1931ApJ....74...43H

THE VELOCITY-DISTANCE RELATION AMONG EXTRA-GALACTIC NEBULAE[1]

By EDWIN HUBBLE AND MILTON L. HUMASON

ABSTRACT

Methods of determining distances of extra-galactic nebulae are discussed, and the mean absolute magnitude is revised on the basis of (1) Shapley's revision of the zero-point of the period-luminosity curve for Cepheids, and (2) more extensive observations of stars involved in nebulae. The revised value is M (vis) $= -14.9$.

The mean color-index of the nearer extra-galactic nebulae appears to be of the order of $+1.1$ mag., hence M(pg) $= -13.8$. A color-excess is suggested which is independent of distance but shows some relation to galactic latitude.

The velocity-distance relation is re-examined with the aid of 40 new velocities, 26 of which refer to nebulae in 8 clusters or groups. Distances of the clusters, ranging out to about 32 million parsecs, have been derived from the most frequent apparent magnitudes. The velocity displacements reduce the apparent magnitudes by amounts which become appreciable for the more distant clusters.

The new data extend out to about eighteen times the distance available in the first formulation of the velocity-distance relation, but the form of the relation remains unchanged except for the revision of the unit of distance. The relation is

$$\text{Vel.} = \frac{\text{Dist. (parsecs)}}{1790},$$

and the uncertainty is estimated to be of the order of 10 per cent.

PART I. DISTANCES OF NEBULAE

Distances of nebulae are derived from the application of absolute-magnitude criteria to stars involved in the nebulae. This is the only direct method available at present, and it plays the same rôle among the nebulae that micrometer measures play among the stars. All other methods are calibrated by means of the small sample collection of nebulae in which stars can be seen and studied. The assumption that this collection is a representative sample is a first approximation, supported in a general way by the consistency of the results to which it leads. The basic data will be materially increased only when larger telescopes and faster plates are available.

Absolute-magnitude criteria can be applied only when stars of familiar types can be identified. Such stars are by no means the brightest in the nebulae, hence the method is restricted to the very

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 427.

nearest of the neighboring systems. There is evidence, however, for a fairly definite upper limit to the absolute luminosity which stars attain, and this permits estimation of the distances of all nebulae in which any stars can be seen, even when particular types cannot be identified. Probable errors are considerable, but statistical results are reliable and are valuable in enlarging the sample collection of nebulae with distances derived from familiar criteria.

When no stars can be seen, a new criterion is required, necessarily calibrated by the data for the sample collection. This is furnished by the absolute luminosities of the nebulae themselves, which exhibit a restricted range about a well-defined most frequent value. The criterion is statistical, but when it is once calibrated, its application appears to be quite general.

DISTANCES DERIVED FROM STARS OF RECOGNIZED TYPES

Stellar types which have been identified in nebulae include novae, Cepheid variables, irregular variables, helium stars (B0 and O, sometimes involved in emission luminosity), and P Cygni stars. The Cepheids furnish the most reliable distances; the other types are important as confirming the general order of distance. Distances are expressed in terms of a unit defined by the zero-point of the period-luminosity relation among Cepheids; hence revisions of the zero-point change the value of the unit without affecting the relative distances of the nebulae. Shapley's[1] recent revision reduces the previously current value of the unit by about 11 per cent. The new value may be accepted as of the right general order, although further revision may be expected from the data on the motions of Cepheids which are now being accumulated.[2]

Including the Magellanic Clouds, there are 8 nebulae in which types of stars have been identified, and these, together with the 2 companions of M 31, give 10 nebulae with distances derived from the criteria of absolute magnitude. These are the most reliable distances available at present, but 3 of them, those for M 81, M 101,

[1] *Star Clusters*, p. 189, 1930. Revised distances are given for the Large Magellanic Cloud, 26,200 parsecs, $m-M=17.10$, and for the Small Magellanic Cloud, 29,000 parsecs, $m-M=17.32$.

[2] Gerasimovič (*Astronomical Journal*, 41, 17, 1931) suggests a correction of the order of 1 mag. on the basis of motions at present available.

and N.G.C. 2403, are less certain than the others.[1] The distances, total absolute visual magnitudes, and, for later use, absolute photographic magnitudes of the brightest stars involved in these 10 nebulae are listed in Table I. The distances and stellar magnitudes, the

TABLE I

DISTANCES OF EXTRA-GALACTIC NEBULAE

System	Distance in Parsecs*	M_n	M_s
LMC	0.290×10^5	−16.6	− 5.8
SMC	0.262	15.8	7.4
N.G.C. 6822	1.92	12.0	5.6
M 33	2.36	14.9	6.3
M 31	2.47	17.0	5.8
M 32	2.47	13.2	....
N.G.C. 205	2.47	12.7	....
M 101	4.0	13.1	6.0
N.G.C. 2403	(6.3)	15.3	6.0
M 81	(7.3)	−16.0	− 5.8
Mean M_s	....	....	− 6.1
$m_s - m_n$ from Table II	....	....	+ 8.9
Mean M_n	....	−14.7	−15.0
Adopted M (vis)	....	....	−14.9
CI	....	....	1.1
M(pg)	....	....	−13.8

* Distances are corrected for Shapley's revision (*Star Clusters*, p. 189, 1930) of the zero-point of the period-luminosity curve for Cepheids.

latter on the international scale, were determined at Mount Wilson, except for the Magellanic Clouds, for which Shapley's results are used. The total visual magnitudes are based upon Holetschek's measures as corrected by Hopmann,[2] except those for the Clouds,

[1] A study of the 20 plates of M 101 available at Mount Wilson indicates 3 probable novae with a mean observed maximum of 18.0± as compared with 17.1 in M 31; 2 short-period variables, probably Cepheids, with maxima about 19.0 as compared with 18.0 for the brightest Cepheids in M 31 and M 33; 3 apparently irregular variables with maxima between 18.0 and 18.5 as compared with maxima of irregular variables in M 31 and M 33 ranging from 15.3 to 18.9. These results suggest a modulus for M 101 about 1 mag. greater than for M 31 and M 33, corresponding to a distance about 60 per cent greater. The distances of M 81 and N.G.C. 2403 are suggested by a comparison of 7 irregular variables in the former and 5 in the latter with those in M 31 and M 33.

[2] *Astronomische Nachrichten*, **214**, 425, 1921.

estimated from various sources, and that for N.G.C. 6822, measured at Mount Wilson.[1]

The range in total absolute magnitude is about 5.0, with an average residual of 1.5 around the mean value of −14.7. The range in the magnitudes of the brightest stars is about 1.8, with an average residual of 0.4 around the mean value −6.1. The scattering among the brightest stars is considerably smaller than that among the nebulae themselves and would be almost negligible were it not for the outstanding case of the Large Magellanic Cloud.

UPPER LIMIT OF STELLAR LUMINOSITY AS A CRITERION OF DISTANCE

These facts lend color to the assumption of a fairly uniform upper limit to the luminosity of stars in the great isolated systems, which may be used as a criterion of distance where stars can be seen but no types can be identified. This assumption is reasonable in the light of current ideas on stellar constitution, and empirical investigations, beyond the range of Table I, indicate that it is fairly reliable, at least in a statistical sense. If the brightest stars have a uniform intrinsic luminosity, the scattering in the differences in apparent magnitude between the nebulae and their brightest stars should represent the scattering in the absolute magnitudes of the nebulae alone. In other words, let the subscripts n and s refer to the nebula as a whole and to the brightest stars, respectively; then if M_s is constant, the scattering in $m_s - m_n$ should represent that in M_n. Conversely, if M_s varies through a considerable range, the scatter in $m_s - m_n$ should be conspicuously greater than the scatter in M_n alone. Actually, the available data support the first of these alternatives. The scattering in the observed $m_s - m_n$ is fully accounted for by that in M_n, as indicated both by Table I and also by the several clusters of nebulae where, regardless of the distance, the scattering in M_n is represented by the scattering in m_n in each cluster.

[1] An additional system, I.C. 1613, is an irregular nebula similar to the Magellanic Clouds and N.G.C. 6822. The nature of this system was discovered by W. Baade at Hamburg, who communicated the information by letter. The nebula is very faint and highly resolved. Several variables, some with Cepheid characteristics, have already been found, but further investigation is necessary in order to settle the matter. Preliminary estimates indicate an upper limit for the stars of the order of −6 and hence are in this respect thoroughly consistent with the previous results.

The relevant data are listed in Table II and exhibited graphically in Figure 1. With one known exception,[1] resolution into stars is restricted to spirals of intermediate and late types, whether normal or barred, and to the irregular nebulae. Only these types were considered. Moreover, in order that the data should be fairly representative and homogeneous, the observations were restricted to nebulae of visual magnitude 11.5 or brighter on Holetschek's scale as corrected by Hopmann. There are about 140 such objects in the entire sky, of which 76 have been measured by Holetschek. Photographs

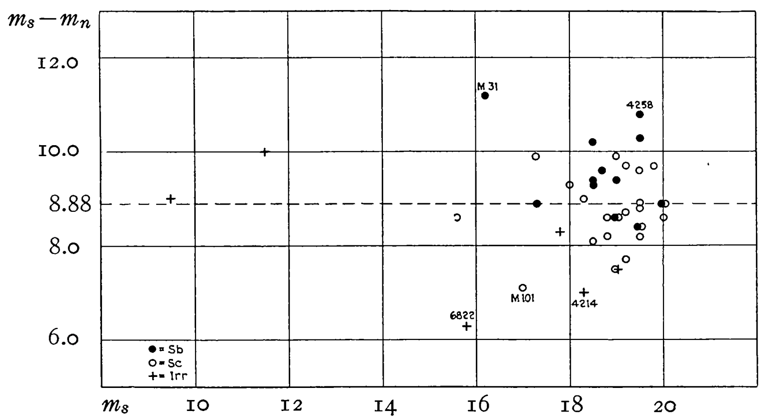

FIG. 1.—Independence of $m_s - m_n$ and m_s, where m_n is the apparent magnitude of a nebula and m_s that of the brightest stars in the nebula. Since the scatter in $m_s - m_n$ is fully accounted for by the known range in the absolute magnitudes of the nebulae, the range in the absolute magnitudes of the brightest stars is negligible, and m_s may be used as a criterion of relative distance.

with the large reflectors at Mount Wilson are available for 67 of these, which, together with N.G.C. 6822 and the Magellanic Clouds, represent about 50 per cent of the entire list. Except for the three objects just mentioned, the visual magnitudes of the nebulae, m_n, in Table II are those of Holetschek,[2] while the photographic magnitudes of the brightest stars, m_s, represent measures or estimates from the plates for the brightest four or five objects which appeared to be individual stars. Stars were found in 40 nebulae, or about 57 per

[1] M 87, an apparently globular nebula in the Virgo cluster. Stars cluster around the periphery, the brightest being about 19.5. A nova has been observed among them.

[2] Actually they are Holetschek's measures with Hopmann's scale corrections applied (see *Mt. Wilson Contr.*, No. 324; *Astrophysical Journal*, **64**, 321, 1926).

TABLE II

Apparent Magnitudes of Nebulae and Their Brightest Stars

N.G.C.	m_s	m_n	$m_s - m_n$
Type Sb			
224	16.2	5.0	11.2
1068	18.7	9.1	9.6
2841	>19.5	9.4	>10.1
3031	18.5	8.3	10.2
3310	>19.0	10.4	> 8.6
3489	>20.0	11.2	> 8.8
3623	>20.0	9.9	>10.1
3627	18.5	9.1	9.4
3628	>20.0	11.4	> 8.6
4030	19.5	11.1	8.4
4192	>19.5	10.9	> 8.6
4216	>19.5	10.8	> 8.7
4258	19.5	8.7	10.8
4438	>19.0	10.3	> 8.7
4450	>20.0	10.6	> 9.4
4565	>19.5	11.0	> 8.5
4736	17.3	8.4	8.9
4826	18.5	9.2	9.3
5055	19.0	9.6	9.4
5746	>19.5	10.4	> 9.1
7331	19.0	10.4	8.6
Mean (10)	....	....	9.58
Type SBb			
3351	>19.5	11.4	> 8.1
3414	>19.5	11.5	> 8.0
3504	>20.0	11.4	> 8.6
4245	>20.0	11.1	> 8.9
4314	>20.0	11.1	> 8.9
4394	>19.0	11.5	> 7.5
4548	20.0	11.1	8.9
4699	>19.5	10.0	> 9.5
4725	19.5	9.2	10.3
5566	>19.0	11.1	> 7.9
Mean (2)	....	....	9.6
Type Sc			
253	18.3	9.3	9.0
598	15.6	7.0	8.6
628	18.8	10.6	8.2
1084	20.0	11.4	8.6
2403	18.0	8.7	9.3
2683	>20.0	9.9	>10.1
2903	19.0	9.1	9.9
3147	>19.0	11.4	> 7.6
3521	19.8	10.1	9.7
3810	19.5	11.3	8.2

TABLE II—*Continued*

N.G.C.	m_s	m_n	m_s-m_n
Type Sc—*Continued*			
4088	19.0	11.5	7.5
4254	19.0	10.4	8.6
4321	19.2	10.5	8.7
4414	>19.5	10.1	> 9.4
4490	18.8	10.2	8.6
4501	>19.0	10.5	> 8.5
4559	19.5	10.7	8.8
4569	>19.5	10.9	> 8.6
4605	19.5	9.9	9.6
4631	19.2	9.5	9.7
5005	19.5	11.1	8.4
5194	17.3	7.4	9.9
5236	18.5	10.4	8.1
5248	19.2	11.5	7.7
5457	17.0	9.9	7.1
6503	>19.0	9.9	>9.1
Mean (20)			8.71
Type SBc			
613	>19.5	10.6	> 8.9
3953	20.0	11.1	8.9
4303	19.5	10.6	8.9
4579	>19.5	9.7	> 9.8
Mean (2)			8.90
Type Irr			
LMC	9.5	0.5	9.0
SMC	11.5	1.5	10.0
3034	>20.0	9.0	>11.0
4214	18.3	11.3	7.0
4449	17.8	9.5	8.3
4656	19.0	11.5	7.5
4753	>20.0	11.4	> 8.6
5363	>20.0	11.1	> 8.9
6822	15.8	9.5	6.3
Mean (6)			8.02

MEANS

Type	No.	m_s-m_n
Sb	12	9.58
Sc	22	8.73
Irr	6	8.02
Total	40	8.88

cent of the number observed. When no stars could be detected, the estimated limit of the photograph is given with a suitable sign indicating that stars, if present, are fainter than the given m_s. In general, these limits are above the extreme limits of the plates on account of the difficulties of detecting stars on a luminous background.

The range in $m_s - m_n$ is 4.9 mag., and the average residual is 0.77 around the mean value 8.88. The range corresponds with that of M_n in Table I and in the various clusters, about 5.0, or slightly more. The average residual is considerably less than that for the nebulae in Table I, but is closely comparable with the mean residuals found in the clusters, about 0.8 mag. The distribution of $m_s - m_n$ is fairly symmetrical about the mean, and forms a curve similar to the distribution of apparent magnitudes in clusters of nebulae. Table I indicates no conspicuous relation between M_n and M_s, and Figure 1 no relation[1] between $m_s - m_n$ and m_s. The extreme values of $m_s - m_n$, 11.2 for M 31 and 6.3 for N.G.C. 6822, refer to systems whose distances are well known and are obviously explained by the extreme values of M_n. These considerations indicate that the range in M_s, if not negligible, is at least much less than the range in M_n, and hence justify the use of M_s as a criterion of distance. The limiting magnitudes, where no stars were detected, are thoroughly consistent with this interpretation.

On account of the restricted data in Table I, the evaluation of M_s is less certain. The simple mean, -6.1, together with the mean $m_s - m_n$, 8.9, in Table II gives a mean $M_n = -15.0$ as compared with -14.7 for the 10 nebulae in Table I. The brighter value may be given double weight because of the small range in M_s and the relatively extensive data for $m_s - m_n$. For this reason the mean absolute visual magnitude of extra-galactic nebulae is adopted as

$$M_n \text{ (vis)} = -14.9 \, .$$

[1] The differences $m_s - m_n$ show a conspicuous correlation with the nebular magnitudes, m_n, fainter than about 9.5, in the sense that the differences decrease as the nebular magnitudes increase. This is an effect of selection due to the considerable range in the absolute magnitudes of the nebulae. Stars can be detected with reasonable certainty down to about 19.5, corresponding to a normal nebula of magnitude about 10.6. Among nebulae which appear fainter, stars can be seen only in those which are intrinsically faint and hence nearer than the average, in which case the differences $m_s - m_n$ are necessarily small.

Hence,

$$M_s\ (\mathrm{pg}) = -\ 6.0\ .$$

The corresponding formula for the distance of a nebula, as indicated by the brightest stars involved, is

$$\log D = 0.2 m_s + 2.2\ ,$$

where the distance, D, is expressed in parsecs. The uncertainty in D is probably of the order of 20 per cent.

One further point should be mentioned in connection with Table II. The nebulae are listed according to type, and the mean differences $m_s - m_n$ appear to vary systematically with the type, the means being 9.6, 8.7, and 8.0 for *Sb*, *Sc*, and Irr., respectively. The data are very limited, and the effect can be attributed largely to 3 *Sb* nebulae, M 31, N.G.C. 4258, and N.G.C. 4725, and to 3 Irr. nebulae, N.G.C. 4214, 4656, and 6822. The former are known or suspected to be unusually bright, and the latter unusually faint. Their elimination would remove most of the systematic effect, but the procedure would be rather arbitrary. Further investigation, including nebulae in the Virgo cluster, supports the systematic variation, but reduces the amount very considerably. The subject will be discussed in a later paper. The evidence indicates both a slight decrease in the average luminosity of nebulae and a slight increase in the luminosity of the brightest stars along the sequence of classification. These, however, are second-order effects and can be ignored in preliminary statistical investigations.

The use of the brightest stars as a criterion of distance has been criticized on the ground that clusters and groups at such remote distances would not be distinguished from single stars. The criticism appears reasonable, but in actual practice there seems to be little or no confusion. The data for the nearer nebulae behave in the same manner as those for the more distant, and the frequency distribution of the differences $m_s - m_n$ is fairly consistent with available information concerning the luminosity function of the nebulae themselves. It is possible that this homogeneity is accidental, but its value for empirical investigations is nevertheless considerable.

TOTAL LUMINOSITIES OF NEBULAE AS A CRITERION OF DISTANCE

For nebulae in which no stars can be seen, the only criteria of distance are the apparent dimensions and magnitudes of the nebulae themselves. The dimensions have only a limited application. They vary systematically throughout the sequence of classification; hence it is necessary that the nebulae be photographed on a scale sufficient to show the detailed structure on which the classification is based. Moreover, since the luminosity in general fades away from the nuclei, dimensions increase systematically with effective exposure and at rates varying, among other things, with type and nuclear luminosity. This applies also to the total luminosities of nebulae—increasing exposure brings new regions above the threshold of the plates—but since the luminosities are largely concentrated in the nuclear regions, the percentage effect is much less serious than in the case of dimensions. It frequently happens that moderate exposures on faint nebulae register diameters only one-third those which full exposures reveal, while two-thirds of the total luminosities are registered. The resulting effects on distances as derived from apparent diameters and from luminosities are about 300 and 20 per cent, respectively. Such considerations indicate that, while the diameter may be used to advantage in special cases,[1] the total apparent magnitude offers much the more reliable criterion of distance. Luminosity, moreover, varies only slightly along the sequence of classification, and, as previously remarked, this second-order effect can be ignored in general statistical investigations. The greatest uncertainty at present arises from the lack of a well-established system of nebular photometry, free from serious systematic errors, over the observed range in apparent magnitude.

VISUAL MAGNITUDES

The range in the absolute luminosities of nebulae, from Tables I and II and from the clusters, appears to be of the general order of 5 mag. with an average residual of about 0.8 mag. around a mean value provisionally adopted as -14.9 on Holetschek's visual scale.

[1] Provisional mean values for the dimensions of nebulae along the sequence of classification are given in *Mt. Wilson Contr.*, No. 324; *Astrophysical Journal*, **64**, 321, 1926. The revised value of M_n reduces the tabulated values by about 11.5 per cent.

In view of his long experience in the visual photometry of comets, Holetschek's measures form the most complete and homogeneous list of nebular magnitudes available at the present time. The scale as revised by Hopmann[1] appears to be reasonably free from serious systematic errors as far as they have been checked, that is, to about 12.0. To the same limit the list of nebulae is reasonably complete or representative. Accidental errors naturally occur, but for statistical investigations these are of slight importance.

PHOTOGRAPHIC MAGNITUDES

Photographic photometry of focal nebular images is unsatisfactory for two reasons. Luminosity determined from such images varies with the effective exposure at a rate depending on several factors, and the photometry of surfaces differs widely from that of point-source images. Approximate estimates may be derived from simple comparisons of focal images when these are comparable in size with the images of stars, but the procedure involves a rather obscure subject, namely, the manner in which the photometry of surfaces merges into that of point-source images. As far as the writers know, this field has never been investigated in detail. The complications are avoided by using extra-focal images larger than the focal dimensions of the nebulae. These images are closely comparable with images of the comparison stars, and the resulting luminosities are then independent of exposure. The method has been used for the Mount Wilson magnitudes, both photographic and photovisual, referred to in the following pages. The magnitudes are on the international scale and were derived with the 10-inch Cooke astrographic camera (f/4.5) and with the large reflectors. With the 60-inch reflector, the Ross zero-power correcting lens, which eliminates coma over a field about 1° in diameter, proved of very great value, especially for the brighter nebulae where suitable comparison stars are rather widely scattered.

Among the very few published magnitudes of nebulae derived from photographic plates are the two lists from Harvard. One[2] con-

[1] Hopmann measured the magnitudes of the comparison stars for which Holetschek had used the *B.D.* magnitudes.

[2] *Proceedings of the National Academy of Sciences*, **15**, 565, 1929.

sists of 47 nebulae for which radial velocities were then available; the second[1] consists of nearly 2800 nebulae in the region of the Virgo cluster, repeating magnitudes of 103 of the brighter objects previously published[2] in *Harvard Circular* No. 294 with a scale revision for a few fainter than 13.0. Both lists purport to give photographic magnitudes on the international scale as derived from direct comparisons between focal images of nebulae and of stars. Except for the fainter nebulae in the Virgo cluster, small-scale plates were used. Five of the Virgo nebulae are common to the two lists, but a systematic difference averaging about 1 mag. indicates some unexplained factor, and the lists must be examined separately.

Of the shorter list, 15 objects have been measured at Mount Wilson on extra-focal plates with a thermocouple photometer. Comparison stars were standardized by reference to the North Polar Sequence and to Selected Areas. The results are given in Table III. The differences $MW-H$ range through some 3 mag. and average about +0.5, which probably accounts for the mean color-index of +0.23 derived by comparing Harvard photographic with Holetschek's visual magnitudes. The results indicate the present unsatisfactory state of photographic nebular photometry and the necessity for standard conventions. The writers believe that the latter must be based on extra-focal methods.

The list of magnitudes in the Virgo cluster appears to be much more homogeneous, at least for the brighter nebulae. Extra-focal magnitudes, both photographic and photovisual, have been measured

[1] *Harvard Annals*, **88**, 1930.

[2] Wirtz in *Publicationen der Sternwarte in Kiel*, No. 15, p. 35, 1927, gives magnitudes of 98 nebulae in the Virgo cluster measured with an electrophotometer from small-scale, focal plates. He derived his zero-point and scale from the Harvard nebular magnitudes in *Circular* No. 294 and hence his results merely confirm the general similarity of the focal images as observed with the eye and with the photometer. The method of calibration accounts for the absence of systematic differences in Wirtz's results from short and long exposures. Baade (*Mitteilungen der Hamburger Sternwarte in Bergedorf*, **6**, 98 [No. 29], 1928) gives photographic magnitudes of 25 nebulae in the Ursa Major cluster, ranging from 16.0 to 18.0, measured on plates with the 1-m. f/3 reflector. The method is not stated, but, since the nebulae are very small and faint, the conditions were suitable for focal comparisons. The Mount Wilson extra-focal measures agree with Baade's results from 16.0 to 17.3, beyond which the former are somewhat brighter There is no marked improvement over Baade's method, whatever that may have been. No other lists of photographic magnitudes are known to the writers.

for about 60 nebulae in the central region of the cluster. The differences $MW-H$ vary systematically, but not linearly, through a range of about 0.6 mag. from 10.5 to 14.0, beyond which the extra-focal measures are not yet sufficiently numerous for comparison. Twenty of the brighter nebulae are in Holetschek's list and furnish the following means:

$$MW(\mathrm{pg})-Harv(\mathrm{pg})=+0.15 \text{ (ave. res., 0.2)},$$
$$MW(\mathrm{pv})-Hol\ (\mathrm{vis})=-0.07 \text{ (ave. res., 0.35)}.$$

TABLE III

MAGNITUDES OF BRIGHT NEBULAE

N.G.C.	MW_{pg}	S_{pg}	H_{vis}	$MW-S$	$MW-H$
205	11.0	11.2	9.3	−0.2	+1.7
221	9.6	8.7	8.8	+0.9	+0.8
278	11.8	11.7	12.0	0.1	−0.2
404	12.0	11.6	11.1	0.4	+0.9
584	11.8	11.8	10.9	0.0	0.9
936	11.6	11.2	11.1	0.4	0.5
1023	11.4	11.2	10.2	0.2	1.2
1068	9.85	9.8	9.1	0.0	0.7
1700	12.7	12.1	12.5	0.6	0.2
4382	10.9	9.7	10.0	1.2	0.9
4472	10.1	9.1	8.8	1.0	1.3
4486	10.8	9.2	9.7	1.6	1.1
4526	11.3	10.3	11.1	1.0	0.2
4649	10.8	9.8	9.5	+1.0	1.3
7619	13.1	14.6	11.8	−1.5	+1.3
Means				+0.45	+0.85

MW = Mount Wilson extra-focal measures.
S = Shapley, *Proceedings of the National Academy of Sciences*, **15**, 569, 1929.
H = Holetschek.

COLOR-INDICES

The mean color-index from extra-focal measures of the 60 nebulae in the Virgo cluster is $+1.10\pm0.02$. This must be compared with $+0.9\pm0.12$ from a comparison of MW(pg) with Holetschek's visual magnitudes for 10 nebulae in Table III, and with values ranging from $+1.05$ to $+1.2$ for the brighter members of four clusters of fainter nebulae derived from extra-focal measures at Mount Wilson, and finally with the value of $+1.3$ for Wolf's cluster in Perseus (gal. lat. -13°). These measures indicate a mean color-index of the order of $+1.1$ for nebulae in general, and hence a mean absolute photographic magnitude of the order of -13.8.

The spectral types exhibit a narrow range about the mean G3, and large-scale spectra of M 31 and M 32 indicate definite dwarf characteristics. Thus the color-indices suggest a color-excess[1] of the order of 0.3 mag. Since the effect exhibits no conspicuous dependence on distance and the largest value is in the lowest galactic latitude, the source is possibly within the galactic system itself. This conclusion must be accepted with appropriate reservations, however, for although there is no evidence whatever for absorption, either selective or general, in extra-galactic space, there still remains the possibility of undetected systematic effects in the measures or even unrecognized characteristics in the nebulae themselves. The apparent velocity displacements, for the nearer nebulae at least, can account for only a small fraction of the observed color-excess.[2] Further investigations are under way which should either correct the measures or throw light on the contents of interstellar space. Meanwhile, provisional values of the mean absolute magnitudes of nebulae, -14.9 pv and -13.8 pg, may be used for distance criteria provided the apparent magnitudes are based upon extra-focal exposures.

The luminosity criterion is purely statistical and is reliable only when large numbers of nebulae are available. One direct application concerns the great clusters of nebulae. The mean or most frequent apparent magnitude of the many members is a good indication of the distance of a cluster, and hence clusters offer the greatest distances that can definitely be assigned to individual objects. If observations of very remote objects are desired, the brightest members of very faint clusters may be selected. This procedure assumes that cluster nebulae are comparable with isolated nebulae, but evidence from several sources indicates that the assumption is well founded.

[1] The color-excess is in general agreement with Humason's observation that the distribution of intensities in the continuous spectra corresponds to spectral types considerably later than those indicated by the absorption lines.

[2] This effect is discussed in a later section of the present paper.

PART II. VELOCITY-DISTANCE RELATION

FIRST FORMULATION

The criteria of distance, approximately in the form described, have been known for several years. With their aid a relation between radial velocity and distance was established among the nebulae for which velocities were known. The relation is a linear increase in the velocity amounting to about +500 km/sec. per million parsecs of distance. This result was published[1] two years ago, together with the announcement that a program was under way having for its purpose the testing of the relation over as great a range in distance as could be covered with existing equipment. The first result of the new program, a velocity for N.G.C. 7619 in the Pegasus cluster which was consistent with the relation already established, was mentioned in the paper.[2]

NEW DATA

About forty new velocities are now available, representing in general very faint and hence distant nebulae, and the velocity-distance relation may be examined in the light of the new material. The spectrograms were photographed by Humason and measured by Humason and Miss MacCormack, with the exception of nine which were obtained by Pease and very kindly placed at our disposal for measurement and discussion. These spectra have been discussed separately,[3] hence only the displacements, expressed as velocities, will be considered here.

Velocities previously available, owing very largely to the great

[1] Hubble, *Mt. Wilson Communication*, No. 105; *Proceedings of the National Academy of Sciences*, **15**, 168, 1929. The revision of M_n increases the coefficient to about 550 km/sec. per million parsecs.

[2] Professor de Sitter (*Bulletin of the Astronomical Institutes of the Netherlands*, **5**, 157 [No. 185], 1930) published a redetermination of the velocity-distance relation in which he arrived at the same numerical result. This was to be expected since he used essentially the same data, together with a few new velocities published from Mount Wilson, some accompanied by estimates of the distances in order to emphasize the consistency of the results from the new program. There are certain criticisms of the treatment of the data (for instance, the use of the brightest members of clusters as nebulae of normal luminosity, etc.), but these are of minor importance. Differences tend to cancel out, and the final numerical result agrees with the Mount Wilson result.

[3] Humason, *Mt. Wilson Contr.*, No. 426; *Astrophysical Journal*, **74**, 35, 1931.

pioneer work of V. M. Slipher at the Lowell Observatory, although naturally restricted to the brighter, nearer nebulae, included five members of the Virgo cluster. This set the limit to the observed range in distance used for the first formulation of the velocity-distance relation. The new program contemplated the use of much greater distances, far beyond the limits at which stars can be detected, and hence included the several brightest members of as many clusters or groups as possible. Isolated nebulae were also included to fill gaps in the observing seasons and to furnish a few mean points that might test the similarity of isolated and cluster nebulae.

The new data are listed in Table IV. Observed displacements, expressed as velocities, are reprinted from Humason's paper.[1] Holetschek's visual magnitudes are available for 6 isolated nebulae; photographic magnitudes from extra-focal measures are given for 10 isolated nebulae; while for 8 clusters or groups the most frequent magnitude is listed. The magnitude for the Virgo cluster is assumed to be 12.5. Corrections for the solar motion were computed from data given in *Mt. Wilson Communication* No. 105,[2] and the corrected apparent velocities of the nebulae were rounded off to the nearest 50 km/sec. There is no advantage in recomputing the solar motion with the new material, for the uncertainties in the distances, although relatively moderate, are numerically so large as to mask the effect which is sought.

ISOLATED NEBULAE

The isolated nebulae have been combined into two groups: the 6 nebulae having Holetschek magnitudes and the 10 having photographic magnitudes (including N.G.C. 4865 for reasons which will be discussed later). These give the mean values $\log v = 3.109$ for $m = 11.75$ vis, or $+1280$ km/sec. for 2.15 million parsecs; and $\log v = 3.534$ for $m = 14.3$ pg, or $+3420$ km/sec. for 4.2 million parsecs, respectively. The first is in fair agreement with the results for the

[1] Three nebulae in Humason's list are omitted: N.G.C. 205, a companion to M 31; N.G.C. 6822; and M 101. The last two were included in the first presentation of the velocity-distance relation, the first being there omitted only by an oversight. Its inclusion with M 31 and M 32 does not affect the results.

[2] *Proceedings of the National Academy of Sciences*, **15**, 168, 1929.

TABLE IV

VELOCITIES AND MAGNITUDES

	v	v_s	v_0	m_{vis}*	m_{pg}	Type	Diam.
Virgo cluster:	km/sec.						
N.G.C. 4192...	+ 1150	+ 20					
4374...	1050	30					
4382...	500	40					
4472...	850	20	+ 890	......	(12.5)	......	......
4486...	800	30					
4526...	580	20					
4649...	1090	+ 40					
Pegasus:							
N.G.C. 7611...	3400						
7617...	3900						
7619...	3800	+ 90	3810	......	15.5	......	......
7623...	3800						
7626...	3700						
Pisces:							
N.G.C. 380...	4400						
383...	4500	+ 60	4630	......	15.4	......	......
384...	4500						
385...	4900						
Cancer:							
N.G.C. 2562...	5100	−130	4820	......	16.0	......	......
2563...	4800						
Perseus:							
N.G.C. 1270...	4800						
1273...	5800	0	5230	......	16.4	......	......
1275...	5100						
1277...	5200						
Coma:							
N.G.C. 4853...	7600						
4860...	7900	+100	7500	......	17.0	......	......
4865†..	5000						
4884...	6700						
Ursa Majoris:							
Baade 24......	11700	+100	11800	......	18.0	......	......
Leo:							
No. 1.........	19700	− 90	19600	......	19.0	......	......
Isolated nebulae:							
N.G.C. 2859...	1500	− 50	1450	11.1	......	SBa	1.9
2950...	1500	+ 60	1560	11.6	......	SBa	1.4
3193...	1300	− 60	1240	12.1	......	E2	1.0
3227...	1150	− 60	1090	12.0	......	Sb	3.0
3610...	1850	+100	1950	11.8	......	E4	1.4
4051...	650	+ 90	740	11.9	......	Sb	4.0
6359...	3000	+250	3250	......	14.3	E2	0.4
6658...	4100	+270	4370	......	14.8	Sa	1.6
6661...	3900	+270	4170	......	14.0	Sa	1.6
6702...	2250	+280	2530	......	14.6	E2	0.6
6703...	2000	+280	2280	......	13.6	E0	0.9
6710...	5100	+280	5380	......	15.0	Sa	0.8
6824...	3200	+240	3440	......	14.0	Sb	1.6
7217...	1050	+200	1250	......	12.3	Sb	3.0
7242...	+ 5000	+200	+ 5200	......	15.5	E2	0.3

* Holetschek visual magnitudes.

† Possibly an isolated nebula superposed on the cluster. $m_{pg}=14.7\pm$. The type is E4; the diameter, 0.′13.

clusters, but the second is discrepant by about 1000 km/sec. or 1 mag. The logarithms of the velocities have been used in deriving the mean values because magnitudes are logarithmic functions of distances.

CLUSTERS OF NEBULAE

Virgo cluster (R.A. $12^h25^m\pm$, Dec. $+12^\circ.5\pm$, 1930; gal. long. 256°, lat. +73°).[1]—The cluster includes several hundred members scattered over an elliptical area about $12^\circ\times10^\circ$. It is the largest and nearest of the known clusters and includes 16 out of the 34 extra-galactic objects in Messier's list. Nebulae of all types except the irregular are represented among its members, but elliptical nebulae and early spirals are relatively much more numerous than among the nebulae at large. The predominance of early types is a conspicuous feature of clusters in general, and the Virgo cluster is exceptional in the considerable number of late-type spirals which are included.

This cluster has been investigated at the Harvard College Observatory, where the 24-inch Bruce camera is especially suited to the problem. The system of magnitudes there established appears to deviate systematically from the Mount Wilson extra-focal magnitudes, but between 12.0 and 12.5 the two systems are nearly the same. For this reason 12.5, adopted at Harvard as the most frequent photographic magnitude, is used in the present discussion. A study of the cluster has been under way with the large reflectors at Mount Wilson, but the angular extent is so great that the data are not yet complete. The distance provisionally adopted is 1.8 million parsecs. This result, which differs widely from Shapley's estimate of 10 million light-years,[2] is derived from the following sources:

a) The most frequent apparent magnitude, 12.5, combined with the adopted mean absolute magnitude, −13.8, gives 1.8 million parsecs.

b) Stars with magnitudes as indicated have been found in the following nebulae:

[1] H. Shapley and Adelaide Ames have recently reported an extension of the Virgo cluster, stretching many degrees to the south (*Harvard Bulletin*, No. 880, 1930).

[2] Or about 9 million light-years on the revised scale.

N.G.C. 4254	Sc	18.8	N.G.C. 4303	Sbc	19.5
4321	Sc	19.0	4567	Sc	20.0
4535	Sc	19.3	4568	Sc	20.0
4294	Sc	19.5	4548	SBb	20.0
4298	Sc	19.5	4486*	E0	19.5
4713	Sc	19.5			

*N.G.C. 4486 is the only known example of an elliptical nebula in which stars can be detected.

Since other late-type nebulae in the cluster show no stars as bright as 20.0, the mean value for the upper limit is probably not brighter than 20.0 and, in view of the restricted range in the upper limit suggested by Tables I and II, is probably not much fainter than 20.0. The adopted $M_s = -6.0$, combined with apparent limits of 20.0 and 20.5 for stars in the cluster, gives distances of 1.6 million and 2.0 million parsecs, respectively. It is probable that the actual distance lies between these limits. From these considerations it is concluded that the stars involved indicate a distance of the order of 1.8 million parsecs.

c) The Virgo cluster offers a special case in which apparent diameters can be used as a criterion of distance with some degree of reliability. The analysis of the cluster is still incomplete, but the mean diameters may be approximated from the known values for the largest nebulae, together with probable values for the smallest as estimated by inspection of the many plates available. This procedure leads to Table V, in which the diameters in parsecs are from a previous investigation,[1] but have been reduced by 11.5 per cent to agree with the revised value for M_n.

The uncertainties in the method are obvious and the precise value of the result has little weight; yet the order of the distance is probably reliable and agrees with the previous indications. When the complete analysis of the cluster with the large reflectors is available, the mean apparent diameters will be revised in the light of the frequency distributions of the diameters and a closer approximation will be made.

The seven velocities now available for the Virgo cluster exhibit a total range of 550 km/sec. about the mean value +890 km/sec.

The Pegasus cluster (R.A. 23^h17^m, Dec. $+7°50'$, 1930; gal. long.

[1] *Mt. Wilson Contr.*, No. 324; *Astrophysical Journal*, **64**, 321, 1926.

55°, lat. −49°).—This cluster consists of about 100 nebulae scattered over an area roughly 1° in diameter about the two brightest objects, N.G.C. 7619 and 7626. Of these, the former is the brighter, with a total photographic magnitude of 13.1. From this upper limit the magnitudes range down to about 18.0, with the most frequent value at 15.5. Since the faint end of the frequency-curve is rather uncertain, the peak, i.e., the most frequent luminosity, is used here, as in all the other clusters, as the criterion of relative distance. Extra-focal measures of the brighter nebulae obtained with the 10-inch

TABLE V

DIAMETERS AND DISTANCES

TYPE	DIAMETERS			DISTANCE
	Largest	Smallest	Mean	
E0–E2	2'.0	0'.1	1'.05 = 380 parsecs	1.3×10^6 parsecs
E3–E5	3.0	0.1	1.55 = 620	1.4
E6–E7	2.4	0.2	1.3 = 1000	2.7
Mean E	....	....	....	1.8×10^6
Sa	5.0	0.2	2.6 = 1280	1.7
Sb	8.0	0.3	4.15 = 1680	1.4
Sc	6.1	0.4	3.25 = 2210	2.4
Mean S	....	....	....	1.8×10^6
Mean E and S	....	....	....	1.8×10^6

camera and the 60-inch reflector are in good agreement, and color-indices derived from photovisual magnitudes are available for N.G.C. 7619 and 7626. These are 1.1 and 1.0, respectively, but are subject to some uncertainty. The distance, as indicated by the most frequent apparent magnitude, is 7.25 million parsecs. Velocities for five nebulae show a total range of 500 km/sec. around the mean value +3800 km/sec.

The Pisces group (R.A. 1^h35^m, Dec. +32°, 1930; gal. long. 96°, lat. −30°).—This is not a cluster, but a group of some 25 elliptical nebulae which stands out from the approximately uniform background of field nebulae. The brighter ones are catalogued as N.G.C. 379, 380, 382, 383, 384, and 385. The luminosities, as indicated by the hump on the otherwise smooth curve expressing the frequency distribution over a field about 1° in diameter, range through some 3 mags. and are rather unsymmetrical about the most frequent magnitude, 15.4. This corresponds to a distance of 7 million parsecs. The small number of objects introduces a corresponding uncertainty into this result and possibly accounts for the restricted range in luminosity. The four velocities available range through 500 km/sec., with a mean around +4630 km/sec.

The Cancer cluster[1] (R.A. $8^h16^m.5$, Dec. +21°20′, 1930; gal. long. 170°, lat. +30°).—This consists of about 150 nebulae distributed over an area of nearly 1 square degree centered near N.G.C. 2562. The cluster has not been analyzed in so detailed a manner as the others, but the range is normal and the most frequent magnitude is about 16.0, a value which is definitely established to the nearest half-magnitude. The corresponding distance is 9 million parsecs. No color-indices have been measured. Velocities of 2 nebulae are available, averaging about +4800 km/sec.

The Perseus cluster (R.A. 3^h15^m, Dec. +41°15′, 1930; gal. long. 118°, lat. −13°).—A cluster of about 500 nebulae scattered over an area nearly 2° in diameter, discovered by M. Wolf[2] in 1905. The center is near N.G.C. 1275, the brightest nebula in the cluster, with a photographic magnitude about 13.8. The nebulae are elliptical for the most part, but include an occasional early-type spiral, either normal or barred. Magnitudes range from 13.8 to about 19.0, with a most frequent magnitude rather sharply defined at 16.4, thus indicating a distance of 11 million parsecs.

Internally consistent color-indices for 16 nebulae have been derived from extra-focal measures, both photographic and photovisual, on two pairs of plates made with the 60-inch reflector. Comparison stars were standardized by means of Selected Areas and the North Polar Sequence. For 15 nebulae the color-indices range from 1.2 to 1.4, with a mean of about 1.3. The photographic magnitudes extend from 14.6 to 16.7 and appear to be independent of the colors. The

[1] The Cancer cluster was discovered independently by Dr. Edwin F. Carpenter, of the Steward Observatory, Tucson, Arizona, who communicated the information by letter and later presented a paper on the cluster at the Pasadena meeting of the A.A.A.S. in June, 1931. The cluster was first photographed at Mount Wilson in January, 1929.

[2] *Astronomische Nachrichten*, **170**, 211, 1905; *Veröffentlichungen der Sternwarte zu Heidelberg*, **6**, 131, 1913.

remaining nebula, N.G.C. 1275, has a color-index of about 0.75, a discrepancy which was probably explained when a strong emission spectrum was registered from the bright stellar nucleus.[1] The unusually large color-indices of the other nebulae appear to be confirmed by an exposure-ratio plate made by the method developed by Seares. The measures suggest excessive colors for all the 60 nebulae recorded, but precise quantitative interpretation requires a careful calibration which has not yet been effected. An abnormally high color-excess appears, however, to be definitely established. In view of the low galactic latitude, -13°, and the position bordering on a known region of calcium absorption, the color-excess suggests scattering by material within the galactic system. In this case the photographic magnitudes should appear fainter than normal and the distance derived from them should be too great. A comparison of this distance, 11 million parsecs, with that derived from the velocity-distance relation and the mean velocity of +5200 km/sec. from 4 nebulae indicates a discrepancy in the proper direction and of about the proper amount.[2] The deviation is small, however, and not definitely outside the limits of certainty or consistency in the correlation of velocities and distances. The results, in short, are suggestive but not definitive.

The Coma cluster (R.A. $12^h55^m.5$, Dec. $+28^\circ20'$, 1930; gal. long. 26°, lat. $+87^\circ$).—The cluster consists of about 800 nebulae scattered over an area roughly $1^\circ.7$ in diameter, centered near N.G.C. 4874 and 4884. Some 25 of the brightest nebulae were observed visually by D'Arrest[3] as early as 1865, but the nature and extent of the cluster were not fully recognized until Wolf[4] photographed the region in

[1] N.G.C. 1275 is classed as an irregular nebula but not of the Magellanic Cloud type. It appears to be somewhere between the elliptical nebulae and the spirals. For purely descriptive purposes it could be classed as an elliptical nebula which has broken up without the formation of spiral arms.

[2] The most frequent photographic magnitude, 16.4, indicates a distance of 11 million parsecs and hence a velocity of about +6000 km/sec. Since the color-index is 0.2 mag. greater than that for nebulae in general, a correction of 0.4 mag. is suggested by the hypothesis of scattering. This reduces the most frequent magnitude to 16.0, the distance to 9.1 million parsecs, and the expected velocity to +5000 km/sec. The observed velocity is +5200 km/sec.

[3] *Astronomische Nachrichten*, **65**, 1, 1865.

[4] *Ibid.*, **155**, 127, 1901.

1901 and recorded 108 nebulae in a circle 30′ in diameter. H. D. Curtis,[1] with the Crossley reflector, counted 304 in an area $40' \times 50'$, and on a plate by J. C. Duncan with the 100-inch reflector, over 400 have been found in about the same area. Photographic magnitudes range from about 14.1 to 19.5, with 17.0 as the most frequent value. The apparent velocity displacement, as will be shown later, indicates a correction of -0.1 to the apparent photographic magnitude. The corresponding distance is then about 13.8 million parsecs. Color-indices from two pairs of extra-focal exposures average 1.12 for 10 of the brighter nebulae.

The analysis of this cluster will be described in some detail as an example of the method used for them all. The adjacent regions were first photographed in order to delineate the cluster and to determine the number of nebulae included. Next, by focal comparisons with Selected Areas and the North Polar Sequence, a standard sequence of comparison stars was established in the cluster. Then, from extra-focal exposures, on which the images were of various dimensions, a sequence of nebular magnitudes was established. Finally, on all the photographs available, from small camera plates to those with the 100-inch reflector, the nebulae were counted in a specified area; in this particular cluster, about 1500 square minutes. For the counts, a long exposure with the 100-inch was used as a guide in order that no nebula might be missed or mistaken for a star on the shorter exposures and the small-scale plates. Each plate thus gave the number of nebulae to a definite limit, which was determined from the sequence of extra-focal magnitudes extending to about 18.1. Fainter magnitudes were estimated by extrapolations guided by previous results from statistical investigations of the limits corresponding to various exposure conditions. The numbers of nebulae to successive limits were corrected for field nebulae by tables derived from unpublished investigations of the relation between number of nebulae and effective exposure or limiting magnitude over the higher galactic latitudes.

The data for the Coma cluster are listed in Table VI and exhibited in Figure 2 as a relation between N_m, the number of nebulae to a given limiting magnitude, and m, the limiting magnitude itself.

[1] *Publications of the Lick Observatory*, **13**, 33, 1918.

From the smooth curve in Figure 2 the numbers of nebulae within successive intervals of magnitude were derived from the differences in the ordinates. These gave the frequency distribution represented in Figure 3, in which the lower curve represents the numbers in successive intervals of 0.5 mag.; the upper curve, on a reduced scale, gives the sums for three such intervals and thus represents a smoothing by overlapping means. In spite of the uncertainty in the fainter

TABLE VI

DISTRIBUTION OF MAGNITUDES IN THE COMA CLUSTER

Limit m_{pg}	Nebulae	Scurce	Interval	Nebulae*
14.5	1	Measured magnitudes	14.0–14.5	1
15.0	4	Measured magnitudes	14.5–15.0	3
15.5	17	Measured magnitudes	15.0–15.5	14
16.2	60	Measured magnitudes	15.5–16.0	29
16.5	80	10-inch focal	16.0–16.5	34
17.2	140	100-inch extra-focal	16.5–17.0	43
17.3	150	60-inch extra-focal	17.0–17.5	38
18.1	202	100-inch extra-focal	17.5–18.0	34
19.2	255	36-inch focal†	18.0–18.5	29
19.5	266	100-inch focal	18.5–19.0	23
20.5	263	100-inch focal	19.0–19.5	15
			19.5–20.0	2
			20.0–20.5	0

* Numbers were read from a smooth curve representing the data in the first two columns.

† Photograph with the Crossley reflector, available through the courtesy of the director of the Lick Observatory.

magnitudes, the curves are very symmetrical. The peak is at 17.0,[1] with an uncertainty of probably not more than 0.1 mag.

Velocities for 4 nebulae are available. Three of these, N.G.C. 4853, 4860, and 4884, show a range of 1200 km/sec. about the mean velocity +7500 km/sec. The fourth, N.G.C. 4865, has a velocity of +5100 km/sec. It is an elliptical nebula, about E4, estimated at 14.7 mag., similar in appearance to the other cluster nebulae. From an inspection of the plates it would be described simply as one of the brightest members of the cluster. Its velocity, however, is the one outstanding discrepancy among some 28 velocities in 8 clusters or groups. This suggests that it is a field nebula, seen in projection

[1] This value is 0.1 mag. brighter than that given for the Coma cluster in the *Annual Report of the Mount Wilson Observatory, 1929–1930*, and represents a revision of the system of magnitudes.

against the cluster. The absolute photographic magnitude, as derived from the velocity-distance relation, would then be −15.1, nearer the mean luminosity of nebulae in general than the value −16.0 which must be assigned if it is considered a member of the cluster. Such cases must occasionally be encountered and this may be an in-

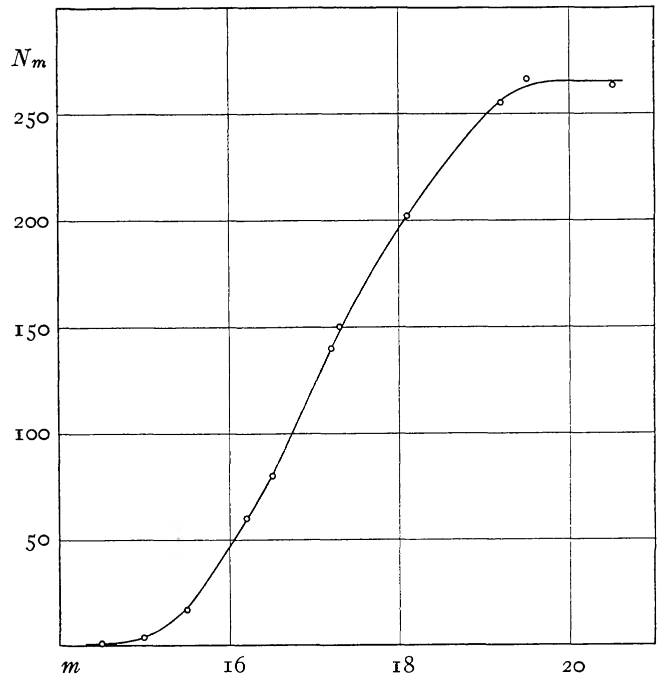

FIG. 2.—Counts of nebulae in the central region of the Coma cluster. N_m is the number of nebulae as bright or brighter than apparent photographic magnitude m.

stance. Observations of the cluster will be continued, however, especially in view of the wide range in the three more normal velocities, in order that the matter may be re-examined in the light of more data.[1]

[1] Humason has since obtained velocities for 5 additional nebulae in the Coma cluster: 6600, 6900, 6900, 7000, and 8500 km/sec. for I.C. 4045 and N.G.C. 4872, 4874, 4881, and 4895, respectively. The uncertainties are rather large, since each velocity is derived from a single spectrogram with the small dispersion of 875 A per millimeter. For this reason the range, 1900 km/sec., may be exaggerated, but it is clearly larger than those observed in the other clusters and raises the question of a possible correlation between range and distance.

The mean velocity of the 8 nebulae is 7360, as compared with 7500 for the 3 nebulae previously observed. The effect of this revision on the correlation of velocities and distances is considerably less than the probable errors.

The Ursa Major cluster (R.A. $11^h43.3^m$, Dec. $+56°8'$, 1930; gal. long. 106°, lat. +59°).—This cluster comprises about 300 nebulae within a roughly circular area about 0°7 in diameter. The cluster was discovered by W. Baade,[1] who published a description, together with photographic magnitudes of the brighter nebulae, in 1928. It was found independently at Mount Wilson in 1924 and delineated by photographs covering the adjacent regions. The magnitudes range from about 16.0 to fainter than 20.0. The most fre-

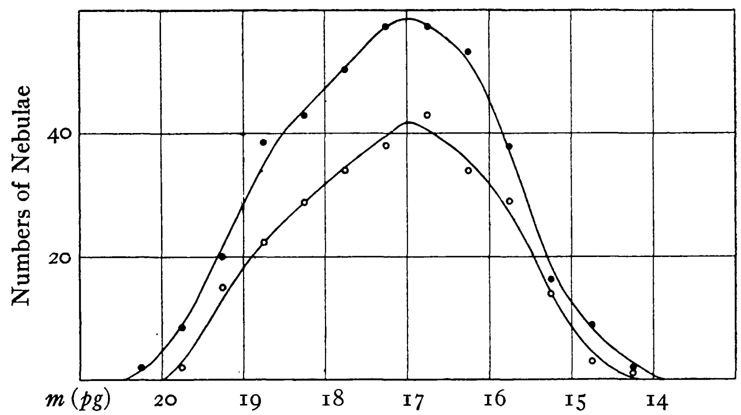

FIG. 3.—Frequency distribution of apparent photographic magnitudes among nebulae in the central region of the Coma cluster. The lower curve (open circles) represents the numbers of nebulae in successive intervals of 0.5 mag. as read from the curve in Fig. 2. The upper curve, on another scale, represents a smoothing by overlapping means.

quent magnitude is about 18.0, which, corrected by −0.15 mag. to compensate for an effect of the red-shift, corresponds to a distance of 22 million parsecs. Extra-focal magnitudes from two pairs of plates give a mean color-index of +1.20 for 10 nebulae; an exposure-ratio plate gives a mean of +1.16 for 20 nebulae. The extra-focal photographic magnitudes agree very closely with Baade's values for nebulae brighter than about 17.3, beyond which the extra-focal measures are somewhat brighter. By comparing the brighter nebulae in the two clusters Baade derived a distance of the order of fifteen times that of the Virgo cluster. This is only 25 per cent greater than the relative distances adopted in the present discussion, although Baade's absolute distance, based upon Shapley's value of 10 million

[1] *Loc. cit.*

light-years for the Virgo cluster, is twice that derived from the most frequent magnitude.

A velocity has been measured for nebula No. 24 in Baade's list. Before the spectrogram was obtained, a velocity of +12,000 km/sec. was predicted on the basis of the distance indicated by the most frequent magnitude. The measures, corrected for solar motion, give +11,800 km/sec.

The Leo cluster (R.A. 10^h24^m, Dec. $+10°50'$, 1930; gal. long. 201°, lat. +54°).—This cluster, which includes about 300 nebulae scattered over an area approximately 0°.6 in diameter, was called to our attention by W. H. Christie, who discovered it on plates taken with the 60-inch reflector in December, 1929. The faintest nebulae are probably beyond the limits of the photographs available, and the form of the frequency-curve of apparent magnitudes is difficult to determine. Both the brightest and the most frequent magnitude, determined independently, are about 1 mag. fainter than those in the Ursa Major cluster, and this agreement lends added confidence to the value assigned to the Leo cluster, namely, 19.0 for the most frequent magnitude. The correction necessitated by the red-shift in the spectrum is about 0.25 mag.; hence the distance is of the order of 32 million parsecs.

One small-scale spectrogram (about 875 A to the millimeter) has been obtained for one of the brightest nebulae in the cluster. The definition is excellent, and the five lines identified, including the all-important H and K lines, establish a displacement corresponding to +19,600 km/sec. beyond reasonable doubt. No color measures are available as yet.

EFFECT OF RED-SHIFTS ON APPARENT MAGNITUDES

If nebular luminosity approximates black-body radiation, the red-shifts produce redistributions of intensities which can be represented by new black-body curves corresponding to lower temperatures and hence to later spectral types. This affects the observed photographic magnitudes by amounts which become appreciable for $d\lambda/\lambda > 0.02$, corresponding to distances greater than, say, 10 million parsecs.

The relation between total radiation and photographic magnitude is obtained as follows: The total radiation of a celestial body is expressed by the bolometric magnitude m_b. Allowance for absorption by the earth's atmosphere, Δm_r, gives the radiometric magnitude, m_r, the quantity measured by a thermocouple:

$$m_r = m_b + \Delta m_r \; .$$

Visual magnitude is related to radiometric magnitude and heat-index, HI, just as photographic magnitude is related to visual magnitude and color-index, CI. Thus

$$m_{\text{vis}} = m_r + HI \; ,$$

$$m_{\text{pg}} = m_{\text{vis}} + CI \; .$$

Therefore

$$m_{\text{pg}} = CI + HI + \Delta m_r + m_b \; .$$

Bolometric magnitudes are derived from the observed radiometric magnitudes by calculating the correction Δm_r on the assumption of black-body radiation. Color- and heat-index are observed quantities which bear known empirical relations to spectral types as derived from absorption lines where the displacement of the lines is negligible. Under these conditions both indices and spectral types are related to temperature. Deviations of the indices from the normal values corresponding to spectral types thus determined are commonly referred to as color-excess, CE, and heat-excess, HE, respectively. They represent abnormal distribution of intensity in the continuous spectrum as compared with the pattern of absorption lines.

A red-shift, by redistributing the radiation to correspond with a lower temperature and hence with a later spectral type, introduces an increment to the photographic magnitude which, by virtue of the last equation, can be expressed as

$$\begin{aligned} \Delta m_{\text{pg}} &= \Delta CI + \Delta HI + \Delta[\Delta m_r] + \Delta m_b \; , \\ &= CE + HE + \Delta[\Delta m_r] + \Delta m_b \; . \end{aligned}$$

The first three terms on the right are evaluated from the advance in spectral type, i.e., they represent the differences between the indices and Δm_r for the type as indicated by the absorption lines, dG3 in the case of the nebulae, and the type corresponding to the lower apparent temperature introduced by the red-shift.

From Wien's law the new temperature is

$$T=\frac{5760}{1+\frac{d\lambda}{\lambda}},$$

where 5760 is the temperature for dG3 and $d\lambda/\lambda$ is the red-shift. The corresponding spectral type and hence the color-excess, heat-excess, and $\Delta[\Delta m_r]$ are then derived graphically from the data in Table VII.

TABLE VII

Spectral Type	Temp.*	Color-Index†	Heat-Index‡	Correction to No Atmosphere‡
F5	6500°	0.62	0.30	0.44
dG0	6000	.72	.32	.43
dG5	5600	.83	.39	.41
dK0	5100	0.99	0.55	.40
dK5	4400	1.26	1.10	.48
dM	3400	1.76(Ma)	1.40(Mo)	0.53(Mo)

* Russell, Dugan, and Stewart, *Astronomy*, **2**, 753, 1927.

† Seares, *Mt. Wilson Contr.*, No. 226; *Astrophysical Journal*, **55**, 165, 1922. Color-indices are in Table XII.

‡ Pettit and Nicholson, *Mt. Wilson Contr.*, No. 369; *Astrophysical Journal*, **68**, 279, 1928. Heat-indices are from Table V; corrections to no atmosphere are from Table IV (in which the temperatures are those in Table V corresponding to water-cell absorptions).

The increment in the bolometric magnitude follows from the decrease in ν introduced by the red-shift. The energy of each quantum of radiation is reduced by a constant factor, namely, $1+d\lambda/\lambda$, and hence the total radiation, i.e., the sum of all the quanta, is reduced by the same factor. The increment in m_b is therefore

$$\Delta m_b=2.5 \log \left(1+\frac{d\lambda}{\lambda}\right).$$

If an actual velocity of recession is involved, an additional increment, equal to that given above, must be included in order to account for the difference in the rates at which the quanta leave the source and reach the observer.[1] This additional increment will be neglected for the present, since the writers desire to emphasize the observational results without discussing the interpretation more than is necessary for the immediate purpose. The increments, moreover, are small; for the largest observed red-shift, Δm_b is about 0.07 mag., and the effect on the final correlation is less than the probable error. Eventually it may be possible to test the matter by counts of nebulae to successive limits of apparent magnitude on the assumption of uniform distribution, an assumption which appears to be well established to about the limits at which the additional increment would be expected to become sensible.

Table VIII gives the temperature, spectral type, color-excess, heat-excess reduced to no atmosphere, and the increments in the bolometric and the photographic magnitudes corresponding to different red-shifts in a normal dG3 spectrum. Approximate distances as indicated by the velocity-distance relation are added in the last column.[2] These results ignore the color-excess, independent of apparent velocity displacements, which the photometric observations appear to indicate. If further investigations confirm the preliminary indications, the quantities in Table VIII must be revised.

The effect of the red-shift on the photographic magnitude is about 0.25 for the Leo cluster, 0.15 for the Ursa Major cluster, and 0.10 for the Coma cluster. These corrections were applied in calculating the distances from the observed magnitudes. For objects nearer than the Coma cluster, the corrections are less than the uncertainties of the magnitudes and have been ignored.

[1] The factor is $\sqrt{\frac{1+v/c}{1-v/c}}$, which closely approximates $1+d\lambda/\lambda$ for red-shifts as large as have been observed. A third effect due to curvature, negligible for distances observable at present, is discussed by R. C. Tolman in *Proceedings of the National Academy of Sciences*, **16**, 511, 1930.

[2] Table VIII extends the calculations of Δm_{pg} out to red-shifts about three times those actually observed in order to indicate the possibility of testing the velocity-distance relation near the limits of direct photography with large telescopes. If the relation is approximately linear, the limit of the 100-inch reflector for normal nebulae should correspond to a red-shift of the order of 40,000 km/sec. The proposed 200-inch may be expected to approach 60,000 km/sec. The values of Δm_{pg} are 0.55 and 0.95, respectively, and should be readily detected in the correlation of numbers of nebulae per plate and exposure times, provided the assumption of uniform distribution is approximately true.

RELATION BETWEEN RED-SHIFTS AND APPARENT MAGNITUDES

The quantities actually observed in the present investigation are red-shifts and apparent magnitudes. The relation between them is so definite and significant that it may be emphasized before interpreting the magnitudes in terms of distance. The fact that the red-shifts are expressed on a scale of velocities is incidental; for the present purpose they might as well be expressed as $d\lambda/\lambda$.

TABLE VIII

EFFECT OF VELOCITY DISPLACEMENTS ON PHOTOGRAPHIC MAGNITUDES

Velocity Km/Sec.	$d\lambda/\lambda$	Temp. Abs. Cent.	Spectral Type	Color-Excess	Heat-Excess*	Δm_b	Δm_{pg}	Distance in Parsecs
1000	0.0033	5740	dG 3.3	0.008	0.004	0.003	0.015	1.8×10^6
4000	.0133	5685	4.0	.02	.015	.015	.05	7.2
8000	.0267	5615	4.7	.04	.03	.03	.10	14.4
12000	.0400	5540	5.8	.06	.05	.04	.15	21.6
16000	.0533	5470	6.5	.08	.06	.06	.20	28.8
20000	.0667	5400	7.2	.10	.08	.07	.25	36
30000	.100	5235	8.8	.16	.13	.11	.40	54
40000	.133	5080	K 0.2	.21	.20	.14	.55	72
50000	.167	4940	1.4	.26	.32	.17	.75	90
60000	0.200	4800	2.3	0.31	0.44	0.20	0.95	108

* Reduced to no atmosphere.

The significant data are listed in Table IX. For comparison with the clusters, the 16 isolated nebulae are combined in a single group, giving a mean log v corresponding to a mean magnitude, the 6 visual magnitudes being reduced to the photographic system by the adopted color-index, +1.1. Another mean for isolated nebulae is furnished by the 21 objects in which no stars can be seen, which are listed in the former discussion of the velocity-distance relation.[1] The visual magnitudes are reduced to the photographic system as above. This group was not used directly in the preliminary formulation, which was based primarily upon nebulae in which stars could be identified. Its inclusion in the present discussion does not materially affect the results, but it permits the statement that the present correlation in-

[1] *Mt. Wilson Communication*, No. 105; *Proceedings of the National Academy of Sciences*, **15**, 168, 1929. The 21 unresolved nebulae are those in Table II, with the exception of N.G.C. 4526, which belongs to the Virgo cluster and was included in the table by an oversight.

cludes all nebulae with observed velocities for which apparent magnitudes are the sole criterion of distance.

TABLE IX

Cluster	Number of Nebulae	Diameter	Mean Velocity	Number of Velocities	Mean m_{pg}	Δm*	Color-Index
Virgo...........	(500)	12°	890	7	12.5	....	1.1
Pegasus.........	100	1	3810	5	15.5	....	1.1
Pisces..........	20	0.5	4630	4	15.4	....	....
Cancer..........	150	1.5	4820	2	16.0	....	....
Perseus.........	500	2.0	5230	4	16.4	....	1.3
Coma...........	800	1.7	7500	3	17.0	−0.10	1.1
Ursa Major.....	300	0.7	11800	1	18.0	0.15	1.2
Leo............	400	0.6	19600	1	19.0	−0.25	....
Isolated nebulae I	....	....	2350	16†	13.8	....	....
Isolated nebulae II............	....	....	630	21‡	11.6	....	....

* Correction due to apparent velocity displacement (see Table VII).

† Isolated nebulae in Table IV. Velocity represents the mean log v. Visual magnitudes are corrected for color-index.

‡ Unresolved isolated nebulae previously observed (Table II in *Proceedings of the National Academy of Sciences*, **15**, 171, 1929). Velocity represents the mean log v; visual magnitudes are reduced to photographic.

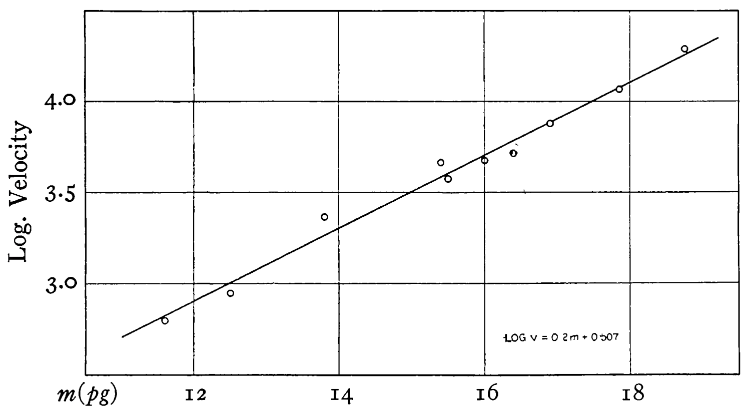

FIG. 4.—Correlation between the quantities actually observed in deriving the velocity-distance relation. Each point represents the mean of the logarithms of the observed red-shifts (expressed on a scale of velocities) for a cluster or group of nebulae, as a function of the mean or most frequent apparent photographic magnitude.

In Figure 4 the mean logarithms of the velocities are plotted against the most frequent or mean apparent magnitudes for the clusters and the two groups of isolated nebulae. Each point is given unit weight. The correlation is closely linear and a least-squares solution leads to the equation

$$\log v=(0.202\pm 0.007)m+0.472\,. \tag{1}$$

This justifies the adoption of the exact decimal 0.2 as the coefficient of m; a new solution then gives

$$\log v=0.2m+0.507\pm 0.012\,. \tag{2}$$

The average deviation from this formula is 0.031 in $\log v$ and 0.15 in m, over a range of 1.5 in $\log v$ and 7.25 in m.

Since distance in parsecs is expressed by

$$\log d=\frac{m-M+5}{5}=0.2m-0.2M+1\,,$$

equation (2) may be written

$$\log \frac{v}{d}=0.2M-0.493\,. \tag{3}$$

Also, since

$$M=c-2.5\log L\,,$$

where L is the intrinsic luminosity,

$$\log \frac{v}{d}=c_1-0.5\log L\,,$$

$$\frac{v}{d}=\frac{c_2}{\sqrt{L}}\,.$$

Hence, independent of theory, it follows that the ratio of red-shift to distance is constant, provided the intrinsic luminosity of nebulae is statistically constant; otherwise the ratio varies inversely as the square root of the intrinsic luminosity.

VELOCITY-DISTANCE RELATION

The constancy of M_n is believed to be well established, hence apparent magnitudes of nebulae are accepted as measures of distance. The interpretation of red-shifts as actual velocities, however, does not command the same confidence, and the term "velocity" will be used for the present in the sense of "apparent" velocity, without prejudice as to its ultimate significance. In this sense the relation between velocity and distance is clearly linear, and the slope is determined by the numerical value of M_n. Introducing the adopted value $M_n = -13.8$ pg into equation (3), we have

$$\log \frac{v}{d} = -3.253 ,$$

$$v = \frac{d}{1790} = 558 \text{ km/sec. per million parsecs.}$$

A probable error computed in the usual way would merely indicate the consistency of the data and not the real uncertainty, which arises mainly from the possibility of systematic errors in the magnitudes, color-indices, and the adopted M_n. Accidental errors and the effect of peculiar velocities, both of individual nebulae and of clusters, are probably small in comparison. It is believed, however, that the uncertainty in the final result is definitely less than 20 per cent and probably not more than 10 per cent.

This result, which may be rounded off to 560 km/sec. per million parsecs, differs from the 500 km/sec. found in the previous discussion only by the amount corresponding to the change in M_n produced by Shapley's revision of the standard unit of distance. Without the revision the new result would differ from the old by less than 1 per cent. Since the two investigations were based upon different criteria of distance, the close agreement emphasizes the internal consistency of our present ideas concerning luminosities of nebulae.

The data used in the two investigations are combined in Figure 5, where velocities are plotted against distances, the latter all reduced to the present scale. The observations now cover a range about eighteen times that available for the preliminary investigation and approach the limit of present instrumental equipment; but the form of the correlation is essentially unchanged, except for the revision in the unit of distance, and hence the velocity-distance relation appears to be a general characteristic of the observable region of space. Aside from its cosmologic significance, the relation offers a new method of determining distances of individual objects in which the percentage errors actually diminish with distance. This opens new possibilities for the investigation of nebulae, some of which may be mentioned at the present time.

THE LUMINOSITY FUNCTION OF NEBULAE

In addition to the evidence on luminosity derived from nebulae in which stars can be identified, absolute magnitudes may be found

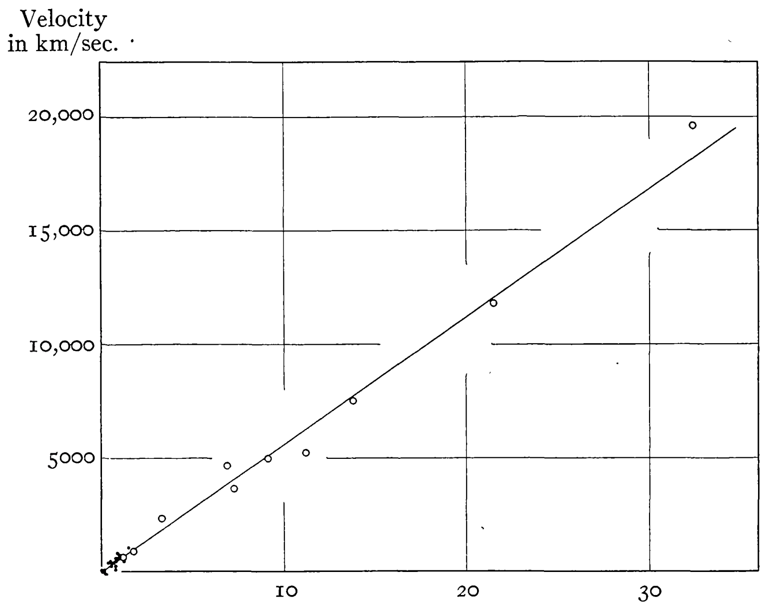

FIG. 5.—The velocity-distance relation. The circles represent mean values for clusters or groups of nebulae. The dots near the origin represent individual nebulae, which, together with the groups indicated by the lowest two circles, were used in the first formulation of the velocity-distance relation.

from the distances indicated by the apparent velocities. The number of known velocities will increase as time goes on and eventually will be sufficient to determine a reliable frequency-curve for isolated objects and even for the different types of nebulae. The study of these curves, together with those for the clusters, may be expected to throw light on several problems, including some of evolutional significance. At present it is only possible to compare the frequency-curve for 56 isolated nebulae of all types with a combined curve for the several clusters. The result is shown in Figure 6. Distances of the 20 isolated nebulae in which stars can be seen are derived from the stars; distances of the remaining 36 objects, from the velocities corrected for the solar motion. One object, N.G.C. 404, has been omitted because the observed velocity is so small that the peculiar motion may be large in comparison with the distance effect. The curves for the clusters are combined on the assumption that the most frequent absolute photographic magnitude is -13.8.

The curves for the clusters and for the isolated nebulae have the same range, but the former is considerably more symmetrical. It is

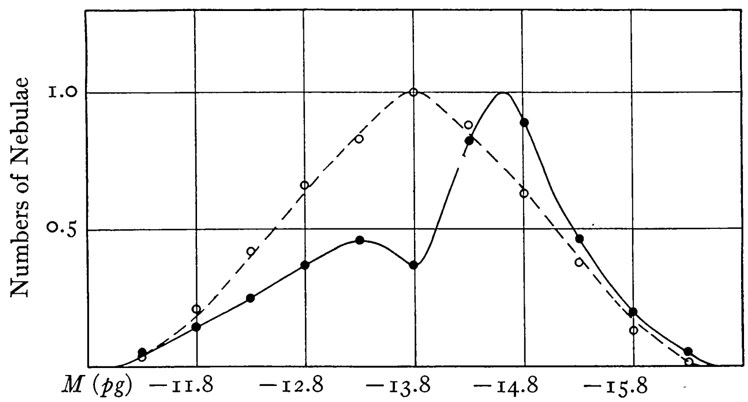

FIG. 6.—Frequency distribution of absolute photographic magnitudes among extragalactic nebulae as derived from clusters (circles) and from isolated nebulae (dots). Distances of the isolated nebulae were derived mainly from the red-shifts. The range in the two curves is the same. The asymmetry in the curve for isolated nebulae is believed to be due, in part at least, to effects of selection.

believed that the asymmetry of the curve for the isolated nebulae is partly accounted for by an effect of selection, but more extensive data will be required to determine the matter.

ABSORPTION OF LIGHT IN LOW GALACTIC LATITUDES

The problem of absorption within the galactic system can be approached statistically by comparing absolute luminosities of nebulae in low latitudes with those in the higher latitudes. The Perseus cluster with its large color-excess and, apparently, its low mean luminosity is very suggestive of obscuration, but it leaves the question open as to whether the material responsible for the obscuration is uniformly diffused throughout the galactic system or isolated in clouds. Several large spirals in low latitudes are known in which the surface brightness appears abnormally faint (I.C. 342 is a good example), and the complete absence of extra-galactic nebulae along the galactic plane is generally attributed to obscuration. On the other hand, extra-galactic nebulae are found in considerable numbers near the galactic plane in longitudes 10°–50° and obscuration is not conspicuous. Several late-type spirals near I.C. 1303, in latitude +8°, for instance, are not particularly faint for their dimensions, and exposure-ratio plates do not indicate excessive colors. The same is true

TABLE X

	M_{pg}	Gal. Lat.
N.G.C. 6710	−15.0	+11°
6658	14.7	+13
6661	15.4	+13
6824	15.0	+15
7242	14.4	−16
6702	13.7	+19
6703	14.5	+19
7217	−14.5	−20
Mean	−14.65	

of the earlier types N.G.C. 6658 and 6661 at +12°. Systematic investigations will be necessary in order to determine the situation, but the velocity-distance relation furnishes the method and a program is already under way. The results so far available do not favor any considerable general obscuration, but the data are meager as yet, and no definite conclusion can be drawn. The 8 nebulae within 20° of the galactic plane given in Table IV average about 0.85 mag. brighter than normal, and the brightest 4 are in the lowest latitudes. The data are given in Table X. A systematic effect of selection is probably involved. At any rate, this group of abnormally bright objects accounts for the large deviation of the mean point from that derived from the velocity-distance relation for 10 nebulae with photographic magnitudes, and for part of the asymmetry in the luminosity function shown in Figure 6.

MASSES OF NEBULAE

The most direct indications of the masses of nebulae are derived from spectrographic rotations. The measures give linear velocities

1931ApJ....74...43H

of rotation at various angular distances from the nuclei, and in order to estimate the masses it is necessary to convert the angular distances into linear measure. This requires a knowledge of the distances of the nebulae themselves. Spectrographic rotations are now available for M 31 and M 33, that for the latter revised on the basis of new spectra of the nucleus, and for N.G.C. 4594. The distance of the last can now be derived from the apparent velocity. Further measures of rotational velocities are very desirable and will be made most easily on the early-type nebulae, E7 and Sa, in which stars are not found. With the aid of the velocity-distance relation, however, the rotation plates will themselves furnish the distances. Since the mass, for a given set of measures, varies linearly with the distance, uncertainties in the latter quantity, arising from the unknown peculiar motions of the nebulae, will not change the order of the result.

The present contribution concerns a correlation of empirical data of observation. The writers are constrained to describe the "apparent velocity-displacements" without venturing on the interpretation and its cosmologic significance. Further observations are desirable and will be carried on, although it seems probable that the general features of the relation are already sketched in outline nearly to the limit of existing equipment. Color investigations and statistical counts of nebulae to successive limits offer possibilities of extending the range of the observations while the spectrograph explores more thoroughly the region already scouted.

CARNEGIE INSTITUTION OF WASHINGTON
MOUNT WILSON OBSERVATORY
March 1931