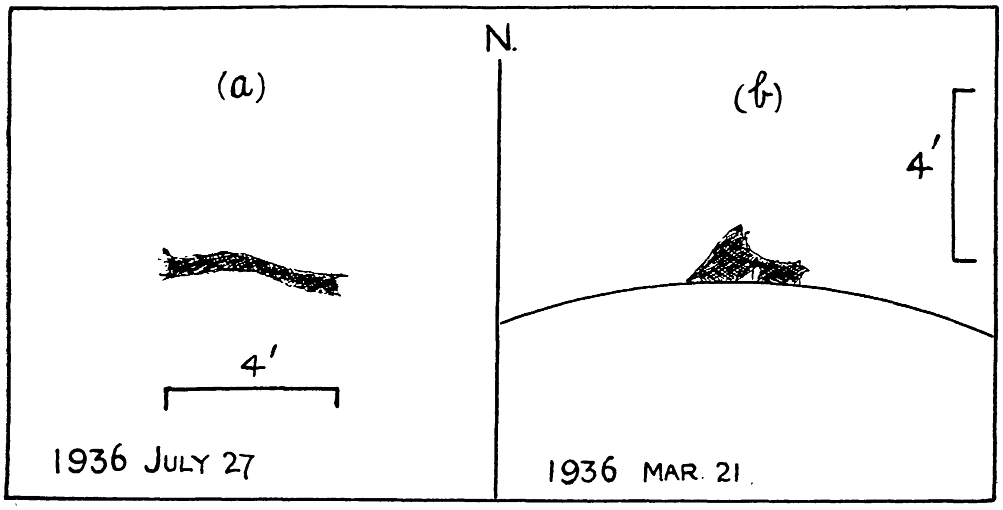

Fig. 3.

and corrected according to the methods described in paragraphs 2 and 3, with $h = 2{\cdot}5$ and $s = 0{\cdot}05$.

The dark flocculi giving the typical contour in Table II were observed on 1936 April 18 (two markings), May 4, May 6 and July 27. One of them is represented in fig. 3 (*a*). They were quite definite objects (necessarily so selected to make a difficult observation possible) and ranged in length from 2′ to $4\frac{1}{2}$′ and from $\frac{1}{3}$′ to 1′ in breadth. (The field of view of the spectrohelioscope is 4′ wide, the height being adjustable by means of a stop.) For purposes of identification the following approximate positions of these five flocculi are given :—

Date 1936	Longitude	Distance from Central Meridian	Latitude
	°	°	°
April 18	36	2 W.	43 S.
18	62	28 W.	30 N.
May 4	232	51 W.	30 N.
6	152	4 E.	44 N.
July 27	134	18 E.	26 S.

A check on the order of accuracy of the contour of $H\alpha$ given in Table II by these five sets of observations is provided by three other sets of measures made by Newton and Barton with the lamp photometer on May 18, August 23 and August 24 respectively. This second method of obtaining the contour is exactly the same as that used in obtaining the contour of the normal $H\alpha$

line described by us in *M.N.*, **96,** 8, 1935, except that in this case the observer views the portion of the dark flocculus selected for measurement against its chromospheric background and matches it with the comparison strip given by the lamp beam which passes through the adjustable wedge. The observation is rather difficult to make and demands a steady sky that occurred only rarely at Greenwich during the summer of 1936. There is rather a scatter shown by the observations, but the following mean values (uncorrected) are in reasonable agreement with those given in Table II by the direct wedge method :—

Distance from Centre	*I* (Continuous Spectrum = 100)
·00 A.	11·5
·31 A.	20
·61 A.	40
·70 A.	52

It will be seen from Table II that the corrected central intensity of the dark markings is 2 per cent. of the continuous background outside the line. About half the correction from $r = \cdot 11$ to $r = \cdot 02$ is due to finite resolving power and half to heterochromatic light. An alternative correction with $h = 3$ and $s = 0{\cdot}03$, which also gives 0·15 for the central intensity on the undisturbed disk, gives the same value 0·02 for the dark marking. One of us (H. W. N.) has the impression that there are narrow markings darker than those described here, but it is very difficult to form an opinion on such a matter where contrast is everything. On the whole we are inclined to assign a probable error of $\pm$ ·01 to the central intensity 0·02, but to admit that there may be an over-correction of ·01 or ·02—solely on the ground that other markings look darker. No doubt the quantity varies from one dark marking to another. As regards the *width* of the contour of the dark markings, it may be noted that measures made at Greenwich during 1936 show variations in the range of wave-length over which the marking is visible in the spectrohelioscope. For eleven markings with a mean radial velocity of $\pm$ 44 km./sec. the measured range was 0·93 A. on either side of the maximum intensity, while for forty-four markings with a mean velocity of $\pm$ 1·5 km./sec. the corresponding quantity was 0·62 A.

5. *Contour of a Typical Stable Prominence.*—The following measures (Table III) show the mean contour of a number of typical stable prominences of 1′ to 2′ in height above the limb. They were measured with the lamp photometer, and the unit of intensity is the light of the continuous spectrum just outside $H\alpha$ at the *centre* of the disk. The observations giving the mean contour represented in Table III were obtained on 1936 February 13, February 20, March 18, March 21, May 19, May 20 and June 17, the respective position angles of the prominences (measured from the north point through E., S., W. and N.) being approximately 110°, 19°, 184°, 9°, 131°, 101° and 146°. Fig. 3 (*b*) shows the type of prominence measured. The contour was usually determined on one side only of the centre of $H\alpha$; the results have been meaned to give one side and the contour has been assumed to be symmetrical.

TABLE III

Contour of Typical Stable Prominence

Distance from Centre in Angstroms	Intensity	
	Uncorrected	Corrected
1·0	0	0
0·9	1	1
0·8	1	1
0·7	2	2
0·6	3	3
0·5	5	5
0·4	6	6
0·3	7	7
0·2	8	9
0·1	9	11
0·0	10	12

Other prominences have been observed at Greenwich during 1935–36 which differed from this mean prominence markedly. Some are, for example, about twice as bright or greater; one of eruptive type having a measured intensity of the maximum ordinate of brightness, displaced by Doppler effect, of nearly 60 per cent. (uncorrected) that of the continuous spectrum 10 A. from $H\alpha$ at the centre of the disk. The mean intensity at the maximum ordinate of brightness given by 61 stable prominences during 1936 is, however, 9 per cent. (continuous spectrum = 100), so that the contour given in Table III may be regarded as an average one for this class of prominence that appears to produce absorption markings on the disk, the typical $H\alpha$ contour of which is given in paragraph 4.

6. *Discussion.*—Our measures show that the average stable dark marking emits less radiation per square second of arc than the average stable prominence, in the wave-length of the centre of $H\alpha$. (Outside the line, of course, the reverse must be the case.) We are aware of a possibility of systematic error, since the observer would tend to pick out the brighter parts of a prominence and the darker parts of a dark marking in considering the object—a dark part of a prominence would be considered as a hole through it. Nevertheless the objects which we have described here were all more or less structureless.

It might be thought that if the hydrogen cloud constituting a prominence when seen at the limb and a dark marking when seen on the disk is emitting radiation isotropically, there is no reason why the prominence (side view of the cloud) should appear brighter than the dark marking (top view of the cloud). There is, however, the consideration that markings vary in intensity according to the absorbing power of the cloud, that is to say, presumably according to its density or thickness; but the denser markings certainly do not give rise to weaker prominences. There is a definite tendency for prominences to give no markings on the disk at solar minimum.*

The observed phenomena seem to be explained by the view that the

* We owe these remarks to Mr. Evershed.

hydrogen cloud does not emit radiation from its interior, but behaves very like an opaque body. The depth of the contour of the dark marking shows that the cloud has an optical depth of 4 or 5 in the wave-length of the centre of $H\alpha$; accordingly the radiation field at the side of the cloud exposed to light from the photosphere is controlled almost entirely by inflowing radiation. This inflowing radiation is selectively scattered, and a certain percentage is scattered backwards, the process being very like reflection of sunlight from the surface of a thin cloud in the sky—except that in the solar cloud the process is monochromatic.

A simple model of the prominence and dark marking may be made by sticking a small tuft of cotton wool to the surface of an opaque 60-watt lamp, and turning the lamp slowly. When the cotton is at the "limb" it projects as an illuminated object like a prominence, and on turning the lamp so as to bring the cotton on to the "disk" it resembles a dark marking. In this case, as in the solar case, the side view of the illuminated projection is brighter than the top view (view on the disk) where the eye perceives light which has had to struggle through the opaque object.

Besides accounting for the appearance of the hydrogen cloud as a prominence outside the limb, and when it is half over the limb, and for the observation that the cloud appears darker near the centre of the disk than when seen at the limb, the view that the radiation from a hydrogen cloud is a cloud reflection of outside radiation accounts for the observed ratio of the intensities of prominences in $H\alpha$ and $H\beta$ determined at Greenwich. If we take as unit the emission in 1 A. of the continuous spectrum from the centre of the disk just outside the line concerned, the ratio found from Greenwich observations of contours of $H\alpha$ and $H\beta$ emission from eight prominences observed during 1936 is 4·6. It has been shown by one of us * that a ratio of 4·4 in these units (differing very widely from the value appropriate to thermodynamical equilibrium at 6000°, namely, 11·4) is to be expected if the outside of the prominence is in polychromatic radiative equilibrium with the radiation from the photosphere, assuming a central intensity of 0·01 for the Lyman alpha line. Other observers, notably Schwarzschild and Perepelkin, have found Balmer decrements in prominences which deviate markedly from thermal equilibrium values; but Menzel and Cillié † find approximately equilibrium values for the hydrogen emission from the chromosphere at a height of 670 km. (at the 1932 eclipse). This difference between observations of chromosphere and prominence seems too great to be attributed to photometric error.‡ On the reflection-opacity view of prominence radiation, there is no reason to suppose that the decrement from the chromosphere and prominences should be the same.

Summary.—Visual photometric observations made during 1936 of the intensities of stable dark markings and prominences in $H\alpha$ light are described.

* Woolley, *M.N.*, **96**, 515, 1936.

† *Ap. J.*, **85**, 88, 1937.

‡ (Note added in proof.) It must be remarked that observations made by A. D. Thackeray on plates taken at the eclipse of 1936 by no means support Menzel and Cillié's value for the ratio $H\alpha/H\beta$.