FORMATION OF GALAXIES, STARS, AND PLANETS*

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ABSTRACT

The *galaxy* is supposed to have been *formed* from a *primordial* gas, extending to distances greater than its present dimensions. The original motions in the gas may be of any kind; in any case, continuous, fluid motion will gradually be established, owing to viscous forces, if the mean free path of the particles of which the gas is composed is smaller than the dimensions of the system. The hydrodynamical laws can then be applied. This implies also that the particles are exceedingly small (atoms or molecules). It is assumed that an approximately steady state of motion is developed, such that the velocities everywhere can be regarded as a finite, continuous, and single-valued function
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of position, and that the terms defining the explicit dependence on the time are small in comparison to the inertia terms. It is shown that, if these conditions are satisfied, and if we follow a portion of the gas in its motion, its scalar velocity does not change with time. The motions—at least in the interior of the system—are such that the system contracts, generally or locally. At the surface there is a loss of matter owing to escape of particles. The contraction proceeds until the gas has developed into one or more bodies. If the system originally had a finite angular momentum, the bodies formed rotate in the same direction as the system.

If the system has symmetry about the axis of rotation, the motions in the gas are circular in parallel orbits. If smaller condensations are formed before the larger ones have been developed, the system is very flat. The stars formed after the steady state has been reached also move in circles about the axis and in a common plane. Stars formed before this state was reached may have any rotation and any motion, but the velocity must be less than that of escape.

If the relative motion of *two massive condensations* is circular, then, as long as the gaseous envelope has sufficient density, the motions in the gas are along "surfaces of zero relative velocity" with constant scalar velocity. Smaller bodies formed from this envelope move in the same way, but only bodies formed close to a primary can retain this motion after viscous forces and pressure gradients are no longer acting. It is suggested that retrograde satellites are either captured or formed from the gas when in a locally stable state of retrograde motion corresponding to a near coalescence of the two loops in the known periodic orbits in the problem of three bodies.

Application of the theory to the *solar system* indicates that the planets were not formed *in situ* but at great distances from the sun. The theory seems to account for the existence of nearly circular orbits for the massive planets and for the general behavior of the motions of the asteroids and the satellites. If the gas from which the planets were formed was once in the interior of the sun, it may have been expelled either during an encounter with another star or during an explosion like that of a nova. On the first alternative, planetary systems are very rare; on the second, they are a very common phenomenon in the universe.

INTRODUCTION

In *Mount Wilson Contribution* No. 492[1] the writer has attempted to explain the observed facts of stellar motions, in particular the

* *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 503.

[1] *Ap. J.*, **79**, 460, 1934.

predominance of circular orbits, the asymmetry in stellar motions, and the relationship between the motions and the physical properties of the stars, as results of viscous forces acting during the general contraction of a very large system, the stars being formed by local condensations in the primordial gas. After the paper had been sent to press, Lindblad's[2] article appeared, dealing partly with the same problem. Lindblad studies the problem from a different viewpoint, but his principal results are the same, namely, that in a system with axial symmetry, in which the density increases toward the center, we may expect circular "planetary" orbits in a common plane of motion. He emphasizes the fact that the escape of particles at the surface contributes greatly to the general contraction of the system. Although he does not explicitly introduce viscous forces, his analysis has certain things in common with that of fluid motion. To the present writer it seems that the motions of the stars are determined by conditions existing *at the time of their formation*, and that the problem can only be studied by regarding the primordial gas as a viscous, compressible fluid and by finding the probable development of such a fluid. In doing this opportunities will arise for a further explanation of some of the results in *Contribution* No. 492.

Let us think of a "gas" consisting of small "particles" freely hovering in space. The particles may be atoms, molecules, or dust particles; but any volume we study must include a great number of them, in order that statistical laws may be applied. Under what conditions may we regard such a gas as a "fluid" in the hydrodynamic sense? The particles are, in general, in motion, owing to the general attractions in the system; and we can certainly apply the concept of viscosity to the gas, since this implies nothing but a mixing of particles in adjacent regions, resulting in a transfer of momenta. The longer the mean free path and the greater the velocity dispersion ("temperature"), the more freely does the mixing take place and the greater is the coefficient of kinematic viscosity. This circumstance may or may not produce "continuous" motion like that in a fluid. If the mean free path is much greater than the dimensions of the system, each particle will describe an independent orbit and no continuous motion will result from the mixing. We must hence assume

[2] *M.N.*, **94**, 231, 1934.

that the mean free path is small compared with the dimensions of the system, which implies the assumption that the particles are exceedingly small, probably of the size of atoms or molecules.[1] The velocity of a particle is then little changed between collisions, which is a necessary condition for the applicability of the hydrodynamic laws.[3] Under these conditions the gradient of the gas pressure is a force acting on the particles themselves.

Darwin[3] was probably the first to realize fully the importance of viscous forces in the development of cosmic systems. On account of the high viscosity, most writers have assumed that the final steady state of the fluid is a rotation as a solid, although Jeans[4] has shown that, in the interior of the stars at least, we must have an angular velocity which decreases outward. The further development of a fluid with constant angular velocity has been extensively studied by many prominent investigators. In this case, the dissipation function, which is never negative, has reached its ultimate minimum, which is zero. But we know from the work of Helmholtz[5] and Korteweg[6] that, if the inertia terms can be neglected, the motions in a viscous, incompressible fluid, with fixed boundary conditions incompatible with rigid-body motion, can reach a steady state in which the dissipation function is a minimum greater than zero. Lord Rayleigh[7] has generalized Korteweg's theorem to include velocities of any size by "introducing" forces parallel to the vector product of the linear and angular velocities. From these results we see that it is important to consider the developments in the fluid previous to the establishment of rigid-body motion.

STEADY MOTIONS IN A COMPRESSIBLE VISCOUS FLUID

In most applications of the hydrodynamics of viscous fluids it has been assumed that the fluid is incompressible and that the coeffi-

[3] Cf. G. H. Darwin, "On the Mechanical Conditions of a Swarm of Meteorites and on Theories of Cosmogony," *Scientific Papers*, **4**, 391, 1889.

[4] *M.N.*, **86**, 328, 444, 1926.

[5] "Zur Theorie der stationären Ströme in reibenden Flüssigkeiten," *Wissenschaftliche Abhandlungen*, **1**, 223, 1882.

[6] *Phil. Mag.* (5th Ser.), **16**, 112, 1883.

[7] *Ibid.*, **36**, 354, 1893.

cient of viscosity is constant throughout the fluid. We cannot make either of these assumptions. Hardly anything has been done on the motions in compressible fluids of variable viscosity. Knowing the mathematical difficulties in studying, for instance, the figures of equilibrium for incompressible fluids of constant angular motion, it may at first sight seem as if we could make little or no headway in the more general case. But in the absence of forces at the boundary the fluid can adapt itself to the gravitational forces, which are the only forces we shall consider, and it will be found that it is then possible to establish some general relations which are important for a study of the development of such a fluid.

Using Stokes's expressions for the stresses in a viscous fluid element, we can write down the equations of motion of an element of unit volume:

$$\left.\begin{aligned}\rho\left(\frac{Du}{Dt}+\frac{\partial G}{\partial x}\right)&=\frac{\partial p_{xx}}{\partial x}+\frac{\partial p_{yx}}{\partial y}+\frac{\partial p_{zx}}{\partial z}=-\frac{\partial p'}{\partial x}+\mu\nabla^2 u\\&\quad+\left(2\frac{\partial u}{\partial x}-\Theta\right)\frac{\partial\mu}{\partial x}+\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\frac{\partial\mu}{\partial y}+\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right)\frac{\partial\mu}{\partial z}\;;\\p'&=p-\frac{\mu\Theta}{3}\;;\qquad\Theta=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\;.\end{aligned}\right\}\quad(1)$$

In this equation, u, v, and w are the velocity components in an inertial reference frame, ρ the density, p the static pressure, μ the coefficient of viscosity, Θ the divergence, and p_{xx}, p_{yx}, p_{zx}, the components of the stress tensor. The factor $1/3$ occurring in p' implies the assumption that the pressure depends only upon the density and temperature of the gas and not upon the rate of expansion.[8] The other two equations of motion can be found by cyclic permutations.

In the beginning the motions in the fluid may be of any type. Large or small vortices, vortex sheets, and vortex rings may develop and be dissipated, and the temperature may be distributed in an irregular fashion, giving rise to convection currents and escape of particles of high velocity. If at any one point the gradient of the angular velocity is abnormally high, a large part of the gross motion will be

[8] Cf. Lamb, *Hydrodynamics* (3d ed., 1906), pp. 534, 586.

converted into heat motion, the temperature will rise at that point, the coefficient of viscosity will increase, and the dissipation of mechanical energy into heat motion may for a time proceed at an accelerated rate. We shall first suppose that all the heat generated by friction is retained as heat motion in the system and that no energy is lost or gained in the form of radiation or by subatomic processes.

When the fluid is compressible, we have no reason to believe that any unique steady state of motion exists. For instance, the fluid may contract uniformly or locally in a way we cannot predict. We shall assume, however, that "continuous" motion will gradually be established. We will also limit ourselves to those cases for which a co-ordinate system exists such that the motions referred to this system are nearly steady. For convenience we will call this a "steady" state of motion. When it is necessary to take into account the change in motion at any particular place, we shall assume that the terms depending on the partial time-derivatives of the properties of the system are everywhere small relative to the inertia terms. The theory is hence applicable to systems in which there are progressive changes in density, temperature, and pressure, as well as to systems the properties of which are subject to periodic changes, although these changes must always be sufficiently slow. On account of the condition of continuity and steadiness of the motion, we can regard the velocity vector, if referred to the proper co-ordinate system, as a continuous, finite, and single-valued function of position, containing also small terms changing slowly with the time.

On account of the progressive change in density, it is convenient to reduce the equations of motion in (1) to unit mass. We write them in the form

$$\frac{Du}{Dt}+\frac{\partial G}{\partial x}=\frac{1}{\rho}\left(\frac{\partial p_{xx}}{\partial x}+\frac{\partial p_{yx}}{\partial y}+\frac{\partial p_{zx}}{\partial z}\right)=f_x\ , \qquad (2)$$

with similar equations for the y and z components, where f_x, f_y, and f_z are the components of the accelerations due to pressure gradients and to viscous drag.

Let us study a portion of the fluid which at time t occupies a rectangular volume element $dxdydz$ having its center at x, y, z. Calculating the rate at which work is being done by the tractions on the pairs of opposite faces, we obtain[9]

$$\left[\frac{\partial}{\partial x}(p_{xx}u+p_{xy}v+p_{xz}w)+\frac{\partial}{\partial y}(p_{yx}u+p_{yy}v+p_{yz}w)+\frac{\partial}{\partial z}(p_{zx}u+p_{zy}v+p_{zz}w)\right]dx\,dy\,dz=(A+B)dx\,dy\,dz\ ,$$

in which A and B are defined by the equations:

$$A=\left(\frac{\partial p_{xx}}{\partial x}+\frac{\partial p_{yx}}{\partial y}+\frac{\partial p_{zx}}{\partial z}\right)u+\left(\frac{\partial p_{xy}}{\partial x}+\frac{\partial p_{yy}}{\partial y}+\frac{\partial p_{zy}}{\partial z}\right)v+\left(\frac{\partial p_{xz}}{\partial x}+\frac{\partial p_{yz}}{\partial y}+\frac{\partial p_{zz}}{\partial z}\right)w\ , \quad (3)$$

$$B=p_{xx}\frac{\partial u}{\partial x}+p_{yy}\frac{\partial v}{\partial y}+p_{zz}\frac{\partial w}{\partial z}+p_{xy}\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)+p_{xz}\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right)+p_{yz}\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right). \quad (4)$$

A represents the rate at which the tractions on the faces of an element of unit volume are doing work in increasing its kinetic and gravitational energy. B is the rate at which work is being done in producing changes in the volume and the shape of the element, which result in changes in its temperature, density, and pressure. We may also say that A is the rate at which the gross mechanical energy increases, while B is the rate at which the microscopic energy increases. Putting $A+B=C$, we see that C is the rate per unit volume at which the total energy is increased.

The "surface" of the body of fluid is defined by a negligible density; and since there are no tractions on this surface, we must have

$$\iiint C\,dx\,dy\,dz=\int\frac{C}{\rho}\,dm=0\ . \quad (5)$$

C is hence positive in some regions and negative in others, and on the average equal to zero. It is known that B is, in general, positive. A is thus a predominantly negative quantity.

Multiplying equations (2) by u, v, and w, respectively, and introducing equation (3), we obtain the equation for the conservation of energy:

$$\left.\begin{aligned}\frac{D}{Dt}(T+G)=uf_x+vf_y+wf_z=\frac{A}{\rho}=-\varphi\ ,\\ 2T=u^2+v^2+w^2\ .\end{aligned}\right\} \quad (6)$$

[9] Lamb, *op. cit.*, p. 540.

The function φ is predominantly positive and represents the rate *per unit mass* at which mechanical energy is dissipated after the steady state has been established. It can vanish everywhere only if there is no dissipation of energy at all; the body can then have no other motions than a general rotation with constant angular velocity, although for the simple case of constant μ we can, in addition, have a uniform contraction or expansion.

We shall now vary the velocity of an element of the fluid and the path it describes under the influence of inertia, gravity, pressure gradients, and viscous forces, first studying the case in which the proper co-ordinate system is an inertial frame.

From the definition of T, u, v, and w, we find

$$\begin{aligned}\Delta T &= \tfrac{1}{2}\Delta(u^2+v^2+w^2) = u\Delta\left(\frac{Dx}{Dt}\right)+v\Delta\left(\frac{Dy}{Dt}\right)+w\Delta\left(\frac{Dz}{Dt}\right)\\ &= u\,\frac{D}{Dt}\,(\Delta x)+v\,\frac{D}{Dt}\,(\Delta y)+w\,\frac{D}{Dt}\,(\Delta z)=\frac{D}{Dt}\,(u\Delta x+v\Delta y+w\Delta z)\\ &\qquad -\left(\frac{Du}{Dt}\,\Delta x+\frac{Dv}{Dt}\,\Delta y+\frac{Dw}{Dt}\,\Delta z\right) .\end{aligned}$$

Hence

$$\int_{t_1}^{t_2}\left(\Delta T+\frac{Du}{Dt}\,\Delta x+\frac{Dv}{Dt}\,\Delta y+\frac{Dw}{Dt}\,\Delta z\right)dt=[u\Delta x+v\Delta y+w\Delta z]_{t_1}^{t_2} . \tag{7}$$

At the fixed times t_1 and t_2 the variations Δx, Δy, and Δz are supposed to vanish, and the integral must hence be equal to zero.

Multiplying equations (2), respectively, by Δx, Δy, and Δz, adding, and using equation (7), we obtain

$$\int_{t_1}^{t_2}(\Delta T-\Delta G+f_x\Delta x+f_y\Delta y+f_z\Delta z)dt=0 . \tag{8}$$

According to our assumptions both G and T can be regarded as continuous, finite, and single-valued functions of x, y, and z, containing terms changing slowly with time. We can then replace T and G by their mean values. We write

$$T=T_m+(t-t_m)\frac{\partial T}{\partial t}\,,$$

$$G=G_m+(t-t_m)\frac{\partial G}{\partial t}\,,$$

where $t_m=(t_1+t_2)/2$, and T_m and G_m indicate average values of the kinetic and gravitational energies, reduced to unit mass.

T and G may depend explicitly on the time, however, without affecting our conclusions. The important thing is that we limit ourselves to the cases in which the motions become more and more steady, so that we can regard T, when referred to a proper co-ordinate system, as being a function of x, y, z, and t, and not of the velocity, as in the usual Lagrangian equations.

Equation (8) can now be written in the form

$$\Delta\int_{t_1}^{t_2}(T-G+xf_x+yf_y+zf_z)dt=0\,, \tag{9}$$

where Δ indicates a space variation, and where f_x, f_y, and f_z are not varied. Equation (9) can be regarded as a condition for a minimum of "action" of the "kinetic potential" $(T-G)$, the viscous forces, and the pressure gradients, all reduced to unit mass.

The partial space derivatives of (9) must vanish; and, since t_1 and t_2 are arbitrary quantities, the integrands themselves must vanish. Hence

$$\left.\begin{aligned}\frac{\partial(T-G)}{\partial x}+f_x&=0\,,\\ \frac{\partial(T-G)}{\partial y}+f_y&=0\,,\\ \frac{\partial(T-G)}{\partial z}+f_z&=0\,.\end{aligned}\right\} \tag{10}$$

Multiplying (10) by u, v, and w, respectively, we find, using (2) and (3),

$$\frac{D}{Dt}(T-G)=-(f_x u+f_y v+f_z w)=-\frac{A}{\rho}=\varphi\ . \tag{11}$$

From (6) and (11),

$$\frac{DT}{Dt}=0\ , \tag{12}$$

$$\frac{DG}{Dt}=-\varphi\ . \tag{13}$$

In interpreting equations (12) and (13) we must remember several things. We have assumed $\partial T/\partial t$ and $\partial G/\partial t$ to be small compared to

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}$$

and to

$$u\frac{\partial G}{\partial x}+v\frac{\partial G}{\partial y}+w\frac{\partial G}{\partial z}\ ,$$

respectively; hence the equations are valid only when the variations in velocity and gravity at any one fixed point are slow. Second, the usual conception of the "velocity following an element of the fluid in its motion" has in our case a statistical meaning only, since we have no means of identifying a "fluid element" in its motion. Consequently, we cannot say anything about what happens when we "follow" an element during an indefinite time. We can only say that at any time the instantaneous velocity vector lies in a surface of constant scalar velocity (equation 12). The gravitational energy decreases as we follow the local movements (equation 13); hence the instantaneous velocity vector cuts the surfaces of constant gravitational potential inward. The surfaces of constant scalar velocity and constant gravitational potential are both supposed to form a set of closed, non-intersecting surfaces. Since the motions are systematically inward, the system contracts, and the surfaces of constant T

and G are slowly changing. In the limiting case when $\varphi = 0$, there is no longer any contraction, and the system has dissolved itself into a single star or a group of stars, all rotating as solids.

The loss in gravitational energy is equal to the gain in heat and pressure energy; none is converted into kinetic energy after the steady state has been established. On account of the rise in temperature, the energy carried away at the surface by escaping particles may be very considerable. We thus see that viscous forces produce at the same time a scattering and a condensation; we confine our study to the contracting system.

It is interesting to note the rapid changes in the moment of inertia of the system. We write the virial theorem in the form

$$\frac{1}{2}\frac{D^2I}{Dt^2} = \int (2T+G)dm ,$$

where I is the moment of inertia. Differentiating with regard to time, we obtain

$$\frac{1}{2}\frac{D^3I}{Dt^3} = \int \left(2\frac{DT}{Dt} + \frac{DG}{Dt}\right) dm .$$

Hence, from (12) and (13),

$$\frac{D^3I}{Dt^3} = -2\int \varphi \, dm = -2\dot{\Phi} . \qquad (14)$$

Φ is the dissipation function, which is always positive when there is internal motion in the system. Dealing, as we are, with the third time-derivative, it is clear that the moment of inertia must decrease rapidly when Φ is finite. Since particles escape at the surface, the decrease is still more rapid than equation (14) indicates.

Since the particles we are considering are built up of electric charges, the shake-up of the particles themselves by collisions produces, in general, radiation, which may be released immediately, or later when atomic readjustments take place. Ionization may also take place, resulting in the liberation of electrons. In a particular element of the fluid, we have then several forms of energy, parts of which are retained in the element, while other parts are transferred to adjacent elements or to the surrounding space. We shall still assume, however, that an approximately steady state of motion like the one previously considered will gradually be established.

Under this condition we find that, if there is no source of energy in the interior, $\iiint C dxdydz$ is negative. B now includes the rate at which microscopic forms of energy in general are produced. Since C is predominantly negative, $\varphi=(B-C)/\rho$ is still positive and numerically larger even than in the case previously considered. We can still regard the average values of T and G as functions of position, and equations (11), (12), and (13) still hold.

The quantity φ is the rate at which mechanical energy per unit mass is dissipated. The difference between the case in which radiation is produced and lost to the surrounding space and that in which no radiation is lost lies in the resultant distribution of temperature, density, and pressure, and in the greater speed at which the contraction proceeds in the first case, as compared with the second. For the first, φ is positive even after B has reached a zero value, and contraction can hence proceed after rotation as a solid has been reached —a fact utilized in Laplace' theory.

NEBULA WITH AXIAL SYMMETRY

Let us now apply these results to the development of a nebula in which the density decreases from a center outward in a more or less regular fashion. Before a steady state is reached, a general contraction occurs owing to the escape of particles of high velocity and to the loss of energy by radiation produced by the collisions. The velocities in the central part increase gradually, and so also do the density, temperature, and pressure.

If the density distribution is such that in the first approximation it has axial symmetry and the density decreases outward from a single center, viscosity will produce motions which, referred to an inertial reference-frame fixed to the center, will ultimately be approximately steady. After this state is established, the velocity of a moving "element" remains constant (like that of a body freely falling in a viscous fluid) and every element has an "inward" velocity component. At any particular point in the fluid the velocity will, however, change with time. For the simple case considered, in which no very large secondary condensations are produced, the system at any single instant will have a shape and a density distribution symmetrical about an axis of rotation coinciding in direction with the original total angular-momentum vector. There will also be a plane of symmetry perpendicular to the axis of rotation. It is also clear that surfaces of equal pressure, temperature, viscosity, gravity, and scalar velocity will all be symmetrical about the axis of rotation and about the equatorial plane.

If the density gradient is not exactly uniform, the general tendency toward contraction must also show as a tendency toward local condensation. Before the steady state is reached, local condensations may form and be dissolved; but after this state has been attained, such condensations no longer dissolve but tend to increase in density and mass, provided the secondary condensations are so small that the disruptive effects of tidal forces can be neglected. The theory can hence be applied to the formation of stars, but caution must be used when applying it to very extended stellar groups. As long as the density around the condensations is everywhere still finite, the motion of an element of the fluid is affected by the viscous forces and the pressure gradients. During the gradual accretion of the condensation, the atmosphere around it receives a net outward momentum corresponding to the decrease in the pressure gradient. When ultimately the density outside the condensation vanishes and the viscous forces and the pressure disappear outside the condensations, equations (12) and (13) are still valid, with a value of φ outside the condensation equal to zero, provided free particle motion can take place along surfaces of constant gravitational energy. Since condensations formed outside the central plane will *gradually* sink into this plane, the final orbits will all lie in this common plane of motion. For a density distribution symmetrical about an axis, the final orbits of stars developed from the gas are, hence, circles in a common plane perpendicular to the axis of symmetry.

The angular velocity ζ at a distance r from the axis of symmetry is given by the equation

$$2\zeta=\frac{dv_2}{dr}+\frac{v_2}{r}\,, \tag{15}$$

where v_2 is the velocity perpendicular to the radius vector. For the limiting case ζ equal to zero, $v_2=c/r$, which represents the usual conditions of motion about a straight vortex core when the fluid is kept immovable at "infinity." But in the present case there are no forces acting on the "boundary" of the system, the velocities are not zero at the "boundary," and the rate at which v_2 decreases with r is less than in the usual case. Putting, approximately, $v_2 = c/r^n$, we have in general $n < 1$, with the result that ζ is everywhere positive—that is, the local rotation is everywhere in the same direction as the general rotation. The exponent n decreases with the time, since in the other limiting case, namely, that of a motion as a solid, we have $n = -1$.

If a star is formed before the steady state has been established, it may have any rotation. It may also have a velocity differing greatly from circular motion; and it may have a velocity component perpendicular to the plane of symmetry. The bodies so formed, as explained in *Contribution* No. 492, are the high-velocity stars in the galaxy and the asteroids in the solar system. The steady state may never be completely established, but may, nevertheless, serve as a useful standard indicating a general tendency.

It is important to note that the rate at which viscous forces can reduce space gradients in the angular velocity depends upon the density and the dispersion in velocity of the particles, that is, upon the "temperature" of the gas, and also upon the mean free path and the dimensions of the system. When the condensations which later become stars are formed, the nebula is supposed to be *many times greater* than after the stars are developed; further, the temperature is supposed to be very low. The low temperature, coupled with the very small density of the system, makes the rate at which viscous forces can reduce the velocity gradient very small, in spite of the long free path. It is not surprising, therefore, that the nebulae do not rotate as solids. The same conclusion applies to the gas forming the solar system. According to the present picture, the dimensions of this system were originally of about the same order as interstellar distances, and the planets were not formed *in situ* but at much greater distances from the sun than they are now.

It is very probable that condensations with masses much smaller than those of stars or planets are first formed, and that these condensations later combine into stars, planets, or satellites. If this is so, the condensations will tend to sink into the central plane and the system will become very much more flattened than would be the case when no small condensations are formed. The small condensations may well be identical with the "planetesimals" of Moulton and Chamberlin. A great number of these condensations have not yet combined with larger bodies; they still exist and are often seen individually as meteors and in groups as comets or showers of meteors. Whether the bodies finally formed will have masses equal to those of the particles in Saturn's ring, or of the order of those of satellites, planets, or stars, cannot be predicted. This detail depends upon the degree of regularity in the original distribution of density and temperature and the extent of the regions of uniform motion at the time these bodies were formed.

TWO CONDENSATION CENTERS

The case involving two condensation centers with masses of the same order of magnitude can also be studied, provided their relative motion is nearly circular. As long as there is still intervening gas, gravitational energy is dissipated and the system contracts. A time will come, however, when the two condensations can be regarded as separate bodies, which still contract although their mutual distance no longer systematically decreases. If a small condensation has been formed slowly and has gradually sunk into the equatorial plane, it will also move, at least approximately, in the common plane of motion. We introduce a co-ordinate system rotating with the constant angular velocity ω of the two large condensations, the origin being at the center of mass of the system. Denoting co-ordinates and velocities referred to the rotating system with subscripts, we find as before:

$$\left.\begin{array}{ll}\dfrac{DT_1}{Dt}=0\;; & \dfrac{D}{Dt}\left(G-\dfrac{\omega^2r^2}{2}\right)=-\varphi\leq 0\\ 2T_1=u_1^2+v_1^2+w_1^2\,, & r^2=x_1^2+y_1^2=x^2+y^2\,.\end{array}\right\}\qquad(16)$$

These equations determine the changes in the kinetic and gravitational energies of an element of the gas, reduced to unit mass. The velocity vector always lies on a surface of constant scalar velocity. The surfaces $G-\omega^2r^2/2=$const. are analogous to the surfaces of "zero relative velocity" in the restricted problem of three bodies. It has been shown by Miss Rein[10] that the structure of these surfaces, when the bodies are surrounded by an attracting mass of gas or dust, is completely analogous to Hill's surfaces. We can say that the motions are such that at a particular instant they nearly follow surfaces defined by the equation $G-\omega^2r^2/2=\text{const.}$, that these surfaces contract, and that the scalar velocity along any one of these surfaces is constant. In the end, all three bodies rotate in the same direction as the system, and the two massive bodies move in fixed circles about one another. The "satellite," however, cannot in general move along a surface $G-\omega^2r^2/2=\text{const.}$, since it has now a free-particle motion, and the only periodic orbits inclosing one of the attracting centers are of another type. Motions along the foregoing surfaces are possible only so long as viscous forces and pressure gradients are still acting. Close to the planet, however, where the perturbations are small, we may have free-particle motions in direct, nearly circular orbits, with no secular changes in the size of the orbit —a well-known theorem in the theory of perturbations. Farther out, a satellite moving freely in a direct orbit would be so greatly disturbed by the sun that it would sooner or later come very close to its primary and be involved in the gaseous envelope and probably drawn in or disrupted. If it moved in a retrograde orbit closely resembling one of the known periodic orbits, it could keep on moving without ever coming too close to its primary. If the orbit were retrograde in the non-rotating system, such a satellite could not have been formed after the steady state described had been established, since the motions in the layer between direct and retrograde motion do not represent minimum action. It would either have been formed before a steady state was reached or have been captured by the aid of frictional forces in the gas surrounding the primary. But these forces must be so adjusted that they would take away just enough kinetic energy from the satellite to make the orbit nearly periodic. Since so fine an adjustment is improbable—just as circular motions are improbable without the aid of viscous forces—and since several retrograde outer satellites exist, it seems possible that a locally stable

[10] "On the Masses of Condensations in Dust Nebulae," *Astron. Jour. Soviet Union*, **10**, 4, 1933.

state of fluid motion exists, corresponding to retrograde motion along surfaces of zero relative velocity. For a limited region we can then also regard the velocity as a continuous, single-valued function of position.

Nothing is known about periodic orbits with high inclinations to the plane of motion of the attracting centers. Hence we cannot say anything about the formation of the satellites of Uranus and Neptune, except that motions parallel to the ecliptic had not yet been established when the satellites were formed.

ROTATION PERIODS IN THE SYSTEM OF SATURN

The present theory may account for the peculiar fact that the inner ring of Saturn has a shorter time of rotation than the surface of the planet. The difficulty of explaining this fact from the standpoint of Laplace' theory has been emphasized by Moulton.[11] In the present picture, when Saturn was greater than at present, its internal motion deviated much more from that of a solid than it does now. It was also surrounded by a flat disk of gas in the equatorial plane before the solid particles in the ring had completely condensed. During the condensation process an outward gas pressure was active, and its disappearance was accompanied by an increase in the circular motion of the ring, which must be retained, according to equations (12) and (13), even though the pressure gradually disappears. The circular velocity must hence increase until free-particle motion is established. This conclusion does not contradict equation (12), since the matter in the "element" whose motion we are following is contained partly in the condensation and partly in the surrounding gas.

On the surface of the planet when still in a gaseous state, we did not have free-particle motion, but a balance between gravitational forces on the one hand and centrifugal forces and pressure gradients on the other. In the present theory the angular momentum is not conserved, since the escape of particles in the atmosphere carries away a large part of this momentum. When the surface has become more or less solidified, we may well have a velocity at the surface considerably smaller than the free circular motion, as is now the case on the earth.

[11] *Ap. J.*, 11, 111, 1900.

CHANGES IN MOTION AFTER THE FORMATION OF THE STARS

The present study forms a complement to the valuable work of Bok[12] about the stability of moving star clusters. He has shown that such clusters tend to disperse, owing to the effect of tidal forces from an attracting central mass in the galaxy. The theory here outlined shows that previous to the formation of the stars and the stellar groups we have a general tendency toward concentration of matter into stars moving in nearly parallel, circular orbits about the center of the galaxy. Bok's theory hence refers to the later changes in the motions of the stars after their formation.

FORMATION OF HEAVY ATOMS AND OF PLANETS

An interesting phase of the present theory may be mentioned here. We are inclined to think that heavy atoms are formed from hydrogen, neutrons, and electrons in the hot interior of the stars. The matter in the earth and the planets is supposed to have been ejected by the sun after the heavier atoms were formed. But in the present picture the matter in the earth was never in the melting-pot of the sun or of any other star after the steady state of motion had been established. But perhaps heavy atoms existed in the primordial gas, or were formed by processes not requiring excessive heat; or the sun may have exploded like a nova long ago, and what we have been studying are the processes following this cataclysm. We have some evidence that nova outbursts are normal phenomena among the stars.

If we admit that heavy atoms can be formed only in the interior of stars, the matter of which the planets are built up must have been ejected by the sun, either as a nova outburst or as a consequence of a close encounter with another star. In the first case, planetary systems like that of the sun are very common. In the second case, they are extremely rare phenomena in the universe. In whatever way the cataclysm happened, it does not affect the present picture of development, which deals only with changes occurring after a steady state of motion was last established.

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[12] *Harvard Circ.*, No. 384, 1934.