

Space Distribution of Planetary Nebulae.

(Studies on the O class stars, planetary nebulae and novae. V note.)

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On the basis of correlations found in the former note the hypothesis of constancy of absolute integrated photographic magnitudes of nebulae M_n is adopted. The proper motion and radial velocity data together with the galactic rotation lead to $M_n = +0.2$. With this value of M_n the space co-ordinates and the dimensions of 119 nebulae were computed. The space density of nebulae is extremely low and falls rapidly with the increasing distance from the Sun. The ring nebulae show traces of expansion. The absolute magnitudes of the nuclei are strongly correlated with their temperature.

1. Introduction. The distances and dimensions of planetary nebulae and the absolute magnitudes of their nuclei are known at present very approximately. The trigonometric parallaxes of individual nebulae failed to give reliable values and all efforts were directed mainly to the determination of mean, statistical parallaxes. The results formerly obtained by GERASIMOVICH¹⁾ seemed to be reliable, indicating that the nuclei of planetary nebulae are „ultra-white dwarfs“, according to the terminology we have proposed.

However, it is very desirable to know the individual data for separate nebulae. The only reliable method for this purpose was recently suggested by ZANSTRA²⁾, who determined the distances of a number of nebulae with known temperatures and apparent magnitudes of their nuclei m_* , adopting the existence of a relation

$$M_* = M_0 + 0.7 (m_* - m_n),$$

where M_* is the absolute magnitude of a nucleus and m_n is the integrated magnitude of the corresponding nebula. The constant M_0 was determined from the galactic rotation of these objects. A similar method is here adopted by the writer. However, this method is applicable to all nebulae with known apparent integrated magnitudes m_n . It was suggested by the writer in his fourth note on the subject³⁾, that a number of dependences known from the statistics of planetary nebulae can be explained best on the assumption, that the absolute integrated photographic magnitudes M_n have small dispersion, and probably are constant⁴⁾.

1) H. B., Nr. 864, S. 9, 1929. — 2) ZS. f. Astrophys. **2**, 329, 1931, Nr. 5.
— 3) Russ. Astron. Journ. **11**, 40, 1934, Nr. 1. — 4) l. c. Sections 9, 12, 13, 14.
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The approximate constancy of M_n is probable mainly because the apparent integrated magnitudes of nebulae m_n are not correlated with any other physical characteristics of these objects, though the latter show many reasonable correlations among themselves.

2. *Determination of absolute integrated magnitudes of nebulae.* The hypothesis of the constancy of absolute integrated photographic magnitudes M_n of nebulae being adopted, the relative distances of all nebulae might be easily calculated, from their apparent magnitudes. However, as we are interested in the absolute distances it is necessary to determine the zero point of these distances. Unfortunately this cannot be done at present with high accuracy.

Using the methods described below we used all the data concerning these nebulae from the general catalogue of planetary nebulae contained in our fourth note¹⁾.

I. The new list of absolute proper motions of planetary nebulae published by VAN MAANEN²⁾ leads to a mean parallax $0''.00068$. The mean apparent magnitude of corresponding 21 nebulae is $11^m.13$, which gives $M_n = 0^m.29$.

II. The method described by ZANSTRA³⁾ could be applied to 51 nebulae with known m_* , and to 68 nebulae (after the rejection of NGC. 6833) adding those, for which the "apparent" magnitudes of their invisible nuclei m_* were determined by the writer theoretically¹⁾. There was found for these nebulae the correlation

$$m_* = a + 0.85 (m_* - m_n),$$

where a is a constant. Interpreting this as an expression of the true correlation $M_* = M_0 + 0.85 (M_* - M_n) = M_0 + 0.85 (m_* - m_n)$ it is possible to determine M_0 from the galactic rotation. In fact

$$5 \lg r = m_* + 5 - M_0 - 0.85 (m_* - m_n).$$

Therefore, the formula for the galactic rotation can be represented in the form

$$v = A \cdot 10^{-0.2 (M_0 - 5)} \cdot \sin 2 (l - l_0) \cos^2 b \cdot 10^{0.2 [m_* - 0.85 (m_* - m_n)]} = C \cdot Q,$$

where

$$C = A \cdot 10^{-0.2 (M_0 - 5)}$$

and

$$Q = \sin 2 (l - l_0) \cos^2 b \cdot 10^{0.2 m_* - 0.17 (m_* - m_n)}.$$

Calculating Q for 51 and 68 nebulae we find for the mean value of C 0.252 ± 0.144 (m. e.) and 0.251 ± 0.109 respectively.

¹⁾ Russ. Astron. Journ. **11**, 40, 1934, Nr. 1. — ²⁾ Aph. J. **77**, 186, 1933, Nr. 3. — ³⁾ ZS. f. Astrophys. **2**, 329, 1931, Nr. 5.

However the agreement of the two figures is accidental, and the value of mean C depends greatly on the rejection of some individual nebulae (C varies from 4.3 to -2.0), so that it is worth little confidence. The same is evident from an inspection of ZANSTRA's data.

We have shown¹⁾ that practically there is the relation

$$m_* = a_1 + 1.0 (m_* - m_n),$$

or m_n does not depend on the temperature of the central star. Therefore finally we have rejected this method.

III. Assuming $M_n = \text{const}$ the effect of galactic rotation can be used for the calculation of M_n or of the mean distance \bar{r} . We have

$$\lg r = 0.2 (m_n - M_n) + 1,$$

hence

$$v = A \cdot 10^{-0.2 M_n + 1} \cdot \sin 2(l - l_0) \cos^2 b \cdot 10^{0.2 m_n} = \Theta \cdot \Phi,$$

where

$$\Theta = A \cdot 10^{-0.2 M_n + 1}.$$

The formula could be applied to 99 nebulae and gave mean

$$\Theta = 0.159.$$

With mean $m_n = 11^m.8$ and $A = 0.0166^2)$ we have

$$M_n = 0.10.$$

However the value of Θ is not stable. It changes considerably if some nebulae are rejected, but the limit of such rejection is indefinit. Therefore we have used the graphical method.

IV. The values of $v \cdot 10^{-0.2 m_n}$ were plotted on a graph as a function of $2(l - l_0)$. 23 nebulae were rejected on sufficient grounds and then maxima and minima of the sine curve were determined from the mean curve. The weighted mean amplitude is 0.162 ± 0.015 , and $M_n = 0.05$ (with $\bar{m}_n = 11^m.8$).

V. Plotting v as a function of $l - l_0$ (correcting for $\cos^2 b$) from 85 nebulae we find the mean amplitude of the resulting sine curve ± 35 km/sec, which corresponds to a mean distance $\bar{r} = 2110$ parsecs and gives $M_n = +0.20$ (with $\bar{m}_n = 11^m.8$).

VI. Dr. MENZEL has given³⁾ an interesting purely astrophysical method for calculating the parallax of planetary nebulae. Unfortunately his method involves too many numerical data which must be taken hypothetically, so that the result with a slight change of adopted numerical

¹⁾ Russ. Astron. Journ. **11**, 40, 1934, Nr. 1. — ²⁾ Publ. Dom. A. O. Victoria **5**, 1933, No. 3. — ³⁾ PASP. **43**, 334, 1931.

data concerning the physical state in the nebulae leads to relative distances from 1 to 10. We could not find any other astrophysical method to calculate the parallax by using less hypothetical data, which could be added to the methods used above. The method of the galactic dip and others of such kind are too rough for the present case.

Therefore, we have finally for M_n from the methods:

- I. Proper motions. 0^m29
- III. Numerically from galactic rotation 0.10
- IV. Graphically from galactic rotation 0.05
- V. Graphically from galactic rotation 0.20

The adopted weighted mean value of M_n is + 0^m2. It was of interest to investigate whether the absolute proper motions as determined by VAN MAANEN show the effect of dispersion in the distances of nebulae.

Adopting for the solar apex the data of WIRTZ¹⁾ we have found the parallax assuming the mean peculiar velocity $\bar{V}' = 29$ km/sec:

	From τ -component	From v -component
10 nebulae brighter than 11 ^m ($\bar{m}_n = 9^m24$)	0'000 82	0'001 26
10 nebulae fainter than 11 ^m ($\bar{m}_n = 13^m06$) . . .	0.000 75	0.000 87

The effect of the distance is clearly shown, but is far too small. It is very likely that here as well as in the other similar cases the measured proper motions of very distant objects are systematically too large.

3. *The effect of light absorption in space.* It is evident that owing to the large distances to the planetary nebulae their light suffers absorption in the interstellar space. However it can be shown, that in the present case this absorption does not change considerably the calculated distances and dimensions of planetary nebulae, and does not modify qualitatively the character of their space distribution.

In fact, as may be seen from the above section the calculated absolute magnitude of nebulae M_n is computed with their mean apparent magnitude m_n , so that they both are affected by the light absorption in space nearly to the same amount. Now, the distances of individual nebulae r are calculated from the formula $5 \lg r = m_n + 5 - M_n$, so that the error in $\lg r$ is proportional to the error in $(m_n - M_n)$. So we have the effect only of a differential light absorption which affects $(m_n - M_n)$ nearly as it affects $(m_n - \bar{m}_n)$, or much less than it affects m_n alone.

¹⁾ Russ. Astron. Journ. 11, 40 1934, Nr. 1.

Let r be the observed distance affected by the absorption, r' the true distance, \bar{r} the mean distance affected by absorption and k the absorption coefficient. Then it is easy to see that as a first approximation we have

$$5 (\lg r - \lg r') = k (r' - \bar{r}).$$

It is a matter of doubt what value of k must be taken for the planetary nebulae with emission spectra. If we take $k = 0^m.5$ pro kiloparsec (interpolation for the nebular green light from TRUMPLER's data¹⁾ on light absorption) we find that for 80 per cent of nebulae ($r' < 4000$ parsecs)

$$\frac{r}{r'} < 1.6.$$

Besides, as can be seen from our data, the majority of the planetary nebulae are outside the thin absorbing layer localized near the plane of the galaxy as suggested by VAN DE KAMP²⁾. Therefore, the differential absorption will affect the calculated distances to a far smaller degree than that mentioned above, since only a small part of the distance to the nebulae is inside the absorbing layer.

The space distribution of planetary nebulae described below must be regarded as a first approximation, and the small influence of light absorption in space is here of second, if not of "third hand" importance.

Besides, the absorbing matter seems to be localized rather irregularly and the present results, though probably erroneous, form nevertheless a quite definite system which can be corrected in future.

4. *Space distribution of planetary nebulae.* Data for individual nebulae 119 in number (92 per cent of all known planetaries) are found in table 1. The distances R are given in parsecs. The rectangular galactic coordinates x , y and z in parsecs are calculated, directing the X axis toward the galactic centrum ($l = 325^\circ$, $b = 0^\circ$), and the Z axis toward the north pole of the galaxy. The linear diameters of nebulae D (minimum and maximum) are expressed in thousands of astronomical units. M_* is the absolute magnitude of the nucleus. When this is put in parentheses, it means that the apparent magnitude of the coresponding nucleus (not observed directly) was calculated theoretically³⁾ from the relative line intensities of the nebular spectrum. These values are reduced to our scale of temperatures, though in our note IV the corresponding m_* were given in ZANSTRA's scale. Other data mentioned here are to be compared with the data of our General Catalogue of Planetary Nebulae³⁾.

¹⁾ Lick Observ. Bull. No. 420, 1930. — ²⁾ A. J. **40**, 945, 1930. —

³⁾ Russ. Astron. Journ. **11**, 40, 1934, Nr. 1.

The space distribution of planetary nebulae in projection on the plane of the galaxy XY is shown in fig. 1. The scale is in kiloparsecs. The Sun is at the origin. One nebula (NGC 6833) is shown outside the frame of the figure. Three distant nebulae (NGC. 6881, 6894, 7139) could not be shown at all in fig. 1. It is easy to see that the centrum of the system of *known* planetary nebulae is not in the direction to the galactic center ($l = 325^\circ$), nor in the direction to the center of the

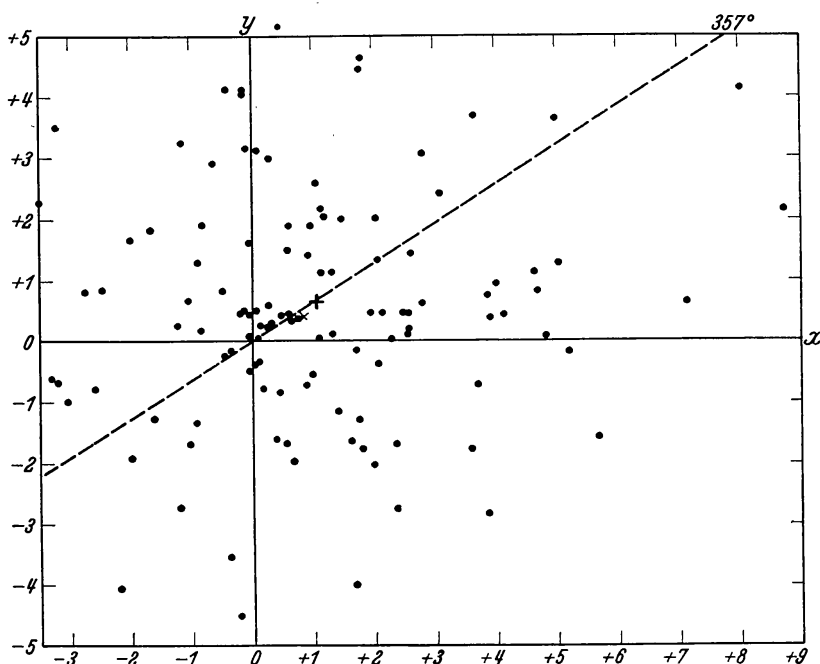


Fig. 1. Planetary nebulae projected on the plane of Galaxy.

local system ($l = 250^\circ$). It is at 1250 parsecs from the Sun in the direction $l = 357^\circ$ (its position is marked by a heavy cross) and has the co-ordinates:

$$X_0 = +1055, \quad Y_0 = +648, \quad Z_0 = -60.$$

If we reject the few isolated nebulae situated farther than 5500 parsecs from the Sun, we obtain the center of the system at the distance of 970 parsecs in the direction 352° with the co-ordinates

$$X_0 = +866, \quad Y_0 = +448, \quad Z_0 = -55.$$

This center is marked in fig. 1 by a light cross. It seems possible that future discoveries of planetary nebulae shall place the center of their system farther away from the Sun, and nearer to the direction to the

galactic center. However, it seems evident that the system of planetary nebulae has no connection with the local system.

Fig. 2 represents the projection of nebulae on the plane ZX' , the X' axis being directed to the center of the system in the plane of the galaxy (marked by a cross). The scale in fig. 2 is the same as in fig. 1. NGC. 6620 with $x' > 8000$ parsec is just outside the frame of the fig. 2, and IC I 1295 is not shown at all.

The galactic concentration of planetary nebulae is evident, though not very strong. Several nebulae are very distant from the plane of the

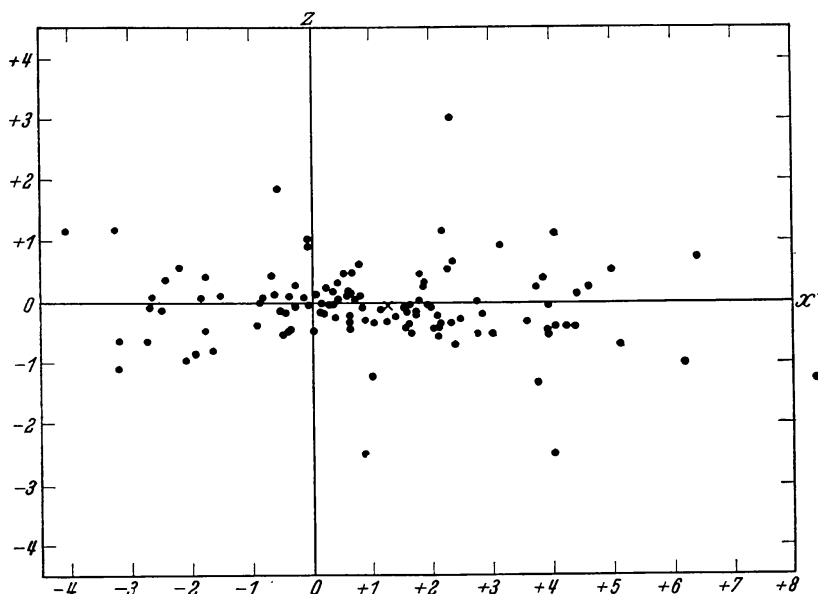


Fig. 2. Planetary nebulae projected on the plane perpendicular to the plane of Galaxy.

galaxy. NGC 6058, 7408 and "star Küstner 648 in Globular cluster M 15" are examples deviating by 3100, 2450 and 2500 parsecs. It is of interest whether the last nebula really belongs to the cluster. Its distance from the Sun is 5250 parsecs, while the cluster according to SHAPLEY is at a distance 13100 parsecs. This, as well as the lack of sufficient agreement in the radial velocity of the cluster and of the nebula, shows them to be unconnected physically.

It can be seen that 80 per cent of the nebulae lie inside a layer 1200 parsecs thick and nearly symmetrical relatively to the plan of the galaxy. The system forms approximately an ellipsoid with the relation of axis 1:7.

The space density of known planetary nebulae is extremely low, and falls very rapidly with the distance from the Sun. The mean space density

of nebulae per cube 10^3 parsecs in a side, within a cylindrical layer 1200 parsecs thick at different distances is as follows:

0 — 1000 parsecs	9.0
1000 — 2000	5.0
2000 — 3000	1.1
3000 — 4000	0.6
4000 — 5000	0.2

This is 8 times smaller than the space density of galactic star clusters, and probably smaller than the density of any other known class of celestial bodies. The rapid fall of density with the distance from the Sun can hardly be real and makes probable that there exists a great number of relatively near but still undiscovered planetary nebulae.

5. *The linear diameters.* The true linear diameters of planetary nebulae are very different, from 1,4 (IC II 2553) to 500 thousand (NGC 7139) astronomical units. Still larger is the nebula IC I 1295, 960000 a. u. or nearly 4 parsecs in diameter. The distribution of the nebulae according to their linear diameters is a follows:

<i>D</i> in 1000 a. u.	<i>n</i>	<i>D</i> in 1000 a. u.	<i>n</i>
0 — 4	7	32 — 64	16
4 — 8	7	64 — 128	14
8 — 16	21	128 — 256	12
16 — 32	27	256 — 512	9
		> 512	2

55 per cent of the nebulae have diameters from 4 to 32 thousand a. u. These diameters do not show any correlation with their distance from the Sun, thus arguing against the great influence of light absorption in space on the calculated distances.

However, the diameters are not correlated to any physical characteristic of the nebulae or of their nuclei, except in two cases. One of these shall be mentioned in the next section, the other shows that some of the nebular forms according to our classification¹⁾ are restricted to definit dimensions. The semi diffused nebulae of class V, five in number, are large, 120000 a. u. on the average. The stellar nebulae (class I) and nebulae with regularly illuminated discs (class II) are the smallest, 12000 (from 1000 to 26000) and 22000 a. u. (from 11000 to 46000) respectively. All other nebulae, including the ring forms (class IV) have shaply varying sizes, from 4000 to 500000 a. u. or more. Nevertheless, on the average the ring

¹⁾ Russ. Astron. Journ. **11**, 40, 1934, Nr. 1.

nebulae and those of class IIIb (irregular discs with a ring effect) are smaller than those of the class IIIa (irregular discs). Their corresponding diameters are 78000, 77000 and 173000 a. u. (IC I 1295 being omitted). If the planetary nebulae really expand, the sequence of forms during their evolution

$$I - II - IV - IIIb - IIIa \text{ or } V$$

seems reasonable. However, it may be noted that the velocity of expansion suggested by ZANSTRA does not show any correlation with the other known data concerning these nebulae. The same result was found by the writer in discussing the other data concerning the nebulae¹).

6. *The ring nebulae.* The latter might be supposed to form a homogeneous class, and were therefore studied once more separately giving the only result, which seems of interest.

Ten ring nebulae for which CURTIS²) has estimated the intensity of the ring relatively to the intensity of its inner part show that, the larger their true diameter D , the larger is this relation J of intensities. Approximately

$$D = 5 \cdot J.$$

The same fact in a less definite degree is shown by other nebulae with traces of a ring structure. Considering the ring nebulae as spherical shells, this might be explained by their expansion. In fact the increase of the radius of the shell, provided its thickness does not increase at the same rate, will provoke a growth of the apparent brightness of the ring. If d is the thickness of the shell expressed as a fraction of its radius

$$J = \sqrt{\frac{2}{d}} - 1.$$

The observed increase of J is proportional to that of D and might mean, that the expansion is accompanied by a decrease of the thickness of the shell. So it is necessary to postulate a quicker expansion for the inner surface of the shell or a slower expansion for its outer surface. However, the thickness of the shell estimated by CURTIS does not agree with his estimates of J for many nebulae, so that the detailed analysis of the fact is now premature. An exact photometric survey of ring nebulae for this purpose is now in progress at Moscow.

7. *Absolute magnitudes of the nuclei.* The absolute magnitude of the nuclei shows a pronounced dependence on their temperature. For 57 nuclei

¹) Russ. Astron. Journ. **11**, 40, 1934, Nr.1. — ²) Publ. Lick Observ. **13**, 1918.

Table 1.

	<i>R</i>	<i>X</i>	<i>Y</i>	<i>Z</i>	<i>D</i>	<i>M</i> _*
40	1000	— 520	840	160	36 (38×60)	1.4
0 ^h 22 ^m 8	4170	— 150	4130	— 560	21	
+ 55 ^o 21'						
246	460	— 60	100	— 450	104	2.7
650—1	2510	— 1700	1810	— 490	105×218 (218×394)	4.6
II 1747	4790	— 3270	3500	20	62	1.3
I 289	2630	— 2040	1660	100	79×118	4.7
I 351	2760	— 2530	820	— 750	19	2.8
II 2003	3020	— 2800	820	— 790	15	(5.3)
1501	4170	— 3480	2260	430	217	0.2 :
1514	1320	— 1240	270	— 360	139	0.2 :
1535	660	— 440	— 250	— 420	12 (24)	2.5
<i>J</i> 320	3470	— 3220	— 740	— 1060	23 (52)	0.2 :
I 418	2290	— 1650	— 1290	— 920	29	0.2 :
2022	3310	— 3080	— 1060	— 620	64 (93)	1.1
II 2149	870	— 840	180	150	5.2×10.4 (8.7×13.0)	4.3
II 2165	2880	— 2030	— 1960	— 590	23	(4.8)
<i>J</i> 900	2760	— 2640	— 810	130	30	
2371—2	3630	— 3350	— 650	1230	207 (196×435)	0.2 :
2392	420	— 370	— 140	130	7,1 (19)	2.7
2438	1660	— 950	— 1350	140	113	5.5
2440	2000	— 1060	— 1690	110	40×108	(4.3)
2452	3020	— 1230	— 2760	— 120	42	>6.7
2610	4790	— 2180	— 4100	1190	168	2.4
II 2448	1820	550	— 1680	— 440	15	
2792	4570	— 80	— 4550	400	46	
2818	3630	— 370	— 3560	610	145	
2867	790	150	— 770	— 70	6,3	
II 2501	1660	370	— 1610	— 130	3,3 :	
3132	400	30	— 390	90	12	2.6
II 2553	360	110	— 340	— 30	1,4	
3211	2090	680	— 1970	— 140	17	
3242	580	— 40	— 480	320	9×15 (20×23)	2.9
3587	2290	— 1080	650	1910	458	2.5+
3918	4400	1710	— 4030	440	44	
4361	1320	440	— 830	930	55 (107)	2.2
II 3568	1900	— 930	1280	1060	34	0.2 :
<i>CD</i> — 50 ^o	2400	1630	— 1630	480	22	
8073						
5315	3630	2380	— 2730	— 220	18	
II 4406	1200	920	— 690	340	24	
5873	4170	3600	— 1750	1180	12	
5882	1140	970	— 560	210	8,0	
15 ^h 26 ^m 2	2880	2360	— 1650	— 100	69	
— 58 ^o 49'						
6058	4170	1070	2610	3070	92	0.2 :
6072	6020	5690	— 1570	1160	181×301	3.6
II 4593	1000	670	360	650	11 (15)	0.2 :
6153	1820	1390	— 1160	180	3,6	
6210	790	430	450	480	10×16 (16×34)	3.0
II 4634	2630	2570	130	560	26 (29×53)	(4.6)
II 4637	4790	3880	— 2820	40	9,6	

	<i>R</i>	<i>X</i>	<i>Y</i>	<i>Z</i>	<i>D</i>	<i>M</i> _*
II 4642	2760	2020	— 1990	— 430	5,6 :	2.7
6309	1900	1800	380	480	19×36	
6326	2510	1790	— 1730	— 310	?	
6369	7240	7180	630	750	203	
17 ^h 35 ^m 8	4790	4650	1130	240	24	
— 24° 38'						
II 4663	3800	3700	— 720	— 520	?	>4.9
CD — 29°	2290	2290	60	— 40	6,9	
139 98						
6439	5250	5070	1280	530	26	
6445	3980	3900	760	260	135 (151×199)	
17 ^h 47 ^m 9	5240	5220	— 150	— 390	26 :	6.1
— 34° 21'						
17 ^h 49 ^m 2	4790	4710	830	170	24	2.5
— 21° 44'						
17 ^h 53 ^m 3	1740	1720	— 120	— 220	3,5 :	
— 38° 49'						
6543	520	— 60	450	250	10	
6537	2880	2800	650	30	14	4.7 :
6563	5250	4840	100	— 670	194×262	
6565	3980	3940	410	— 320	36	
6572	760	600	450	130	11	
6567	2000	1940	480	— 20	18	
II 4699	2160	2070	— 370	— 510	11 :	0.2 :
18 ^h 13 ^m 0	3470	2580	220	690	23	3.5
+ 10° 6'						
6620	9110	8970	1260	— 1260	46	0.2
6629	2190	2130	450	— 200	33	2.4
6644	2510	2440	470	— 320	7,5	(2.6)
II 4732	4170	4030	930	— 480	8,3 :	
II 4776	2880	2760	490	— 670	17	
18 ^h 45 ^m 4	2510	1460	2000	390	?	
+ 20° 43'						
— 32° 14' 73	1380	1330	140	— 310	5,6	5.6
I 1295	9120 :	8090	4120	— 810	958	
6720	660	270	580	150	39×55	
6741	2000	610	1900	— 110	16	
18 ^h 58 ^m 7	4370	4160	420	— 1330	22	
— 33° 19'						(4.5)
6751	2510	2080	1320	— 280	53	1.3
6772	6310	5020	3680	— 1010	353×473	4.1
II 4846	3020	2590	1460	— 530	6,0 ?	(3.2)
6778	3980	3110	2430	— 500	64 (88)	1.6
6781	2880	2030	2030	— 180	305	2.7 :
6790	1740	1300	1130	— 220	3,5 :	(6.5)
6803	1740	1140	1310	— 150	9,6	2.9
6804	4170	2780	3080	— 380	138 (234)	0.2 :
6807	5250	3680	3680	— 690	10 :	(5.0)
+ 30° 36' 39	760	300	300	50	3,8	5 3
6818	870	730	390	— 280	13×19	
6826	520	30	510	200	13	
6833	5250	450	5140	940	10 :	
6842	4790	1790	4440	— 20	225	
						(6.0)
						0.2 :

	R	X	Y	Z	D	M _*
6853	300	130	270	— 20	72 × 144	6.2
6879	2400	1180	2050	— 410	12	(2.6)
6881	6610	1490	6440	— 140	34	
6884	3020	260	2990	320	23	(5.8)
6886	2510	1160	2190	— 380	18	(7.9)
6891	1740	920	1420	— 390	12 (26)	0.2 :
6894	6920	2130	6570	— 440	304	2.4
II 4997	1740	830	1490	— 350	3,5 :	(1.8)
6905	2190	940	1900	— 400	88	2.4
7008	4170	— 430	4140	320	288 × 359	0.2 :
7009	440	280	230	— 240	5 × 11 (11 × 13)	3.5
7026	3160	— 110	3160	— 30	25 × 79	2.5
7027	1100	1100	40	— 1210	12 × 19	(6.2)
7048	1660	— 30	1660	— 80	91	7.2
* Küstner	5250	1810	4620	— 2480	?	
648 in M 15						
II 5117	4170	— 140	4040	— 430	8,3 :	(4.5)
21 ^h 29 ^m 1	3160	60	3110	— 180	16	
— 39° 11'						
7139	6310	870	6210	770	423 × 543	
II 5217	3020	— 670	2930	— 340	21	(4.2)
7293	180	80	60	— 150	130 × 162	7.0
7354	3470	— 1190	3260	80	69 (111)	3.9
7408	3310	1760	— 1280	— 2490	238	
I 1470	2090	— 840	1900	— 30	94 × 146	
7635	520	— 200	480	— 10	94 × 107	
7662	550	— 170	490	— 180	8,2 (16)	3.8

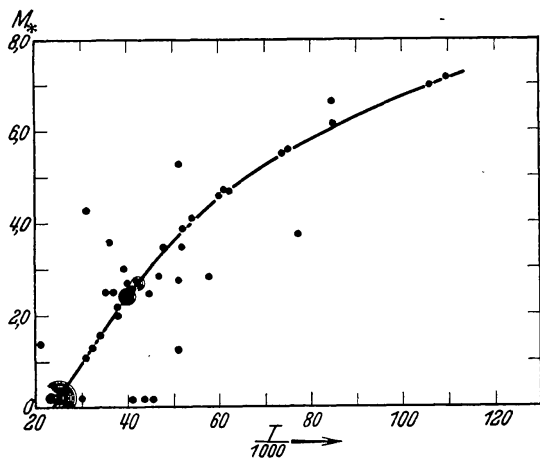


Fig. 3. Correlation between the absolute magnitudes of the nuclei and their temperatures.

the observed dependance is represented in fig. 3, where the circles signifie that in their center two, three or more dots are confluent. The mean curve can be represented by the formula

$$M_{*ph} = - 0.000875 T^2 + 0.1988 T - 4.21,$$

T being expressed in thousands of degrees. Computing the bolometric absolute magnitudes we have the following table:

$T/1000^{\circ}$	26	30	40	50	60	70	80	90	100	110
M_{*ph}	+ 0.2	1.0	2.5	3.6	4.6	5.3	5.9	6.4	6.8	7.2
M_{*bol}	— 1.6	— 1.1	— 0.4	+ 0.1	+ 0.6	+ 0.8	+ 1.0	+ 1.1	+ 1.2	+ 1.3

The dispersion for the photographic absolute magnitudes is 7^m0 , but only 2^m9 for the bolometric magnitudes. Assuming for the nuclei at the two ends of the foregoing table the Sun's mass, we obtain a range of densities from 3 to $9 \cdot 10^5$ g/cm³. So among the nuclei we have both stars of high though not abnormal density and such which are hundreds of times denser than the ordinary white dwarfs. The author believes however, that the relativity red shift in their spectra can be small if the lines in their spectra originate in the vast, extended atmospheres surrounding these stars themselves, where the intensity of gravitation is much lower, than at the surface of such stars. This may explain the apparently negative result obtained in 1931 at Simeis by Mrs. SHAIN, GERASIMOVICH and the author for the nuclei of NGC. 7635 and $+30^\circ 3639$. However, it is to be noted that these nuclei are of relatively high absolute luminosity ($+0,2$ and $+0,6$), so that the red shift in their spectra must be still smaller.

8. *Peculiar velocities.* The peculiar radial velocities v' corrected for the effect of galactic rotation shows nothing particular. For 90 nebulae with $v' < 100$ km/sec the mean peculiar velocity is 27,6 km/sec, and 38,8 including all 100 nebulae. The corresponding numbers not corrected for the effect of galactic rotation were 28,7 and 37,0 km/sec.

The peculiar space velocities were computed with our values for the parallaxes. They range from 3,8, ... to ... 147,176 km/sec with a mean 70 km/sec. The mean peculiar radial velocity of these 20 nebulae is 24,0 km/sec. Here again it seems that the proper motions of these objects are measured systematically too large. The peculiar space velocities increase on the average with the increase of the distance to the nebula. This can be in part due to the effect of galactic rotation. Besides this shows once more that the measured proper motions are too large for distant nebulae.

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