small quantity, and this requires an approximation to spontaneous generation of energy for which α and β both vanish, at least very approximately. The whole discussion now proceeds precisely as before. The effective condition which discriminates between stable and unstable configurations is condition (3), which remains unaltered from the original investigation.

The Rotation of the Galaxy. By J. S. Plaskett, F.R.S.

The theory of a rotation of the galactic system recently developed by Lindblad * in a series of papers constitutes an important and interesting hypothesis which serves to explain some of the observed stream and other motions in the stellar system. According to this hypothesis, the galactic system is composed of a number of sub-systems, each in approximately dynamical equilibrium, and each rotating about a common axis perpendicular to the galactic plane. The system with the highest speed of rotation, assumed to be that containing the Milky Way clouds, will be the most flattened towards the plane, will have a high star density and small residual velocities, so that the orbital motion will be nearly circular. Other sub-systems, all supposed to extend to about the same boundary in the plane, with a lower speed of rotation will be less flattened and have higher residual velocities. The globular cluster system, nearly spherical, is an example of a slowly rotating sub-system and, as is well known, has high residual velocities, while the sun and surrounding stars lie in a flattened system with a high speed of rotation. The asymmetry in stellar velocities observed by Stromberg † can be explained satisfactorily on this hypothesis, which requires the direction of the axis of asymmetry, $61^\circ{\cdot}5 \pm 5^\circ$ galactic longitude according to Stromberg, to be at right angles to the direction of the centre of rotation, hence also of the centre of the system of globular clusters which lies in longitude $325^\circ \pm 3^\circ$, the deviation being within the limits of error of the determinations. Kapteyn's stream motions are also explained, though not so directly, and the hypothesis, though still offering many difficulties, seems a very promising one.

The question has, however, been placed on a much firmer basis by two recent important papers by Oort ‡ and a third by Schilt, § which strongly confirm the presence of a rotational effect in the galaxy by an analysis of the observed radial velocities and proper motions of distant stars. The simple method of Oort for dealing with the radial velocities of distant galactic stars, which will be used in this paper, may be briefly summarised. Consider a star or group of stars at a distance r from the sun, and let the sun be at a distance R from the centre of the

* *Arkiv f. Mat.*, etc., Bd. 19A, Nos. 21, 27, 35; 19B, No. 7; and *M.N.*, **87**, 553.

† *Ap. J.*, **59**, 228 and **61**, 363.

‡ *B.A.N.*, No. 120, 132.

§ *Proc. Nat. Ac. Sc.*, **13**, 642.

galactic system, and suppose that $\frac{r}{R}$ is so small that terms of the second and higher orders may be neglected. The gravitational force K due to the whole system is obviously directed towards the centre and is some function of the distance R to the centre. The circular velocity of rotation at the sun will be

$$V = \sqrt{RK}.$$

It can be readily shown by a simple geometrical construction that the residual radial velocity, ρ', in the galactic plane caused by this rotation is

$$\rho' = rA \sin 2(l - l_0),$$

where l is the galactic longitude of the star, l_0 of the centre of the system, and where A has the value

$$A = \frac{V}{4R}\left(1 - \frac{R}{K}\frac{\delta K}{\delta R}\right).$$

" A " may be otherwise defined as the residual radial component, in kilometres per second due to the galactic rotation, for stars at a distance of one parsec. The relation may be stated more generally for stars at all latitudes by the expression

$$\rho = V_0 \cos \lambda + rA \sin 2(l - l_0) \cos^2 b,$$

where ρ is the radial velocity of the star, b its galactic latitude, V_0 the velocity of the sun towards, and λ the distance from, the solar apex for the group of stars considered. By means of this equation and a similar one for the proper motions, Oort was able to show that the distant stars exhibited a very definite residual effect which is most probably and simply explained as due to a rotation of the galactic system.

In view of the interest and importance of the matter, it seems worth while to present some further evidence on a galactic rotation. In this connection I am indebted to a suggestion by Dr. Schlesinger to observe here the radial velocities of a number of faint " *c* " stars, probably very suitable for testing the matter, and which is being undertaken. In the meantime, however, it seemed that the velocities of the fainter B0 to B5 stars, which are under investigation here by Mr. J. A. Pearce and the writer, would be useful in testing the rotational effect. While the investigation of the B stars, which includes all those of classes B0 to B5 brighter than visual magnitude 7·5 and north of − 11°, is still incomplete there are available, with the kind co-operation of Mr. Pearce, the final radial velocities of 250 B0 to B5 stars and the probable velocities of 140 more. With the addition of the velocities previously determined for the O to B5 stars, there are available, in the region of the sky within observational reach at Victoria, which includes the galactic region between longitudes from about 335° to 205°, the radial velocities of 610 O to B5 stars.

In the course of the investigation of the B stars, it has been found that some 25 stars classified as B in the Henry Draper Catalogue are in reality absorption line O-type, and consequently, with this new material, it was decided to determine the rotation effect from both B and O stars. The positions of these 610 stars were transformed to galactic co-ordinates, and for this purpose and for obtaining the distance to the apex a set of graphs recently computed and drawn by Messrs. Pearce and Hill, and now being published, much reduced the labour. The positions were tabulated in the order of galactic longitude, and all stars whose galactic latitude was greater than 30°, some 45 in number, were rejected. The numbers were further reduced by the rejection of all O stars with a radial velocity greater than 60, and all B stars of a greater velocity than 50 km. per second. This seemed desirable in view of the relative smallness of the peculiar velocities, as one large velocity would have a greater effect in the solutions than five to ten average velocities. The question of group velocities in the B stars offered some difficulty, but it was finally decided, in view of the fact that the group motion is generally in the direction and of the order of the rotation effect, that only the Perseus Clusters need be considered. The velocities are high, about − 45 km. per second, and there are a considerable number in a small region. Finally rather more than half of these high-velocity group motion stars were rejected, leaving not more than one cluster star in the smaller class and magnitude groups and not more than two in the larger groups.

There remained for determining the galactic rotation the radial velocities of 65 O-type stars, of 132 B0 to B2 stars, and of 352 B3 to B5 stars, of which probably 90 per cent. are within 15° of the galactic plane and none at a greater distance than 30°. The distribution in galactic longitude between the limits observable here, 335° to 205°, is only moderately uniform, as the B stars are considerably less numerous between the longitudes of 80° and 150°. Further, the southerly longitudes are entirely unrepresented and the solutions are consequently subject to this disadvantage, partially compensated, however, by the fact that the longitudes available include the two positive maxima of the rotation effect at longitudes 10° and 190°, with one intervening negative maximum at longitude 100°.

From preliminary work, it was seen that the main factor for satisfactory results was a large number of stars in each group so as to reduce the effect of one or two large or discrepant peculiar velocities. Consequently, it was decided to use a relatively small number of groups in galactic longitude, eight being finally chosen, and to select these longitude regions so that the number of stars in each group rather than the longitude interval was nearly equal. The B-type stars were divided into two magnitude groups of nearly equal numbers, 243 stars $<6{\cdot}21$ vis. mag. and 241 $>6{\cdot}20$, and the O-type stars into 33 stars $<6{\cdot}51$ and 32 $>6{\cdot}50$. The B-type stars were also divided into two spectral divisions, thus forming two groups, bright and faint, of B0 to B2 stars and two groups of B3 to B5 stars, thus making four groups of B stars and two of O stars. The longitude intervals, the number of stars, the mean longitude, and the mean observed residual radial velocity are shown in the following tables :—

TABLE I.

Group Divisions of B Stars.

Limits of l.	Bo–B2 < 6·21. No.	$\bar{l}$.	$\bar{\rho}'$.	Computed ρ'. l=325°.	l=332°·7.	B3–B5 < 6·21. No.	$\bar{l}$.	$\bar{\rho}'$.	Computed ρ'. l=325°.	l=308°·5.
347°–24	2	6°·0	+ 3·8	+7·8	+8·7	23	5°·6	+3·7	+5·1	+5·2
26–42	4	34·5	+10·3	+6·0	+8·2	21	33·6	+2·7	+3·8	+1·7
43–64	6	49·2	+10·6	+3·5	+6·3	20	54·2	+1·7	+1·2	−1·7
65–80	10	71·3	− 3·7	−0·5	+1·5	14	70·9	−4·9	−1·0	−3·5
81–112	8	94·8	− 2·8	−2·9	−2·1	15	95·1	−2·2	−2·9	−3·5
113–149	7	130·0	+ 2·9	−0·3	−1·0	35	127·8	+0·3	−1·2	+0·8
150–171	12	161·8	+ 1·0	+5·4	+5·1	31	160·4	+5·9	+3·2	+5·2
172–193	17	177·0	+ 7·9	+7·3	+7·7	18	180·8	+6·0	+4·9	+5·5

Limits of l.	Bo–B2 > 6·20. No.	$\bar{l}$.	$\bar{\rho}'$.	Computed ρ'. l=325°.	l=328°·5.	B3–B5 > 6·20. No.	$\bar{l}$.	$\bar{\rho}'$.	Computed ρ'. l=325°.
347°–24	1	24°·0	+24·4	+14·6	+15·2	37	15°·5	+3·7	+4·8
26–42	8	39·9	+12·6	+ 8·4	+ 9·8	31	31·2	+3·8	+3·3
43–64	11	51·4	0·0	+ 2·3	+ 0·9	23	50·0	+0·9	−0·1
65–80	11	71·7	− 5·7	− 8·7	− 7·8	26	73·3	−5·5	−4·8
881–112	16	100·4	−17·6	−16·0	−17·0	23	91·6	−6·9	−7·0
113–149	2	116·5	−12·2	−13·4	−15·4	13	129·2	−3·8	−2·0
150–171	2	163·5	+13·4	+10·0	+ 8·0	13	163·7	+3·6	+2·5
172–193	15	175·5	+12·1	+14·5	+13·2	9	178·4	+3·5	+4·4

TABLE II.

Group Divisions of O Stars.

Limits of l.	All O's. No.	$\bar{l}$.	$\bar{\rho}'$.	Comp. ρ'. l=322°·3.	O's < 6·51. No.	$\bar{l}$.	$\bar{\rho}'$.	Comp. ρ'.	O's > 6·50. No.	$\bar{l}$.	$\bar{\rho}'$.	Comp. ρ'.
334°–357	6	340°·7	+28·9	+15·7	3	344°·0	+32·2	+15·2	3	337°·3	+25·6	+13·5
24–46	8	38·4	+21·4	+13·5	2	35·0	+25·6	+15·5	6	39·5	+20·1	+15·7
48–67	11	58·2	+ 6·5	+ 2·2	9	60·2	+ 6·9	+ 6·3	2	49·0	+ 4·9	+ 8·9
70–92	7	76·0	−10·8	− 6·0	5	74·0	− 7·9	+ 1·5	2	81·0	−18·0	−13·3
103–117	9	110·0	−14·8	− 9·6	1	111·0	+ 9·3	− 2·0	8	110·0	−17·8	−16·7
137–170	7	154·1	+18·7	+12·4	3	157·7	+28·3	+13·1	4	151·5	+11·5	+ 9·2
174–177	12	175·0	+14·7	+20·9	6	175·7	+ 8·7	+18·2	6	174·3	+20·7	+23·2
190–206	5	197·4	+ 6·4	+21·4	4	199·2	+ 8·5	+18·9	1	190·0	− 3·6	+26·5

The additional columns headed "computed ρ'" will be referred to later. In obtaining the observed residual velocity from ρ, the radial velocity, the solar apex was taken as 271°, + 28°, the mean of Campbell's * and Wilson's † values. For the B-type stars a solar velocity of 20 km. per second was accepted, but, as it seemed likely to be larger for the O stars, a solution for K and V_0 was made from the 65 O stars, giving $V_0 = -22{\cdot}5$, $K = +8{\cdot}5$. If the seven high velocity O's previously rejected were included, the values became $V_0 = -27{\cdot}6$, $K = +5{\cdot}9$. However, as these stars were not used in the solutions for the rotation effect, the residual velocities of the O stars were obtained from the value of $V_0 = -22{\cdot}5$ km. per second.

The above longitude, type, and magnitude groups of B and O stars were then solved for the rotational effect, Oort's equation of condition becoming, as the stars are all close to the Galaxy and the $\cos^2 b$ term may be neglected,

$$K + \bar{r}A \sin 2(l - l_0) = \bar{\rho}'.$$

Solutions were made for the three unknowns, K, $\bar{r}A$, and l_0, and also, assuming $l_0 = 325°$, the longitude of the centre of the system of globular clusters, for K and $\bar{r}A$ alone. The values of the unknowns with their probable errors are shown in the following table :—

Table III.

Values of l_0, K, and $\bar{r}$A.

Class and Mag. Limits.	No.	$\bar{m}$.	Solutions including l_0.			Solutions with $l_0=325°$.	
			l_0.	K.	$\bar{r}$A.	K.	$\bar{r}$A.
B0–B2 < 6·21	66	5·00	332°·7±9°·8	+3·2±1·0	+ 5·9±0·9	+2·4±1·0	+ 5·4±1·4
B0–B2 > 6·20	66	6·94	328·5±3·0	−0·4±0·9	+16·7±1·1	+0·2±0·8	+16·2±1·1
B3–B5 < 6·21	181	5·16	308·5±2·6	+0·9±0·3	+ 4·8±0·5	+1·1±0·6	+ 4·0±0·8
B3–B5 > 6·20	175	6·79	325·0±2·1	−1·2±0·3	+ 5·9±0·3	−1·2±0·2	+ 5·9±0·3
O all	65	6·43	322·3±7·2	+5·8±2·9	+17·1±4·0		
O < 6·51	33	5·55	..	..	..	+8·3±2·9	+11·2±4·5
O > 6·50	32	7·34	..	..	..	+4·2±2·0	+22·2±2·8

In addition, in order to see whether the relatively large range of longitude in the first, fifth, and sixth longitude groups introduced any systematic effect in the solutions, the group of B3 to B5 stars brighter than 6·21 mag., where the value for l_0 is discrepant, was rearranged into fifteen instead of eight longitude divisions, no one of which covered more than 15° of longitude. The solution gave practically identical values with that for the eight groups, l_0 being changed only 0°·2, thus justifying the small number of longitude groups.

The Values of l_0.—The values of l_0 obtained by the solutions are consistent except in the case of the brighter group of B3 to B5 stars,

* *A.S.P.*, **38**, 255.
† *Ast. J.*, **36**, 141.

where the value $308^\circ\cdot5$ is more than 15° lower than the mean of the other groups. A comparison of the run of the mean residual velocities in this group in Table I. shows that the negative values begin and end at lower latitudes than the other groups, thus lowering the value of l_0. While the run of the residual velocities produces a low value of l_0, a comparison with the computed values of ρ' for $l_0 = 325^\circ$, in the next column, shows that a change of two or three kilometres in the mean residual velocities at longitudes 50° and 130°, easily produced by the addition of two or three high-velocity stars, would bring the value of l_0 into agreement with the other groups. In view of this consideration, and of the further fact that for the brighter and nearer stars the rotational effect is necessarily not so definite, it does not seem unreasonable to consider this deviation as an accidental error due to chance combination of the peculiar velocities and to give it less weight than called for by its derived probable error. As a fair estimate the weight corresponding to a doubled probable error was given to this group, which even then makes its weight greater per star than that given to the allied group of the brighter B0 to B2 stars. Weighting the other four groups according to their probable errors, we have as the mean value of the longitude of the centre of rotation

$$l_0 = 324^\circ\cdot5 \pm 1^\circ\cdot8,$$

a value in almost exact agreement with the longitude, 325°, of the centre of the system of globular clusters and with Oort's * value of $324^\circ \pm 2^\circ$. The value obtained by Schilt † of 340° is based on the proper motions, hence of lower weight, with unknown systematic errors, of the Gyllenberg stars of the zone 35° to 40°, a narrow small circle of the sphere only crossing the galactic region at an angle of about 60°, at longitudes 40° and 140°. This value is obviously of lower weight than a determination from the radial velocities of the B stars, which are practically all in the galactic region.

It may hence be definitely concluded that the longitude of the centre of the galactic rotation, as determined from the radial velocities of the O and B stars, coincides with the longitude of the centre of the globular clusters, thus providing a confirmation of Lindblad's hypothesis of a common centre of rotation for the sub-systems of the Galaxy.

The Value of the K-term.—Only a few remarks need be made here about the value of the K-term, as it can be discussed more profitably when our investigation of the B stars is completed. It is certainly not a constant, seeming to be much larger for the hottest and brightest stars. It decreases from + 8 for the brighter to + 4 for the fainter O stars, from + 3 for the brighter to zero for the fainter B0 to B2 stars, and from + 1 for the brighter to − 1 for the fainter B3 to B5 stars. That this is not due to combining it in a solution with the rotation term can be seen from comparing the values thus obtained with those of the K-term derived from the residual velocities alone of the same B and O stars, where the variation is in the same direction but the values generally higher.

* *B.A.N.*, No. 120, p. 132.
† *Proc. Nat. Ac. Sc.*, **63**, 644.

TABLE IV.

The K-term from Residual Velocities.

Types.	Mag. Limits.	K.
B0–B5	<5·5	+ 3·0
B0–B5	5·5–6·65	+ 1·7
B0–B5	>6·65	− 0·1
B0–B2	All	+ 3·2
B3–B5	All	+ 0·8
O	<6·51	+10·8
O	>6·50	+ 6·2

While it is true that the velocities of the brighter B stars were determined at other observatories and of the fainter B's here, that does not apply to the O's nearly all determined at Victoria, where K has the greatest positive value and varies markedly, so that it can hardly be considered an instrumental or institutional effect. However, the question has not much bearing on the rotation effect as a solution, for $\bar{r}$A alone gave values differing little from those when K was included, and the discussion can well be postponed until fuller data are available.

The Rotation Term, $\bar{r}A$.—As will be seen at once from Table III., the rotation term $\bar{r}$A is always positive, and in the fainter and more distant stars is at least ten times its probable error. Further, for the stars of higher temperature especially, it increases markedly and nearly in the corresponding ratio as faintness and presumably increasing distance. Its increasing lower relative and absolute probable error in the more distant stars is to be expected as the rotation effect becomes the preponderating one and any irregular distribution of the peculiar velocities of the stars has proportionally less effect. Altogether the evidence given by these additional radial velocities of the B and O stars strongly supports the hypothesis of a rotation of the Galaxy. It seems worth while to redetermine the value of the constant A, the magnitude of the rotation effect in kilometres per second for stars at a distance of one parsec, from this new material. The following table, Table V., gives the computed value of A for the different groups of stars from the best available values of their mean parallaxes.

TABLE V.

The Value of A.

Class.	No.	$\bar{m}$.	Mean Parallax. Gerasimovic.	Oort.		Abs. Mag.	$\bar{\pi}$.	$\bar{r}$.	A.	π'.
B0–B2	66	5·00	″·0019	″·0030	..	″·0032	″·0027	460	·0117	″·0035
B0–B2	66	6·94	·0015	..	..	·0013	·0014	885	·0183	·0012
B3–B5	177	5·16	·0052	·0053	..	·0053	·0053	232	·0172	·0048
B3–B5	175	6·79	·0032	..	..	·0025	·0029	430	·0137	·0033
			Wilson.	Oort.	Plaskett.					
O	33	5·55	·0014	·0020	·0013	·0015	·0016	775	·0144	·0017
O	32	7·34	·0015	..	..	·0007	·0011	1130	·0196	·00086

In Table V. the first three columns contain the spectral class, the number of stars, and the average magnitude of the different groups. The next group of four columns contains various estimates of the mean parallax of the different groups. The last of these under the heading Abs. Mag. is the computed parallax on the assumption of an absolute magnitude of $-2{\cdot}5$ for B0 to B2, $-1{\cdot}2$ for B3 to B5, and $-3{\cdot}5$ for the O-types. This was added for the purpose of partially counteracting the effect of the less accurately determined proper motions of the faint stars, which are likely to give spuriously high values of their mean parallax. The next column, $\bar{\pi}$, is the simple mean of the previous estimates, while $\bar{r}$ is the average distance of the groups on the basis of Oort's * determination of $\bar{r}\bar{\pi} = 1{\cdot}24$ for the B-type stars and assumed the same for the O's. The next column gives the values of A for the different groups obtained by dividing the $\bar{r}$A's of Table III. by the $\bar{r}$ of the previous column. It was considered preferable to use the second set of values of $\bar{r}$A obtained by assuming $l_0 = 325°$, as if the peculiar distribution of the radial velocities of the B- and O-type stars is due to a rotation of the Galaxy, this rotation is probably about a common centre, and does not shift from longitude 332° to 308° as we go from bright B2 to bright B3 stars.

There is as satisfactory an accordance between the different values of A as can reasonably be expected from the uncertainties of the mean parallaxes and of the different values of $\bar{r}$A itself, and this agreement forms a decided confirmation of the reality of the rotational effect. The mean value of A obtained by weighting the individual values according to the number of stars in the group is

$$A = +{\cdot}0155 \pm {\cdot}0007 \text{ km. per sec. per parsec,}$$

and is about 18 per cent. less than Oort's † revised value of ·019, but as it is based on larger and more homogeneous material seems equally reliable, and in any case the difference is insignificant in view of the rather heterogeneous nature of Oort's data. Schilt's ‡ value of 0″·0106 or ·050 km./sec. is three times larger, but the relative unsuitability of proper motions for determining this constant and the non-galactic narrow zone of stars concerned make his determination one of smaller weight. As a matter of interest, what might be called the rotational values of the mean parallax of the various groups have been obtained from the mean value of A and the respective values of $\bar{r}$A, and are given in the last column of Table V. under π'. A comparison of these values with the otherwise determined mean parallaxes shows good agreement and indicates the usefulness of the rotational analysis in obtaining the distance of very remote objects. The differences between the rotational parallaxes and those determined from proper motions lie mainly in the groups of fainter stars in which the proper motions are uncertain. The parallaxes determined from the rotation term seem at least equally reliable as requiring a smaller dispersion in absolute magnitude as we go from bright to faint stars.

As a final check on the reality of the rotational effect, the computed values of the residual radial velocities have been entered in adjacent

* *B.A.N.*, **132**, 81. † *Ibid.*, **132**, 82. ‡ *Proc. Nat. Ac. Sc.*, **13**, 644.

columns to the observed $\bar{\rho}$'s in the various groups of B's and O's in Tables I. and II. The columns next to the observed velocities in Table I. and in the two magnitude subdivisions of Table II. have been computed with the values of $\bar{r}$A and K obtained, assuming $l_0 = 325°$, while the additional columns in the first three groups of the B's where the l_0 obtained in the solution differed more than 3° from 325° have been obtained by using the value of l_0, K, and $\bar{r}$A obtained in the complete solutions. In view of the nature of the material, and, in many cases, the small numbers of stars to balance any irregularities in the peculiar velocities, the agreement between the observed and computed velocities is remarkably good. This is particularly the case in the fainter and more distant stars where the rotational term becomes more pronounced, and where the probable errors of this term become so relatively small as to make the rotational effect very probable. Oort's * statement that his results " leave hardly any doubt as to the reality of this sin $2(l-325°)$ term." " It is possible, of course, that this term may be explained by systematic motions arising from another cause than rotation, but these systematic motions must bear a remarkably close resemblance to a rotation," is then emphatically strengthened by this investigation.

In conclusion, the probability of galactic rotation seems to be materially increased by this analysis of the radial velocities of the fainter B- and O-type stars. It will be of interest to see whether this conclusion is confirmed and strengthened when the remaining velocities of the B stars are completed, and when the velocities of a number of the faint and distant " *c* " stars are obtained.

The System of Procyon. By H. Spencer Jones, M.A., Sc.D., H.M. Astronomer.

In his work on *The Internal Constitution of the Stars*, Eddington remarks (p. 152), in reference to the testing of the mass-luminosity relation by observational data, that " Procyon is reckoned second-class because the semi-axis of the orbit is not well-determined ; observations of this star are greatly to be desired and would form a most important check on the theory."

Three elliptic orbits have been computed for Procyon, the derived elements being given below for purposes of comparison.

	See.	Auwers.	Boss.
P	40*y*.	40·0*y*.	39·0*y*.
T	1891·0	1883·0	1886·5
e	0·45	0·227	0·324
a	0″·94	0″·956	1″·00
Ω	108°·3	267°	330°·7
ω	286°·35	72°·6	36°·8
i	33°·1	38°·3	14°·2
m_1/m_2	1 : 0·2	..	1 : 0·33

* *B.A.N.*, No. 120, 279.