On the continuous spectrum of the corona and its polarisation.

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Scattering of the photospheric light by the electrons in the corona. The electrons are assumed to be free and to scatter according to the classical formulae. An exact expression for the light scattered by one volume element is derived, taking account of the darkening at the sun's limb. The intensity and the polarisation of the observed light are calculated for a corona in which the electron density is proportional to r^n; they appear to be nearly independent of wave-length.— *Absolute numbers concerning the emission and scattering in the corona.* The measurements of Young are analysed by a new graphical method; absolute values are found for the number of scattering electrons and the light emitted by the corona itself at different distances from the limb (table VIII). — *Is the scattering due to free or to bound electrons?* Against the argument of Schwarzschild, it is pointed out that only combined measurements of the intensity and the polarisation in different colors are conclusive as to the kind of scattering material. The available observations seem to support the hypothesis of the bound electron; with this assumption, the temperature of the corona comes out as $3000^0 - 4000^0$. On the contrary, atomic theory would ascribe the chief role to the free electrons. New observations are needed. — *The emission of the corona itself due to recombinations?* If the corona is supposed to be composed of electrons and positive ions, due to the ejection from the photosphere, the numbers derived from the scattering are sufficient to explain also the emission as a recombination spectrum. — *The emission and the "scattering" in the corona may be described as one single process,* consisting in a scattering according to a law $a + b \cos^2 \vartheta$; the coefficients a and b are given in table X. *The Raman hypothesis* of Mecke and Wildt would correspond to such a type of scattering; but is criticised and found improbable. — *The age of the sun.* With the ordinary laws, the existence of the corona as a mixture of ions and electrons is only possible if the particles are continuously ejected. This ejection of material from the sun, calculated for electrons and for a mixture with ions, is consistent in both cases with the age of the sun. — *The scattering and deflection of starlight by a corona of electrons or ions* are calculated and appear to be too small with any assumption to be detected by the observations.

The determination of the scattering and of the emission in the corona.

Scope of the paper. How the continuous spectrum of the corona originates is still imperfectly understood. Befor we make new hypotheses, it seems desirable to start from the simplest assumption, that of a corona composed of free electrons, and to show how a number of important data concerning the radiation and the structure of the corona may be directly calculated from observations of the intensity and the polarisation of the coronal light. Partly such calculations have been made already by others but we will attempt to reach a greater accuracy and to give absolute values for all quantities involved. Guided by the results obtained, we will then discuss other hypotheses and show how our results can be transferred to them.

We will begin by assuming that in every volume element of the corona there are free electrons which scatter the light from the photosphere, and we will calculate the brightness of the corona as seen by a terrestrial observer.

Such a calculation has to proceed by the following steps:

1. The law must be given according to which a ray of light is scattered by a volume element of the corona. If the scattering is due to free electrons, the intensity of the scattered light at a distance R from the particle is given by the classical formula:

$$\frac{e^4}{R^2 C^4 m^2} \frac{1 + \cos^2 \vartheta}{2}; \tag{1}$$

e = electrostatic charge, m = mass of the electron, C = velocity of light, ϑ = angle between the incident ray of light and the direction of observation.

In the expression $1 + \cos^2 \vartheta$, the first term concerns the electric vector perpendicular to the plane: incident ray — direction of observation; the second term concerns the electric vector in that plane. This scattering coefficient is independent of wave-lenght.

WOLTJER* has shown that the reaction of the motion of the electron induced by the incident radiation on the process of scattering is ineffective, except for those parts of the corona which we see projected on the solar disc and therefore cannot be observed.

2. One determinate volume element of the corona receives light from a considerable part of the solar surface. The light scattered in the direction of the earth is to be found by an integration over all the incident rays.

This was first done by SCHUSTER, in a beautiful paper** published as long ago as 1879, in a time when proper observations were almost com-

* B. A. N. **3**, No. 103, 163, 1926.

** M. N. **40**, 35, 1879. A number of errors are found in this paper; some of them are corrected by YOUNG, Lick Obs. Bull. **6**, 169, 1911. Others are: page 36, fig. 1: the angle ω has its top in P, not in M. Page 38, formula (6): for $\frac{IB^2}{\pi}$ read πIB^2; on next line, put $\pi IB^2 = \frac{1}{2}$. Page 42, 2nd and 3rd line from below: the limits of the integrals are 0 and $\pi/2$, not 0 and 2π. Page 45, 3rd line from below: for $k - 1 = c$ read $1/k = c$, and for $k - 1$ read $1/k$. Page 54, 3rd line from below: read

$$I_0 = \frac{1}{3}\left[k^{n+1}P_n - 4k^{n-1}P_{n-2} + 4k^{n-1}\frac{1 \cdot 3 \cdot \cdots (n-3)}{2 \cdot 4 \cdot \cdots (n-2)}\frac{\pi}{2}\right].$$

YOUNG's correction is wrong here.

pletely lacking. Howewer, he assumed that the radiation which a volume element of the corona gets from the photosphere is the same in every direction.

The influence of the darkening at the limb of the sun was considered for the first time by WOLTJER*, but only with some approximations; moreover, he considered only the total intensity of the light scattered, and not the polarisation. An exact expression for the light scattered by a volume element quite close to the photosphere was given by PETTIT and NICHOLSON**.

3. The brightness of the corona at a certain point, in so far as scattered light is considered, is the sum of the light scattered by all the particles in the line of sight. These integrals have been calculated by SCHUSTER (l. c.) for different possible distributions of the coronal materia, and by YOUNG for another one (l. c. p. 179) but again without considering the darkening at the limb.

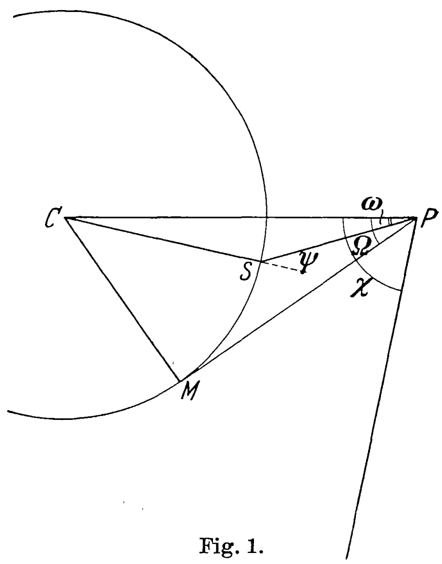

Fig. 1.

4. A practical application of SCHUSTER'S theory to the interpretation of observations has been made by YOUNG, who showed how to determine the radiation of the corona itself***.

Following SCHUSTER, but taking into account the darkening at the limb, we will first derive without approximations an expression for the light scattered by a volume element, and this for the two vibrations separately. By integrating we will then determine the scattered light observed in any point of the corona, and its polarisation. The results will be applied to different types of corona and to direct observations.

The light scattered by one determinate volume element of the corona (Fig. 1). Let P be the volume element considered of the corona, C the centre of the sun, PM the tangent throug P at the solar sphere, Ω the angle CPM, χ the angle between CP and the direction in which the scattered light is observed. The incident light coming from a point S under an angle

* B. A. N. **3**, 103, 1926, No. 94.
** Ap. J. **64**, 136, 1926.
*** Lick Obs. Bull. **6**, 166, 1911.

$\omega = CPS$ with CP has the intensity I per unit solid angle and per sq. cm. SCHUSTER shows, that the light scattered towards the earth and vibrating tangentially has the intensity

$$g_t = B^2 \pi \int_{\cos \Omega}^{1} (1 + \cos^2 \omega)\, I\, d(\cos \omega), \tag{2}$$

and the light vibrating radially:

$$g_r = B^2 \pi \int_{\cos \Omega}^{1} [(1 + \sin^2 \chi) + \cos^2 \omega\, (1 - 3 \sin^2 \chi)]\, I\, d(\cos \omega).$$

As I is given per sq./cm of the corona, g_t and g_r refer also to the intensity on a sq./cm on earth. For a single scattering electron the constant B^2 which was unknow to SCHUSTER, is equal to $\frac{e^4}{2\, R^2 C^4 m^2}$, according to (1).

Instead of g_r it is somewhat simpler to calculate the difference

$$g_t - g_r = \sin^2 \chi \cdot B^2 \pi \int_{\cos \Omega}^{1} (-1 + 3 \cos^2 \omega)\, I\, d(\cos \omega). \tag{3}$$

We will take into account the darkening at the sun's limb by putting

$$I = I_0\, (1 - U + U \cos \psi), \tag{4}$$

U being the darkening coefficient and ψ the angle between SP and the radius CS. The $\cos \psi$ is easily expressed in terms of $\cos \omega$; for $CP = \frac{CS}{\sin \Omega}$,

$\sin \psi = \frac{\sin \omega}{\sin \Omega}$, and

$$\cos \psi = \frac{\sqrt{\sin^2 \Omega - \sin^2 \omega}}{\sin \Omega} = \frac{\sqrt{\cos^2 \omega - \cos^2 \Omega}}{\sin \Omega}. \tag{5}$$

Thus, putting $\cos \omega = x$, the scattered intensity is given by:

$$\frac{g_t - g_r}{I_0 B^2 \pi \sin^2 \chi} = (1 - U) \int_{\cos \Omega}^{1} (-1 + 3\, x^2)\, dx + U \int_{\cos \Omega}^{1} (-1 + 3\, x^2)\, \frac{\sqrt{x^2 - \cos^2 \Omega}}{\sin \Omega}\, dx$$

and by

$$\frac{g_t}{I_0 B^2 \pi} = (1 - U) \int_{\cos \Omega}^{1} (1 + x^2)\, dx + U \int_{\cos \Omega}^{1} (1 + x^2)\, \frac{\sqrt{x^2 - \cos^2 \Omega}}{\sin \Omega}\, dx.$$

The integrals are resolved by elementary calculation; we will write:

$$I = \int_{\cos\Omega}^{1} \sqrt{x^2 - \cos^2\Omega}\, dx = \frac{\sin\Omega}{2} - \frac{\cos^2\Omega}{2} \log \frac{1+\sin\Omega}{\cos\Omega},$$

$$II = \int_{\cos\Omega}^{1} x^2 \sqrt{x^2 - \cos^2\Omega}\, dx = \frac{\sin\Omega}{8} + \frac{\sin^3\Omega}{8} - \frac{\cos^4\Omega}{8} \log \frac{1+\sin\Omega}{\cos\Omega}.$$

Then

$$\frac{g_t - g_r}{I_0 B^2 \pi \sin^2\chi} = (1-U)\cos\Omega\sin^2\Omega + U\,\frac{-I+3\,II}{\sin\Omega}$$

$$= (1-U)\cos\Omega\sin^2\Omega - \frac{U}{8}\left[1 - 3\sin^2\Omega - \frac{\cos^2\Omega}{\sin\Omega}(1+3\sin^2\Omega)\log\frac{1+\sin\Omega}{\cos\Omega}\right], \quad (6)$$

$$\frac{g_t}{I_0 B^2 \pi} = (1-U)\left(\frac{4}{3} - \cos\Omega - \frac{\cos^3\Omega}{3}\right)$$

$$+ U\frac{I+II}{\sin\Omega} = (1-U)\left(\frac{4}{3} - \cos\Omega - \frac{\cos^3\Omega}{3}\right)$$

$$+ \frac{U}{8}\left[5 + \sin^2\Omega - \frac{\cos^2\Omega}{\sin\Omega}(5-\sin^2\Omega)\log\frac{1+\sin\Omega}{\cos\Omega}\right]. \quad (7)$$

The expressions (6) and (7) give complete information concerning the light scattered by an electron of the corona. (If from them we calculate the total light from an electron near the limb, putting $\omega = \Omega = \chi = 90^0$, the formulae reduce to that given by PETTIT and NICHOLSON). — They may be written:

$$\frac{g_t - g_r}{I_0 B^2 \pi \sin^2\chi} = (1-U)\,A + U B, \quad (8)$$

$$\frac{g_t}{I_0 B^2 \pi} = (1-U)\,C + U D. \quad (9)$$

The functions A, B, C, D, are tabulated in table 1. It would be of little use to discuss completely the result obtained, because the radiation of one

Table 1.

Ω	A	B	C	D
90°	0	0,250	1,333	0,750
80	0,168	0,275	1,158	0,704
75	0,241	0,292	1,069	0,670
70	0,302	0,304	0,978	0,624
60	0,375	0,311	0,792	0,515
50	0,377	0,283	0,602	0,398
40	0,316	0,225	0,416	0,276
30	0,216	0,148	0,251	0,169
20	0,110	0,074	0,117	0,078
10	0,030	0,018	0,030	0,019
0	0	0	0	0

single volume element is not accessible to observation. Still it may be interesting to calculate (as SCHUSTER does) the fraction $q = \frac{g_t - g_r}{g_t}$ which is closely related to the polarisation $p = \frac{g_t - g_r}{g_t + g_r}$ and increases or decreases together with it. The values of q are given in table 2 for several

Table 2.

	Ω	$U = 0$	$U = 0{,}3$	$U = 0{,}4$	$U = 0{,}5$	$U = 0{,}6$	$U = 0{,}7$	$U = 1$
0′	90°	0	0,065	0,091	0,120	0,152	0,189	0,333
2,5	60	0,473	0,502	0,512	0,524	0,539	0,557	0,604
8,9	40	0,760	0,775	0,779	0,784	0,790	0,796	0,815
25,9	20	0,939	0,941	0,942	0,943	0,944	0,945	0,946
∞	0	1	1	1	1	1	1	1

values of U, corresponding to different wave-lenghts, and for different values of Ω, corresponding to different distances from the sun's centre. For $U = 0$ (the extreme infrared), the results become identical with those of SCHUSTER; for other values of U the polarisation is larger; for $U = 1$ (the extreme ultraviolet), we find the greatest values of the polarisation which are theoretically possible. The difference is considerable near the limb of the sun where the isotropic radiation would give no polarisation and the darkening at the limb makes q to increase up to 0,333; but farther on this difference quickly decreases, and at a distance of about 9′ (corresponding with $\Omega = 40^0$) it amounts hardly to some procents. It is clear indeed, that for a point at a considerable distance from the sun it will make no difference whether the radiation is isotropic or not, the source of light being seen as a point.

In comparing our values for g_r and g_t with these of SCHUSTER, we must take into account that our factor I_0, the *central* intensity, is not the same as his factor $\bar{I}$, which is the *mean* intensity over the disc. The relation between them is given by

$$2\pi I_0 \int_0^{\pi/2} (1 - U + U \cos\psi) \sin\psi \cdot \cos\psi \cdot d\psi = \pi \bar{I},$$

which reduces to

$$I_0 = \frac{\bar{I}}{1 - U/3}. \tag{10}$$

We substitute this in (6) and(7) or in (8)and (9), and compare now the expression

$\frac{g_t - g_r}{\bar{I} B^2 \pi \sin^2\chi} = [(1-U)A + UB]\frac{1}{1-U/3}$ with A (the result of SCHUSTER), and the expression

$$\frac{g_t}{I B^2 \pi} = [(1 - U)\, C + U D]\, \frac{1}{1 - U/3} \text{ with } C. \tag{11}$$

The old and the new results will be identical if

$$B = 2A/3 \quad \text{and if} \quad D = 2C/3;$$

we expect that B and D will converge towards these values as soon as the point P considered is at some distance from the sun. We will write the correct value for all distances in the form:

$$B = 2A/3 + \delta, \quad D = 2C/3 + \varepsilon, \tag{12}$$

where δ and ε are functions of the distance to the sun, which must quickly decrease from the limb outwards.

From (11) and (12) it is seen now that the correct formulae for the scattered radiation may be written:

$$\left.\begin{aligned} \frac{g_t - g_r}{I B^2 \pi \sin^2 \chi} &= A + \frac{U}{1 - U/3}\,\delta, \\ \frac{g_t}{I B^2 \pi} &= C + \frac{U}{1 - U/3}\,\varepsilon. \end{aligned}\right\} \tag{13}$$

The first term is the value of SCHUSTER, the second one is the correction for darkening at the limb (the mean radiation remaining the same).

The calculation of δ and ε from (13) and table 1 gives the following results.

Table 3.

Ω	δ	ε	Ω	δ	ε
90°	0,250	— 0,139	50	0,032	— 0,004
80	0,163	— 0,068	40	0,014	— 0,001
75	0,131	— 0,043	30	0,004	—
70	0,103	— 0,028	20	0,0005	—
60	0,061	— 0,013			

It is clear that the values of δ and ε decrease quickly as we had expected, which proves that SCHUSTER'S approximation is nearly correct as soon as the electron considered is at some distance from the photosphere. At smaller distances the tangential vibration is decreased when the radiation is distributed according to the "darkened limb scheme"; whereas the radial vibration is increased.

The formulae (8) and (9), (11), (13), are equivalent; all of them give the components of the light vibration scattered by one element of the corona towards the earth.

The total radiation scattered by all the electrons in the line of sight. To get a general idea of the polarisation and of the decrease in intensity of the corona as a whole, we first imagine an idealised corona in which the density of the electrons is decreasing according to the n^{th} power of the distance r to the centre of the sun, and write $N = N_0/r^n = N_0 \sin^n \Omega$, N_0 being the number of electrons pro ccm near the sun's limb. By variing n we can obtain various types of corona. We will integrate the radiation scattered by all the electrons along the line OP (Fig. 2), which are seen projected on 1 sq./cm of the corona at a distance from the centre equal to c times the solar radius ϱ; the radius is perpendicular to OP in T, and the distance of every electron from T is measured by the coordinate $h = c\varrho\cot\chi$. The number of electrons in every volume element is then

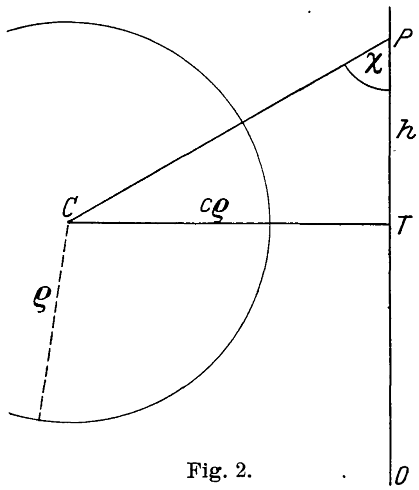

Fig. 2.

$$N\,dh = N_0 c\varrho \frac{\sin^n \Omega}{\sin^2 \chi}\,d\chi.$$

The total intensity of the scattered light with tangential vibration will be called J_t, and that of the light with radial vibration J_r. We have

$$\left.\begin{aligned}\frac{J_t - J_r}{I B^2 \pi N_0 \varrho} &= c\int_0^\pi \left(A + \frac{U}{1-U/3}\,\delta\right)\sin^n \Omega\,d\chi \\ &= 2\,(S_t - S_r) + \frac{U}{1-U/3}\,c\int \delta \sin^n \Omega\,d\chi, \\ \frac{J_t}{I B^2 \pi N_0 \varrho} &= c\int_0^\pi \left(C + \frac{U}{1-U/3}\,\varepsilon\right)\frac{\sin^n \Omega}{\sin^2\chi}\,d\chi \\ &= 2\,S_t + \frac{U}{1-U/3}\,c\int \varepsilon\,\frac{\sin^n \Omega}{\sin^2 \chi}\,d\chi.\end{aligned}\right\} \qquad (14)$$

S_r, S_t are the integrals calculated by SCHUSTER in terms of elliptic functions*.

We determine now the correction by calculating the integrals in δ and ε by mechanical integration. In table 3 δ and ε are given as functions of Ω,

* Our factor 2 arises from the fact that SCHUSTER puts $I B^2 \pi N_0 \varrho = \frac{1}{2}$ and calculates J_t, $J_t - J_r$.

so it is easy to calculate $\delta \sin^n \Omega$ and $\varepsilon \sin^n \Omega$ as functions of Ω, for any value of n. These are transformed into functions of χ by the relation:

$$\sin \Omega = \sin \chi / c.$$

By drawing δ and ε as functions of χ for several values of c and n and integrating the curves, we find the integrals

$$\varDelta (n, c) = c \int_0^\pi \delta \sin^n \Omega \, d\chi, \quad \text{and} \quad E (n, c) = c \int_0^\pi \varepsilon \frac{\sin^n \Omega}{\sin^2 \chi} \, d\chi.$$

Finally the scattered intensity emitted by 1 sq./cm of the corona towards 1 sq./cm on earth, is

$$\left.\begin{aligned} J_t - J_r &= \bar{I} B^2 \pi N_0 \varrho \left[2 (S_t - S_r) + \frac{U}{1 - U/3} \varDelta (c, n) \right], \\ J_t &= \bar{I} B^2 \pi N_0 \varrho \left[2 S_t + \frac{U}{1 - U/3} E (c, n) \right], \end{aligned}\right\} \quad (15)$$

where $B^2 = \dfrac{e^4}{2 R^2 C^4 m^2}$; S is given by SCHUSTER'S and by YOUNG'S tables, $\varDelta$ and E are given by us in table 4*.

Measurements of the polarisation in the corona have been made by visual examination or by photography, but probably always in integrated light and never by means of a spectrograph. It will therefore be sufficient to calculate $J_t - J_r$ and J_t for the two corresponding effective wave-lengths, λ 4300 and λ 5700, for which the darkening coefficients are about $U = 0{,}81$ and $U = 0{,}60$.

From the formulae (15) and table 4 we derive table 5; moreover in table 6 we have given for several distances from the sun the calculated values for the two quantities which are directly determined by observation, the total intensity and the polarisation. It is seen that:

1. The intensity of the scattered light compared with the mean solar radiation of the same wave-length is nearly the same for blue and for yellow light. That was to be expected, the scattering particles being free electrons; the anisotropy of the radiation near the sun, due to darkening at the limb, is different for the different w.-l., but the influence of this on the intensity of the coronal light as a whole is to be neglected. A corona of free electrons will be white. This result was already obtained by WOLTJER for one volume element.

* The values of S_t for $n = 4$ are determined by interpolation from SCHUSTER's values. His number for $\vartheta = 52^\circ_5$ is wrong. The values of YOUNG for S_t and $n = 8$ are quite wrong and have been recalculated; his values for $S_t - S_r$ ($n = 4$) and others are also slightly irregular.

Table 4.

Distance from limb.	$2S_t - 2S_r$			Δ		
	r^{-4}	r^{-6}	r^{-8}	r^{-4}	r^{-6}	r^{-8}
0′	0,286	0,222	0,182	0,1382	0,1284	0,1187
1	0,329	0,247	0,192	0,0667	0,0541	0,0450
2	0,299	0,201	0,140	0,0394	0,0295	0,0214
3	0,254	0,155	0,0968	0,0253	0,0176	0,0113
4	0,213	0,118	0,0666	0,0166	0,0110	0,0060
5	0,178	0,0880	0,0460	0,0113	0,0071	0,0034
6	0,147	0,0676	0,0318	0,0078	0,0045	0,0021
7	0,120	0,0512	0,0216	0,0056	0,0029	0,0012
8	0,100	0,0390	0,0148	0,0040	0,0020	0,0008
9	0,084	0,0300	0,0106	0,0031	0,0011	0,0005
	$2S_t$			E		
0′	1,339	1,133	1,002	— 0,0549	— 0,0535	— 0,0520
1	0,920	0,694	0,534	— 0,0167	— 0,0138	— 0,0116
2	0,678	0,456	0,314	— 0,0079	— 0,0060	— 0,0045
3	0,518	0,306	0,192	— 0,0042	— 0,0029	— 0,0019
4	0,396	0,210	0,1192	— 0,0020	— 0,0011	— 0,0006
5	0,306	0,149	0,0756	— 0,0009	— 0,0004	— 0,0001
6	0,240	0,1062	0,0490	—	—	—
7	0,193	0,0764	0,0326	—	—	—
8	0,166	0,0566	0,0220	—	—	—
9	0,126	0,0428	0,0156	—	—	—

Table 5.

Distance from limb.	$J_t - J_r$ (blue)			J_t (blue)		
	r^{-4}	r^{-6}	r^{-8}	r^{-4}	r^{-6}	r^{-8}
0′	0,439	0,365	0,314	1,278	1,073	0,944
1	0,404	0,307	0,242	0,902	0,679	0,521
2	0,343	0,234	0,164	0,669	0,449	0,309
3	0,282	0,164	0,109	0,513	0,303	0,190
4	0,231	0,130	0,0732	0,394	0,209	0,118
5	0,191	0,0959	0,0498	0,305	0,149	0,0755
6	0,156	0,0726	0,0341	0,240	0,1062	0,0490
7	0,126	0,0544	0,0229	0,193	0,0764	0,0326
8	0,104	0,0412	0,0157	0,166	0,0566	0,0220
9	0,087	0,0312	0,0112	0,126	0,0428	0,0156
	$J_t - J_r$ (yellow)			J_t (yellow)		
0′	0,390	0,318	0,271	1,298	1,093	0,963
1	0,379	0,287	0,226	0,908	0,684	0,525
2	0,329	0,223	0,156	0,672	0,452	0,311
3	0,273	0,168	0,105	0,515	0,304	0,191
4	0,225	0,126	0,0711	0,394	0,209	0,118
5	0,187	0,0933	0,0485	0,305	0,149	0,0756
6	0,153	0,0710	0,0334	0,240	0,106	0,0490
7	0,124	0,0534	0,0224	0,193	0,0764	0,0326
8	0,103	0,0405	0,0154	0,166	0,0566	0,0220
9	0,086	0,0308	0,0110	0,126	0,0428	0,0156

Table 6.

Distance from limb.	$J_t + J_r$ (blue)			$p = \frac{J_t - J_r}{J_t + J_r}$ (blue)		
	r^{-4}	r^{-6}	r^{-8}	r^{-4}	r^{-6}	r^{-8}
0′	2,117	1,782	1,575	0,207	0,205	0,200
2	0,995	0,664	0,454	0,344	0,353	0,362
5	0,419	0,202	0,101	0,456	0,475	0,492
9	0,165	0,054	0,020	0,53*	0,58*	0,56*
	$J_t + J_r$ (yellow)			p (yellow)		
0′	2,207	1,867	1,655	0,176	0,171	0,164
2	1,015	0,681	0,466	0,324	0,327	0,334
5	0,423	0,205	0,103	0,442	0,455	0,470
9	0,166	0,055	0,020	0,52*	0,56*	0,55*

2. The polarisation also shows only a slight dependance on w.-l. Whereas the radiation of a single element of volume was markedly differently polarised according to the w.-l. (see table 2), the difference for the light emitted by all the electrons in the line of sight is at most only 0,03.

3. Plotting the results on double logarithmic paper shows that, roughly speaking, a decrease of the electron density from the limb outward according to r^{-4}, r^{-6}, r^{-8}, corresponds to a decrease in total scattered intensity according to $r^{-5,7}$, $r^{-7,8}$, $r^{-9,7}$.

The analysis of observations. — A mixture of electron scattering and emission of the corona itself. A rough comparison between the numbers for the polarisation, found in the preceding paragraph, and the results of the observations, shows that there is a considerable disagreement between both sets. The polarisation is much less pronounced than it was expected, for in the photographic observations the proportion of polarized light is not more than 37%, and for the visual observations it reaches hardly 11%.

It is clear that the simplest hypothesis is: to assume that the corona does not only scatter the light, but radiates also itself. It would be possible to assume for the emission different functions of r, and to combine them with various possible distributions of the scattering material. SCHUSTER has calculated for some typical cases the intensity and the polarisation of the emerging light.

But it seems more easy to obtain an exact description of the facts by analysing directly the numbers given by the observations.

* Small errors in the last decimal of S_t and $S_t - S_r$ have here a considerable influence on p.

When *the intensity* and *the polarisation* of the corona are observed as a function of the distance from the limb, the problem is quite determined and it is possible to calculate directly *the electron density* and the *emission of the corona itself* in every volume element and a number of other data.

The only consistent set of data available is that published by YOUNG*, from his measurements on coronal photographs made in 1901, 1905 and 1908. It is true that his reduction from photographic density to intensity is not very trustworthy, density markings being entirely lacking on the original plates. Still his data are of the utmost value, and it is important to derive already now the numerous consequences involved in them; a definitive confirmation can only be obtained by new observations with modern methods.

The argument of YOUNG is this: call J_t and J_r the tangential and radial component of the scattered light, $2A$ the emission proper of the corona. The total intensity of the light observed is then $J_t + J_r + 2A$ and the polarisation is $p = \frac{J_t - J_r}{J_t + J_r + 2A}$. The product $p(J_t + J_r + 2A)$ is therefore equal to $J_t - J_r$, and does only involve the scattering; it may be compared at once with our theoretical curves of table 5.

Table 7.
The analysis of YOUNG's *results.*

Distance from limb.	$J_t + J_r + 2A$	p	$J_t - J_r$ calculated	$J_t - J_r$ observed	$J_t + J_r$	$2A$	$2A$ calculated	Density of scattering material	Emission in every volume element
1′	585	0,145	87	85	297	288	288	226	550
2	398	0,230	93	92	265	133	132	300	245
3	269	0,310	82	83	202	67	67	346	99,2
4	168	0,363	58	61	126	42	41	265	45,4
5	108	0,377	40	41	80	28	31	183	30,8
6	74**	0,372	27	27	51	23	24	125	22,7
7	52**	0,368	19	19	29	23	19	85	17,4
8	40**	0,362	15	14	25	15	16	64	13,3
9	30**	0,357	11	11	20	10	13	50	10,2
10	—	—	—	—	—	—	—	39	7,9
11	—	—	—	—	—	—	—	30	6,2
12	—	—	—	—	—	—	—	23	4,9
13	—	—	—	—	—	—	—	17	3,8
14	—	—	—	—	—	—	—	13	3,1
15	—	—	—	—	—	—	—	10	2,5
16	—	—	—	—	—	—	—	7	2,0
17	—	—	—	—	—	—	—	4	1,6

* Lick Obs. Bull. **6**, 166, 1911.

** Owing to some uncertainty in the correction from moon's limb to sun's limb applied by YOUNG, it is possible that the four last numbers of this column are slightly different from the original numbers.

Now it appears that a decrease of the coronal density according to a law r^{-4}, r^{-6}, r^{-8} does not account in a satisfactory way for the observed $J_t - J_r$ curve, although a formula in r^{-6} gives a rough representation of the facts from 2′ outwards. YOUNG gives a development in a series, which he practically reduces to a term in r^{-6} and r^{-8}. In order to obtain a more exact description of the observations, the density distribution in the corona was determined by a trial and error method, beginning from the outside, and progressing step by step towards the limb. After every step, we determine by a graphical integration if the light observed at that distance has the right proportion to the intensities observed at greater distances. In table 7 column 9, the density curve obtained by that method is given in arbitrary units, while in column 4 the intensities $J_t - J_r$ calculated from it are written down. A comparison with the observed values (column 5) shows a good agreement, which could easily have been made more perfect by a little more care if justified by the accuracy of the observations.

The same density curve is used now for the calculation of J_t by graphical integration. From this and $J_t - J_r$ we derive $J_t + J_r$ (column 6), and by substracting from the observed total intensities (column 2) we find the emission $2A$ (column 7).

By the same argument used for the scattered light, we can find now the radiation proper of every ccm of the corona. If $a(r)$ is that radiation, the emitted light $2A$ observed at a distance c from the centre of the sun is

$$2A = 2c\varrho \int_0^{\pi/2} \frac{a}{\sin^2\chi} d\chi = \frac{2\varrho}{c} \int_0^{\pi/2} \frac{a}{\sin^2\Omega} d\chi. \tag{16}$$

We determine again the function $a/\sin^2\Omega$ by a trial and error method so that the integrals for the several distances from the limb are proportional to $2Ac$. From this function, a itself is found. The results are given in table 7, column 10; the emission calculated with these numbers (column 8) is in satisfactory agreement with the numbers $2A$ derived from the observation (column 7)*.

The net result of our analysis is the determination of the density of the scattering and the emitting material in the corona, as tabulated in columns 9 and 10. Outside 9′ the determination is only rough, as observations at large distances from the sun are lacking. Before discussing these results, we will first convert the numbers found into absolute units.

* A. PANNEKOEK and N. W. DOORN have shown how the same problem may be solved by another graphical method, that which Plummer used for the analysis of globular clusters (Verh. Akad. Amsterdam 14, 1930, Nr. 2).

Absolute values concerning the scattering and emission in every element of volume of the corona.

1. The number of scattering electrons. It is easy to convert the number of electrons pro ccm thus obtained into absolute units. We will follow two different methods of calculation.

a) According to PETTIT and NICHOLSON*, the brightness of the corona at a distance of 4,6′ from the limb is equal to $5{,}4 \cdot 10^{-7}$ times the mean brightness of the sun. By interpolating in table 7, we see that at that same distance a total intensity called 128 corresponds to a radiation difference $J_t - J_r$ called 46; the absolute value of $J_t - J_r$ is therefore $5{,}4 \cdot 10^{-7} \cdot 46/128$ times the mean brightness of the sun.

Now $J_t - J_r$ was the light received from 1 sq./cm of the corona; the "brightness of the sun" means therefore also the light from 1 sq./cm of the photosphere; and as the light from the unit solid angle was called $\bar{I}$, we may write:

$$J_t - J_r = 5{,}4 \cdot 10^{-7} \cdot \frac{46}{128} \frac{\bar{I}}{R^2}.$$

We have shown that a number of electrons N per ccm, being a function of the distance from the limb, results in a radiation difference

$$J_t - J_r = 2\,\bar{I} B^2 \pi \varrho c \int_0^{\pi/2} \left(A + \frac{U\delta}{1 - U/3}\right) N \, d\chi.$$

When the absolute value of N is that given in column 9, the absolute values of c times the integral are given in column 5. By multiplying all these electron densities by an unknown factor x, the numbers $J_t - J_r$ must assume their right values. This gives the following equation:

$$2 \cdot 46 \cdot \bar{I} B^2 \pi \varrho x = \frac{46}{128} \cdot 5{,}4 \cdot 10^{-7} \frac{\bar{I}}{R^2},$$

and

$$x = \frac{5{,}4 \cdot 10^{-7}}{B^2 \pi \varrho R^2 \cdot 2 \cdot 128} = \frac{5{,}4 \cdot 10^{-7} \cdot C^4 m^2}{\pi \varrho e^4 \cdot 128} = 2{,}44 \cdot 10^5.$$

It follows that the number of electrons pro ccm at 1′ from the limb is $226 \cdot 2{,}44 \cdot 10^5 = 5{,}50 \cdot 10^7$.

b) More data are available concerning the *total* intensity of the corona, which appears to be equal to about $1{,}0 \cdot 10^{-6}$ the total solar radiation. The intensity of the corona is $2\pi \int (J_t + J_r + 2A)\, r\, dr$. The absolute

* Ap. J. **62**, 202, 1925.

values of $J_t + J_r + 2A$ are obtained just as the absolute values of $J_t + J_r$ and J_t by multiplying the numbers of the table by $2\,\bar{I}\,B^2\pi\varrho\,x$; the integral is computed by graphical means after extrapolation of the J-curve up to the limb, and gives $2\,\pi \cdot 41200 \cdot 2\,\bar{I}B^2\pi\varrho$ over one square minute of arc, or $\left(\frac{R}{3438}\right)^2 2\,\pi \cdot 41200 \cdot 2\,\bar{I}B^2\pi\varrho$ over 1 sq./cm. Equating this to 10^{-6} times the solar radiation (which is seen under an angle $\pi\varrho^2/R^2$), gives

$$2\,\pi \cdot 41\,200 \cdot 2\,\bar{I}\,B^2\pi\varrho\left(\frac{R}{3438}\right)^2 = \bar{I}\cdot\frac{\pi\varrho^2}{R^2}\cdot 10^{-6}.$$

Substituting the value of B^2 and solving for x gives $x = 1{,}8 \cdot 10^5$. This is in satisfactory agreement with the determination according to method a). We have assumed that the coronal radiation is equal to $1{,}0 \cdot 10^{-6}$ the sun's intensity; as a matter of fact, this is precisely the number found by PETTIT and NICHOLSON. The HARVARD observers find a magnitude difference of 15,01 (mean between visual and photographical results) which is again equivalent to a ratio $1{,}01 \cdot 10^{-6}$. If ABBOT'S value is adopted, which is $0{,}75 \cdot 10^{-6}$, x is reduced in the same ratio.

The different values thus found for the number of electrons in 1 ccm at a distance of 1′ from the sun's limb are:

$5{,}50 \cdot 10^7$
4,0
4,0
3,0

(1,9) found by ANDERSON
(4,7) found by WOLTJER (mean up to 3′ from the limb).

The number of ANDERSON, derived from the same datum as our first determination, but without taking into account the anisotropy of the diffusion nor the polarisation, is about a factor 3 smaller. The number of WOLTJER, estimated by a somewhat similar approximation, corresponds very closely to our results, especially if it is noticed that for the first 3 minutes of arc from the limb our numbers are increasing, vid. 4,0; 5,3; 6,1.

For further calculations, we will use as a mean value: $4{,}0 \cdot 10^7$. With this number are calculated the absolute values for all distances from the limb: table 8, column 2.

At a temperature of 5000^0, this would correspond to an electron pressure $P_\varepsilon = 2{,}6 \cdot 10^{-11}$ atmosphere, gradually decreasing to about $1 \cdot 10^{-11}$ atm.

Table 8.

Absolute values concerning the scattering and the emission in the corona.

Distance from limb	Number of scattering electrons pro ccm	Total light scattered by 1 ccm	Emission of 1 ccm compared with emission 1 sq./cm of photo-sphere	Total light emitted by 1 ccm	Total emission / Total scattering	Total emission pro electron
1′	$4{,}00 \cdot 10^7$	$1{,}01 \cdot 10^{-16}\bar{I}$	$1{,}64 \cdot 10^{-17}$	$2{,}05 \cdot 10^{-16}\bar{I}$	2,04	$5{,}12 \cdot 10^{-24}\bar{I}$
2	5,31	1,12	0,730	0,914	0,818	1,72
3	6,13	1,11	0,296	0,370	0,33	0,60
4	4,69	0,74	0,135	0,169	0,23	0,36
5	3,24	0,46	0,092	0,115	0,25	0,36
6	2,21	0,28	0,068	0,085	0,30	0,38
7	1,50	0,17	0,052	0,065	0,38	0,43
8	1,13	0,12	0,040	0,050	0,42	0,44
9	0,88	0,08	0,031	0,038	0,41	0,43
10	—	—	—	—	—	—
11	—	—	—	—	—	—
12	0,40	—	0,015	—	—	—
13	—	—	—	—	—	—
14	—	—	—	—	—	—
15	0,18	—	—	—	—	—
16	—	—	—	—	—	—
17	0,07	—	—	—	—	—

at 7′ from the limb. The electron density at 1′ is $4 \cdot 10^{-20}$; the total density, if the gas is neutral and has an atomic weight 2, is $1{,}5 \cdot 10^{-16}$. This is sufficiently low compared with other estimates.

2. *The light scattered in all directions by every ccm of the corona.* We first compute the total intensity of the sunlight falling on our volume element. This intensity in units I_0 or $\frac{\bar{I}}{1-U/3}$ is given by the integral

$$2\pi\bar{I}\int_0^{\Omega}\sin\omega\,(1-U+U\cos\psi)\,d\omega = 2\pi(1-\cos\Omega)(1-U) + \frac{2\pi U}{\sin\Omega}\int_0^{\Omega}\sin\omega\sqrt{\sin^2\Omega-\sin^2\omega}\,d\omega,$$

which is solved by elementary calculation (so as on p. 5) and gives:

$$2\pi(1-U)(1-\cos\Omega) + 2\pi U\left[1+\frac{\cos^2\Omega}{2\sin\Omega}\log\frac{1-\sin\Omega}{1+\sin\Omega}\right].$$

For $U = 0{,}81$ the following values are found for the total solar radiation on an element of volume at distances r from the limb:

$r = 1' \ldots 3{,}83\,\bar{I}$	$4' \ldots 2{,}42$	$7' \ldots 1{,}71$
$2' \ldots 3{,}20$	$5' \ldots 2{,}14$	$8' \ldots 1{,}56$
$3' \ldots 2{,}75$	$6' \ldots 1{,}91$	$9' \ldots 1{,}43$

Every free electron scatters a fraction

$$\frac{8\,\pi e^4}{3\,m^2 C^4} = 0{,}66 \cdot 10^{-24}$$

of the incident light. From this, the number of electrons at every place in the corona and the intensity of the incident light, we find the light scattered by every element of volume (Table 8, column 3).

3. The light emitted by the corona itself in every ccm. We determine now in absolute value the emission of 1 ccm of the corona itself towards 1 sq./cm on earth, the unit of intensity being the corresponding emission of 1 sq./cm of the photosphere, that is the mean brightness of the sun in integrated white light.

We compare the integral (16) with the brightness at 4,6′ from the limb, just as for our calculation (*b*) of the scattering; however we have reduced the values obtained to the same mean value of the coronal brightness used also for the scattering (see p. 223).

The absolute values of the emission of every ccm of the corona are given in table 8, column 4.

In order to find *the sum total of the light emitted by 1 ccm of the corona in all directions*, we consider that the mean brightness of the sun is $\bar{I}/R^2$; the total radiation of our element at 1′ from the limb is $164 \cdot 10^{-19} \cdot 4\pi R^2 \cdot \bar{I}/R^2 = 2{,}05 \cdot 10^{-16}\,\bar{I}$.

The numbers for other distances are given in our table 8, column 5.

4. The proportion between the light emitted by the corona itself and the light scattered in all directions is found by dividing the numbers of column 5 by these of column 3. The result is found in column 6.

5. The light really emitted per scattering electron is obtained from the columns 5 and 2 and given in column 7.

The numbers $\bar{I}$ correspond to the numbers F_λ calculated by us in an earlier publication*, they are of the order $3 \cdot 10^{14}\,d\lambda$ erg per sec. and per sq./cm; for integrated light, $\bar{I}$ is about 475 cal. per sec. or $1{,}98 \cdot 10^{10}$ erg per sec.

* M. Minnaert, Bull. Astr. Instit. Netherl. 2, 78, 1924; Likewise, the numbers I_λ (1924) correspond to the numbers I_0 used here (1930).

Discussion of the numerical results. — The origin of the coronal continuous spectrum.

Is the scattered light of the corona due to scattering by free electrons or by bound electrons? There is a reason a priori to expect that free electrons must play the chief role in the scattering of the corona. WOLTJER has already remarked that the scattering coefficients for a free and for a bound electron are in the proportion $\left(\frac{\nu^2}{\nu^2 - \nu_0^2}\right)^2$. Laboratory measurements and quantum mechanics show that ν_0 must be in the remote ultra-violet, so that the proportion becomes very small; if therefore free electrons and ions or atoms are intermixed, by far the greatest part of the scattering will be due to the first ones.

Moreover, the fact that the corona is *white* and not *blue* has been considered by SCHWARZSCHILD and others as a proof that the scattering electrons are free, for only such electrons would scatter all wave-lengths in the same proportion. *But this inference is not convincing.* It could be that bound electrons in the corona are scattering chiefly blue light according to RAYLEIGH'S law, but that the emission of the corona itself corresponds to a much lower temperature than that of the sun and gives a preponderance to the red, the sum of the two being nearly white. *A real decision can only be obtained by determining for different w.-l. the products* $p\,(J_t + J_r + 2\,A) = J_t - J_r$ which are truly proportional to the scattering coefficient; if these products are constant, the scattering is due to bound electrons.

Now the results of the observations are these:

a) The total light $J_t + J_r + 2\,A$, compared for every w.-l. with the mean solar spectrum $\bar{I}$ for that same w.-l. is about a constant (LUDENDORFF);

b) at a distance of say 9′, where the polarisation p is a maximum, it amounts to 11% for visual light, to 37% for photographic light.

Thus if the observations are trustworthy, the numbers $J_t - J_r$ for blue and yellow light are in the proportion $37/11 = 3{,}36$; this being about equal to the fourth power of the w.-l. ratio 3,09, the scattering must be due to bound electrons, belonging to ions or atoms.

This result may be somewhat unexpected, but I can't see any escape between the two alternatives: either the theory on the scattering coefficient of free and of bound electrons is inapplicable for some reason or another; or the observations are quite wrong. It is therefore of the utmost importance to determine again the polarisation of the corona for different wave-lengths.

Let us show that ordinary RAYLEIGH-scattering, combined with a low temperature radiation, could really give a white corona. From the observed scattered blue light and RAYLEIGH'S law we calculate the scattered yellow light; from the knowledge that the corona is withe, we know the total yellow light; substracting the two gives the yellow emission; a comparison with the blue emission will give us the temperature. Using dashes for the yellow radiation, the condition for a white corona is written:

$$\frac{J_t + J_r + 2A}{\overline{I}} = \frac{J'_t + J'_r + 2A'}{\overline{I'}}. \tag{17}$$

For RAYLEIGH scattering, we have:

$$\frac{J'_t + J'_r}{J_t + J_r} = \frac{\overline{I'}}{\overline{I}}\left(\frac{\lambda}{\lambda'}\right)^4 (1 + \mu); \tag{18}$$

the small term μ is due to the unequal darkening at the limb; from table 6 (r^{-6}), it is found that μ decreases from about 0,035 at 1′ from the limb to 0,000 at 8′.

ABBOT'S tables give

$$\frac{\overline{I'}}{\overline{I}} = 1{,}02, \quad \left(\frac{\lambda}{\lambda'}\right)^4 = \frac{1}{3{,}088}.$$

Combining the equations (17) and (18), we get:

$$\frac{2A'}{2A} = 1{,}02\left[1 + \frac{J_t + J_r}{2A}\left(1 - \frac{1+\mu}{3{,}088}\right)\right].$$

The numbers $2A$ and $J_t + J_r$ are found in table 7; the emission $2A'$ of every volume element in yellow light is now given in table 9, column 2. An analysis of the curve $2A'$, plotted as a function of the distance from the limb, by the same method followed on p. 221 for the analysis of the curve $2A$, gives the emission of yellow light in every volume element.

Let us assume now that the radiation of the corona itself by some mechanism or the other is distributed according to PLANCK'S law. Then the comparison of the blue emission $2a$ (table 8) and the yellow emission $2a'$ (table 9) gives the temperature of every element of volume. The temperatures thus found are given in table 9, column 4; on the whole they are by no means improbable, diminishing from about 5000 to 3000°; the subsequent increase seems more doubtful and perhaps due to errors of measurement.

Table 9.

Distance from limb	2 A'	Emission in every volume element	Temperature
1′	483	690	4930°
2	311	440	4070
3	202	283	3310
4	126	173	2960
5	82	106	3080
6	57	63	3340
7	42_5	45	3440
8	32	31	3600
9	23_5	21	3810

Concluding, we find that in the observations themselves there is nothing against the assumption that the scattered light of the corona would be due to ions or atoms. There is only the theoretical argument against it, which, however, is very strong.

We have not yet considered the apparently well established fact, that the FRAUNHOFER lines are absent in the inner corona. This may be understood if the scattering particles are free electrons, for then a strong DOPPLER effect will tend to wash out the lines. This argument therefore tends to support the theoretical view attributing the chief role to free electrons.

Is the light emitted by the corona itself due to recombination? As it is difficult to give a really good theory for that part of the coronal light which is ascribed to the emission of the corona itself, we must carefully consider all possible suggestions, and the following possibility may be examined.

WOLTJER has shown that the transitions from one hyperbolic orbit to an other do not contribute appreciably to the temperature radiation. But let us compute the energy liberated in the highly ionised material of the corona by the recombinations between the free electrons and the positive ions, which must be intermixed with them if we assume that the corona as a whole is neutral. To apply KRAMER's formula on these captures might seem very dangerous, for this is ordinarily used only for the stellar interior. But perhaps it makes not so much difference whether the high ionisation is due to temperature or to low pressure. Let us attempt and see the result.

The number of captures per second is*:

$$SN \frac{64\,\pi^4}{3\sqrt{3}} \frac{Z^4 e^{10} f}{C^3 h^4 m V};$$

* A. S. EDDINGTON, The Internal Constitution of the Stars, p. 236, equation (163,1).

multiplying this by $h\nu$ gives the energy liberated per ccm. In this formula N is the number of free electrons as given in table 8; S is the number of ions, and equal to N/q, assuming that the gas is neutral and that every atom is in the q-stage of ionisation. The value of f is uncertain, it would be 7,1 for a K-electron; 2,3 for an L-electron; 1,43 for an M-electron; uncertain is also Z the atomic number of the ions. The velocity V of the electrons at 4000^0 is $3{,}9 \cdot 10^7$ cm/sec. Let us tentatively assume $S = N/2$, $f = 2$, $Z = 2$ (helium). Then the energy liberated per second becomes $4{,}43 \cdot 10^{-23} N^2$ erg per second and per ccm.

At a distance of $1'$ from the limb this must be equal to $2{,}05 \cdot 10^{-16} \bar{I} = 4{,}06 \cdot 10^{-16}$ erg, and so for the other distances. The number N of free electrons per ccm necessary to give this intensity of recombination spectrum becomes:

$1'$	$N = 30 \cdot 10^7$	$4 \cdot 10^7$	scattering electrons
4	$N = 2{,}5 \cdot 10^7$	$4{,}7 \cdot 10^7$	,, ,,
9	$N = 0{,}6 \cdot 10^7$	$0{,}9 \cdot 10^7$	,, ,,

The number of free electrons found giving the recombination spectrum is remarkably close to the number of scattering free electrons determined on p. 223, except perhaps very close to the limb.

This may be interpreted in two ways. We could say that the recombination of the electrons, which must be compensated by a corresponding photoelectric ejection, is only a more special description of the scattering by bound electrons when radiation of higher frequency than the series limit falls in. Then our calculation would prove, that with the special assumptions here adopted the bound electrons would not be much less effective than the free ones.

As this last conclusion seemes theoretically unsatisfactory an alternative explanation is preferable. It could be that a considerable fraction of the electrons is not produced by the radiation, or by collisions with particles in equilibrium with that radiation, but by high speed electrons or ions ejected from the sun. In this case the emission of light by these recombining electrons is not included in the dispersion formulae, but must be estimated as we have done it. The free electrons ejected by the radiation and those ejected by the electron bombardment are alltogether operative in scattering, but only the last ones give a recombination spectrum which could be „the emission of the corona itself“.

There remains the difficulty, that such a recombination spectrum should consist of series of monochromatic lines with a continuum at the end, and not of a real continuous spectrum.

The „emission“ and the „scattering“ in the corona considered as one single process. It is possible that there is no reason to separate the continuous spectrum of the corona in one part which is unpolarised, and another which is scattered with total polarisation under 90°. There are numerous types of scattering where the light is only partly polarised, e. g. the scattering of large solid particles, the RAMAN scattering, the resonance scattering of gases.

If the coronal light is to be attributed to one of these processes, it would be comprehensible at once why the „scattered“ and the „emitted“ part of the radiation are so nearly of the same order of magnitude.

In order to facilitate the comparison between such effects and those in the corona, we will describe for blue light the total continuous radiation coming from every volume element, as simply photospheric radiation scattered for every elementary ray according to a law $a + b \cos^2 \vartheta$, the part $a - b$ being equally strong in all directions and unpolarised, the part $b(1 + \cos^2 \vartheta)$ being anisotropic and polarised so as RAYLEIGH scattering or free electron scattering. This formula is probably a good approximation for any sort of scattering or emission except for very swift electron- or X-radiation.

The first part is found as follows. At 1′ from the limb 1 ccm „emits“ $1{,}64 \cdot 10^{-17} \bar{I}$ towards the unit solid angle (table 6, column 4). For that element, the total incident radiation is $3{,}83\, \bar{I}$ (p. 225). So 1 ccm at 1′ from the limb may be considered as „scattering isotropically“ towards the unit solid angle $1{,}64/3{,}83 \cdot 10^{-17} = 0{,}426 \cdot 10^{-17}$ of every ray of light incident.

Calculating now the second part, we refer to formula (1), according to which the fraction of the incident light „anisotropically scattered“ by N electrons towards the unit solid angle is $\frac{N e^4}{m^2 C^4} \cdot \frac{1 + \cos^2 \vartheta}{2} \cdot$ For 1 ccm at 1′ from the limb we have found (p. 223) that $N = 4{,}00 \cdot 10^7$. That gives for the scattered fraction:

$$4{,}00 \cdot 10^7 \cdot 0{,}0784 \cdot 10^{-24} \frac{1 + \cos^2 \vartheta}{2} = (0{,}156 + 0{,}156 \cos^2 \vartheta) \cdot 10^{-17}.$$

The coefficients a and b of the general scattering formula $a + b \cos^2 \vartheta$ become now $a = 0{,}156 + 0{,}426 = 0{,}582$, and $b = 0{,}156$. The corresponding coefficients for other distances from the limb are found by the same method and are given in table 10. The last column of this table shows how the quotient a/b is decreasing from the limb outward till a more or less constant value is reached.

Table 10.

The radiations of the corona considered as scattering of the type $a + b \cos^2 \vartheta$.

Distance from limb	a	b	a/b
1′	$0{,}582 \cdot 10^{-17}$	$0{,}156 \cdot 10^{-17}$	3,74
2	0,435	0,207	2,10
3	0,347	0,239	1,45
4	0,239	0,183	1,30
5	0,169	0,126	1,34
6	0,122	0,086	1,42
7	0,089	0,059	1,51
8	0,070	0,044	1,59
9	0,057	0,035	1,63

We will not venture to give an hypothesis concerning the peculiar kind of scattering process for which the coefficients a and b would have the values here derived from the observations. Every hypothesis of that kind will have to account also for the fact that a and b are independent of the wave-lenght.

Is the continuous light due to Raman scattering? This hypothesis has been raised lately by MECKE and WILDT*. It would account at once for the fact that the polarisation is much less than RAYLEIGH's formula gives, and the scattering and the emission could be considered as one single process. After a discussion of that possibility together with Dr. PLACZEK, who was so kind to give me very valuable information concerning the theory of the RAMAN-effect, I come to the conclusion that the intensity of this effect would be too small compared with the other types of scattering. It is true that direct measurements of the RAMAN intensities in the laboratory have been made only in the case of vibrating and rotating molecules, not for jumping electrons. But the fact that the RAYLEIGH scattering has been observed several times in monoatomic gases (as they may occur on the sun), and that the RAMAN scattering has been constantly overlooked or been below the threshold of the measuring instrument, seems to prove that it is at least several times smaller than the ordinary RAYLEIGH scattering; it could at most give a small contribution

* ZS. f. Phys. **59**, 503, 1930.

to the coronal light. Moreover, there remains the fact that scattering by free electrons must be much more important than any type of scattering by bound electrons.

Another question arises: call ν_r the frequency of the (in general forbidden) transition in the atom, which is added to or substracted from the incident radiation ν. Now where is this frequency ν_r in the spectrum? MECKE and WILDT ascribe the line spectrum of the corona to the effect of frequencies in the extreme ultraviolet; but this is impossible for the continuous spectrum: unexcited atoms give no RAMAN lines if $\nu < \nu_r$, excited atoms would give a spectrum in the remote ultraviolet of frequency $\nu_r - \nu$. It becomes then necessary to postulate the existence of hypothetical frequencies in the red or infrared.

Then we have to consider the intensity distribution of the scattered light over the spectrum. MECKE and WILDT assert that „on thermodynamic grounds" it would be the same as for the incident light. It is difficult to understand how such a conclusion could be reached; it is true that in thermodynamic equilibrium with black body radiation the RAMAN light scattered out of an incident ray is exactly compensated by the RAMAN light scattered into the ray other wave-lengths. But the case of the corona differs in two respects from the equilibrium conditions.

1. The corona is only observed sideways, and so only the light scattered out of the incident beam is measured; this may be quite different in composition from the incident light. Is is the same case as for the light scattered sideways by RAYLEIGH scattering, which will be different from the incident light, although in *equilibrium* conditions both are the same.

2. The excitation of the atoms in the corona will certainly not correspond to a temperature of 5740°, which is the temperature of the incident radiation, but will be much lower, perhaps 4000° or still less.

It is difficult to say how the scattered light will be changed, compared with the incident light, because we have no information on the kind of scattering atoms or ions. To a certain extent the two deviations from thermodynamic equilibrium could balance; suppose as a limiting case that the scattering atoms have strong lines in the remote ultraviolet, the incident radiation being ordinary light, then the first effect would mean that the scattered light becomes nearly proportional to ν'^4 and gives a blue corona. But the second effect would make every anti-STOKES line, compared with the STOKES line, weaker than in temperature equilibrium, and this gives a reddening. One would think that the first effect predominates, but exact calculations are nearly impossible.

The dynamics of the corona and the age of the sun. The dynamics of the corona are not considered in this paper. ANDERSON has shown* that the corona cannot be in equilibrium if the ordinary physical laws are valid. Instead of assuming, as he does, that very hypothetical modified laws must be applied, we may attempt to account for the corona by assuming that it really is not in equilibrium, and that its particles are continually projected towards space. The question arises if this loss of material is consistent with the age of the sun. Now the DOPPLER displacement of the FRAUNHOFER lines in the outer parts of the corona amounts to 26 km/sec at 20′ from the limb. From our calculations we estimate that the number of electrons there is about 10^6. This gives a stream of $1{,}3 \cdot 10^{13}$ electrons per second through every sq. cm of the solar surface**. It represents a mass of 10^{-14} g if only electrons are escaping, $2 \cdot 10^{-11}$ g or more if the electrons are accompanied by positive ions, for one year $3 \cdot 10^{-7}$ g or $6 \cdot 10^{-4}$ g. Now under every sq./cm of the solar surface there is a mass of $3 \cdot 10^{10}$ g. After 10^{12} years, which may be the order of magnitude of the age of the sun, only 10^{-5} or $2 \cdot 10^{-2}$ of the mass would have escaped. It is possible that the escape has been much more rapid in earlier stages of the sun's developement, but the mass available then was more considerable than it is now.

Our conclusion is that this argument gives no decision between the hypothesis of a neutral corona or of a pure electron corona.

The scattering of starlight by an electron corona. In a short note***, EINSTEIN has emitted some doubt on the possibility of a free electron corona, on account of the "absorption" which starlight would undergo in it. We are now in a position to calculate quantitatively this absorption, which as a matter of fact is only the loss which the beam of light suffers by scattering. Every electron scatters a fraction $\frac{8 \pi e^4}{3 m^2 C^4} = 0{,}66 \cdot 10^{-24}$ of the light incident on a sq./cm, and the number of electrons N at any distance from the limb is given in table 8, which may be extrapolated for a distance 0. The total number of electrons in a column of 1 sq./cm cross section, tangential to the sun's limb, is therefore cf. (16):

$$2 c \varrho \int_0^{\pi/2} \frac{N}{\sin^2 \chi} d\chi = 2 \varrho \int_0^{\pi/2} \frac{N}{\sin^2 \omega} d\omega = 7{,}3 \cdot 10^{18}.$$

* ZS. f. Phys. **35**, 770, 1926.

** W. ANDERSON, making a similar calculation, finds about 4 times this number (ZS. f. Phys. **41**, 70, 1927).

*** A. N. **219**, Nr. 5233.

And the extinction of starlight amounts to

$$0{,}66 \cdot 10^{-27} \cdot 7{,}3 \cdot 10^{18} = 4{,}8 \cdot 10^{-6}.$$

This is negligibly small, and can play no role in the observations. On the contrary, a small fraction of the same order scattered from the extremely intense photospheric light and seen against a dark background is quite clearly visible. This explains at once why the scattering of sunlight by the corona is so important, while the scattering of starlight is to be neglected.

It is to be noticed, that, if the scattering is due to atoms and ions instead of free electrons, this result would be unchanged. The scattering coefficient being smaller, more particles would be necessary to account for the observed brightnesss of the corona, and this would precisely compensate for the fact that every particle scatters less of the starlight.

The deflection of starlight by refraction in the corona. ANDERSON has inquired whether possibly the observed EINSTEIN effect could be explained by refraction of the starlight in an electron corona. Now EINSTEIN and PAGE have pointed out that an electron gas would have an index of refraction smaller than one, and should therefore give precisely the reverse effect*. However, there remains to investigate if this reverse effect is appreciable; if it is, after substraction of this correction the observed EINSTEIN effect would be greater than the theoretical prediction.

We consider first the case of a pure electron corona, of which the refractive power is given by

$$n - 1 = -2\pi \frac{N e^2}{m \nu^2} = \frac{N e^2 \lambda^2}{2\pi m} = 8{,}3 \cdot 10^{-23} N,$$

for $\lambda = 0{,}4300$ mm. From table 8, col. 2, we find the values of N: in the interval from 3′ to 9′, it decreases about as $r^{-7,1}$, and the refractive power is found to be

$$n - 1 = 1{,}25 \,.\, 10^{-14} \,.\, r^{-7,1},$$

the solar radius being the unit in which r is measured. We take rectangular coordinates with the origin in the centre of the sun (Fig. 3). The ray of light from the star is parallel to the y-axis and passes at a distance x from the sun's centre. In a point P of the ray, corresponding to an angle $\vartheta = xOP$ at the origin, the steepest gradient of the density is directed according to OP, and the component of the gradient normal to the ray of light is

$$n' = \frac{dn}{dr} \cos\vartheta = -8{,}9 \cdot 10^{-4} \cdot r^{-8,1} \cos\vartheta.$$

* A. N. **219**, Nr. 5233; **220**, 205; **221**, Nr. 5300, 329, 1924.

The radius of curvature ϱ of the ray is then given by the wellkwown expression $1/\varrho = n'/n$, where n may be put equal to 1. The deflection of the ray from the y-direction will be called φ; its increase near P is then

$$d\varphi = \frac{dy}{\varrho} = \frac{1}{\varrho}\,\frac{X}{\cos^2\vartheta}\,d\vartheta = -\frac{8{,}9\cdot 10^{-14}X}{r^{8{,}1}\cos\vartheta}\,d\vartheta = -\frac{8{,}9\cdot 10^{-14}(\cos\vartheta)^{7{,}1}}{X^{7{,}1}}\,d\vartheta.$$

And the total deflection is given by

$$\varphi = \frac{8{,}9\cdot 10^{-14}}{X^{7{,}1}}\int\limits_{-\pi/2}^{+\pi/2}(\cos\vartheta)^{7{,}1}\,d\vartheta.$$

The integral is equal to

$$\frac{2\pi}{2^{8{,}1}}\cdot\frac{\Pi(7{,}1)}{\Pi(3{,}55)^2} = 0{,}90;$$

and at a distance of 3′ from the limb, where the steepest gradient begins, φ comes out $2{,}3 \,.\, 10^{-14} = 5'' \,.\, 10^{-9}$.

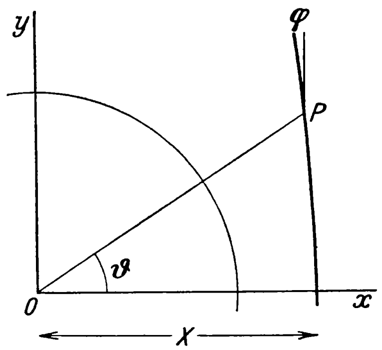

Fig. 3.

So it is shown that an electron density sufficient to account for the light scattered by the corona, is much too small to give any appreciable deflection to the rays of a star, especially because the density gradient is so small.

As yet we have only considered the case of a corona in which the scattering is due to free electrons. Let us see how things would be if the scattering is to be attributed to bound electrons only, ν_0 being their own frequency and ν that of the incident light. The scattering coefficients for bound and for free electrons are in the proportion

$$\frac{\sigma_b}{\sigma_f} = \left(\frac{\nu^2}{\nu_0^2 - \nu^2}\right)^2,$$

and the numbers of electrons N wanted to give the observed scattering are in the reverse proportion. The refractive powers $(n - 1)$ are proportional to $\frac{N}{\nu_0^2 - \nu^2}$, so

$$\frac{(n-1)_b}{(n-1)_f} = -\left(\frac{\nu_0^2 - \nu^2}{\nu^2}\right)^2 \frac{\nu^2}{\nu_0^2 - \nu^2} = -\frac{\nu_0^2 - \nu^2}{\nu^2} = 1 - \left(\frac{\nu_0}{\nu}\right)^2.$$

The fraction ν_0/ν is certainly greater than 1, probably of the order 10.

The deflection by a corona composed exclusively of atoms and ions whit their bound electrons, would thus be greater than that of a corona in which free electrons play the principal role, and probably by a factor of the order 100. But this is still much too small to give a perceptible deflection.

I owe thanks to Prof. Dr. L. S. ORNSTEIN for his kind interest in this paper, and for his suggestion about the possibility of explaining the continuous emission of the corona as a recombination spectrum.

Utrecht, Heliophysical Institute of the Physical Laboratory, May 1930.