been verified experimentally. Nevertheless, it must be remarked that the new equation involves a number of terms of the second order (which we have neglected) which would give rise to perturbations of the atom not derivable by the method of general relativity. Unfortunately these terms are so very small as to preclude any hope of their giving rise to observable effects. To this extent, therefore, Schroedinger's equation must remain unverified.

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THE COEXISTENCE OF STELLAR AND INTERSTELLAR CALCIUM LINES IN THE MASSIVE B9 STAR H.D. 698.

J. A. Pearce, M.A., Ph.D. (Plate 16.)

(Communicated by Sir Arthur Eddington, F.R.S.)

I. *The Interstellar Cloud*

The theory of the nature and physical properties of the diffuse matter in interstellar space, as proposed by Eddington * in the Bakerian Lecture before the Royal Society, 1926, readily accounted for several phenomena connected with the so-called "stationary" lines of calcium and sodium. Thus the widespread distribution of the calcium cloud, relatively at rest with respect to the stars, as observed by Slipher † in 1909 from widely distributed Class B stars, and independently observed by Plaskett ‡ in 1924 from his observations of the Class O stars, naturally follows from the theory that this interstellar matter is uniformly distributed throughout the galactic plane. The observation of Young § that the "stationary" H and K lines were not associated with stars of lower temperature than Class B3 was readily explained by the fact that only the highly luminous stars of Classes O and B, whose absolute magnitudes exceeded $-2^{m}.0$, had been observed at distances sufficient to produce the interstellar lines.

The theory accounted for the presence of ionised atoms of calcium and sodium in space. The earlier explanation that these atoms were ionised by the excitation of neighbouring high-temperature stars was now abandoned in favour of the view that the ionisation was produced by the general stellar radiation. Under the existing conditions of a density of the order of 10^{-24} gm./cm.^{3} and a temperature of 10,000° K. most of the calcium present would be doubly ionised, and sodium singly ionised, leaving a sufficient amount of neutral sodium and singly ionised calcium to cause the observed absorption.

The theory, however, required the confirmation of several points before its complete acceptance. Observational evidence of a quantitative nature

* *Proc. Roy. Soc.*, 111, 424, 1926.
† *Low. Obs. Bull.*, 2, 1, 1909.
‡ *Publ. D.A.O.*, 2, 335, 1924.
§ *Ibid.*, 1, 219, 1920.

was necessary to show the uniform distribution of the interstellar matter. The non-occurrence of the interstellar lines in the spectra of late-type stars was a second objection. Thirdly, the anomalous strength of the interstellar D lines of sodium required explanation, for according to Eddington there would only be one part in two million of neutral sodium available for the absorption of the interstellar D lines.

The first of these objections was partially removed by the investigations of Struve,* who showed that the intensities of the interstellar lines increased with the distances of the stars. . Further confirmation was obtained by Gerasimovič and Struve,† who showed that the calcium cloud was rotating about the distant centre in Sagittarius, the uniform distribution being evident from an approximate two-to-one correspondence between the distances of the stars and cloud. A more complete discussion of "The Motions and Distribution of Interstellar Matter" was made by J. S. Plaskett and the writer ‡ in 1929 from their extensive material of the radial velocities of the O- and B-type stars. A general solution was first made for the solar motion, the K term and the galactic rotation of the interstellar cloud from the radial velocities of the K lines appearing in 261 stars of Classes O to B2, uniformly distributed in the galactic zone, approximately $l = 350°$ to $190°$, $b = 0°$ to $\pm 20°$. The analysis showed that the calcium cloud was relatively at rest with regard to the stars, having a K term of zero and a solar motion of 19·9 km./sec. directed towards an apex 20° away from the normal position, the discrepancy being due to the unsymmetrical distribution of the stars. Further, like the stars, the calcium cloud was in rotation about the distant galactic centre in the direction of Sagittarius.

A second solution for the elements of the galactic rotation, assuming a normal solar motion and a zero K term, gave the longitude of the galactic centre as $331°{\cdot}7 \pm 5°{\cdot}7$ coincident with the centre of the globular cluster system, while the rotational term of $\bar{r}A = 7{\cdot}9 \pm 0{\cdot}8$ km./sec. denoted a mean solar distance of approximately 465 parsecs for the cloud. Simultaneous solutions for the distances of the stars and the cloud, dividing the stars into (*a*) five groups according to apparent magnitudes, and (*b*) five groups according to the intensities of the K lines, resulted in a remarkable relationship of a two-to-one correspondence, the average distances of the stars being almost exactly double those of the clouds. When it was shown that this relationship held for star groups varying from 600 to 1600 parsecs in distance, no reasonable doubt remained regarding Eddington's hypothesis of an approximately uniform distribution of the cosmic matter.

A further research by the authors § from all available material, including 308 stars and 6 novæ showing interstellar sodium and calcium lines, has been recently completed. The analyses of the radial velocities confirm and amplify the findings of the preliminary investigation. A new method of estimating the intensities of the K lines was developed, and a linear relationship expressing the distances of the stars in terms of the intensities of the K lines established. The mean parallaxes of the several groups of stars

* *Ap. J.*, **65**, 163, 1927 ; **66**, 353, 1928. † *Ibid.*, **69**, 7, 1929.
‡ *M.N.*, **90**, 243, 1930. § *Publ. D.A.O.*, **5**, No. 3, in press.

were determined from proper motions.* These parallaxes were used with care to derive a value of the differential rotational constant $A = 0{\cdot}017$ km./sec. per parsec from the Sun. The investigation furnishes definite evidence in favour of both the rotation of the galaxy and the uniform distribution of the diffused matter throughout the region inhabited by the Classes O to B2 stars.

The second objection, "the non-appearance of the interstellar lines in the spectra of stars later than Class B5," has now been satisfactorily removed by the study of the massive binary system H.D. 698, of spectral type B9sek, discussed in this paper. The survey of the interstellar D lines now being made at the Mount Wilson Observatory by Dr. P. W. Merrill will supply the accurate and extensive observations on sodium which are urgently needed to explain the anomalous intensities of these lines and establish the theory for interstellar sodium.

II. *The Coexistence of Stellar and Interstellar Calcium Lines in the Spectra of the Stars of Classes* B2 *to* B5

It follows from the theory of a uniform distribution of the cosmic cloud that the interstellar calcium and sodium lines should be superimposed upon all stellar spectra as soon as the stars are at sufficient distances to show the interstellar absorption. They have been observed in practically all the O5 to B2 stars fainter than $5^{m}{\cdot}0$ and in many spectra of the faint B3 stars, but have not been observed in stars later than Class B5. The absence of the stellar calcium lines in the spectra of the O5 to B2 stars makes the identification of the interstellar calcium lines in these types certain. Stellar calcium makes its marginal appearance in Class B2, slowly increases in intensity through the later divisions of Class B, continually increases throughout Classes A, F and G, reaching a maximum intensity in Class M. In spectral types B3 to B5 the intensity of the stellar K line is comparable with that of the interstellar K line, so that for these subdivisions of Class B the coexistence of these two calcium lines of different origin is first observed. For the bright B3 to B5 stars nearer than 300 parsecs the measures of the faint K line show that it is entirely stellar in origin, but for the fainter B3 to B5 stars the measured velocities of the line indicate a blended origin whether the stars be spectroscopic binaries or possess a constant velocity.

The coexistence of stationary and oscillating K lines among the early Class B spectroscopic binaries has been known for many years. While suspected and reported for several stars, there are only 3 stars for which the coexistence has definitely been shown. The first case detected was that of H.D. 23180, *o* Persei, type B2k, a spectroscopic binary investigated by Jordan † in which both spectra appear and oscillate with large amplitudes. In addition to strong narrow H and K lines of constant velocity, Jordan observed a weak stellar K line oscillating with the same amplitude and in

* Unpublished proper motions for the faint O to B5 stars has been kindly placed at our disposal by the Director of the Dudley Observatory.

† *A.O.*, **2**, 63, 1910; **3**, 195, 1916.

the same period as the hydrogen and helium lines of the primary star. Somewhat similar results were found by Boothroyd * for the double-lined binary H.D. 218440, type B3k. In addition to a fairly strong K line of constant position, he found the presence of oscillating stellar K lines for both stars upon four spectrograms, these plates thus showing three components for the K line and giving indisputable evidence of the coexistence of stellar and interstellar calcium. This star has recently been observed by the writer,† who has determined the mass ratio of the two components. The three K lines appear on many of the spectrograms taken at the time of maximum separation of the two components.

The third case is that of the eclipsing variable U Ophiuchi, type B5nk, the orbit of which was investigated by Plaskett.‡ The spectrum contains double lines of the elements hydrogen, helium, carbon and calcium, typical of a normal B5 star. All the lines are diffuse and faint, lacking in contrast. The K lines oscillate in harmony with the other stellar lines. Recently Struve § has detected the presence of a weak interstellar line of stationary position between the oscillating calcium components. His observation has been confirmed by an inspection of our plates.

Four additional examples of this coexistence have been found by the writer. In the two double-lined B3k stars, H.D. 29376, vis. mag. 6·89, and H.D. 39698, vis. mag. 5·89, whose spectroscopic orbits and mass ratios have been determined, || faint stellar K lines were observed in addition to strong interstellar lines. On those spectrograms taken near the γ axes the measures of the K lines indicate a blended origin, but on plates taken near the times of maximum separation three components of the K line were observed. Similar results were found for two spectroscopic binaries previously investigated at this institution. A remeasurement of 7 spectrograms of H.D. 25833, type B3k, vis. mag. 6·61, orbit determined by Plaskett,¶ and of 9 spectrograms of H.D. 214240, type B3k, vis. mag. 6·20, orbit by Hill,** showed the presence of oscillating calcium lines for both components in each star, in addition to the interstellar lines.

It is thus evident from these seven examples that interstellar and stellar calcium coexist in the spectra of the Class B stars of types B2 to B5, and may be separately distinguished in those spectroscopic binaries of sufficiently large amplitude and whose distances are sufficiently great to indicate the interstellar absorption, *i.e.* exceeding 400 parsecs.

III. *Interstellar Calcium in Late-type Spectra*

The detection of an interstellar K line in the spectra of stars later than Class A is a difficult observational problem owing to the presence of the stellar calcium, which blends with and obliterates the absorption due to the intervening cosmic cloud. The great width and strength of the winged

* *Publ. D.A.O.*, **1**, 281, 1921.
† *Idid.*, **6**, No. 6, in press.
‡ *Ibid.*, **1**, 138, 1919.
§ *Ap. J.*, **6**, 199, 1930.
|| *Publ. D.A.O.*, **6**, No. 9, in press.
¶ *Ibid.*, **3**, 184, 1925.
** *Ibid.*, **3**, 358, 1926.

H and K lines in Classes F and G practically precludes the possibility of detecting a relatively faint interstellar K line in the spectra of any of these stars.

The only possibility of showing the coexistence of stellar and interstellar calcium for these classes would be to find a distant giant star with a radial velocity large enough to displace the stellar K line and reveal the narrow interstellar line projected upon the wing of the broad stellar line. This imposes very severe conditions upon the observer. The minimum distance at which an interstellar K line has sufficient intensity to be positively detected is, according to our recent work, some 400 parsecs. For the line to be seen upon the wing of the stellar line it would require an intensity corresponding to a distance of at least 1000 parsecs. A G0 giant of absolute magnitude zero at this distance would be of the 10th apparent magnitude, requiring a very long exposure for a dispersion sufficient to differentiate the two lines. Furthermore, this 10^m G0 giant must show a velocity shift, orbital or constant, of 250–300 km./sec., a necessary and essential condition, which so restricts the problem that the probability of finding a Class F or G star which exhibits the coexistence of stellar and interstellar calcium is very small indeed.

For stars of Class B8 to A5, however, the conditions are decidedly more promising, all factors uniting to favour the observation of the two K lines. The stellar K lines for these stars are usually narrow, so that a velocity displacement of only 100 km./sec. would suffice to resolve the two K lines completely, a velocity not uncommon among binary stars of this class. Since the interstellar line is outside the wing of the stellar line and seen against the background of the spectrum, a weaker intensity and hence a considerable less distance of the star would be required. Furthermore, the increased luminosity of these earlier type stars increases the apparent magnitudes, so that our search is among the seventh to ninth magnitude Classes B8 to A5 stars having radial velocities exceeding 100 km./sec. So stringent, however, are the conditions that only one spectrum later than type B5, that of H.D. 698, is known which shows the double calcium lines of different origin.

IV. *The Spectroscopic Orbit of H.D.* 698

This star in Cassiopeiæ, $\alpha = 0^h\ 06^m.3$, $\delta = +57^\circ\ 39'$ (1900), vis. mag. 7·08, spectral Class B9sek, was observed by Merrill * in his survey of Class B emission stars in 1920–24. He found the Hα line to be a strong emission line and classified the spectrum as B8seα. Four spectrograms in the ordinary blue-violet region were obtained, which gave a range of 69 km./sec. in radial velocity, and accordingly its binary nature was announced.

A total of 40 spectrograms have been obtained at Victoria during the past six years. The earlier plates were obtained with the short-focus camera, having a linear dispersion of 49 A. per mm. at Hγ, but the later observations, taken near the descending node to show the triple calcium lines, were obtained with the medium-focus camera, having a dispersion of 29 A. per mm.

* *Ap. J.*, **61**, 389, 1925.

The spectrum of the primary star is typical of Class B9 stars of high luminosity, containing, in addition to a strong helium spectrum, a well-developed metallic spectrum, usually associated with early Class A stars. The relative intensities of the λλ 4481/4471 lines assigns the star to subclass B9. All the lines are narrow and strong, of excellent quality for measurement. The helium and metallic lines undoubtedly arise in the same atmosphere as is shown by the orbital motions of the star. The abnormal association of a helium spectrum with a metallic one, combined with the "*c*" quality of the lines, and the emission character of $H\alpha$ and $H\beta$ all point to a star of high intrinsic luminosity for Class B9.

The secondary spectrum is faint relative to the primary, and only measureable at times of maximum separation of the lines. It is a normal B5 spectrum, as near as can be estimated, containing only the usual lines of helium I, hydrogen, magnesium II, carbon II, and calcium II of this subclass. It was measured on 24 spectrograms.

Portions of the last two spectrograms obtained are illustrated in Plate 16. In (*a*) the double lines of the *Mg* II/*He* I pairs, λλ 4481/4471, are shown. The camera used was the 1M, dispersion 34 A. per mm. at this region, with a slit width of 0·0015 inch, exposure 2 hours, emulsion Eastman 40. Spectrogram (*b*) was obtained with the 1L camera, dispersion 14 A. per mm. at the K region, slit width 0·002 inch, exposure 3 hours 22 minutes on a Barnet plate. The three calcium lines clearly exhibit the coëxistence of stellar and interstellar calcium in this interesting spectrum. The central sharp line is due to interstellar absorption, the strong violet component is the stellar K line of the primary star, while the faint redward component is the stellar K line of the secondary star. Compare this illustration with the radial velocity curves, fig. 2, on which stellar calcium velocities are shown as filled circles.

The orbital elements were determined from 30 observations of the primary star alone, owing to the relatively high probable error of the secondary observations.

Orbital Elements and Minimum Masses of H.D. 698

Period	$P=$	55·904 days.	
Eccentricity	$e=$	0·033	± 0·007.
Periastron passage	$T=$	J.D. 2425634·780.	
Longitude of apse	$\omega=$	313°·3	± 0°·41.
Semi-amplitude, primary	$K_1=$	85 ·5	± 0 ·46 km./sec.
Semi-amplitude, secondary	$K_2=$	215 ·5	± 3 ·0 km./sec.
Velocity of system	$\gamma=$	−24 ·5	± 0 ·4 km./sec.
Velocity of calcium Ca^+	=	−13 ·9	± 0 ·6 km./sec.
$a_1 \sin i$	=	65,700,000 km.	
$a_2 \sin i$	=	165,600,000 km.	
$m_1 \sin^3 i$	=	113·2⊙,	$\frac{m_2}{m_1}=0{\cdot}40$.
$m_2 \sin^3 i$	=	44·9⊙	

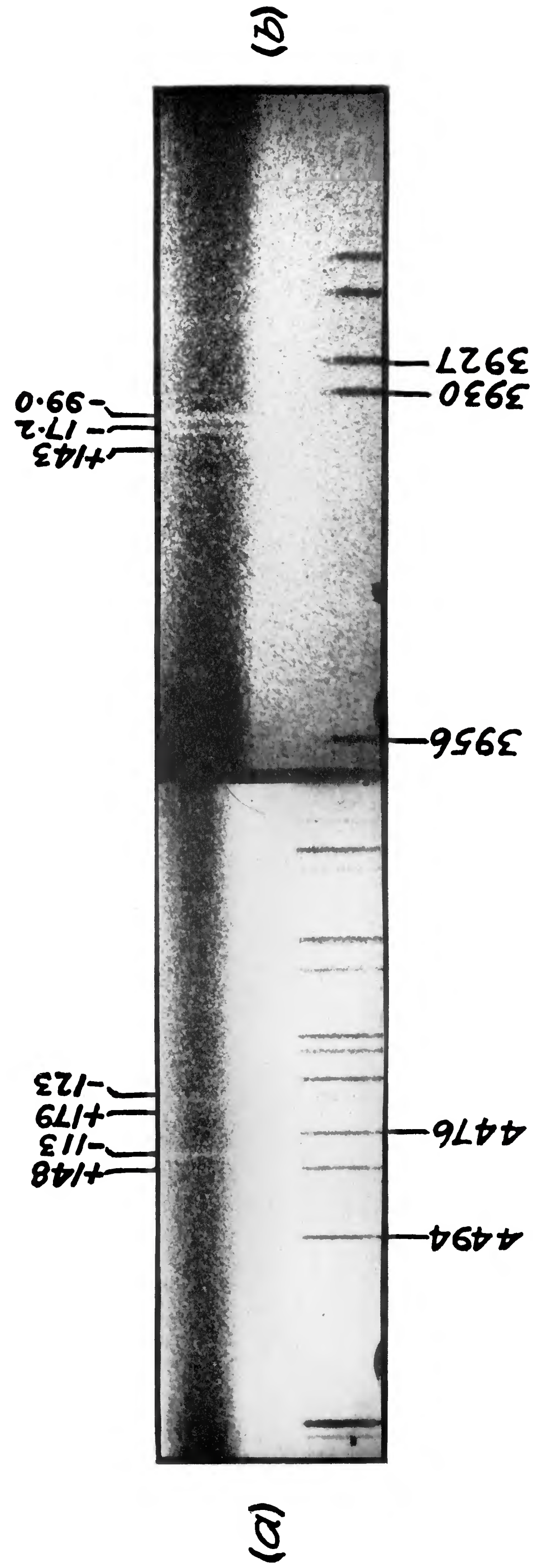

Fig. 1.—Spectra of H.D. 698.

(a) G.C.T. 1931 Oct. 10·288, Phase 39·640.
Obs. Vel. I. −104·2 km./sec.; II. +172·6 km./sec. Red. ⊙ = +4·5 km./sec.
(b) G.C.T. 1931 Oct. 11·370, Phase 40·722.
Obs. Vel. I. −95·6 km./sec.; II. +163·4 km./sec. Red. ⊙ = +4·0 km./sec.

The detailed investigation of this interesting system will appear in the Publications of the Dominion Astrophysical Observatory, and in this section it is only desirable to refer to the unusual masses of the stars. Of those

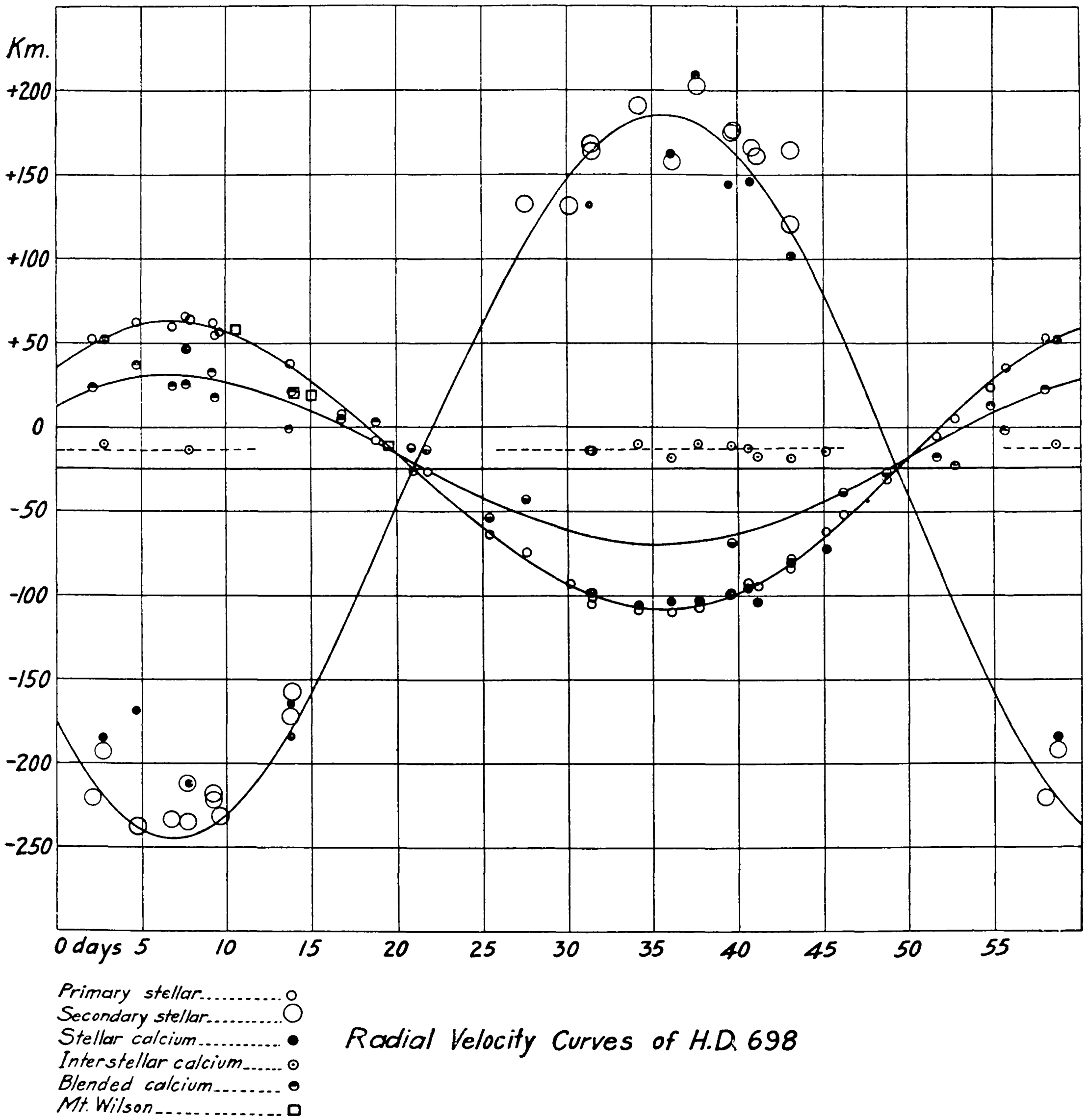

FIG. 2.

binary systems whose orbital systems have been definitely established, this remarkable system, H.D. 698, with a minimum mass, $M \sin^3 i = 158{\cdot}1\odot$, is the most massive thus far discovered.

Next in order are :

Star	Type	$M \sin^3 i$	Investigator
H.D. 47129	O8ek	138·9 ☉	J. S. Plaskett
H.D. 1337	O8nk	70·1 ☉	J. A. Pearce
H.D. 57060	O7sfk	56·5 ☉	J. A. Pearce

V. *The Calcium Lines in H.D.* 698

The most interesting feature of this system is the unique behaviour of the calcium lines. A single K line was measured on 18 of the 40 plates. An inspection of the velocity diagram, fig. 2, will show that from a phase of 46 days to 31 days, *i.e.* for 41 days out of the 56-day period, these measures follow a curve unmistakably indicating a blend of a stellar K line of the primary star with another line of constant velocity. The blended velocities vary from +38 km./sec. to −52 km./sec. about the constant axis of −13·9 km./sec. For a few days during the two-month period, when the stars are passing through the nodes and have maximum relative velocities, this blended line divides into three components as illustrated in fig. 1 (*b*). Eleven secondary calcium, twelve primary calcium and twelve interstellar calcium lines were observed on fifteen spectrograms taken at these favourable epochs. All the observations are shown on the velocity diagram, in which the origin of the various calcium components, whether blended, stellar or interstellar, are clearly indicated.

The twelve observations of the interstellar lines give a velocity of −13·9 ± 0·6 km./sec. The line is narrower than the primary stellar K and has an estimated intensity of 0·6 that of the stellar K line. This intensity ratio would indeed be expected from the relative displacements of the primary star and the blended curve about the interstellar calcium axis, the former being 1·7 times the latter, indicating the relative intensities of the stellar and interstellar lines.

The absolute magnitude of the star may be estimated from the intensity of the interstellar calcium line by a method recently employed by J. S. Plaskett and the writer in our investigation of the interstellar calcium lines in the O5 to B3 stars, the results being in press. The intensity of the line in H.D. 698 is 6·2 on our scale, corresponding to a parallax of 1220 parsecs. This leads to an absolute magnitude of −3·4 for the system, or −3·1 for the B9 component and −1·6 for the B5 component. The stars are outside the region considered by Eddington's Mass-Luminosity relation, and it does not seem justifiable to extrapolate the curve to find their theoretical absolute magnitude. It would be anticipated, however, that such large masses would have greater luminosities than we have found, and it would seem to be difficult to interpret their masses upon the luminosity theory.

Summary

The theory of a uniform distribution of diffuse matter in interstellar space as proposed by Eddington requires that the interstellar lines of calcium and sodium be superimposed upon the spectra of all stars more distant than 400 parsecs, irrespective of their spectral class.

The lines have been generally observed in the stars of spectral Classes O5 to B2 fainter than visual magnitude 5·5, in the spectra of Novæ and Wolf-Rayet stars, and in many of the distant stars of Classes B3 to B5, but have not previously been detected in spectra later than Class B5.