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TWO K-TYPE SPECTROSCOPIC BINARIES

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33 PISCIUM

ABSTRACT

Fifteen single-prism spectrograms secured at Victoria in 1920 and 1926 serve to determine the orbit of this spectroscopic binary of type K0. The early observations of the Lick and Cape observatories fit in with the period of 72.93 days as herein determined. The semi-amplitude is 16.43 km./sec., the eccentricity .272 and the probable error of a plate has the satisfactorily low value of ± 0.8 km./sec.

The star 33 Piscium (1900 $\alpha = 0^{\text{h}} 00^{\text{m}}.2$, $\delta = -6^{\circ} 16'$), visual magnitude 4.68 and type K0 was announced as a spectroscopic binary in the *Lick Observatory Bulletin*, No. 199. Their first six plates showed but little range in the measures as they all happened to fall near the phase of maximum velocity, as shown by the table herein. The seventh plate, however, showed unmistakably the binary character of the star. Three plates were also made by the observers at the Cape Observatory in 1917, the data of these and the Lick plates being given for convenience of reference.

Three plates were obtained here in 1920 and measured by Dr. R. K. Young before leaving the Observatory; the rest of the work is due to the writer who took over the star at Young's suggestion. All the Victoria plates were made with the single-prism equipment, the dispersion at H γ being about 29 angstroms per millimetre. The plates were measured on the spectro-comparator against Arcturus No. 2702 as a standard. As the plates are in many cases weaker in the violet than usual, by reason of the star's southern declination, it was customary to measure every region between $\lambda 4236$ and $\lambda 4528$ instead of alternate regions between $\lambda 4005$ and $\lambda 4528$ which is the usual procedure. The plate of September 8 is poorer than the average and should have been given slightly less weight than the others, but no appreciable error is introduced by considering them all of equal weight, as was done. Had this observation been weighted less the probable error of a plate would have been slightly reduced, but even as it stands now it has the exceptionally low value of ± 0.8 km./sec.

In the observations which follow, Greenwich Mean Time is used in the sense understood prior to 1925. The phases are reckoned from the periastron passage, finally adopted, using the period 72.93 days. This period, suggested by our own observations, satisfied reasonably well all the early observations and it was felt that no improvement would result by including it in the least-squares solution.

OBSERVATIONS AT OTHER OBSERVATORIES

Observatory	Date	Julian Date	Phase	Vel.	O-C
Lick.....	1905 Aug. 9.....	2,417,067.983	7.40	+10.5	+0.3
".....	Oct. 19.....	7,138.805	5.30	+12.5	-0.2
".....	1908 Aug. 4.....	8,158.964	4.43	+14.0	+0.6
".....	Dec. 24.....	8,300.628	0.24	+13.8	+1.0
".....	1909 Aug. 1.....	8,520.995	1.82	+13.7	-0.2
".....	1910 Aug. 5.....	8,889.945	6.12	+15.3	+3.4
".....	Oct. 24.....	2,418,969.739	12.98	+ 3.9	+1.5
Cape.....	1917 Nov. 10.....	2,421,543.360	33.08	-17.9	-1.7
".....	Nov. 19.....	1,552.343	42.07	-20.1	-1.3
".....	Dec. 4.....	1,567.304	57.03	-13.4	0.0

VICTORIA OBSERVATIONS

Plate	Date	Julian Date	Phase	Vel.	O-C
4705	1920 July 25.....	2,422,531.965	1.64	+14.6	+0.7
5134	Sept. 29.....	2,597.866	67.54	+ 5.1	+1.5
5247	Oct. 21.....	2,619.729	16.47	- 2.6	+0.2
13823	1926 July 19.....	4,716.966	71.67	+11.5	+0.2
13957	Aug. 5.....	4,733.986	15.76	- 1.8	-0.1
14003	" 18.....	4,746.976	28.75	-15.0	-1.1
14015	" 20.....	4,748.992	30.76	-13.3	+1.7
14099	" 30.....	4,758.940	40.71	-20.0	-1.4
14158	Sept. 8.....	4,767.853	49.62	-15.7	+2.3
14197	" 11.....	4,770.874	52.64	-17.8	-1.3
14249	" 19.....	4,778.855	60.63	-10.1	-1.0
14260	" 22.....	4,781.854	63.62	- 3.6	+0.6
14290	" 25.....	4,784.846	66.62	+ 0.4	-1.3
14318	Oct. 4.....	4,793.817	2.66	+12.9	-1.1
14327	" 11.....	4,800.795	9.63	+ 7.8	+0.5

NORMAL PLACES

—	Mean Phase		Vel.	Wt.	O-C	
	Prel.	Final			Prel.	Final
1.....	3.30	2.15	+13.8	2	-0.2	-0.2
2.....	10.78	9.63	+ 7.8	1	-0.1	+0.5
3.....	17.26	16.11	- 2.2	2	-1.2	-0.3
4.....	30.90	29.75	-14.1	2	+0.4	+0.5
5.....	41.86	40.71	-20.0	1	-0.5	-1.4
6.....	52.28	51.13	-16.8	2	+1.9	+0.6
7.....	61.78	60.63	-10.1	1	-0.6	-1.0
8.....	64.77	63.62	- 3.6	1	+0.5	+0.6
9.....	67.77	66.62	+ 0.4	1	-1.8	-1.3
10.....	68.69	67.54	+ 5.1	1	+0.9	+1.5
11.....	72.82	71.67	+11.5	1	-0.1	+0.2

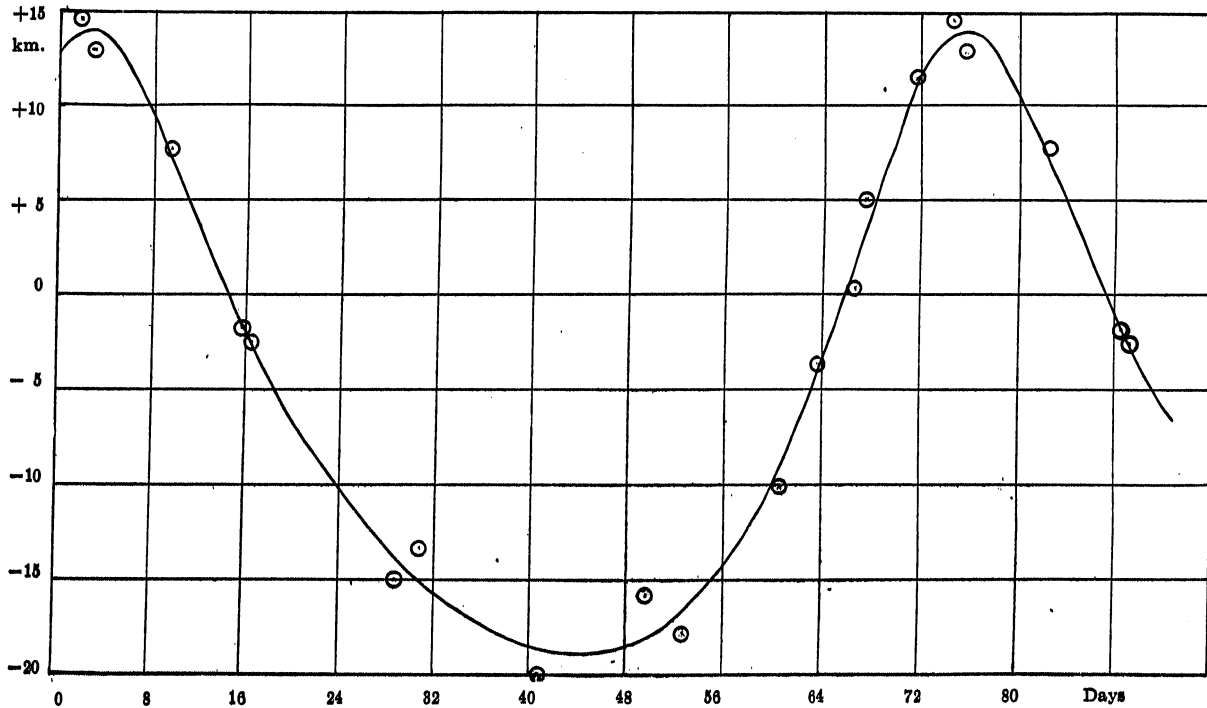


Fig 1. Velocity Curve of 33 Piscium, Showing Individual Observations.

PRELIMINARY ELEMENTS

- P = 72.93 days
- e = 0.25
- ω = 330°
- γ = -6.68 km./sec.
- K = 17.0 km./sec.
- T = J.D. 2,422,529.180

These preliminary elements were obtained in the usual graphical manner and then observation equations were built up connecting the residuals for the normal places with the elements. In these equations the following substitutions were made:—

$$\begin{aligned}x &= \delta \gamma \\y &= \delta K \\z &= K \delta e \\u &= -K \delta \omega \\v &= +16.135 \delta T\end{aligned}$$

OBSERVATION EQUATIONS

1.....	1.000x	+1.215y	+0.901z	-0.167u	-0.063v	+0.2 = 0
2.....	1.000	+0.855	-0.785	+0.644	+0.835	+0.1
3.....	1.000	+0.333	-0.984	+0.868	+0.806	+1.2
4.....	1.000	-0.463	+0.458	+0.609	+0.425	-0.4
5.....	1.000	-0.757	+0.990	+0.107	+0.134	+0.5
6.....	1.000	-0.707	+0.265	-0.509	-0.276	-1.9
7.....	1.000	-0.168	-1.120	-1.048	-0.983	+0.6
8.....	1.000	+0.150	-1.149	-1.123	-1.230	-0.5
9.....	1.000	+0.520	-0.628	-1.078	-1.338	+1.8
10.....	1.000	+0.637	-0.361	-1.032	-1.315	-0.9
11.....	1.000	+1.073	+0.844	-0.640	-0.804	+0.1

These resulted in the following normal equations:—

$$\begin{aligned}15.000x + 3.066y - 0.929z - 2.568u - 2.917v - 0.100 &= 0 \\7.784 - 0.320 - 1.099 - 1.423 + 4.343 & \\9.526 + 0.853 + 1.171 - 3.777 & \\8.245 + 8.496 + 2.439 & \\9.179 + 1.490 &\end{aligned}$$

from which the following corrections were obtained:—

$$\begin{aligned}\delta \gamma &= +0.12 \text{ km./sec.} \\ \delta K &= -0.57 \text{ km./sec.} \\ \delta e &= +0.022 \\ \delta \omega &= +7^\circ.71 \\ \delta T &= +1.150 \text{ days}\end{aligned}$$

The value of Σpvv for the normal places was reduced from 15.43 to 9.03, or about 35 per cent.

The following, then, are the final values of the elements with their corresponding probable errors attached:—

FINAL ELEMENTS

$$\begin{aligned}P &= 72.93 \text{ days} \\ e &= 0.272 \pm 0.017 \\ \omega &= 337^\circ.71 \pm 4^\circ.60 \\ \gamma &= -6.56 \text{ km./sec.} \pm 0.23 \text{ km./sec.} \\ K &= 16.43 \text{ km./sec.} \pm 0.31 \text{ km./sec.} \\ T &= \text{J.D. } 2,422,530.330 \pm 0.809 \text{ day} \\ a \sin i &= 15,856,000 \text{ km.}\end{aligned}$$

The graph shown, Figure 1, represents these final elements.

71 AQUILAE

ABSTRACT

Twenty-six spectrograms secured at Victoria during the years 1920 to 1926 are used to determine the orbit of this binary, announced twenty-one years ago from the Lick Observatory. The early observations of the Lick, Bonn and Cape observatories fit in with the period of 205 days as determined here. The range is small, the semi-amplitude being 9.78 km./sec., but as the star is of K-type the measures are reliable and the orbit is satisfactorily determined. The probable error of a plate is ± 1.0 km. per second.

The star 71 Aquilae (1900 $\alpha = 20^h 33^m.2$, $\delta = -1^\circ 27'$) visual magnitude 4.51 and type K0, was announced as a spectroscopic binary by Curtis from the Lick Observatory in *Lick Observatory Bulletin*, Volume III, page 86. The third plate showed a definite variation in the radial velocity. Their five plates together with three taken at Bonn and three also from the Cape Observatory appear in the table of early observations. Aside from slight systematic differences the residuals, as computed from the Victoria elements, are satisfactory for all three observatories.

Of the twenty-six plates obtained at Victoria between the years 1920 and 1926 fifteen were secured by Dr. R. K. Young and these, with four others, were measured by him before leaving this Observatory. At his suggestion this spring the writer undertook to secure the additional plates that were necessary to determine the period and other elements of the orbit. Four of his plates were remeasured by the writer, the results agreeing within 0.5 km. and the means of the two have been used. Another plate, number 8282, is missing from the collection, otherwise it would have been remeasured as it gives a large residual. From the limited number of regions measured on it by Young it is evidently a weak plate and accordingly has been given half weight.

The plates were all taken with the single-prism equipment giving a dispersion at $H\gamma$ of 29 angstroms to the millimetre. They were measured on the spectrocomparator against Arcturus No. 2702 as a standard and in general alternate regions from $\lambda 4005$ to $\lambda 4528$, or twelve in all, were measured.

In the observations which follow Greenwich Mean Time is used in the sense understood prior to 1925. The decimal of a day is assumed for the Lick observations.

OBSERVATIONS OF 71 AQUILAE AT OTHER OBSERVATORIES

Observatory	Date	Julian Date	Phase	Vel.	O-C
Lick.....	1900 June 18.....	2,415,189.8	53.2	- 2	-0.5
"	July 10.....	5,211.8	75.2	- 5	+2.3
"	1903 Sept. 22.....	6,380.8	14.2	+ 3.0	-1.5
"	1904 Aug. 7.....	6,700.8	129.2	-12.7	+2.2
"	" 30.....	6,723.8	152.2	-11.3	+1.9
Bonn.....	1910 Sept. 18.....	8,933.4	106.8	- 9.1	+4.2
"	" 21.....	8,936.4	109.8	-10.1	+3.6
"	1911 Sept. 1.....	9,281.4	44.8	- 1.0	-1.5
Cape.....	1917 Sept. 19.....	1,491.4	204.8	+ 2.5	0.0
"	1918 Aug. 21.....	1,827.4	180.8	-13.7	+1.2
"	" 23.....	2,421,829.3	132.7	-15.0	-0.2

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VICTORIA OBSERVATIONS OF 71 AQUILAE

Plate	Date	Julian Date	Phase	Vel.	Wt.	O-C
4516	1920 June 30.....	2,422,506.899	195.35	+ 1.5	1	+1.8
4697	July 25.....	2,531.827	15.28	+ 4.2	1	-0.8
5026	Sept. 3.....	2,571.739	55.19	- 1.2	1	+1.0
5244	Oct. 21.....	2,619.624	103.07	-10.5	1	+2.3
6007	1921 May 3.....	2,815.992	94.44	-12.2	1	-0.8
6210	July 10.....	2,881.886	160.34	-11.5	1	+0.1
6562	Sept. 14.....	2,947.715	21.17	+ 4.1	1	-0.4
6827	Nov. 8.....	3,002.591	76.04	- 9.8	1	-2.2
7637	1922 June 7.....	3,213.956	82.41	-10.1	1	-1.1
8010	Aug. 28.....	3,295.756	164.21	-11.6	1	-1.0
8212	Oct. 9.....	3,337.670	1.12	+ 5.0	1	+2.3
8282	Nov. 2.....	3,361.578	25.03	+ 8.8	½	+4.6
8457	Dec. 1.....	3,390.563	54.01	- 0.8	1	+1.1
9514	1923 Oct. 11.....	3,704.649	163.10	-10.5	1	+0.4
10140	1924 June 22.....	3,959.927	8.38	+ 5.5	½	+1.6
10218	July 3.....	3,970.885	19.33	+ 1.4	1	-3.2
10235	" 6.....	3,973.921	22.37	+ 3.5	1	-0.9
10291	" 13.....	3,980.875	29.33	+ 5.4	1	+1.8
10326	" 20.....	3,987.892	36.34	+ 2.2	1	-0.3
13748	1926 July 9.....	4,706.901	140.35	-14.0	1	+0.5
13915	Aug. 2.....	4,730.859	164.31	-11.3	1	-0.7
13998	" 18.....	4,746.813	180.26	- 5.3	1	+0.4
14091	" 30.....	4,758.706	192.16	- 1.7	1	-0.3
14092	" 30.....	4,758.718	192.17	- 2.3	1	-0.9
14148	Sept. 6.....	4,765.750	199.20	- 1.0	½	-2.0
14206	" 18.....	2,424,772.749	1.20	+ 2.6	1	-0.1

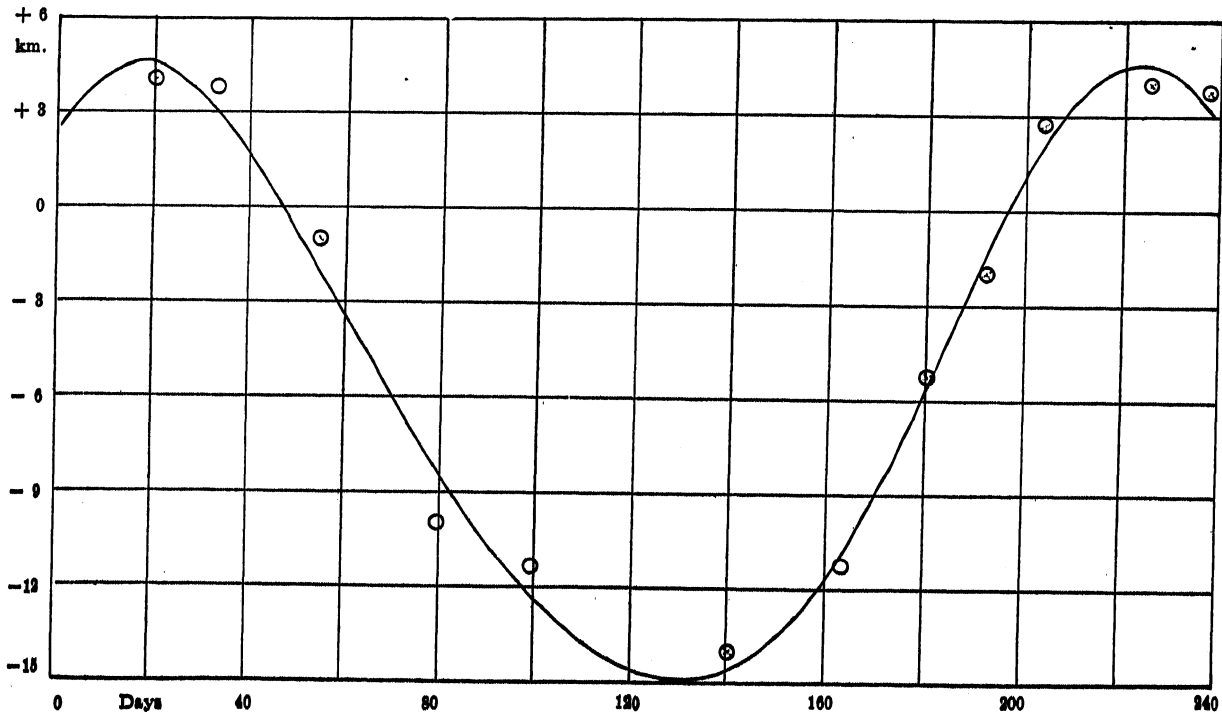


Fig. 2. Radial Velocity Curve of 71 Aquilae Showing Grouped Observations.

With the aid of the early observations the period was determined by the writer as 205.0 days and this value is considered as definite without any recourse to least-squares. The Victoria observations were grouped according to phase into ten normal places and a set of preliminary elements obtained graphically. No further use was made of the early observations.

NORMAL PLACES

	Mean Phase		Velocity	Wt.	O-C	
	Prel.	Final			Prel.	Final
1.....	49.81	54.60	- 1.00	2	+0.98	+1.01
2.....	74.44	79.23	- 9.95	2	-1.50	-1.64
3.....	93.96	98.75	-11.35	2	+1.06	+0.83
4.....	135.56	140.35	-14.00	1	+0.39	+0.54
5.....	158.20	162.99	-11.22	4	-0.55	-0.23
6.....	175.47	180.26	- 5.30	1	+0.37	+0.38
7.....	187.38	192.17	- 2.00	2	-0.24	-0.61
8.....	198.70	203.49	+ 2.71	4	+1.23	+0.56
9.....	15.36	20.15	+ 4.00	4½	-0.09	-0.61
10.....	28.04	32.83	+ 3.80	2	+0.86	+0.67

PRELIMINARY ELEMENTS

$$\begin{aligned}
 P &= 205.0 \text{ days} \\
 e &= 0.10 \\
 \omega &= 330^\circ \\
 \gamma &= -6.22 \text{ km./sec.} \\
 K &= 9.5 \text{ km./sec.} \\
 T &= \text{J.D. } 2,422,316.34
 \end{aligned}$$

In the observation equations, built up according to the usual Lehmann-Filhés form, the following substitutions were made:—

$$\begin{aligned}
 x &= \delta \gamma \\
 y &= \delta K \\
 z &= K \delta e \\
 u &= -K \delta \omega \\
 v &= .2956 \delta T
 \end{aligned}$$

OBSERVATION EQUATIONS FOR 71 AQUILAE

1.....	1.000x	+0.447y	-0.982z	+0.883u	+0.903v	-0.98 = 0
2.....	1.000	-0.234	-0.349	+0.897	+0.811	+1.50
3.....	1.000	-0.651	+0.588	+0.624	+0.551	-1.06
4.....	1.000	-0.859	+0.385	-0.376	-0.284	-0.39
5.....	1.000	-0.468	-0.806	-0.882	-0.822	+0.55
6.....	1.000	+0.061	-0.955	-1.050	-1.098	-0.37
7.....	1.000	+0.470	-0.316	-0.974	-1.076	+0.24
8.....	1.000	+0.811	+0.523	-0.740	-0.830	-1.23
9.....	1.000	+1.036	+0.809	-0.001	+0.058	+0.09
10.....	1.000	+0.965	+0.018	+0.429	+0.530	-0.86

These resulted in the following normal equations:—

$$\begin{array}{r}
 24.500x + 7.455y - 0.144z - 4.200u - 4.291v - 5.395 = 0 \\
 13.218 + 5.028 - 1.027 - 0.972 - 5.900 \\
 10.763 + 1.155 + 1.073 - 4.367 \\
 12.757 + 12.906 + 0.667 \\
 13.192 + 0.883
 \end{array}$$

from which the following corrections were obtained:—

$$\begin{array}{l}
 \delta \gamma = +0.13 \text{ km./sec.} \\
 \delta K = +0.28 \text{ km./sec.} \\
 \delta e = +0.028 \\
 \delta \omega = -0^\circ.57 \\
 \delta T = -4.79 \text{ days}
 \end{array}$$

The value of Σpvv for the normal places was reduced from 17.83 to 13.97, or about 22 per cent, and all differences between equation and ephemeris residuals were less than 0.1 km.

The probable error of a plate obtained from the last two columns of the table of Victoria observations is ± 1.0 km. per sec., a value very satisfactory for single-prism dispersion.

The following are the final elements with their probable errors attached:—

FINAL ELEMENTS		
P	= 205.0 days	
e	= 0.128	± 0.042
ω	= $321^\circ.43$	$\pm 18^\circ.88$
γ	= -6.09 km./sec.	± 0.26 km./sec.
K	= 9.78 km./sec.	± 0.39 km./sec.
T	= J.D. 2,422,311.55	± 10.40 days
$a \sin i$	= 27,347,000 km.	

The graph shown, Figure 2, represents these elements with the observations as grouped.

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