6. The computed date of the next Standard Epoch is 1945·2 when the NEB. will be at or near a minimum of redness and the SEB. at or near a maximum.

Observations of the redness of the two equatorial belts in 1945, and during a year or two before and after that date, are much to be desired, and it is to be hoped that such will be forthcoming from other observers. The work is quite simple and interesting. All that is necessary is to estimate the redness of the two belts, if not on the adopted scale, in simple words, such as colourless (or bluish), slightly red, red, deep red, intense red, and so on. Fancy descriptions of the colours had best be avoided. The early descriptions of the colours of double stars demonstrate the inutility of this.

Note on the Red Spot

The great plainness of the red spot during the past opposition will have been noticed by many. It was first brought to my knowledge in a letter from Mr. Peek (Director of the Jupiter Section of the B.A.A.), and it was soon confirmed by direct observation with the telescope. According to my recollection the spot has been, in fact, decidedly plainer than at any time previously since about the year 1883. The red colour, too, has been very conspicuous, attaining to 5 or even 6 on the scale of redness (which ranges from 0 to 10). Antoniadi at Meudon describes the colour as a very intense "rose vermillion." It usually seemed to me less red than the NEB. Quénisset's interesting photographs (*l'Astronomie*, **50**, 45, 1936) give a very good idea of the appearance of the planet and the spot as viewed with a very low power on a very bad night. Under such conditions the appearance was almost exactly the same as depicted in an old drawing by Cassini, dated about the year 1665.

Polvarth, St. Mawes :
1936 *November* 2.

ON THE POSSIBLE RESULTS OF AN ENCOUNTER OF PLUTO WITH THE NEPTUNIAN SYSTEM.

Raymond A. Lyttleton.

Introduction.—The changes which occur in a planetary orbit due to the action of the other planets indicate that, except in the case of Pluto, the general features and extent of the solar system remain more or less unaltered over a very long interval. But Pluto may at times be nearer to the Sun than Neptune, so that as the nodes and apses of their orbits move the corresponding elliptic paths may ultimately intersect. Thus encounters of Pluto are clearly relevant to the development, both past and future, of the solar system.

1. During an encounter of the two planets the action of the Sun will be neglected and the presence of Triton ignored. The validity of this approxi-

mation to the motion is manifest, but it may also be used as a definition of a close encounter.

On this basis the relative velocity of the two bodies is unaltered in magnitude by an encounter, and hence the effects will be greatest (*ceteris paribus*) when the relative velocity is largest. Since the orbit of Neptune is so nearly circular this maximum value depends almost entirely on the angle of intersection of the orbits and occurs when this angle is greatest.

For Neptune: Mass $= \odot \div 19{,}314$
$a = 30{\cdot}07$ units, $e = 0{\cdot}0085$, $i = 1^\circ\ 47'$.

For Pluto: Mass $= \odot \div 3 \times 10^6$ approximately,
$a = 39{\cdot}60$ units, $e = 0{\cdot}2537$, $i = 17^\circ\ 9'$.

Taking Neptune's orbit to be circular, i the angle between the orbital planes and ψ the angle which Pluto's path makes with a coplanar circle through the point in question, the cosine of the angle between the paths is $\cos i \cos \psi$. The maximum values of i and ψ are $18^\circ\ 56'$ and 5° approximately, so that for the present values of the eccentricity and inclination the maximum angle is about $19^\circ\ 24'$. (The error due to the neglect of Neptune's orbital eccentricity is about $\frac{1}{2}^\circ$.)

Consider first an encounter near the aphelion of Neptune. The velocity of Neptune at aphelion is

$$v \text{ (Neptune)} = 0{\cdot}1808 \text{ astron. unit of velocity} = 3{\cdot}344 \text{ miles per sec.}$$

At this distance

$$v \text{ (Pluto)} = 0{\cdot}2018 \text{ unit} = 3{\cdot}731 \text{ miles per sec.,}$$

while the velocity of escape at this distance is

$$v \text{ (escape)} = 0{\cdot}2568 \text{ unit} = 4{\cdot}749 \text{ miles per sec.}$$

And hence the maximum relative velocity is

$$v \text{ (relative)} = 0{\cdot}0676 \text{ unit} = 1{\cdot}253 \text{ miles per sec.}$$

Now if d is the distance of closest approach of two bodies in hyperbolic orbits and W their relative velocity before encounter, the eccentricity of the orbits is $1 + W^2 d/(\text{Mass})$. Taking d to be 20,000 miles, the value of the eccentricity is less than $1{\cdot}05$, so that the orbit can be almost parabolic and the relative velocity may be turned through nearly 150°.

The greatest velocity referred to the Sun which Pluto may have after such an encounter corresponds to the relative velocity being in the same direction as that of Neptune in its orbit, and neglecting the mass of Pluto this value is

$$3{\cdot}344 + 1{\cdot}253 = 4{\cdot}597 \text{ miles per sec.,}$$

and this is less than the escape velocity $4{\cdot}749$ miles per sec. The corresponding angle through which the relative velocity must be turned is easily found to be

$$\sin^{-1}\left(\frac{3{\cdot}731}{1{\cdot}253} \sin 19^\circ\ 24'\right) \text{ or } 81^\circ\ 18'.$$

Hence the eccentricity of the encounter orbit is cosec 41° 9′ or 1·52. Accordingly the requisite distance to which Pluto must approach is $\frac{\text{Mass} \times 0{\cdot}52}{W^2}$, which is

$$0{\cdot}006 \text{ astron. unit} = 550{,}000 \text{ miles}$$

or about two and a half times the present distance of Triton from the planet.

The greatest effect on the major axis of the orbit of Pluto therefore takes place for an encounter at quite a considerable distance from Neptune. It is also of interest to notice that, as far as Pluto is concerned, the mass of Neptune is not involved so long as it is large compared with Pluto. However, the distance at which the greatest effect is produced increases with the mass so that a larger planet is more likely to produce this greatest effect. The new semi-major axis is found to be 240 astronomical units, corresponding to a period of over 3700 years.

Similarly if Pluto encounter Neptune at the perihelion of the latter,

$$\begin{aligned} v \text{ (Neptune)} &= 3{\cdot}404 \text{ miles per sec.,} \\ v \text{ (Pluto)} &= 3{\cdot}781 \text{ miles per sec.,} \\ v \text{ (escape)} &= 4{\cdot}788 \text{ miles per sec.,} \end{aligned}$$

and the maximum relative velocity is 1·269 miles per sec. Hence the velocity of Pluto after encounter cannot exceed 4·673 miles per sec. The semi-major axis of the orbit may therefore be as great as 314 units and the period as long as 5560 years.

The maximum and minimum distances from the Sun can also be shown to occur only for the velocities in the direction of the orbital motion of Neptune—that is, for the maximum and minimum velocities (see § 2). The nearest approach to the Sun corresponds to the smallest velocity and is found to be 7·3 astronomical units, while the greatest distance is 598 astronomical units.

Thus the orbit of Pluto may come within that of Saturn, and it can be shown that it is possible for an encounter with Saturn to effect the ejection of Pluto from the system. But such an encounter is of little interest on account of its extreme unlikelihood.

These remarks indicate the possibilities of Pluto's orbit so long as it is affected only in the manner here suggested. In so far as these direct changes due to Neptune are concerned it is particularly to be noticed that, however many such changes occur in the orbit of Pluto, the relative velocity of the two bodies (before or after an encounter) is unaltered in magnitude to the order of approximation used here. Hence at no time may Pluto be ejected from the system and in all probability is therefore an original member. But the orbit of Pluto may slowly undergo changes of eccentricity and inclination with a possible increase of the relative velocity to an extent permitting Pluto to be expelled.

2. To find the maximum and minimum distances from the Sun of a particle projected with relative velocity U from a point moving with velocity V perpendicular to the direction of the Sun and at a distance R from it.

Supposing U to make an angle θ with V, the energy and angular momentum give at once

$$\frac{1}{a}=\frac{2}{R}-(V^2+2VU\cos\theta+U^2)=x$$

$$a(1-e^2)=R(V+U\cos\theta)$$

for the subsequent orbit. Hence

$$a(1-e)=\frac{1-\sqrt{1-\lambda x(\mu-x)^2}}{x}=L,$$

where

$$\lambda=\frac{R^2}{4V^2}\quad\text{and}\quad\mu=\frac{2}{R}+V^2-U^2.$$

For this distance to be least $\frac{dL}{d\theta}=0$, and this occurs if

$$\frac{dx}{d\theta}=0\quad\text{or}\quad\sin\theta=0,$$

giving the critical values corresponding to the greatest and least velocities, or $\frac{dL}{dx}=0$ giving

$$x^2(\mu-x)[(\mu-x)(\mu+x)^2-8/\lambda]=0.$$

In the present connection, for Pluto $V>U$ and hence $0<x<\mu$. Putting

$$x+\mu=y\quad\text{then}\quad\mu<y<2\mu\quad\text{and}\quad y^2(y-2\mu)=-8/\lambda.$$

The numerical value of μ is 0·091 and of $8/\lambda$ is 0·036, so that

$$y^2(y-0\cdot192)=-0\cdot036,$$

and this equation clearly has no root between 0·091 and 0·192. Accordingly critical values of the distance of the apses occur only for the maximum and minimum velocities.

More generally the left-hand side has stationary values when $3y^2-4\mu y=0$. There will therefore be two positive roots if $4\lambda\mu^3>27$, and these will lie on opposite sides of $4\mu/3$, and one at least of these necessarily gives a real value of θ. The negative root does not correspond to a real θ.

The hyperbolic case may be discussed by writing $-x$ for x.

3. So far the mass of Pluto has been ignored. The similarity in brightness of Pluto and Triton represents almost the only certain physical knowledge of these bodies, and according to Nicholson the mass of the latter is about one-tenth of that of the Earth. Adopting a similar value for Pluto,

$$\text{Pluto}=\text{Earth}\times0\cdot1=\text{Neptune}\times0\cdot006.$$

After an encounter at the perihelion of Neptune the extreme velocities which Neptune may have are approximately

$$3{\cdot}404 \pm (1{\cdot}269 \times 0{\cdot}006) = 3{\cdot}412 \text{ and } 3{\cdot}396 \text{ miles per sec.,}$$

corresponding to semi-major axes,

$$30{\cdot}26 \text{ and } 29{\cdot}98 \text{ units,}$$

while the periods are

$$166{\cdot}5 \text{ and } 164{\cdot}2 \text{ years.}$$

Thus the period could be changed by over a year.

4. Encounters between Pluto and Triton.

The relative velocity of Pluto and Neptune before an encounter has been seen to be at most 1·26 miles per sec. The orbital velocity of Triton is 2·72 miles per sec. and the escape velocity at its distance is 3·84 miles per sec. For a particle in an elliptic orbit near Neptune

$$v^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right),$$

so that for Triton $\mu/a = (2{\cdot}72)^2$. For a particle in a hyperbolic orbit with the velocity 1·26 at infinity

$$v^2 = 2\mu/r + (1{\cdot}26)^2,$$

so that at the distance of Triton (due to the action of Neptune only) the maximum velocity of Pluto would be

$$\sqrt{2 \times (2{\cdot}72)^2 + (1{\cdot}26)^2} = 4{\cdot}05 \text{ miles per sec.,}$$

while the least velocity would exceed 3·84 miles per sec.

The investigation of the encounters will be made by neglecting the action of Neptune and of course of the Sun. On account of the small mass of Pluto and Triton this is not a very good approximation.

The eccentricity of the relative orbit is $1 + \frac{W^2 d}{\text{Mass}}$, where d is the closest distance of approach of the two bodies and W the relative velocity before or after. Taking d to be 5000 miles and the mass = 0·2 Earth, then $\frac{d}{\text{Mass}} = 90$ (using astron. units). W has limits $4{\cdot}05 + 2{\cdot}72$ and $3{\cdot}84 - 2{\cdot}72$ or 6·77 and 1·12 miles per sec. Hence in astronomical units W^2 ranges from 0·134 to 0·0037, thus e ranges from 13 to 1·33. If the masses be equal the changes of velocity (vectorially) which may be communicated may vary from $W/13$ for large relative velocities to $W/1{\cdot}33$ for small relative velocities. While the angle through which the direction of the velocity relative to Neptune can be turned lies between 9° and 97°, so that the asymptotes are inclined at not less than 83°.

The following diagrams indicate some of the more interesting encounters which could occur. The circles represent the spheres of equal velocity relative to Neptune. Thus the outermost sphere has radius equal to the velocity of Pluto, just exceeding the escape velocity, which is the radius of the next sphere, while the smallest sphere has radius equal to the velocity of

Triton in its orbit. The diagrams are to be interpreted as three-dimensional, but it is to be noticed that the extreme cases occur in the plane of Triton's motion. T and P are the ends of the vectors representing the velocities relative to Neptune of Triton and Pluto before encounter in a typical case. The velocity of the centre of mass G of Pluto and Triton is unaltered by the

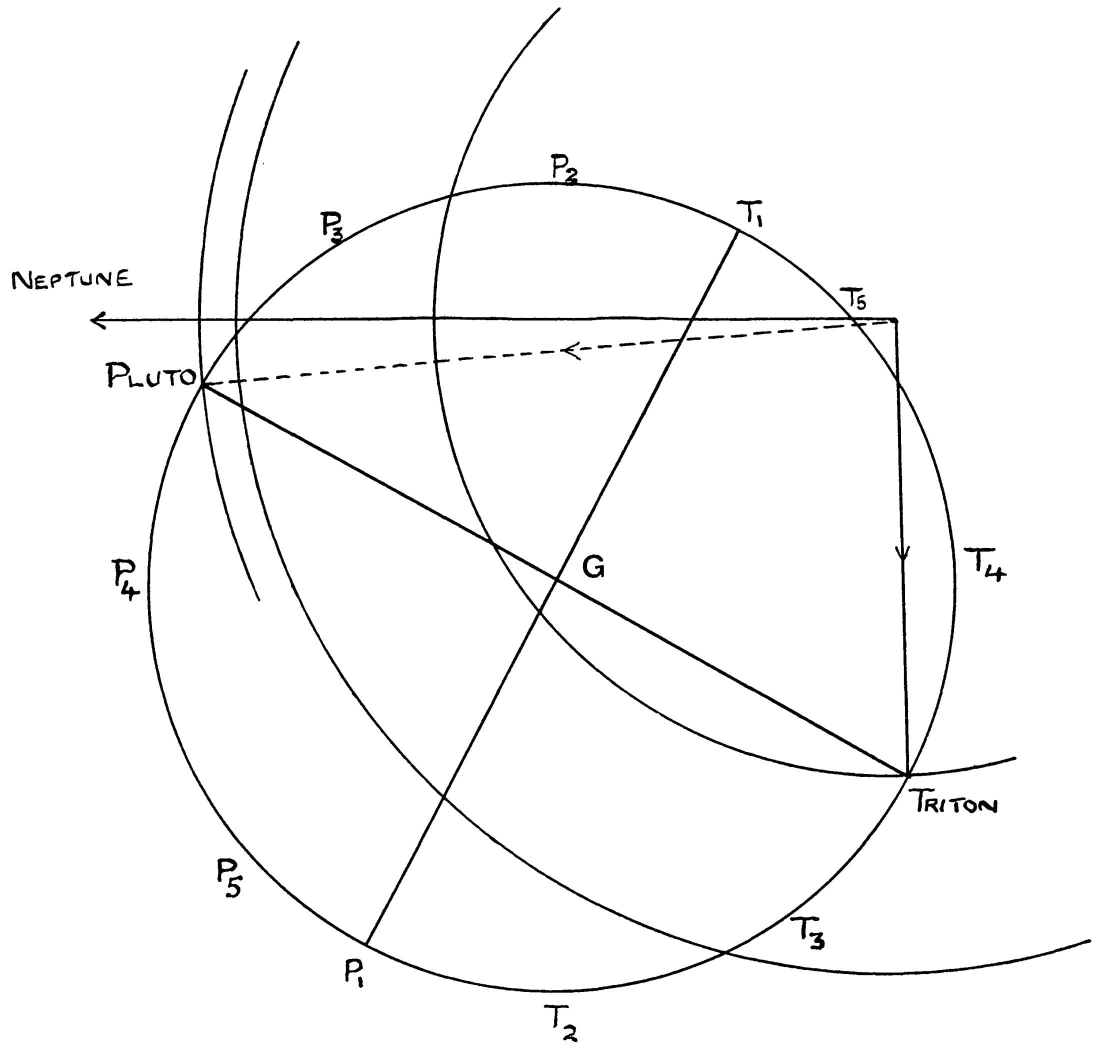

FIG. 1.

encounter, while the relative velocity of the bodies is changed only in direction. Hence the subsequent velocities may be derived from the initial motion simply by a rotation of TP about any line through G.

The following results may occur (they are shown in fig. 1 by the corresponding suffix):—

(i) Triton sent into a smaller direct orbit of high eccentricity T_1 and Pluto not captured P_1.

(ii) Triton ejected and Pluto captured in a direct orbit.

(iii) Triton's orbit disturbed but not radically altered T_3 and Pluto captured in a direct orbit P_3.

(iv) Triton's orbit disturbed T_4 and Pluto not captured.

(v) The angular momentum of Triton reduced to zero T_5 so that it falls into Neptune, Pluto not captured P_5.

(vi) It is also possible that if Pluto had initial velocity in the same general direction as that of Triton, Pluto could be captured in a retrograde orbit with little change in that of Triton. (Not shown in figure.)

5. There is a further very remarkable case. If a closer encounter is

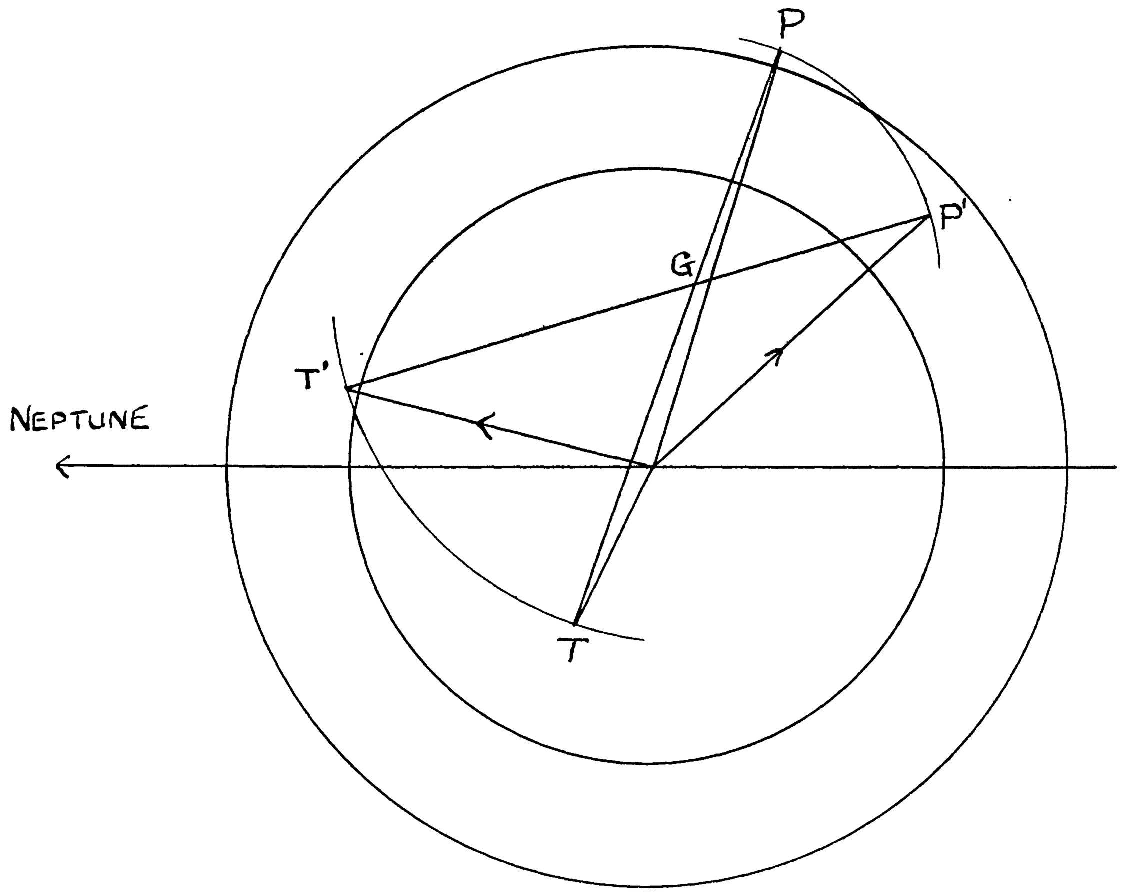

FIG. 2.

permissible the value of the eccentricity of the encounter orbit may be reduced even though the relative velocity is fairly large. The eccentricity is also considerably reduced if the encounter occurs at a distance from Neptune greater than the present distance of Triton. In these circumstances if the velocity of Pluto is in practically the opposite direction to that of Triton the second figure shows that an encounter may result in the capture of Pluto in a direct orbit and the reversal of the general direction of Triton's motion, so that Neptune would have two direct satellites. Now such a motion is clearly capable of complete reversal, so that the interesting possibility arises that Pluto may have been a satellite of Neptune. The encounter which would effect this is rather extreme, but only if it is required that Triton be thrown into a circular retrograde orbit from the start; a relaxation of this condition facilitates the reversal of the orbital motion. The figure is drawn for such a case and also with a view to demonstrating that if Pluto's mass exceeds that of Triton the result is more readily accomplished. The eccentricities of the direct satellite orbits are seen to be large, but if two satellites were describing orbits which permitted a very close encounter there seems little doubt that the perturbations would be very great long before an encounter as close as is envisaged here actually occurred, and these might give rise to eccentric orbits.

The suggestion is supported by the following considerations. Supposing Neptune to have had two direct satellites, a mechanism has to be provided to bring them to a stage where encounter might occur. Now tidal friction, if it were very great, might cause the mean distances of the satellites to approach equality with a resulting encounter. If Pluto had always been an independent planet it would be natural to expect that its orbit would lie completely outside that of the others. (Jeffreys's theory of the rounding-up of orbits by a resisting medium requires a much longer time for a given effect to be produced for Pluto than for Neptune; but the theory is uncertain for massive planets.) Thus Pluto may have been comparatively recently ejected from the Neptunian system. This is again accordant; for tidal friction may have taken place late in the history of the planet, and, furthermore, it is noticeable in this connection that the rotation period of Neptune is by far the longest of the major planets.

Summary.—(i) Encounters of Pluto with the Neptunian system may eventually occur and may have occurred in the past. Their investigation throws some light on the anomalies at the edge of the solar system.

(ii) Under Neptune's action Pluto cannot be ejected from the system, but the extent of its orbit may be greatly altered.

(iii) If Pluto should encounter Triton several interesting cases arise, but these are of slight interest owing to their improbability.

(iv) Nevertheless it appears from these considerations that Pluto may originally have been a direct satellite of Neptune, and that the encounter which gave it an existence as an independent planet also reversed the general direction of motion of Triton, thus giving a second, though rather speculative, possible explanation of the retrograde motion of a true satellite.*

It is a pleasure to accord my thanks to Professor H. N. Russell for the benefit of many discussions of the present work.

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* *M.N.*, **96**, 566, 1936.