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THREE SPECTROSCOPIC BINARY ORBITS

BY

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ABSTRACTS

21 Cassiopeiae.—The orbit of this eclipsing variable undertaken at the suggestion of its discoverer, Stebbins, is of special interest on account of the opportunity given by its fine-lined spectrum and the practical coincidence of photometric and spectroscopic observations to test the differences in phase obtained by the two methods, previously observed by Schlesinger and the writer in other Algol variables. The paper is in two parts on account of the first part being completed before the photometric data were available. The first part contains the elements determined by 25 spectra well distributed over the velocity curve and corrected by least-squares for K, e, ω and γ. The difference between spectroscopic and photometric minimum is 24 minutes, considerably too large apparently to be chargeable to accidental error. It was, therefore, considered desirable to observe further around minimum and 13 additional plates were obtained about a year later, all within a quarter of a day of minimum. As the final photometric orbit showed negligible eccentricity a second and third solution, assuming circular orbits and with and without the additional plates, were made. These reduced the difference of phase to about 4 minutes, well within the accidental error. The reasons for the change and the probable effect of similar procedure on the earlier observed differences are discussed. The spectra around minimum show a rotation effect which appears to indicate identical values of the periods of rotation and orbital revolution.

Boss 3354.—This double-lined A-type binary discovered in 1919 by the writer was further observed in the summer of 1924. The orbital work followed the usual course resulting in the elements $P=3\cdot28655$ days, $e=\cdot0408$, T = J.D. 4226·669, $\omega=211^\circ\cdot05$, $K_1=108\cdot34$, $K_2=128\cdot863$, $\gamma=-0\cdot05$. The masses are $m_1 \sin^3 i=2\cdot47\odot$, $m_2 \sin^3 i=2\cdot08\odot$ while the projected semi-axis major is 11,880,000 km.

H.D. 191201.—The first plate obtained showed double lines of separation 360 km., and the orbit was determined from 36 plates. The star is of type O9 to B0 and the lines, especially of the secondary, are very difficult to measure indicated by the high probable errors of $\pm9\cdot9$ and $\pm25\cdot2$ km. per sec. Least-squares corrections for all the elements were carried through and a satisfactory orbit obtained. The star is unusual for its high eccentricity of 0·26 and the masses also are large, the minimum masses being 13·8 and 12·9 times the sun. From probable estimates of density and calculated values of surface brightness it seems certain that this star must be at least 5,000 light years distant.

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THE ORBIT OF 21 CASSIOPEIAE

PART I

The spectroscopic observation of 21 Cassiopeiae, R.A. 0h 39·0m, Dec. +74° 26′, Vis. Mag. 5·59, Spectrum A2, was undertaken at the kind suggestion of Professor Stebbins who had discovered its variation in light by the photo-electric photometer. The nearly simultaneous observation photometrically and spectroscopically, with the fine quality of the spectrum for measurement, offered a good opportunity of testing the coincidence of spectroscopic and photometric phases of which many puzzling discrepancies in the Algol variables have been found.

The spectrum of type A2 is of fine quality for measurement, containing numerous sharp metallic lines, of which about 20 were measured on each plate. Consequently, the measures are reliable as indicated by the low probable error for single-prism plates of ±1·24 km. per second. For the measures of 19 of the 25 plates used in the orbit, the writer is indebted to Mr. S. N. Hill, computer at the observatory, and the accuracy of his measurement is well shown by the low probable error and the close fitting of the observations to the velocity curve. There seems, in the plates obtained for this solution, to be no evidence of any rotational effect near the time of eclipse, and apparently any deviation from simple elliptic motion thereby produced is negligible so far as these first observations are concerned. Dean B. McLaughlin, of the Detroit Observatory, proposed to examine 21 Cassiopeiae with this end in view, but at the time of writing no resuts are at hand.

The first spectrum was obtained on June 29, 1924, and between that date and October 18 inclusive, 25 spectra were obtained of which all but one by the short-focus camera were made with the medium-focus camera giving linear dispersion at Hγ of 29 A per millimetre. These observations arranged in order of final phase are given in the following table, which is self-explanatory. It should, however, be mentioned, owing to the importance of careful timing in comparing photometric and spectroscopic phases, that the average exposure time was about 12 minutes, so that there is little room for uncertainty in assuming the mid-exposure as the true time of the observation. Further, although normally the observing clock may have a correction of two or three minutes, unimportant in most cases, the correction, on each night on which 21 Cassiopeiae was observed, was obtained by computing from the recorded initial hour angle of every observation of the night and averaging. Hence the clock correction and the time of each observation of 21 Cassiopeiae was determined certainly within a minute which, when spread accidentally over 25 observations, would make any uncertainty on this score a matter of a few seconds only. The velocities given were very carefully determined and are in most cases the mean of two, and sometimes of three, individual measures.

TABLE I.—OBSERVATIONS OF 21 CASSIOPEIAE

Plate Number	Julian Date	Final Phase	Velocities		Residuals	
			Obs'd.	Comp.	Prel.	Final
10672..........	4039·892	0·020	−52·2	−51·41	+0·93	−0·79
10877..........	4071·760	0·617	− 2·0	+ 0·16	−0·62	−2·16
10602..........	4031·894	0·956	+36·6	+34·41	+3·36	+2·19
10692..........	4040·903	1·031	+40·7	+41·36	+0·43	−0·66
10496..........	4009·961	1·359	+65·8	+66·28	+0·38	−0·48
10519..........	4018·943	1·408	+70·1	+69·01	+1·93	+1·09
10815..........	4063·663	1·455	+70·7	+71·44	+0·06	−0·74
10209..........	3969·951	1·554	+77·9	+75·87	+2·72	+2·03
10277..........	3978·960	1·628	+76·1	+77·86	−1·14	−1·76
10391..........	3996·961	1·761	+78·7	+80·14	−1·03	−1·44
10461..........	4005·968	1·833	+80·4	+80·40	+0·27	0·00
10880..........	4077·684	2·074	+74·5	+76·31	−2·15	−1·81
10854..........	4068·851	2·175	+72·0	+72·42	−1·07	−0·42
10507..........	4010·953	2·351	+66·7	+62·88	+2·45	+3·82
10221..........	3970·954	2·557	+45·2	+47·81	−4·64	−2·61
10284..........	3979·947	2·615	+45·9	+42·95	+0·73	+2·95
10831..........	4064·828	2·620	+40·5	+42·53	−4·46	−2·23
10400..........	3997·937	2·737	+35·4	+32·06	+0·77	+3·34
10187..........	3966·959	3·029	+ 2·3	+ 3·71	−4·50	−1·41
10259..........	3975·960	3·096	− 2·9	− 2·90	−3·11	0·00
10442..........	4002·981	3·313	−25·5	−23·51	−4·84	−1·99
10531..........	4020·967	3·431	−33·6	−33·71	−2·43	+0·11
10792..........	4056·753	3·479	−37·8	−37·54	−2·65	−0·26
10473..........	4007·971	3·836	−56·6	−58·60	+1·25	+2·00
10450..........	4003·961	4·293	−60·2	−60·11	+1·19	−0·09

The preliminary period, given by Stebbins from the photometric measures as 4·4672 days, was accepted and, from the velocity curve plotted with this period, the usual method of trial and error determined preliminary elements given in Table II. From these preliminary elements observation equations were formed by Schlesinger's method including as unknowns e, K, γ, and ω, but not P or T, the former being considered as determined by the photometric work and, the latter, owing to the small eccentricity, being indeterminate when corrections for both T and ω are attempted.

From the observation equations the following normal equations were obtained in the usual way:

$$25{\cdot}000\Gamma + 7{\cdot}242\kappa + 3{\cdot}758\pi - 9{\cdot}901\epsilon = -16{\cdot}17$$
$$+12{\cdot}842\kappa - {\cdot}925\pi - 1{\cdot}366\epsilon = +{\cdot}890$$
$$+12{\cdot}155\pi - 1{\cdot}971\epsilon = -24{\cdot}392$$
$$+6{\cdot}355\epsilon = +11{\cdot}311$$

Their solutions gave for the unknowns:—

$$\Gamma = +{\cdot}4287 \quad \text{whence} \quad \delta\gamma = -{\cdot}232$$
$$K = -{\cdot}1374 \quad \delta K = -{\cdot}137 \pm 0{\cdot}43$$
$$\pi = -2{\cdot}2695 \quad \delta\omega = +1^\circ{\cdot}805 \pm 0^\circ{\cdot}318$$
$$\epsilon = +1{\cdot}6506 \quad \delta e = -{\cdot}0100 \pm {\cdot}0057$$

and the final values of the elements.

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TABLE II.—ELEMENTS OF 21 CASSIOPEIAE

Element	Preliminary	Final
P Period..	4·4672	4·4672 days
e Eccentricity....................................	0·03	0·0200 ±·0057
ω Longitude of apse..............................	210°	211°·805 ±0°·318
T Time of periastron..............................	3963·930	3963·930
K Semi-amplitude..................................	72·0	71·86 ±0·43 km.
γ Velocity of system..............................	+10·0	+9·77 km.

The sum of the squares of the residuals, all the observations being given equal weight, was reduced by the solution from 151·7 to 81·5, and the probable error of a single plate becomes ±1·24 km. per second. From these elements are readily obtained

$$a \sin i = 4{,}414{,}300 \text{ km.}$$

$$\frac{m_2^3 \sin^3 i}{(m_1+m_2)^2} = 0{\cdot}172\odot$$

The times of principal and secondary minima when u equals 90° and 270° respectively, correspond to phases in the orbit from periastron of 2·980 and 0·698 or to Julian times 3966·910 and 3964·628. According to the preliminary value kindly sent by Stebbins the time of principal photometric minimum is 3966·893 days, a difference of ·017 days or 24 minutes. The probable error of the determination of ω is ±0°·318 or translated into time ±·004 days, about 6 minutes, so that the difference appears large to be considered accidental. In order to check whether a different method would change this difference, a new solution, assuming ω fixed and correcting for T, was made, resulting in practically identical values of the time of spectroscopic minimum.

PART II

Shortly after the above spectroscopic orbit was completed and the results sent to Stebbins, he sent to us a final value of the period and preliminary values of the times of principal and secondary minima as obtained by Mr. Huffer. The final period is 4·46718 days instead of 4·4672, while a comparison of spectroscopic and photometric minima is given below:—

	Spectroscopic	Photometric	Diff. Days	Minutes
Primary minimum t_1.....	2,423,966·910	3,966·8931	+·017	+24
	±·004	±·0006		
Secondary minimum t_2...	3,969,095	3,969·1260	−·031	−45
	±·004	±·0024		
Difference t_2-t_1.........	2·185	2·2329		
Half the period..........		2·2336		

As the half period differs by less than a thousandth of a day from the interval between primary and secondary minima and as this interval has a low probable error and is a very sensitive method of determining the eccentricity, there seems to be little doubt that the orbit is practically circular and the eccentricity obtained in the above solution arises from the grouping of the errors of observation and their adjustment by least-squares. Consequently, it was decided to make a new solution with eccentricity zero but before doing so to obtain additional plates near principal minimum.

Thirteen additional plates in the interval between ·125 days before and ·252 days after principal photometric minimum were obtained and measured, their dates, phases, observed and computed velocities and residuals along with those of the earlier plates being given in the table of observations, Table III, below. Preliminary circular elements were obtained from the whole series of 38 plates using the corrected period of 2·46718 days and initial phase T_0 corresponding to the time when the velocity curve crosses the γ line with increasing velocity or when the bright component is nearest the earth and hence corresponding to secondary minimum. These are given in comparison with the final elements in Table IV below.

TABLE III.—SECOND TABLE OF OBSERVATIONS

Plate Number	Julian Date	Final Phase	Velocities		Residuals		
			Observed	Computed	Preliminary	Final	First Solution
10602........	4031·894	0·229	+36·6	+32·61	+3·61	+3·99	+2·19
10692........	4040·903	0·304	+40·7	+39·62	+0·65	+1·08	−0·66
10496........	4009·961	0·632	+65·8	+65·46	−0·22	+0·34	−0·48
10519........	4018·943	0·680	+70·1	+68·38	+1·15	+1·72	+1·09
10815........	4063·663	0·728	+70·7	+71·03	−0·91	−0·33	−0·74
10209........	3969·951	0·827	+77·9	+75·60	+1·72	+2·30	+2·03
10277........	3978·960	0·901	+76·1	+78·18	−2·67	−2·08	−1·76
10391........	3996·961	1·034	+78·7	+80·97	−2·84	−2·27	−1·44
10461........	4005·968	1·106	+80·4	+81·44	−1·59	−1·04	0·00
10880........	4077·684	1·348	+74·5	+77·71	−3·66	−3·21	−1·81
10854........	4068·851	1·449	+72·0	+73·79	−2·20	−1·79	−0·42
10507........	4010·953	1·624	+66·7	+64·02	+2·39	+2·68	+3·82
10221........	3970·954	1·830	+45·2	+48·41	−3·33	−3·21	−2·61
10284........	3979·947	1·888	+45·9	+43·38	+2·45	+2·52	+2·95
10831........	4064·828	1·893	+40·3	+42·93	−2·70	−2·63	−2·23
10400........	3997·937	2·010	+35·4	+32·09	+3·32	+3·31	+3·34
12371........	4417·951	2·109	+22·7	+22·45	+0·34	+0·25	
12372........	4417·979	2·137	+21·2	+19·74	+1·57	+1·46	
12373........	4418·007	2·167	+21·1	+16·70	+4·56	+4·40	
12374........	4418·041	2·199	+15·2	+13·46	+1·91	+1·74	
12518........	4435·954	2·243	+11·2	+ 9·05	+2·35	+2·15	
12519........	4435·983	2·272	+10·6	+ 6·12	+4·69	+4·48	
12443........	4427·051	2·274	+ 3·0	+ 5·93	−2·71	−2·93	
12444........	4427·064	2·287	+ 0·6	+ 4·62	−3·79	−4·02	
12520........	4436·011	2·300	− 1·6	+ 3·32	−4·39	−4·92	
10187........	3966·959	2·302	+ 2·3	+ 3·13	−0·59	−0·83	−1·41
10259........	3975·960	2·369	− 2·9	− 3·54	+0·92	+0·64	0·00
12474........	4431·626	2·382	− 3·7	− 4·82	+1·41	+1·12	
12475........	4431·670	2·426	− 9·9	− 9·12	−0·45	−0·78	
12476........	4431·701	2·457	−12·2	−12·11	+0·26	−0·09	
12477........	4431·728	2·484	−12·3	−14·67	+2·73	+2·37	
10442........	4002·981	2·587	−25·5	−24·09	−0·99	−1·41	−1·99
10531........	4020·967	2·704	−33·6	−33·92	+0·80	+0·32	+0·11
10792........	4056·753	2·752	−37·8	−37·62	+0·32	−0·19	−0·26
10473........	4007·971	3·109	−56·6	−57·39	+1·36	+0·79	+2·00
10450........	4003·961	3·567	−60·2	−58·18	−1·58	−2·02	−0·09
10672........	4039·892	3·760	−52·2	−49·94	−1·93	−2·26	−0·79
10877........	4071·760	4·358	− 2·0	− 0·94	−1·19	−1·06	−2·16

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From these observations and the preliminary elements simple observation equations for the corrections $\delta\gamma$, δK and δT were formed. Normal equations were obtained from these 38 observation equations in two ways. First—the first sixteen and the last twelve were combined, omitting the ten plates whose phases were between 2·109 and 2·302 days inclusive, those within 0·13 days from the minimum and in which a rotation effect might be present. Second—all the observation equations were combined but the ten above mentioned with possible rotation effect were given weight of one-third the others. The first set corresponds practically to the solution in part I except that the orbit is circular, and the second includes all the observations. If a rotation effect is present the orbit and the phase will probably not be affected as the observations, and hence the residuals from the rotation effect, are symmetrically distributed about the minimum and indeed this seems to be shown by the very small differences between the two solutions.

First set Normal Equations—omitting those within ·13 day of minimum:—

$$\begin{aligned} 28{\cdot}000\delta\gamma + 5{\cdot}435\delta K + 6{\cdot}321\delta T &= -2{\cdot}970 \\ +13{\cdot}048\delta K - 2{\cdot}180\delta T &= -8{\cdot}246 \\ +14{\cdot}949\delta T &= +3{\cdot}660 \end{aligned}$$

Second set Normal Equations—all plates included:—

$$\begin{aligned} 31{\cdot}600\delta\gamma + 5{\cdot}447\delta K + 9{\cdot}905\delta T &= -0{\cdot}430 \\ +13{\cdot}079\delta K - 2{\cdot}169\delta T &= -7{\cdot}688 \\ +18{\cdot}517\delta T &= +6{\cdot}199 \end{aligned}$$

The solution of these two normal equations gave for the corrections:—

	1st Normals		2nd Normals	
$\delta\gamma$ =	−0·030	±0·307 km.	−·026	±0·297
δK =	−0·591	±0·447 km.	−·544	±0·425
δT =	+0·00169	± ·00379 days	+·00221	± ·00355 days

As is obvious from Table III, the residuals of the 10 plates near minimum are affected by the rotation of the bright component and are abnormally high, positive before and negative after minimum. Hence the probable errors of the second set have been determined by using the probable error of a plate of unit weight obtained from the first set and are, as they should be, from the larger number of observations, slightly smaller than those of the first set. There result then the following corrected elements.

TABLE IV

Element	Preliminary	First Normals	Second Normals
P Period	4·46718	4·46718	4·46718
K Semi-amplitude	72·0	71·41 ±0·45 km	71·46 ±0·42
γ Vel. system	+10·0	+9·97 ±0·31 km	+9·97 ±0·30
T Nearest approach primary	3964·655	3964·656 +·0038 days	3964·657 ±·00355
Primary minimum	3966·889	3966·890	3966·891
Secondary minimum	3969·122	3969·123	3969·124

The photometric phases of primary and secondary minima are 3966·894 and 3969·127 from the final T given by Huffer 3716·732 which makes the spectroscopic phases as determined from the second set of normals of greater weight, ·003 day or about 4 minutes ahead of the photometric, slightly smaller than the probable error of the spectroscopic determination as compared with the 24 minutes, four times the probable error of the earlier solution. It seems hence practically certain, in this case where the spectra are capable of accurate measurement, that there is no observed difference in the times of spectroscopic and photometric minima, just as there is no known physical cause to produce such a difference. The cause of the difference of phase between the earlier and the two later solutions can reasonably be ascribed to the fact that the least-squares solution simply takes the observations as they stand and changes the elements to make the sum of the squares of the residuals a minimum. The difference of a kilometer or so, well within the range of accidental error, in a few of the observations, would be sufficient to produce such a change in the elements. This is shown by a comparison of the residuals from the earlier and later solutions given in the last two columns of Table III where the average change without regard to sign is 0·84 km. and the average change when signs are taken into account is 0·13 km., and is further shown by the fact that the probable error of a single plate of ±1·24 km. per second in the first general solution is increased only to ±1·40 km. per second when the orbit is assumed circular as required by the photometric work.

The relatively large change produced in the spectroscopic phase by a change in the method of solution furnishes some clue as to the cause of the differences between spectroscopic and photometric phases first observed at Allegheny and later at Victoria. The following list contains the data upon this matter readily available.

Star	Reference	*t* spec.-*t* phot.	P.E. Single Plate
Algol	P.A.O. I, 32	−·070	±1·6
δ Libræ	" I, 123	−·078	±4·7
u Herculis	" II, 6	−·016 ±·003	±8·0
U Sagittae	" III, 11	−·022 ±·0116	±8·3
λ Tauri	" III, 23	−·098 ±·011	±7·2
R Can. Maj	" III, 49	−·054 ±·039	±4·8
RZ Cassiopeiae	" III, 137	−·035 ±·080	±7·1
TX Herculis	D.A.O. I, 211	+·040 ±·005	±2·6
RS Vulpeculae	" I, 149	+·128 ±·010	±1·8
TW Draconis	" I, 149	+·019 ±·010	±2·6
Z Vulpeculae	" I, 253	−·014 ±·008	±6·0
TV Cassiopeiae	" II, 144	−·0045 ±·0035	±4·1
21 Cassiopeiae	Full solution	+·017 ±·004	±1·24
21 Cassiopeiae	Circular orbit	−·003 ±·003	±1·40

In this list Algol and λ Tauri are three-body systems with changing orbits so that the values are uncertain, while in RS Vulpeculae the photometric minima were obtained uncertainly several years earlier. When the other differences are compared with those obtained for the fine lined spectrum of 21 Cassiopeiae, repeated at the foot of the table it seems evident, especially when the relative plate errors given in the last column are considered, that they can be similarly explained and that the observed differences are probably due to the method of solution coupled probably in some cases with errors in the photometric phase or period.

The accompanying velocity curves (Fig. 1) show the agreement of the observations with the curves and the slight differences involved in the two solutions. The second shows unmistakably the presence of a rotation effect[1] near minimum reproduced on twice the scale in the inset curve, the rotationally displaced curve being drawn as smoothly as

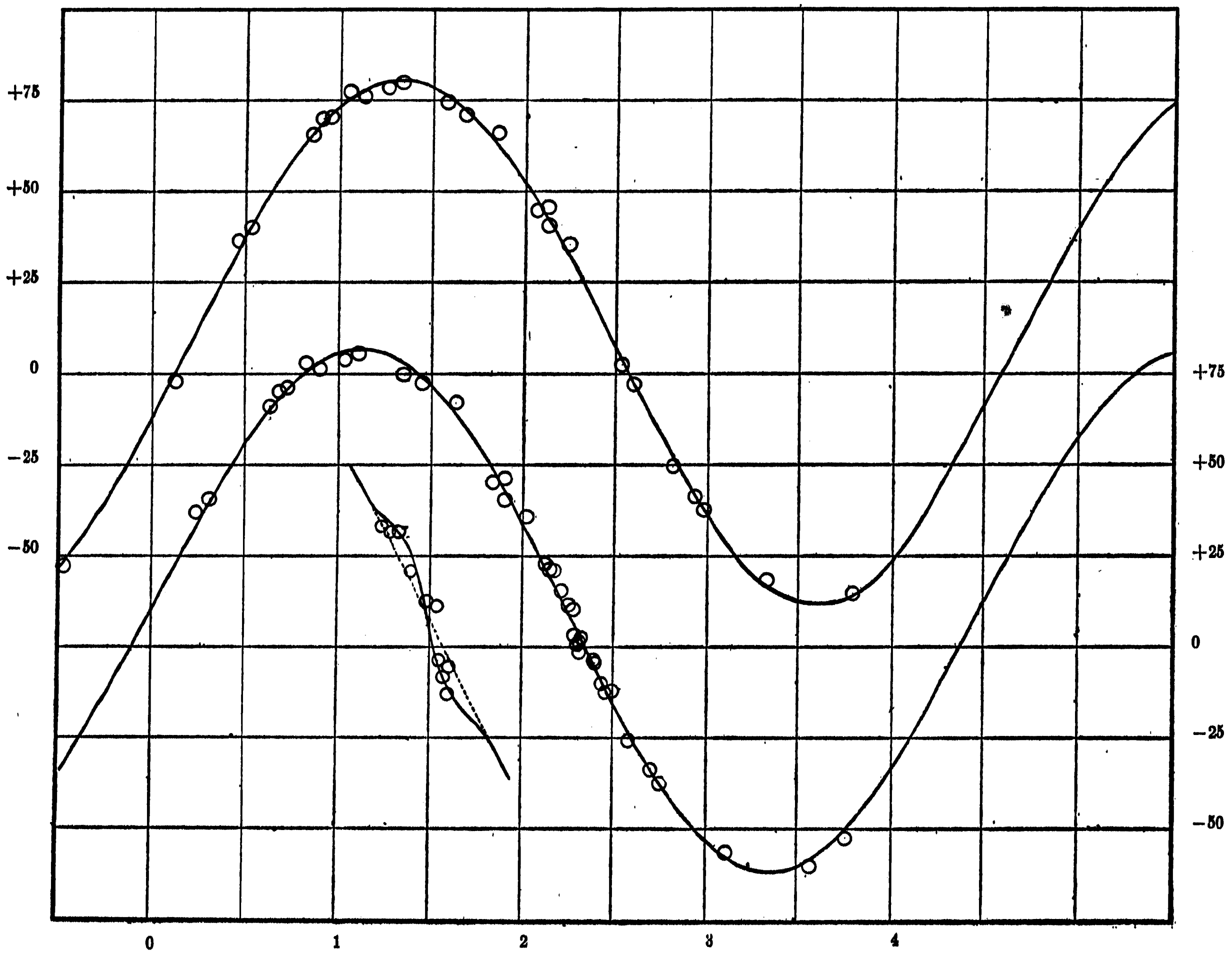

FIG 1—VELOCITY CURVES OF 21 CASSIOPEIAE
UPPER ECCENTRIC—LOWER CIRCULAR ORBIT

possible through the observations. Owing to uncertain weather when these were obtained and to the loss of brightness due to eclipse, these plates are not of as good quality as the others but serve to give an approximate value of the rotation effect. The deviation from the sine curve is, at maximum, apparently slightly greater than 3 km. If we take Huffer's values of the relative radii, 0·156 for the bright, 0·076 for the faint star, times the separation of the pair, with an inclination of 87°·4, and assume rotation and revolution periods the same, the effective displacement roughly calculated has a maximum value of about 4 km. The difference between the observed and calculated values, considering the character of

[1] P.A. 32, pp. 21, 225, 558.

the data, is not greater than should be expected and so far as the results go they confirm the probable hypothesis that tidal effects in such a close pair would equalize the periods of rotation and revolution.

THE ORBIT OF BOSS 3354

The star Boss 3354, R.A. 12^h $48 \cdot 3^m$, Dec. $+83°$ $58'$, spectral type A0, magnitude 5·81, was one of the stars of the first radial velocity programme undertaken here and on the second plate obtained by the writer on March 25, 1919, the spectrum lines were doubled, indicating its binary character. Four plates were obtained in 1919 and one in 1920, but it was not until last season that further observations were secured. Then it was decided to complete the orbit as between 7 hrs. and 15 hrs. of right ascension there are very few stars in the programme of B's now under investigation. Of the 32 spectra obtained altogether, only 27 were used in the determination of the orbit as the others were in such relation to the crossing points that the lines were not separated and yet were too diffuse to be satisfactorily used.

The spectrum is of type A0 with fairly good hydrogen lines frequently well measurable and a number of sharp metallic lines of which 4481 Mg and 3933 Ca are the best. The number of lines measured varied from 6 to 10 and for a double-lined spectrum the measures seemed fairly reliable and certain so that the relatively high probable error per plate of $\pm 4 \cdot 3$ km. per second was disappointingly large. Although there is not much difference in the intensity of the component lines it was generally possible to distinguish between primary and secondary spectrum.

In the table of observations below are given the serial number, the Greenwich date and the Julian date of the observations with the measured velocities of the primary and secondary stars. The phase and the residuals, observed minus computed, from the final corrected orbit are given in the fourth and the seventh and eighth columns.

TABLE V—OBSERVATIONS OF BOSS 3354

Plate Number	Date	Julian Date	Phase	Velocities		Residuals	
				Primary	Secondary	Primary	Secondary
1526	1919 Mar. 8	2,412,026·907	2·227	+ 0·1	....	+ 5·6	− 6·3
1698	Mar. 25	043·813	2·700	− 88·8	+106·2	+ 0·9	− 0·3
1734	April 1	050·812	3·126	−104·8	+112·8	+ 5·0	−17·5
1795	April 13	062·782	1·949	+ 44·8	− 56·2	− 2·2	− 0·2
11330	1925 Mar. 18	2,424,227·896	1·227	+107·9	−120·0	+ 5·5	+ 1·9
11348	Mar. 23	232·985	3·029	−108·2	+130·8	+ 3·9	− 2·5
11363	Mar. 26	235·925	2·683	− 78·6	+106·7	+ 8·9	+ 2·8
11376	Mar. 30	239·942	0·126	− 78·0	+100·8	+ 1·1	+ 7·0
11394	April 5	245·834	2·734	−103·2	+115·7	− 9·2	+10·8
11406	April 6	246·907	0·519	+ 8·8	....	+ 9·0	+ 8·7
11436	April 8	248·795	2·424	− 48·1	+ 60·7	− 3·8	+ 8·1
11449	April 9	249·812	0·137	− 76·9	+ 89·8	+ 0·4	− 1·9
11467	April 10	250·940	1·265	+108·9	−120·3	+ 5·2	+ 3·1
11476	April 13	253·794	0·833	+ 66·6	− 72·1	+ 5·2	+ 0·9
11507	April 17	257·873	1·625	+ 91·6	−110·3	+ 0·9	− 2·2

TABLE V.—OBSERVATIONS OF BOSS 3354—Concluded.

Plate Number	Date	Julian Date	Phase	Velocities		Residuals	
				Primary	Secondary	Primary	Secondary
11516	April 20	260·861	1·327	+106·3	−131·9	+ 1·8	− 7·5
11535	April 23	263·675	0·854	+ 59·7	− 65·9	− 5·1	+11·2
11540	April 23	263·809	0·988	+ 81·2	− 99·0	− 1·1	− 1·0
11546	April 23	263·958	1·137	+ 96·5	−115·5	− 1·1	+ 0·6
11552	April 24	264·847	2·026	+ 40·0	− 46·2	+ 6·7	− 6·4
11566	April 27	267·706	1·598	+106·2	−104·7	+12·8	+ 6·4
11575	April 27	267·944	1·836	+ 53·2	− 84·4	−10·3	− 8·7
11716	June 15	316·724	1·318	+ 96·6	−125·2	− 7·8	− 0·8
11730	June 18	319·800	1·108	+ 90·4	−111·6	− 5·1	+ 2·0
11752	June 21	322·826	0·847	+ 67·4	− 83·2	+ 3·8	− 7·5
11774	June 25	326·818	1·553	+ 86·2	−121·3	−10·5	− 6·2
11837	July 6	2,424,337·783	2·658	− 82·6	+101·7	+ 1·4	+ 1·9

From these observations the period was relatively easily obtained and so adjusted by the fitting of the four early observations that it was considered as definitely determined at 3·28655 days and the period was not included in the least-squares solution. By the usual method of trial and error from the velocity curve plotted with this period the preliminary elements given in Table VII below were obtained. The 27 observations were then grouped into twelve normal places for each component, an ephemeris from the preliminary elements computed and 24 observation equations, formed by Schlesinger's method in the usual way, are given in Table VI below. As the eccentricity is small and both ω and T cannot be accurately determined, the latter was considered fixed and correction made for ω. The last column of this table contains the final residuals computed from the final elements so that preliminary and final residuals can be directly compared.

TABLE VI—OBSERVATION EQUATIONS FOR BOSS 3354

No.	Wt.	Γ	K_1	K_2	π	ϵ	δV	Final Residual
1	2	1·000	−·695	0	− ·719	−·183	− 2·05	+0·78
2	1	1·000	+·032	0	− ·999	−·807	+ 6·42	+9·00
3	3	1·000	+·619	0	− ·785	−·700	− 0·64	+2·83
4	1	1·000	+·804	0	− ·594	−·487	− 3·87	−1·08
5	2	1·000	+·925	0	− ·380	−·268	− 4·55	−2·40
6	4	1·000	+·995	0	− ·103	−·053	− 0·53	+1·08
7	3	1·000	+·907	0	+ ·421	+·033	− 1·35	+1·05
8	3	1·000	+·511	0	+ ·860	−·372	− 7·66	−2·97
9	1	1·000	+·019	0	+1·000	−·765	+ 1·10	+4·31
10	1	1·000	−·339	0	+ ·941	−·834	−10·78	−3·79
11	4	1·000	−·762	0	+ ·647	−·553	− 8·76	+0·57
12	2	1·000	−·992	0	− ·123	+·044	+ 0·68	+4·48
13	2	1·000	0	+·695	+ ·719	+·183	+10·22	+2·44
14	1	1·000	0	−·032	+ ·999	+·807	+14·53	+8·68
15	3	1·000	0	−·619	+ ·785	+·700	+ 5·97	+0·39
16	1	1·000	0	−·804	+ ·594	+·487	+ 0·33	−1·02
17	2	1·000	0	−·925	+ ·380	+·268	+ 3·78	+0·47
18	4	1·000	0	−·995	+ ·103	+·053	+ 1·72	−0·80
19	3	1·000	0	−·907	− ·421	−·033	+ 3·06	−0·60
20	3	1·000	0	−·511	− ·860	+·372	+ 3·33	−3·94
21	1	1·000	0	−·019	−1·000	+·765	+ 4·70	−7·63
22	1	1·000	0	+·339	− ·941	+·834	+20·06	+8·12
23	4	1·000	0	+·762	− ·647	+·553	+14·12	+2·02
24	2	1·000	0	+·992	+ ·123	−·044	− 0·46	−10·08

From these observation equations the following normal equations were obtained:—

$$
\begin{array}{rrrrrcr}
54 \cdot 000\Gamma & + 6 \cdot 030K_1 & - 6 \cdot 030K_2 & + \cdot 000\pi & + \cdot 000\epsilon & = & -102 \cdot 288 \\
 & +16 \cdot 080K_1 & + \cdot 000K_2 & - 1 \cdot 647\pi & - \cdot 644\epsilon & = & +13 \cdot 875 \\
 & & +16 \cdot 080K_2 & - 1 \cdot 647\pi & - \cdot 644\epsilon & = & +73 \cdot 806 \\
 & & & +21 \cdot 808\pi & - 1 \cdot 350\epsilon & = & -57 \cdot 945 \\
 & & & & +11 \cdot 004\epsilon & = & +25 \cdot 508
\end{array}
$$

The solution of these normals gave the following corrections:—

$$
\begin{array}{lcl l lcl}
\epsilon & = & +2 \cdot 363 & \text{whence} & \delta e & = & - \cdot 0092 \ \pm \cdot 005 \\
\pi & = & -2 \cdot 119 & & \delta\omega & = & + \cdot 0183 = +1^\circ \cdot 05 \ \pm 0^\circ \cdot 44 \\
K_1 & = & +3 \cdot 863 & & \delta K_2 & = & +3 \cdot 863 \ \pm 1 \cdot 07 \\
K_2 & = & +1 \cdot 345 & & \delta K_1 & = & +1 \cdot 345 \ \pm 1 \cdot 05 \\
\Gamma & = & -1 \cdot 613 & & \delta\gamma & = & -1 \cdot 68
\end{array}
$$

and hence the final elements, Table VII.

TABLE VII.—ELEMENTS OF BOSS 3354

Element	Preliminary	Final
P Period	3·28655	3·28655 days
e Eccentricity	0·05	0·0408 ±·005
ω Longitude of node	210°	211°·05 ±0°·44
T Time of periastron	2,424,226·669	2,424,226·669 J.D.
K_1 Semi-amplitude primary	107	108·34 ±1·05 km.
K_2 Semi-amplitude secondary	125	128·86 ±1·07 km.
γ Velocity of system	+1·63	−0·05 km.
$a_1 \sin i$ } Projected semi-axis major		5,426,000 km.
$a_2 \sin i$ }		6,454,000 km.
$m_1 \sin^3 i$ } Masses		2·47⊙
$m_2 \sin^3 i$ }		2·08⊙

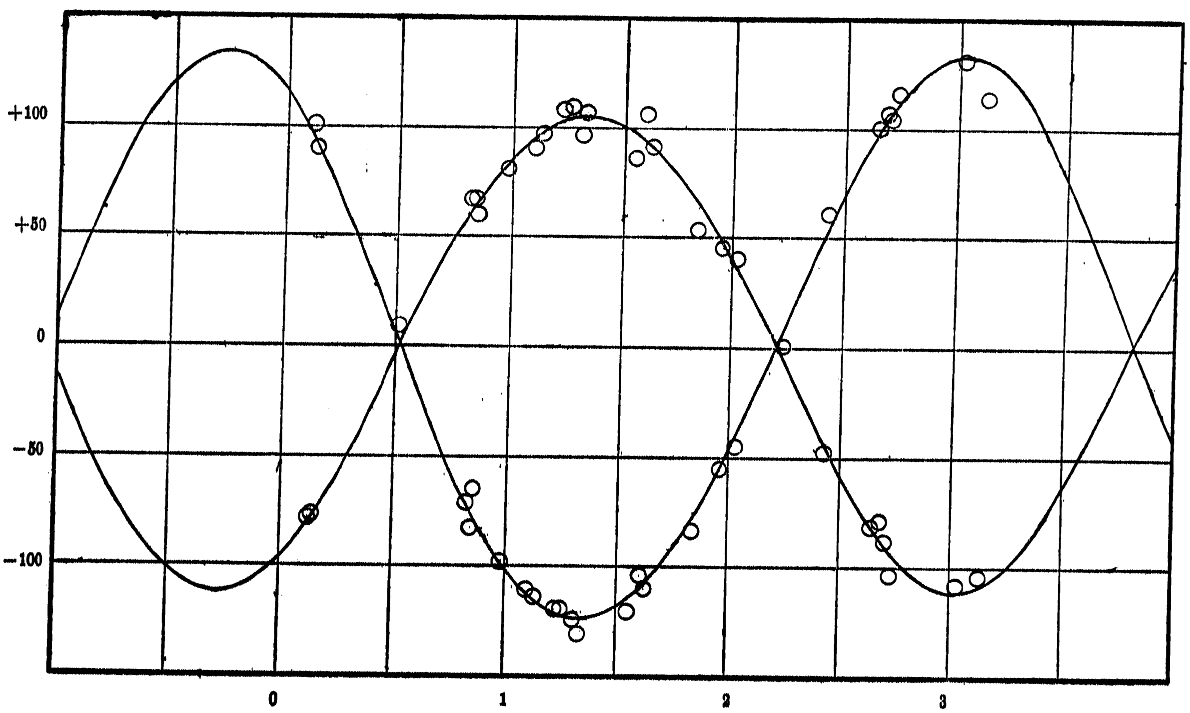

Fig 2—Velocity Curves of Boss 3354

The sum of the squares of the residuals was reduced from 1457 to 711, while the probable error of a single plate became $\pm 4 \cdot 3$ km. per second for the primary and $\pm 4 \cdot 4$ km. per second for the secondary. The accompanying velocity curve (Fig. 2) has the individual velocities indicated by circles.

THE SPECTROSCOPIC BINARY H.D. 191201

The star H.D. 191201, R.A. 20^h $03 \cdot 6^m$, Dec. $+35° 26'$ (1900), visual magnitude $7 \cdot 12$, spectral type by H.D. catalogue B0, is one of the stars in the B-type programme under investigation here by Mr. J. A. Pearce and the writer. It is one of those allotted to me in the division of the stars between the two observers and on the first plate, obtained on September 18, 1924, the spectrum showed double lines with a relative separation of 360 km. Owing to the interest and value of double-lined spectra for obtaining values of mass and estimates of absolute magnitude and parallax it was decided to investigate the orbit. Spectra of this object were obtained as frequently as possible until December 24, after which it was too near the sun to observe. As the spectra were difficult and the behaviour puzzling, observations were carried on after it had passed conjunction beginning on March 26, and continued until October 1, 1925. Altogether 48 spectra were obtained, of which 40 were measured and 36 used in the orbital work.

Although the writer has measured the spectra and determined the orbits of a considerable number of B and O-type double-lined binaries, in none of them were the measures so difficult and uncertain as in H.D. 191201. This is due to two reasons. First, the lines, even of the primary, are diffuse and weak, while those of the second component have only about half the intensity of the primary, and unless there is a relative separation of nearly 200 km., the pair are blended with resultant uncertainty of position of the maxima of intensity, especially in the case of the secondary. Second, the relatively high eccentricity $0 \cdot 26$ of the orbit with ω nearly zero, makes the proportion of the time when the relative separation of the double lines reaches 200 km. only about half the period. Hence, in a considerable number of the plates, the lines appeared broad, diffuse, weak and unsymmetrical and, especially in the early stages, were measured as single lines. It was only after the period, and hence the position of these spectra on the curve, were approximately known that it was seen these unsymmetrical lines might be considered as the blend of a strong primary and a weak secondary. Considering these spectra as double the plates were remeasured, and while the positions of the primary component were fairly well determined, probable error of single plate $\pm 9 \cdot 9$ km. per second, the uncertainty of the positions of the secondary, even with the most careful measurement, is clearly shown by the high probable error per plate, determined from all the plates measured, of $\pm 25 \cdot 2$ km. per second.

The number of lines measured in the spectra varied from 3 to 24, with an average of about 8 lines. The type of spectrum given as B0 in the Henry Draper Catalogue is in reality somewhat "earlier" as, especially in the single-lined spectra, enhanced helium is conspicuous and its ratios to 4471 and $H\gamma$ and the ratios of 4088 to 4097 make the type between O9 and B0. In the single-lined plate of October 12, 1924, in which 24 star lines were measured, the lines measured include Hβ, Hγ, Hδ: He 4922, 4713, 4472, 4388, 4143, 4026; He+ 4686, 4542, 4339, 4200; Si 4574, 4567, 4552, 4116, 4088; N+ 4097; O+ 4651, 4642, 4076; C+ 4267; S? 4254; Ca+ 3967, 3933. As in similar high temperature stars the lines of ionized calcium H and K are of an entirely different character, are sharp and do not share in the displacements of the other lines.

Of the 48 spectra obtained, 8 were of such poor quality, due to faulty exposure or to blend effects, as to be unsuitable for measurement. Of the 40 spectra measured only 36 were used, the other 4 being so near the crossing points that the blended lines could not be resolved and yet too far away to be safely employed as single-lined spectra. The 36 spectra employed in the orbit are given in the table of observations, Table VIII, below, which contains the Greenwich and Julian dates and the final phases of the observations. The observed velocities are in many cases the means of two independent measures of the spectra while the residuals are obtained from an ephemeris computed from the final elements. The velocities from H and K of calcium are given in the last column. The mean calcium velocity is −13·3 ±0·7 km. per second while the probable error of a measure is ±3·5 km. per second. The component of the solar motion in the direction of this star is 18·0 km. per second and the residual velocity +4·7 km. per second of the calcium cloud is the same as the average for the eight other O-type stars within a degree of this region previously investigated by the writer.[1]

TABLE VIII.—OBSERVATIONS OF H.D. 192101

Plate Number	Date G.M.T.	Julian Date	Final Phase	Observed Velocities		Residuals O-C		Calcium Velocity
				Primary	Secondary	Primary	Secondary	
		2,424						
11556	1925 April 24	264·983	0·018	+189·4	−166·3	− 3·5	+52·0	−15·0
10904	1924 Oct. 22	081·637	0·025	+192·7	−206·6	− 0·1	+11·6	
11380	1925 Mar. 30	240·042	0·080	+196·8	−165·0	+ 4·9	+52·3	−20·8
12259	Sept. 13	406·806	0·159	+176·6	−185·8	−12·5	+28·5	
11897	July 17	348·872	0·564	+145·6	−127·2	− 6·5	+47·4	−10·0
11442	April 8	249·024	0·727	+141·4	−105·6	+11·3	+45·3	−13·5
12272	Sept. 14	407·739	1·092	+121·3	− 85·3	+34·7	+ 8·2	
10922	1924 Nov. 9	099·622	1·342	+ 8·0	....	−33·3	....	− 9·1
12407	1925 Oct. 1	424·815	1·499	− 4·8	....	−25·6		
10954	1924 Nov. 26	116·568	1·619	+ 6·9	....	− 0·4	....	−13·6
11062	Dec. 21	141·580	1·629	+ 14·0	....	+ 8·9	....	−12·4
11457	1925 April 9	250·005	1·708	− 2·2	....	+ 1·7		
10916	1924 Nov. 2	092·631	2·685	− 99·2	+ 62·2	−15·7	−16·2	− 5·6
12207	1925 Aug. 31	393·768	3·789	−107·4	+102·0	+11·5	−14·3	−11·6
10930	1924 Nov. 20	110·622	4·008	−133·3	+100·7	−12·6	−17·6	
10878	Oct. 18	077·603	4·326	−113·5	+130·2	+ 7·0	+12·2	−15·7
11011	Dec. 7	127·610	4·328	−118·4	+156·0	+ 2·1	+38·0	−12·5
11367	1925 Mar. 26	236·042	4·414	−120·9	+189·3	− 1·1	+71·9	
12237	Sept. 9	402·804	4·491	−141·7	+137·5	−22·6	+20·9	
10992	1924 Nov. 29	119·560	4·612	−110·2	+140·7	+ 7·3	+25·9	−11·7
11113	Dec. 24	144·580	4·629	−100·5	+134·1	+16·7	+19·5	−27·2
11732	1925 June 18	319·894	4·923	− 89·6	+126·0	+21·5	+18·1	−21·3

[1] Pub. D.A.O., II, p. 337.

TABLE VIII.—OBSERVATIONS OF H.D. 192101—Concluded.

Plate Number	Date G.M.T.	Julian Date	Final Phase	Observed Velocities		Residuals O-C		Calcium Velocity
				Primary	Secondary	Primary	Secondary	
10932	1924 Nov. 21	111·564	4·950	−102·6	+108·6	+ 7·7	+ 1·4	− 2·1
11994	1925 July 30	361·867	5·225	− 94·6	+130·1	+ 6·9	+32·5	−13·2
10881	1924 Oct. 19	078·617	5·340	−113·2	+179·3	−16·3	+86·6	−12·3
12244	1925 Sept. 10	403·733	5·420	− 95·2	+140·3	− 1·9	+51·4	
10754	1924 Sept. 24	053·781	5·507	− 99·3	+ 90·7	−10·1	+ 6·2	−10·5
11482	1925 April 13	253·981	5·684	− 99·4	+131·8	−19·6	+57·4	
11934	July 23	354·876	6·568	+ 9·5	....	+17·2	....	−20·9
10871	1924 Oct. 12	071·670	6·727	− 3·0	....	−13·4	....	− 9·5
11033	Dec. 18	138·594	6·977	+ 22·5	....	−19·5		
11547	1925 April 23	263·982	7·351	+105·2	−128·3	+10·1	−15·0	−17·4
10738	1924 Sept. 18	047·715	7·775	+163·8	−200·5	+ 9·3	−23·3	− 8·7
11757	1925 June 21	322·931	7·960	+186·8	−163·0	+11·9	+36·0	−12·5
11047	1924 Dec. 19	139·610	7·993	+159·9	−140·7	−18·4	+61·8	− 9·9
10823	Oct. 5	064·661	8·052	+179·4	−212·9	− 3·1	− 5·7	−14·6

Considerable difficulty was experienced in obtaining the period of this star, probably mainly due to the shape of the velocity curve, to the large proportion of the plates where the lines appeared single and of course also to the difficult and uncertain measures. Periods of 8·33 and 8·34 days appeared to fit the observations about equally well with a slight preference for 8·33, and as the whole interval covered was only 43 periods, it was obvious that a correction for the period would be required as well as to the other elements. From the velocity curves plotted with a period of 8·33 days preliminary elements were obtained by the usual graphical method, which are given in Table X below.

As a correction to the period was to be included it was impossible to combine generally the observations into normal places. However, where they were grouped closely together in the flat parts of the curve, and where the intervals between them were small, three groups composed of three, two and two plates, plate numbers 10930, 10878, 11011; 10992, 11113; 10881, 10754 respectively, were combined into observation equations **15**, **17**, **22**, thus reducing the number of observation equations to 32.

Owing to the high residuals in many plates it was necessary to weight the observations carefully, else an abnormally high residual, if given undue weight, would too greatly influence the solution. The weights of the individual plates were then based jointly on the number of lines measured, on their interagreement, and on the apparent quality as determined by eyepiece estimates. They were finally limited to three weights, $\frac{1}{2}$, $\frac{2}{3}$ and 1, while the three groups were given the combined weights of their individual plates. The weights of the measures of the second component, on account of their difficulty and uncertainty, were given only half the weights of the primary and hence were $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$.

While it was felt that it would be desirable to use both primary and secondary measures in the solution, the uncertainty of the secondary was so high with the further effect of appearing to have a γ about 20 km. more positive than the primary, that it appeared desirable to restrict the solution to the primary. However, for the interest of the matter and as the work would not be greatly increased, a set of 24 observation equations was also formed for the secondary in addition to the 32 for the primary, the 8 omitted being near the crossing points where, owing to the weakness of the secondary, it was considered preferable to exclude them. It may be stated at once that the least squares solution of the whole set resulted in values of the elements that gave poorer agreement for the primary than the assumed preliminary values and hence the original opinion in regard to omitting the secondary was confirmed. The 32 observation equations for the primary introducing corrections for γ, K, e, ω, T and P were formed by Schlesinger's method and are given in Table IX. The weights assigned are given in the second column and the final residuals in the last column for comparison with the preliminary residuals in the second last column. The computed values of m, the last unknown, were divided by 100 to make them homogeneous with the others.

TABLE IX.—OBSERVATION EQUATIONS OF H.D. 191201

No.	Weight	Γ	κ	π	ε	τ	m	v	Final Residual
1	1·0	1·000	+1·000	−·002	+·000	−·002	− ·001	−1·300	− 3·46
2	0·67	1·000	+ ·994	+·106	+·021	+·105	+ ·236	−3·71	− 0·10
3	0·67	1·000	+ ·986	+·167	+·042	+·165	+ ·332	+5·01	+ 4·91
4	1·0	1·000	+ ·932	+·362	+·160	+·347	+1·272	− 6·76	−12·51
5	1·0	1·000	+ ·692	+·722	+·540	+·619	+1·911	− 0·08	− 6·51
6	1·0	1·000	+ ·601	+·799	+·652	+·660	+1·381	+10·01	+11·34
7	1·0	1·000	+ ·216	+·976	+·888	+·678	+2·483	+40·40	+34·71
8	0·67	1·000	+ ·137	+·990	+·899	+·658	+ ·393	−50·52	−33·26
9	1·0	1·000	− ·091	+·996	+·867	+·594	+ ·455	−15·84	− 0·39
10	0·67	1·000	− ·107	+·994	+·861	+·588	+ ·639	− 6·14	+ 8·86
11	0·67	1·000	− ·118	+·993	+·857	+·582	+2·240	−23·21	−25·65
12	1·0	1·000	− ·205	+·979	+·817	+·551	+1·161	− 7·06	+ 1·72
13	1·0	1·000	− ·693	+·721	+·390	+·315	+ ·166	−27·46	−15·68
14	1·0	1·000	− ·985	+·171	+·012	+·065	+ ·230	+10·29	+11·50
15	3·0	1·000	− ·998	+·054	−·002	+·020	+ ·013	− 1·94	− 1·23
16	0·5	1·000	− ·996	−·088	+·012	−·033	− ·064	− 1·51	− 1·06
17	2·0	1·000	− ·987	−·161	+·032	−·062	− ·057	+12·54	+11·96
18	1·0	1·000	− ·986	−·165	+·033	−·064	− ·232	−23·87	−22·62
19	1·0	1·000	− ·949	−·314	+·100	−·124	− ·089	+ 9·47	+ 7·74
20	1·0	1·000	− ·936	−·352	+·122	−·141	− ·395	+20·38	+21·46
21	1·0	1·000	− ·864	−·503	+·236	−·212	− ·682	+ 4·10	+ 6·86
22	1·67	1·000	− ·851	−·524	+·256	−·223	− ·059	− 9·53	−12·98
23	1·0	1·000	− ·799	−·601	+·330	−·264	− ·960	− 6·69	− 1·86
24	1·0	1·000	− ·731	−·683	+·422	−·314	− ·672	−21·66	−19·62
25	1·0	1·000	− ·192	−·981	+·870	−·606	−1·909	+ 2·64	+17·17
26	0·67	1·000	− ·181	−·983	+·874	−·614	− ·195	−11·58	−13·37
27	0·67	1·000	+ ·054	−·998	+·909	−·694	− ·685	−23·04	−19·47
28	1·0	1·000	+ ·459	−·888	+·723	−·736	−1·651	− 3·83	+10·15
29	1·0	1·000	+ ·734	−·679	+·412	−·625	− ·049	+11·58	+ 9·29
30	0·67	1·000	+ ·910	−·414	+·139	−·404	− ·403	−20·00	−18·38
31	1·0	1·000	+ ·921	−·390	+·122	−·383	− ·095	− 2·15	− 3·13
32	1·0	1·000	+ ·939	−·344	+·090	−·339	− ·959	+ 2·38	+11·93

From these observation equations the following normal equations were formed in the usual way.

$$
\begin{array}{rrrrrrcr}
32\cdot273\Gamma & -\ 6\cdot056\kappa & -\ 0\cdot707\pi & +11\cdot224\epsilon & -0\cdot187\tau & +\ 2\cdot793m & = & -88\cdot016 \\
 & +19\cdot821 & +\ 2\cdot271 & -\ 0\cdot195 & +0\cdot782 & +\ 4\cdot506 & = & +21\cdot103 \\
 & & +12\cdot359 & +\ 1\cdot510 & +7\cdot991 & +15\cdot314 & = & -11\cdot320 \\
 & & & +\ 7\cdot549 & +0\cdot963 & +\ 3\cdot196 & = & -62\cdot313 \\
 & & & & +5\cdot458 & +10\cdot456 & = & -\ 0\cdot625 \\
 & & & & & +28\cdot453 & = & +69\cdot490
\end{array}
$$

The solution of these normal equations resulted in the values of the unknowns and the corresponding value of the corrections below.

$$
\begin{array}{lcl l lcl}
\Gamma & = & -\ \cdot8714 & \text{whence} & \delta\gamma & = & -4\cdot898 \text{ km.} \\
\kappa & = & -\ \cdot027 & & \delta K & = & -\ \cdot027 \text{ km.} \\
\pi & = & -15\cdot078 & & \delta\omega & = & +5^{\circ}\cdot50 \\
\epsilon & = & -\ 8\cdot755 & & \delta e & = & +\ \cdot0238 \\
\tau & = & +\ 4\cdot044 & & \delta T & = & +\ \cdot0203 \text{ days} \\
m & = & +10\cdot145 & & \delta\mu & = & -\ \cdot0003845
\end{array}
$$

These corrections give for the final elements with their probable errors the values of Table X.

TABLE X.—ELEMENTS OF H.D. 191201

Element	Preliminary	Final
e Eccentricity	0·24	0·2638 ±·0154
ω Longitude of apse	355°	0°·50 ±4°·83
K_1 Semi amplitude primary	157 km.	156·97 ±2·97 km.
K_2 Semi amplitude secondary	165 km.	168·5 ±7·7 km.
γ Velocity of system	−0·54 km.	−5·44 km.
T_0 Time of periastron	J.D. 4039·92	J.D. 4039·940 ±·108
P Period	8·33 days	8·33425 ±·0016 days
$a_1 \sin i$ Semi axis major primary		17,351,000 km.
$a_2 \sin i$ Semi axis major secondary		18,626,000 km.
$m_1 \sin_3 i$ Mass primary		13·85☉
$m_1 \sin_3 i$ Mass secondary		12·90☉

The corrections for K_2 were obtained by accepting the elements obtained from the primary solution and obtaining by least-squares the most probable correction for K_2. As will be seen below, the probable error of a single average plate of K_2, ±25·2 km., is very high, nearly three times that of K_1. If the γ for K_2 were assumed 23 km. more positive than for K_1, a quite impossible condition, the probable error per plate is reduced to ±18·6 km. It is obvious then that the measures of the secondary have not only a very high accidental error, but they are systematically affected also, the lines displaced redward on the plates on the average over 20 km. No explanation, except the difficulty of determining the intensity maximum in such faint diffuse lines frequently blended with the primary, can be advanced for this curious systematic displacement. The velocity curves with the separate observations plotted as circles are shown in Fig. 3.

The solution reduced Σpvv from 8013 to 6607 and gave the probable error of an average plate of the primary as $\pm 9 \cdot 9$ km. per second and of the secondary as $\pm 25 \cdot 2$ as stated above. These probable errors are not unreasonably high when the difficulty of the spectra are taken into account and, as the amplitude is large, the elements may be considered as fairly reliable. Probably the values of the mass functions are not in error more than five per cent and it will be of interest, as in the previous B- and O-type binaries, to compute the probable absolute magnitude and parallax.

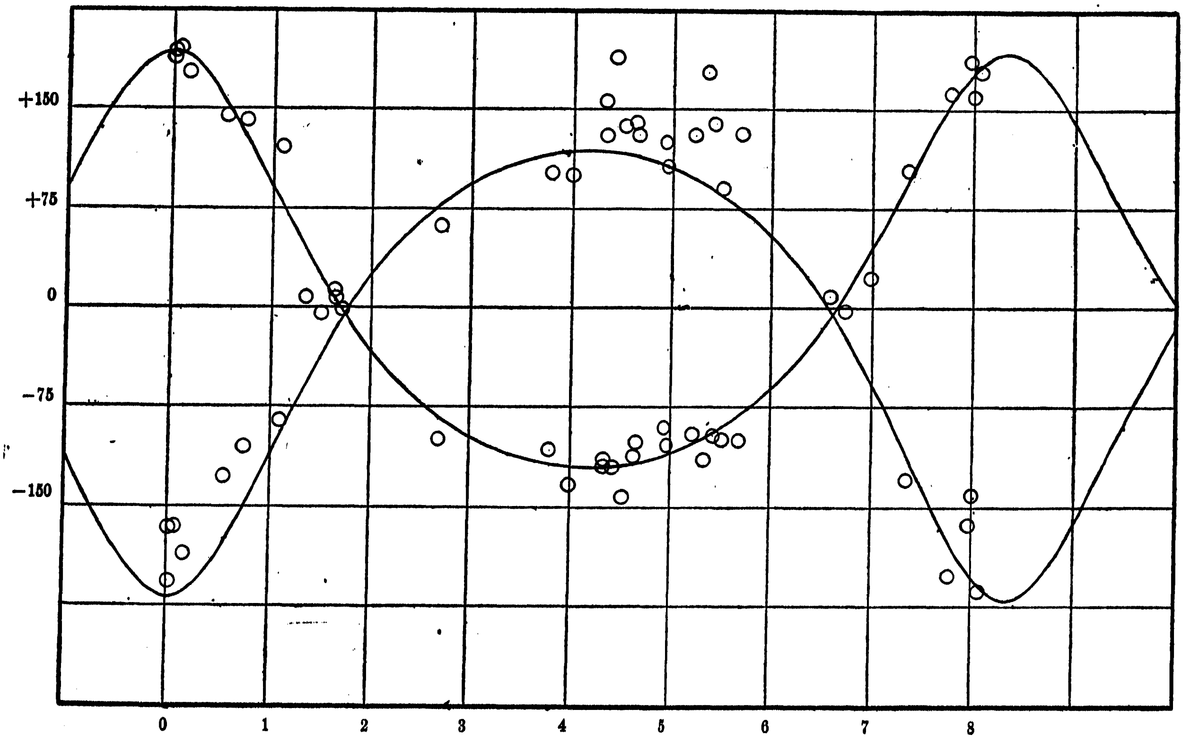

FIG 3—VELOCITY CURVES OF H.D. 191201

If we take the minimum value of the mass of the brighter component $13 \cdot 85 \odot$ and assume a temperature for this O9-B0 star of 26,000° T, a value according with Fowler and Milne's latest determination,[1] we obtain a surface brightness by Hertzsprung's method[2] of $-3 \cdot 84$ magnitudes, 34 times the sun, with which Russell's colour index method practically agrees. Some recent work of Mr. Pearce clearly shows that the density of these massive high temperature stars is of the order of $0 \cdot 01$ the sun. However, the star H.D. 1337, for which this was determined, has obviously only relatively recently become a double as the photometric orbit shows the two stars are nearly in contact and it is probably considerably more diffuse than H.D. 191201.

[1] M.N., 84, 509.
[2] Mt. W. Cont., 10, 399.

The eccentricity of H.D. 191201 of 0·26 indicates a considerable advance in the evolutionary stage and hence probably an increase of density. The eclipsing binary Y Cygni, which is of nearly the same type, has a density of 0·17 so that the density of this star probably lies between the extreme values of 0·01 and 0·2. With a surface brightness of −3·84 magnitudes compared to the sun and a mass 13·85 times, the absolute magnitude is readily calculated as −4·2 for a density 0·01; −3·0 for the probable density of 0·05; −2·5 for density of 0·1 and −2·0 for density 0·2.

The absolute bolometric magnitude of a star of mass 13·85⊙ by Eddington's latest method[1] is −3·17, the correction for a temperature of 26,000° being −1·40, making a total of −4·57. The correction to visual magnitudes for this temperature is +2·53, making the calculated visual magnitude −2·04, corresponding to a density of 0·2. As is shown by Mr. Pearce in his discussion of H. D. 1337 where the density is determined by the photometric orbit, the absolute magnitude of the brighter component obtained from Eddington's method is about 2·3 magnitude fainter than that computed from the known mass, density, and surface brightness of that star, and obviously it does not fit on Eddington's curve which appears to suit better the dwarf than the giant stage at these high temperatures and masses.

The apparent magnitude of the pair is 7·12 and the second component, from the relative intensity of the spectra and also considering the relative masses, must be at least 0·2 magnitudes fainter than the primary. This will make the apparent visual magnitude of the primary 7·77.

The well known formula connecting absolute and apparent visual magnitude with parallax gives a parallax of 0″·0011 for absolute magnitude of −2·0, of 0″·0007 for absolute magnitude −3·0 corresponding to the probable density of 0·05, and a parallax of 0″·004 for magnitude −4·2 and density 0·01, distances respectively of 3500, 4700 and 8000 light years. If we take into account the fact that the inclination must be considerably less than 90° else the star would appear variable, and that hence the mass must be increased, while the high density of 0·2 is unlikely, it seems certain that the distance of this star is at least 5000 light years, making it at a greater distance than the average O-type and several times farther than previously ascribed to the B-type stars.

[1] M.N., 84, 310.

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