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COMMUNICATION FROM THE OBSERVATORY AT LEIDEN.

Epochs of Minimum of AB Andromedae, by *P. Th. Oosterhoff.*

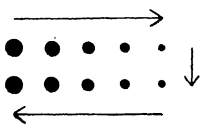
The variability of this star has been discovered by GUTHNICK and PRAGER. They found that the variable is of the W Ursae Majoris type, the period being $^{d}\cdot 33190$ [1]).

At present about one dozen of variable stars of this type are known. From the form of the light-curves, even when accurately observed, it is practically impossible to derive even approximately the shape of the system. Generally the difference between the extréme possible proportions of the two bodies is rather large. It therefore seems that greater accuracy of the photometry of these systems, will not increase our knowledge of their form considerably. On the other hand, the rapid variation of their brightness allows accurate determination of epochs of minimum, so that interesting information as to variation of period may eventually be found within a reasonable time. For this reason I have arranged my observations of this and other objects of the same kind with the particular aim of obtaining epochs of minimum as accurately as possible.

The observations have been made with the 34 cm. photographic refractor of this observatory, the plates used are Eastman 40, 16 cm. × 16 cm., the time of one exposure being 5 min., the time between two exposures half a minute. All observations of one minimum were made on the same plate.

It is evident that a determination of the epoch of minimum depends essentially on the comparison of observations made nearly at the same brightness on the descending and ascending branch of the lightcurve. As shown in figure 1, the images which are equidistant in time are placed in two rows in such a way, that the turning point is chosen as close to the minimum as possible. The images on the comparison of which the determination of the epoch of minimum depends are thus found near to each other, which, in view of the differences in sensitiveness of the photographic plate in different parts, is an important advantage.

FIGURE 1.



Six minima have been observed in this manner, the plates having the following numbers: 1511, 1515, 1518, 1525, 1533 and 1536. During each minimum 23 exposures were made, with the exception of plate 1515, where the last two exposures were prevented by clouds. Three other plates have been taken with a grating in front of the objective, giving an adopted difference in brightness of $^{m}\cdot 92$ between the central and the first order images. The time of exposure of these three plates, was 30, 45 and 45 min. respectively. From these plates the differences in magnitude between the comparison stars was determined. If all exposures could be considered as taken under the same conditions, atmospheric transparency, seeing, etc. no comparison stars would be needed. Practically this is not the case, at least not in the climate of Leiden. Though, as it appears from the above, there is no reason for using an exact scale of magnitude for the simple determination of the epoch of minimum, the use of such a scale is of some interest to determine a fair mean lightcurve near the minimum by the aid of a number of observed minima.

The comparison stars used are:

star	B. D.	Δm
		m
a	+ 36° 5023	
b	+ 35 4972	·57
c	+ 35 4975	·89
e	anonyma	1·33

Δm has been taken relatively to the brightness of comparison star *a*. Star *e* is 8'·5 following and 17'·6 south of the variable.

The plates have been measured in the Schilt microphotometer. The reduction of these measurements into differences in brightness has been made graphically, comparing each exposure of the variable with the exposures of the comparison stars taken simulta-

[1]) *Kleinere Veröffentlichungen der Universitätssternwarte zu Berlin-Babelsberg* (Nr. 4, 1927). As far as I know no other publications concerning this star have appeared up to now.

TABLE 1.

J. D. H. M. T. Gr. — 2420000	Phase	Δm between AB And. and comp. star *a*.	J. D. H. M. T. Gr. — 2420000	Phase	Δm between AB And. and comp. star *a*.	J. D. H. M. T. Gr. — 2420000	Phase	Δm between AB And. and comp. star *a*.
plate *1511*		m	plate *1515*		m	plate *1518*		m
5497·43606	·8666	+ ·22	5498·43682	·8820	+ ·23	5502·41559	·8703	+ ·24
·43987	·8781	·27	·44063	·8935	·32	·41939	·8818	·29
·44368	·8896	·31	·44444	·9049	·36	·42320	·8932	·30
·44749	·9010	·35	·44825	·9164	·40	·42701	·9047	·39
·45130	·9125	·39	·45206	·9279	·56	·43082	·9162	·44
·45511	·9240	·45	·45586	·9393	·61	·43463	·9277	·53
·45892	·9355	·54	·45967	·9508	·69	·43844	·9392	·64
·46273	·9470	·65	·46348	·9623	·81	·44225	·9506	·70
·46654	·9584	·73	·46729	·9738	·92	·44606	·9621	·78
·47035	·9699	·82	·47110	·9853	1·01	·44987	·9736	·90
·47415	·9814	·89	·47491	·9967	·98	·45368	·9851	·97
·47796	·9929	·92	·47872	·0082	·96	·45748	·9965	·94
·48177	·0043	·98	·48253	·0197	·91	·46129	·0080	·98
·48558	·0158	·88	·48634	·0312	·83	·46510	·0195	·97
·48939	·0273	·84	·49015	·0427	·75	·46891	·0310	·86
·49320	·0388	·76	·49395	·0541	·65	·47272	·0424	·78
·49701	·0503	·68	·49776	·0656	·58	·47653	·0539	·66
·50082	·0617	·61	·50157	·0771	·48	·48034	·0654	·58
·50463	·0732	·50	·50538	·0886	·47	·48415	·0769	·48
·50844	·0847	·40	·50919	·1000	·35	·48796	·0884	·42
·51224	·0961	·37	·51300	·1115	·32	·49177	·0998	·39
·51605	·1076	·31				·49557	·1113	·28
·51986	·1191	·25				·49938	·1228	·22
plate *1525*		m	plate *1533*		m	plate *1536*		m
5503·40941	·8648	+ ·22	5504·41016	·8801	+ ·34	5506·39780	·8690	+ ·16
·41322	·8762	·29	·41397	·8916	·37	·40161	·8805	·23
·41703	·8877	·34	·41777	·9030	·41	·40541	·8919	·29
·42084	·8992	·41	·42158	·9145	·49	·40922	·9034	·34
·42464	·9107	·41	·42539	·9260	·50	·41303	·9149	·42
·42845	·9221	·48	·42920	·9375	·57	·41684	·9264	·45
·43226	·9336	·62	·43301	·9490	·70	·42065	·9379	·55
·43607	·9451	·66	·43682	·9604	·78	·42446	·9493	·65
·43988	·9566	·79	·44063	·9719	·90	·42827	·9608	·76
·44369	·9681	·90	·44444	·9834	·95	·43208	·9723	·81
·44750	·9795	·89	·44825	·9949	·99	·43589	·9838	·91
·45131	·9910	·95	·45206	·0063	1·04	·43970	·9953	·93
·45512	·0025	·94	·45586	·0178	·93	·44350	·0067	·98
·45893	·0140	·95	·45967	·0293	·91	·44731	·0182	·93
·46273	·0254	·98	·46348	·0408	·76	·45112	·0297	·87
·46654	·0369	·85	·46729	·0522	·63	·45493	·0411	·78
·47035	·0484	·74	·47110	·0637	·60	·45874	·0526	·63
·47416	·0599	·62	·47491	·0752	·51	·46255	·0641	·54
·47797	·0713	·55	·47872	·0867	·43	·46636	·0756	·41
·48178	·0828	·44	·48253	·0982	·38	·47017	·0871	·37
·48559	·0943	·35	·48634	·1096	·35	·47398	·0985	·27
·48940	·1058	·30	·49015	·1211	·27	·47779	·1100	·26
·49321	·1173	·25	·49395	·1326	·28	·48159	·1215	·23

neously. Figure 2 shows the six minima obtained in this manner. Abscissae are phases, calculated by the formula:

$$\text{phase} = 3{\cdot}013076\,(\text{J. D. H. M. T. Gr.} - 2425497{\cdot}48033) + 2221$$

2221 is the number of periods in the interval between the epoch given by GUTHNICK and PRAGER and the mean epoch of the Leiden observations. Ordinates give Δm relatively to comparison star a.

In Table 1 the J. D., the phase calculated from the above formula, and the Δm of each exposure of the variable, are given. The reduction to the sun has been applied throughout.

To find the epoch of each minimum, the method, explained by HERTZSPRUNG in *B. A. N.* 147 p. 179 has been used. Since the observed points of the lightcurve are equidistant in time in this case, it is not necessary to draw a curve through the observed points. If we assume as approximate phase of minimum the phase of an observed point, or the mean phase of two consecutive observed points, of the lightcurve, the observed values of the magnitude of the variable may be arranged in pairs symmetrical to this approximate phase. Then the sum of the squares of the differences in magnitude within each pair is formed.

For each of the six minima of AB And. six sums of squares have been calculated, the assumed approximate phases of minimum being alternately the phase of an observed point and the mean phase of two neighbouring observed points of the lightcurve. The phase of the minimum was then found by the fourth formula given below. The sums of squares are indicated by y in these formulae and the indices to those y's have been taken symmetrically. The phase of the minimum is then given by:

$$\text{the phase of } y_0 + x_{(y=\text{min.})},$$

the unit of x being the distance in phase between two consecutive y's. For use in other cases the formulae for 3, 4, 5 and 7 sums are also given. The formulae have been derived, supposing the sum of the squares to be a quadratic function of the assumed phase of minimum. The constants of this quadratic function have been derived by the method of least squares.

3 sums:

$$x_{y=\text{min.}} = \frac{1}{2}\cdot\frac{y_{-1}-y_{+1}}{y_{-1}-2y_0+y_{+1}}$$

4 sums:

$$x_{y=\text{min.}} = \frac{1}{5}\cdot\frac{3y_{-1\frac{1}{2}}+y_{-\frac{1}{2}}-y_{+\frac{1}{2}}-3y_{+1\frac{1}{2}}}{y_{-1\frac{1}{2}}-y_{-\frac{1}{2}}-y_{+\frac{1}{2}}+y_{+1\frac{1}{2}}}$$

5 sums:

$$x_{y=\text{min.}} = \frac{7}{10}\cdot\frac{2y_{-2}+y_{-1}-y_{+1}-2y_{+2}}{2y_{-2}-y_{-1}-2y_0-y_{+1}+2y_{+2}}$$

6 sums:

$$x_{y=\text{min.}} = \frac{4}{5}\cdot\frac{5y_{-2\frac{1}{2}}+3y_{-1\frac{1}{2}}+y_{-\frac{1}{2}}-y_{+\frac{1}{2}}-3y_{+1\frac{1}{2}}-5y_{+2\frac{1}{2}}}{5y_{-2\frac{1}{2}}-y_{-1\frac{1}{2}}-4y_{-\frac{1}{2}}-4y_{+\frac{1}{2}}-y_{+1\frac{1}{2}}+5y_{+2\frac{1}{2}}}$$

7 sums:

$$x_{y=\text{min.}} = \frac{3}{2}\cdot\frac{3y_{-3}+2y_{-2}+y_{-1}-y_{+1}-2y_{+2}-3y_{+3}}{5y_{-3}-3y_{-1}-4y_0-3y_{+1}+5y_{+3}}$$

As an example, the determination of the epoch of the minimum taken on plate 1511, is given below.

The values of Δm, found for this plate and given in Table 1, are arranged in the following manner.

		difference			diff.			diff.			diff.			diff.			diff.
·31	·40	·09	·31	·37	·06	·35	·37	·02	·35	·31	·04	·39	·31	·08	·39	·25	·14
·35	·50	·15	·35	·40	·05	·39	·40	·01	·39	·37	·02	·45	·37	·08	·45	·31	·14
·39	·61	·22	·39	·50	·11	·45	·50	·05	·45	·40	·05	·54	·40	·14	·54	·37	·17
·45	·68	·23	·45	·61	·16	·54	·61	·07	·54	·50	·04	·65	·50	·15	·65	·40	·25
·54	·76	·22	·54	·68	·12	·65	·68	·03	·65	·61	·04	·73	·61	·12	·73	·50	·23
·65	·84	·19	·65	·76	·11	·73	·76	·03	·73	·68	·05	·82	·68	·14	·82	·61	·21
·73	·88	·15	·73	·84	·11	·82	·84	·02	·82	·76	·06	·89	·76	·13	·89	·68	·21
·82	·98	·16	·82	·88	·06	·89	·88	·01	·89	·84	·05	·92	·84	·08	·92	·76	·16
·89	·92	·03	·89	·98	·09	·92	·98	·06	·92	·88	·04	·98	·88	·10	·98	·84	·14
				·92						·98						·88	
$y_{-2\frac{1}{2}} = ·2654$			$y_{-1\frac{1}{2}} = ·0941$			$y_{-\frac{1}{2}} = ·0138$			$y_{+\frac{1}{2}} = ·0179$			$y_{+1\frac{1}{2}} = ·1222$			$y_{+2\frac{1}{2}} = ·3169$		

The difference in magnitude within each pair of observed points, is given in the third column and the y's have been calculated by forming the sums of the squares of these differences. Substituting the values of y in the formula for 6 sums, we find:

$$x_{y=\text{min}} = \frac{4}{5}\cdot\frac{5\times·2654+3\times·0941+·0138-·0179-3\times·1222-5\times·3169}{5\times·2654-·0941-4\times·0138-4\times·0179-·1222+5\times·3169} = -·1077$$

FIGURE 2.