to maintain the constant radiation from Sun and stars. So far all attempts to solve this problem have raised more difficulties than they have cleared away, and Eddington, whose name is intimately associated with such progress as has been made in its investigation, is forced to conclude his most recent series of lectures on "the source of stellar energy" with the paragraph:

> I should have liked to close this course by leading up to some great climax. But perhaps it is more in accordance with the conditions of scientific progress that it should fizzle out, ending with a glimpse of the obscurity which marks the frontiers of present knowledge. I do not apologize for the lameness of my conclusion, for it is not a conclusion. I wish I could feel confident that it is even a beginning.

Eddington's words are not words of discouragement. Rather, they are a stimulus to more strenuous efforts. There is so much still to be learned, there are so many problems still to be solved. We need more data, data secured by the execution of carefully planned programs of observation and experiment. Data relating to stars and nebulae, but also data relating to the Moon, the planets, the asteroids and the comets of our own solar system. They all hold secrets, secrets which can be unveiled, and in Professor Whitehead's words, "It is this instinctive conviction vividly poised before the imagination, which is the motive power of research."

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THE PLANETARY NEBULAE

By Donald H. Menzel

One of the most interesting of the unsolved puzzles of stellar spectroscopy is the problem of the planetary nebulae. Much material has been gathered together by the different observers, but there are still many difficulties which lie in the way of anything like a clear understanding of their intricate structure. It is the purpose of this paper to examine as critically as is possible at present the work of others, discussing the points of agreement and disagreement, and finally to present a provisional solution for a few of the problems. It appears that the planetaries are far more complex than has generally been assumed. Furthermore, there are many gaps in the observational material and the present discussion, therefore, is unavoidably fragmentary. It should be regarded as suggestive rather than rigidly complete.

(1) *Parallaxes.* The most reliable trigonometric parallaxes of the planetary nebulae are probably those of Van Maanen,[1] derived with the 60 and 100-inch Mt. Wilson reflectors. The absolute magnitudes of the nuclei, computed from his parallaxes, average about +7.7.

The various indirect methods of estimating parallaxes do not, in general, confirm Van Maanen's results. For example, the proper motions of the planetaries are small when compared with their rather large mean radial velocities. Van Maanen gives the proper motions in right ascension determined as a by-product of his parallax determinations. While it is well known that values so derived are usually not very reliable owing to the short period of time elapsing between the plates, he points out that the probable error of the results is small.

Wirtz, in discussing them,[2] finds some indication of a drift *toward* the apex. Since this would be a contradiction of the results obtained from the radial velocities, which show the solar motion strongly, he makes the suggestion that the comparison stars are nearer than the nebulae. Unfortunately, the material is not well-suited to the discussion. By far the greater number of the nebulae under consideration are located in the 18-hour region near the apex, where only a small fraction of the solar motion is reflected, and the results of the computation are uncertain. Furthermore, if the parallaxes could be so in error, little reliance should be placed on the proper motions derived as a by-product. There is a great need at present for well-determined proper motions, more, perhaps, than for further trigonometric parallaxes. Sometime in the future, we may hope to derive accurate parallaxes from a study of the internal proper-motions of the planetaries. If the observed radial velocity effects are, as is probable, due to true expansion or con-

[1] *Mt. Wilson Contrib.*, **11**, 172, 1917; **13**, 230, 1923.
[2] *A. N.*, **219**, 165, 1923.

traction, these, combined with the foregoing, will give the parallax.

(2) *The Nuclear Luminosities.* The researches of Wright,[3] Hubble[4] and others have established the fact that the nuclear stars are of spectral type O and Oe which, together with their extreme negative color indices and the well-known extension of their spectra into the ultra-violet, leaves no doubt but that their surface temperatures are very high. Plaskett[5] has derived a mean absolute magnitude for the O-type stars not in planetary nebulae of —4.0, that is, the central stars are about 12 magnitudes or so fainter than the average O's, with individual cases where the range is somewhat greater.

Plaskett and Gerasimovic[6] attribute part, at least, of the discrepancy to systematic errors in Van Maanen's parallaxes but this does not remove all the difficulties for, as Plaskett states.[7] ". . . . even if the same order of distance for these two classes is assumed, the difference is about six magnitudes." Furthermore, if galactic concentrations are at all significant, the planetaries are nearer than the normal O's. Gerasimovic[8] in solving for the galactic co-ordinates of the Sun for each type, found for the dispersion, σ, in galactic latitude only 6°.42 for the O's as compared with 20°.0 for the planetaries. He, however, does not take this to signify relative nearness because of the lack of correlation between galactic latitude and the apparent diameters of nebulae.

In the absence of a positive correlation it appears to be sufficient merely to call attention to a statement by Curtis,[9] ". . . . the smallest objects of the class are almost invariably in or very close to the Milky Way. On the other hand, the larger planetaries, and the giants of the class, while somewhat more frequent in the vicinity of the galactic plane, are, on the whole, fairly uniformly distributed over the entire sky." What-

[3] *Pub. Lick Obs.*, **13**, 248, 1918.
[4] *Ap. J.*, **56**, 186, 1922.
[5] *Pub. D. A. O.*, **2**, 330, 1924.
[6] *A. N.*, **225**, 97, 1925.
[7] *Op. Cit.*, p. 355.
[8] *A. N.*, **226**, 327, 1926.
[9] *Pub. Lick Obs.*, **13**, 60, 1918.

ever conclusion one may draw from these facts, it is almost certain that no one will argue that the planetaries are farther away than the O's not involved in nebulosity. There still remains the discrepancy of at least six magnitudes to be accounted for. This difficulty will be considered in § (4).

The fainter planetaries appear to be concentrated on the borders of the black rift in the Galaxy from 18 to 20 hours right ascension. This fact probably indicates that they are obscured by, and are farther away than the dark nebulosity. The distance of this object derived from star counts, should give an upper limit to the size of the planetary parallaxes.

Wilson[10] lists eight nebulae in the Large Magellanic Cloud as planetary. Their average apparent magnitude is about 10.5. Using Shapley's value, 0″.0000317, for the parallax[11] their absolute magnitudes[12] come out —7. If all planetaries were of the same brightness, it would be a simple matter to compute their distances. Hopmann[13] and Wirtz[14] have measured both the surface and total brightness of many classes of nebulae. Holetschek[15] also has measured the apparent magnitudes. Referring to the catalogue of Wirtz, which is more extensive than the others, it will be seen that by far the majority of the planetaries are of the tenth magnitude and fainter. The several of eighth magnitude stand out as exceptions. Parallaxes small enough to permit brightnesses of the order found in the Magellanic Cloud are inadmissable, for they would locate many of the nebulae outside the limits of the Galaxy.

The most probable explanation for the apparent discrepancy is that the nebulae in the Large Magellanic Cloud listed as planetaries are, in actuality, of the diffuse variety. The intensities of the lines in their spectra bear out the truth of this statement. Following Wilson,[16] the relative intensities of N_1, N_2, and $H\beta$, for ten nebulae in the cloud are 10:2.8:4.4. The

[10] *Pub. Lick Obs.*, **13**, 188, 1918.
[11] *Harv. Coll. Obs. Cir.*, **268**, 1924.
[12] *Harv. Coll. Obs. Cir.*, **271**, 1925.
[13] *A. N.*, **214**, 425, 1914.
[14] *Lund Medel.*, **2**, No. 29, 1923.
[15] *Ann. K. K. Sternwarte in Wien*, **20**, 39, 1907.
[16] *Op. Cit.*, p. 189.

average value for seventeen typical Galactic planetaries are 10:3.3:1.1. For both the *Orion* and η *Carinae* nebulae the ratios are 10:3:5, agreeing with the values for the Magellanic Clouds. There is still room for doubt, however, since several Galactic planetaries have approximately the latter ratio.

(3) *Masses.* Campbell and Moore[17] have computed masses for the central stars from their observed spectroscopic velocities of rotation, on the assumption that the outer portions of the nebulae are in orbital equilibrium. For circular orbits, the formula is $M = a'' v^2/930\pi''$, where M is the mass of the star in terms of the Sun as unity, a'' the radius of the orbit, measured in seconds of arc, v the linear velocity in km. per sec., and π'' the parallax of the nebula. The masses, so derived, are minimal values. When other shapes of orbits, the orientation, and radiation pressure are taken into account, the masses would, very likely, have to be increased.

Van Maanen's parallaxes, employed in the above formula, give masses for the nucleus varying from five to a hundred and fifty times that of the Sun. Since, on Eddington's theory,[18] large mass is to be associated with high absolute magnitude, the abnormal faintness of the nuclei again stands out. Nor, as Jeans points out,[19] will any adjustment of the parallaxes remove the difficulty for, if they are changed to fit the known luminosities of the ordinary O-type stars, the masses would become correspondingly greater, inversely proportional to the ratio of the parallaxes. That stars could exist with masses a thousand or more times that of the Sun is, at least without revision of current theories, quite difficult to believe. We have the choice of accepting one of two horns of the dilemma—either Van Maanen's parallaxes are of the right order of magnitude, which gives luminosities for the nucleus incompatible with those of the normal O's; or they are too large, which requires extreme values for the nuclear masses. There is, apparently, no satisfac-

[17] *L. O. B.*, **No. 278.** *Pub. Lick. Obs.*, **13,** 177, 1918.
[18] *M. N.*, **84,** 308, 1924.
[19] *M. N.*, **83,** 483, 1923.

tory way to hedge between the two points of view. Of course, if the nebular atmosphere is so dense that considerable viscosity is present, the computed masses would mean nothing. It would be like trying to calculate the mass of the planet *Jupiter* from its rotation period; but considerations to be presented in §(4) will establish the tenuity of the nebular envelope and render such high viscosity improbable.

(4) *Light Absorption Within the Nebula.* Gerasimovic, in an important paper, *On the Mechanical Equilibrium of Spherical Planetary Nebulae,* after assuming parallaxes of the order of the normal O's, attempts to account for the remaining discrepancy of six magnitudes by attributing it to absorption in the nebular shell; *i. e.,* only one two hundred and fiftieth of the light originally emitted by the star comes directly through. He develops equations for the transmission of radiation through a spherical shell of gas surrounding a star and then tries to evaluate the optical thickness for this shell by comparing the relative intensities of the surface at the edge and center of the ring. It is clear that the greater the edge-center ratio, the greater is the absorption within the nebula. From the absorption coefficient so derived, he proceeds to calculate the theoretical magnitude of the nucleus if it were freed from the surrounding nebulosity, finding it to be, on the average, brighter by the required six magnitudes.

While Gerasimovic's analysis may hold for the monochromatic light emitted by the nebular envelope, it is obvious that the absorption coefficient he has derived is for that light only. His application of it to the opacity for radiation of all wavelengths is certainly not justifiable.

The rate of generation of energy in the depths of a star could scarcely be affected by the presence of a surrounding shell. Of the radiation which once leaves a star's surface, the portion which would be returned by the nebula is negligible. Since a steady state, at least approximately, exists, an absorption of 99.6 per cent of the visual light, as demanded by Gerasimovic, would require the re-radiation of the same energy. The monochromatic lines of the nebula cannot be traced to that source. The blueness of the central star indicates that scattering is inappreciable, which precludes the possibility of a shell of dust, absorbing in the visual and emitting in the infra-red. Measurements of the heat indices and water-cell transmissions would be interesting, but they are not available.

Another point in the argument for the transparency of the nebulae lies in the fact that several small non-galactic nebulae are visible through the large planetary in *Aquarius*.[20] The symmetry of so many of them—the Owl, the Dumb-bell, the various ring nebulae, and others is further evidence. It is not reasonable to suppose that this arrangement in space is accidental. Finally, the doubling of many lines in their spectra as observed by Campbell and Moore[21] is most easily explained by light coming from both sides of a contracting or expanding shell. The discussion of this last point will be deferred until later.

The above evidence seems definitely to establish the transparency of the planetaries and rule out the chance that the faintness of the involved stars is due to nebular absorption. The Crab nebula, and others which may show a continuous spectrum, would be, of course, more opaque than the general run of planetaries.

(5) *The Physical State of the Nuclear Stars.*

(a) I wish to point out, however, that, should subsequent investigations uphold Van Maanen's parallaxes, the anomaly is by no means serious. The joint characteristic of high temperature and low luminosity is found in another class of stars—the so-called white dwarfs. Eddington has solved the difficulties which these stars present, in a surprisingly clear fashion. If their faintness is due merely to the small radius, the computed density would be about 50,000. At the high temperatures which prevail within, the atoms would be completely ionized or, at least, ionized as far as the K-ring, and therefore much smaller than at normal terrestrial temperatures, thus permitting considerably greater compression. The density of the companion of

[20]Hubble, *Ap. J.*, **56**, 179, 1922.
[21]*Pub. Lick Obs.*, **13**, 178, 1918.

Sirius is still some distance from the absolute theoretical limit. The density of the electron is about 10^{10}, while that of atomic nuclei is certainly not less than 10^5 and probably is of the order of 10^6.

If it is assumed, just as in the case of the white dwarfs, that the planetary and normal O's have similar temperatures and that the difference in luminosity is to be attributed to the small surface area of the former, the ratio of the radii would be about 1:250. Plaskett states[22] that it seems unlikely that the densities of the giant O's can be much greater than 0.01 (Sun=1). The densities of the planetary nuclei, therefore, would come out about 200,000 times that of water, that is, they are to be classed with the white dwarfs rather than with the giant stars.

Other evidence seems to lend an air of plausibility to the above conception. The spectra of all dwarfs are known to be stronger in ultra-violet radiation than giants of the same spectral type. This is true even for the white dwarfs. Lindblad,[23] in commenting upon the faint companion of o_2 *Eridani,* mentions a faint extension of the continuous spectrum much farther into the ultra-violet than is the case for the ordinary A-type star. The strong continuous ultra-violet spectrum of the planetary nuclei is one of their distinguishing characteristics.

If we are to give credence, as it seems we must, to the spectroscopic masses of Campbell and Moore, it is probable that the stars are not to be assigned, as their densities would indicate, to the final stage in the ordinary evolutionary sequence. The masses are far too great.

The determinations of the mass of the companion of *Sirius* have ranged, according to Eddington,[24] from 0.75 to 0.95 that of the Sun. Krueger 60, an M dwarf, has a mass of only 0.25 and has not reached the white dwarf stage through which many, if not most, stars will pass. This may be taken as evidence of

[22] *Op. Cit.,* p. 239.
[23] *Ap. J.,* **55,** 85, 1922.
[24] *M. N.,* **84,** 322, 1924.

some dispersion in the masses of stars in the later evolutionary stages.

The theory that the heat of a star is due to annihilation of its mass has apparently met with wide acceptance. Russell[25] has made a suggestion which may have its application here. He assumes the existence of three types of matter which are transformed into energy at three definite temperatures. The first, which is released at a relatively low temperature, accounts for the giant stage. As this supply is diminished, the central temperature increases slowly until as a value of ". . . . some 30,000,000° is approached, a process comes into play which leads to the actual annihilation of the main stellar material, with a correspondingly great liberation of energy." This self-cannibalism takes place while the star is passing down the main sequence, the star, meanwhile, growing denser and more opaque.

"Finally, to account for the white dwarfs, we must believe that there exists a certain residue of refractory material, immune to transformation at 30,000,000°. As the main constituents become exhausted, this will preponderate, and at last be almost exclusively present. If this residue were incapable of transformation, rapid gravitational contraction would ensue until even the ionized atoms were jammed close together. The considerable abundance of white dwarfs per unit volume suggests, however, that further energy liberating changes occur and delay the last act."

The nature of this refractory residue can only be conjectured. It probably consists of the nuclei of certain atoms more stable than the rest and which resist the transmutation. Comparing the masses of the companion of *Sirius* and Krueger 60, the ratio is more than 3 to 1. The amount of refractory material is certainly much greater in the case of the former star, indicating either that the stars, when in the previous stages, contained or developed unequal percentages of this material, or that the star which developed into the white dwarf was originally the more massive. The latter postulate is, perhaps, some-

[25] *Nature,* **116,** 211, 1925.

what more reasonable. The white dwarf of a B or O-star should be more massive than the similar offspring of an A. Russell says in this connection,[26] " . . . the level at which a star would break away from the main sequence and become a white dwarf would depend upon the quantity of 'refractory' material originally present (or perhaps formed as a by-product of other transformations), and any combination of absolute magnitude and spectral type is possible." There should be, then, no great objection to the idea that dwarf B's and O's as well as dwarf A's and F's might exist. In fact, Joy[27] has called attention to the fact that the B8 companion of *Mira Ceti* (absolute magnitude +6) must be classed with the dwarfs.

(b) There is, apparently, no way to fit the nuclei into the normal run of stars. On Eddington's theory,[28] the total light from a star at a given temperature varies approximately as the seven-fifths power of the mass. It is easily shown that an impossibly large value for the parallax would have to be assumed in order to satisfy the mass-luminosity relation, above, and the apparent and absolute magnitudes. Whatever the parallax of the nebula, the central star is deficient in the rate of production of energy per unit of mass.

(c) It is interesting to consider the case from the standpoint of Jeans' theory.[29] Jeans believes that the transformation of matter into energy takes place practically independently of temperature and pressure. He postulates the existence of super-radioactive elements of much higher atomic weight than those on the Earth. In the absence of convective mixing, these would sink into the center of the star and, in the event of fission, there is a considerable chance that an unequal division of the energy-producing material would take place. The smaller star, radiating more energy than is being generated within, will be forced to draw upon its gravitational supply and rapidly contract until the density is so great that Boyle's law no longer holds for its

[26]*Op. Cit.*, p. 211.
[27]*Ap. J.*, **63**, 338, 1926.
[28]*M. N.*, **84**, 310. 1924.
[29]*Nature*, **117**, 18, 1926.

atoms and electrons. Hence the different conditions of *Sirius* and its companion.

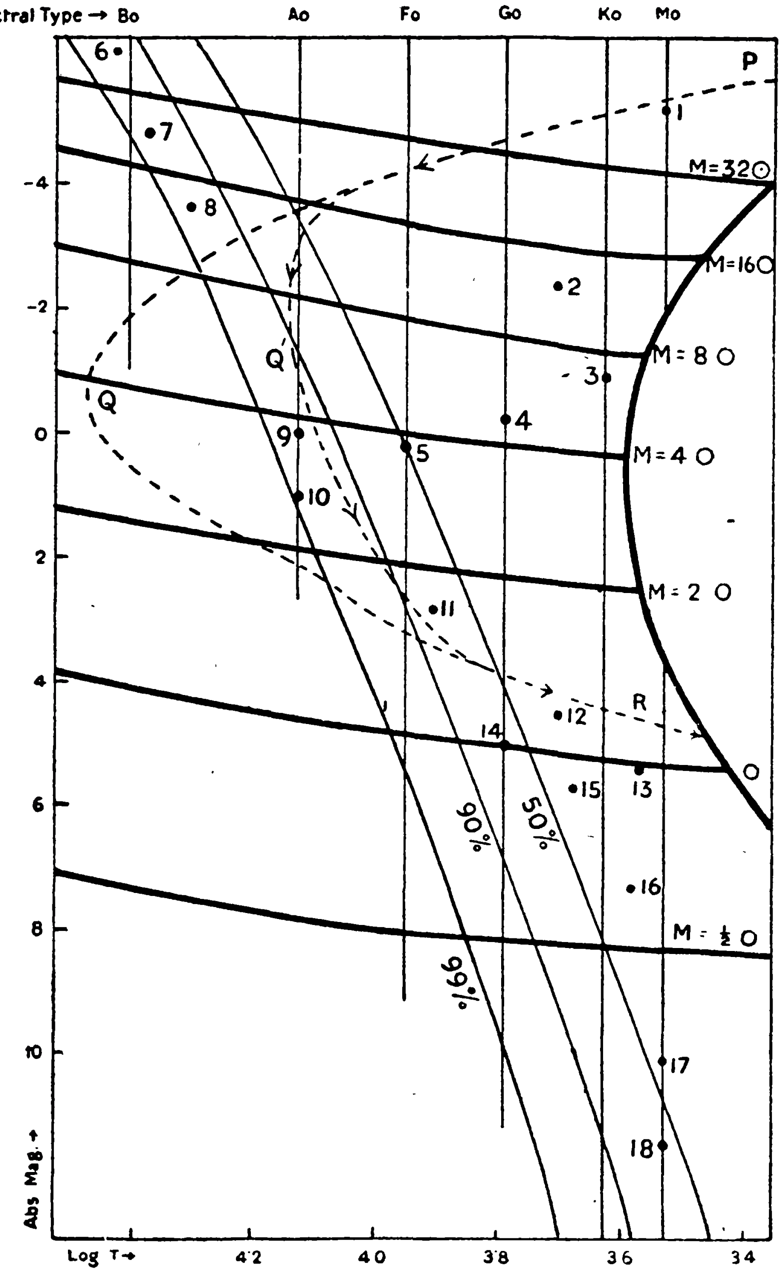

FIG. 1.—The stars, which are those shown by Russell in his diagram in NATURE (August 8, 1925), are as follows: 1, Antares; 2, δ Cephei; 3, Arcturus; 4, 5, Capella; 6, Plaskett's star; 7, V Puppis; 8, Y Cygni; 9, β Aurigæ; 10, Sirius; 11, Procyon; 12, 13, α Centauri; 14, Sun, 15, 16, ξ Bootis; 17, 18, Kruger 60.

The equilibrium of a star is governed by its mass and the rate at which energy is produced in the interior, i. e., its absolute magnitude. Figure 1 is a reproduction of that of Jeans.[30] The abscissae are spectral type (log T) and the ordinates are absolute magnitude. The heavy lines, slanting upward toward the left mark the stable configuration for a star of given mass and given absolute magnitude. The curve on the right marks the boundary between stable and unstable configurations. When a star has such a mass and absolute magnitude that its representative point falls within the region of instablity, equilibrium is impossible. As explained above, it will become a white dwarf. The various lines of thought all point to this conclusion as the most reasonable, no matter what the values found for the parallaxes.

In order to satisfy the law of the conservation of angular momentum, the star's rotational velocity would greatly increase as the star diminished in size. Part, at least, of the widening of the bright lines in the nuclear spectrum may be attributed to Doppler effect.

(6) *Planetary Nuclei, Novae, and Wolf-Rayet Stars.* Other than certain similarities of spectrum, together with the fact that several of the novae have temporarily developed a nebular envelope, the relation between these three classes of objects is not clear. I am inclined to agree with Lundmark[31] that too much stress has been laid upon the fact that some novae have been observed to pass through a kind of planetary stage, for scarcely any nova is known which has remained a typical planetary.

With the recent revision of the time scale in the problem of stellar evolution, it is no longer possible to regard the nova as a normal stage in the life of a star. The frequency of novae compared with the total number of stars in the universe indicates that a star probably experiences these outbursts many hundred times during its history. It follows, almost without question, that the star must return, approximately at least, to its previous state for, otherwise, the well-known main sequence relation would be upset.

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[30] *Op. Cit.*, p. 19.
[31] *Pub. A. S. P.*, **35**, 110, 1925.

Novae arise from giants and dwarfs alike and their cause is probably external. The evidence indicates that the star returns to its former magnitude usually within a few years after the outburst. A star, originally a dwarf, which returns to its previous low absolute magnitude but which still retains a Wolf-Rayet type of spectrum, presents a difficulty analogous to the planetary nuclei—high temperature and low luminosity. The writer suggests that local eruptions of small area on the surface of the star may be partly responsible for the phenomenon. This, combined with the rotation of the star, may account for the observed semi-periodic variability as the star fades in brightness. Further discussion of the nova problem does not come within the scope of this paper.

(7) *The Source of Luminosity.* Observation definitely locates the source of excitation for the planetary nebulae in the nuclear stars, which are usually located near the geometrical center. Hubble[32] has made an extended study of the problem and finds a correlation of 0.65 between m, the magnitude of the central star, and A, the diameter of the nebula on a uniform exposure scale. The equation connecting the two was found to be $m+5.40 \log A=17.88$.[33] The coefficient of the second term is within the experimental error of 5.00, the value required if the inverse-square law is one of the important factors in determining the luminosity of the nebula. In many cases, the total photographic brightness of the nebula is a hundred times or more greater than that of the central star. Since the total energy radiated by the nebula cannot exceed that of the star, one naturally looks to the ultra-violet.

If we assume that, approximately, the stars radiate according to Planck's law, which is probably true as regards order of magnitude, the energy distribution curves may be drawn and the amount of light which normally affects the photographic plate compared with the ultra-violet energy beyond our vision. The former was taken from $\lambda 3500$—4800 A. For hydrogen to emit the Balmer series requires the electron to be raised at least

[32] *Ap. J.*, **56**, 162, 400, 1922.
[33] *Op. Cit.*, p. 432.

to the 3- quantum stage. On the simple theory, this requires the absorption of λ1025, 972, ...911. It was immediately evident, no matter what temperature may be assumed for the star, that even the total energy lying between λ1025 and λ911 falls far short of equalling that radiated by the nebula. The case did not yet appear hopeless for the absorption of λ911 corresponds to ionization of the atom, the removed electron possessing zero velocity. The ionizing power of ultra-violet radiation is well known and it is quite possible that radiation short of λ911 may forcibly cause the ejection of an electron from the atom, the excess energy over that necessary for ionization going into the kinetic energy of the electron in accord with Einstein's familiar law of the photo-electric effect, $\frac{1}{2}\, mv^2 = h\nu - I$. Let us assume the most favorable case, i. e., (a) where all the energy of the star short of λ911 is effective in ionization, (b) the surplus kinetic energy of the electrons is used in further exciting the atoms or else radiated at the head of the Balmer series, and (c) the returning electron stops only in the 2-quantum state before falling to the lowest level. This last condition favors the emission of the Balmer series, but from quantum considerations, not more than one-fourth of the energy originally absorbed can go toward the production of those lines. The remaining three-fourths produce λ1215.

Mechanical quadratures show that for a stellar temperature of 20,000° the ratio of the energy λ3500—λ4800 and λ911—λ0 comes out about 10:1. For a temperature of 40,000° it is 1:22. It is evident that no reasonable temperature could be assigned to the star which would be able to explain more than a small fraction of the nebular radiation as fluorescent. To allow for possible altering of the energy curve by electron scattering in the stellar atmosphere merely serves to increase the discrepancy.

It is necessary to fall back on the theory of corpuscular excitation. Hubble,[34] Russell[35] and others have suggested it before. The hot star is emitting electrons which fly out into

[34] *Op. Cit.*, p. 437.
[35] *Obs.*, **44**, 72, 1921.

space in all directions causing ionization by collision. The intensity of the electron radiation will vary according to the inverse-square law. It is probable that the positive charge acquired by the star will eventually slow the electrons down and therefore decrease their effective power for doing work. The latter quantity is proportional to the electron's kinetic energy, $\frac{1}{2}mv^2$, and since the parabolic velocity depends on the square-root of the distance, the power will vary inversely as the distance of the electron from the nucleus. The total effect will, therefore, be approximately as the inverse cube, with a sudden fall in intensity when the energy of the electrons falls below the critical potential for the line in question. The assumption of parabolic velocity is justified by the fact that the star will develop a sufficient positive potential to draw back the electrons before they escape entirely.

As Russell has pointed out,[36] the sizes of the various monochromatic images in the spectra of the planetary nebulae show a tendency to arrange themselves in the order of excitation potentials for the different elements. The inverse-square law would prescribe a simple fading out of the images but the decrease in the speed of the electrons as well would require the effect noted by Russell.

In this connection the writer has measured the diameters of various images in Wright's photographs taken with the slitless spectrograph. The nebula N. G. C. 7009 was chosen because it presented measurable images of both neutral and ionized helium as well as hydrogen. The means of the longitudinal diameters of $He+$, He, and H are in ratio 1 : 1.20 : 1.56, respectively.

The ionization potentials of the three atoms are approximately in the ratio of 4 : 2 : 1, and if the ionizing force varied inversely as the distance, the ratio of the diameters would be 1 : 2 : 4. For an inverse-square law we would have 1 : 1.4 : 2 and for an inverse-cube, 1 : 1.26 : 1.58. The almost exact agreement of the observations with the figures last given is probably fortuitous and it is by no means certain that all nebulae

[36] *Op. Cit.*

will give the same result. A cursory examination of some of the photographs strengthens the opinion that they will not, but it appears probable that further observational material will more nearly confirm the inverse-cube than the inverse-square law. Differences in the distribution of elements would tend to introduce many irregularities and the inverse-square law would hold only if a negligible fraction of the emitted energy is absorbed. Furthermore, the atomic weights of the elements may possibly enter into the problem of the distribution.

It is possible, however, that we are avoiding one difficulty by introducing another. The energy of the expelled electrons does not come from nothing and it is just about as difficult to find this source of energy as it is to find the etherial radiation. If, as the above indicates, the total radiation, corpuscular and etherial, emitted by the star is insufficient to account for the absolute magnitude of the nebula, it would appear that there is some generation of energy within the gaseous envelope. The problem is much more complicated than has hitherto been supposed and considerably more data are needed.

(8) *The Form and Physical Condition of the Nebulosity.*

Jeans[37] has investigated the possible forms of gaseous shells in equilibrium with rotation, gravity, and gas and radiation pressure. He apparently has been quite successful in explaining many of the varied forms which the planetaries assume. Gerasimovic[38] has studied the equilibrium of a spherical shell of gas, and Milne[39] the calcium atmosphere. In view of the probable insufficiency of the ultra-violet radiation, as indicated in §(7), any conclusions based on radiation pressure are to be regarded as doubtful.

Here, again, the problem is more complicated than the simple assumptions. A spherical shell is certainly far from the actual condition—an ellipsoidal shell of varying thickness would probably be a closer approximation. The photographs, however, show that the shells contain many filaments which inter-

[37] *M. N.*, **83**, 481, 1923.
[38] *A. N.*, **225**, 89, 1925.
[39] *M. N.*, 84, 354, 1924; **85**, 111, 1924; **86**, 8, 1925; *Obs.*, **48**, 317, 1925.

lace intricately and the brightness of the surface varies greatly from place to place. Furthermore, the nebulae are undoubtedly far from a quiescent condition. The radial velocities of Campbell and Moore show the curious doubled lines which apparently permit of no other explanation than Doppler effect—expansion or contraction, probably the latter, since it is the red component which is the stronger.

Wright[40] has proposed a classification for the planetaries on an empirical basis. It appears that the classification has a true physical basis as well. While it depends, in part, upon the intensity variations of the nebular lines of unknown origin, it is consistent with the behavior of the lines of H, He, and He+. It is reasonable to suppose that there might possibly be some correlation between spectral type and surface brightness. Wirtz's values for the latter, when plotted against the former, however, gave no positive correlation. The absolute magnitudes of the central stars (Van Maanen) appear to show considerable correlation with spectral type. The results, however, are just the reverse of what would be expected from elementary considerations, i. e., the nebulae which show the greatest ionization are associated with the intrinsically fainter stars. This casts additional doubt on Van Maanen's parallaxes.

The writer has also examined Wright's observations[41] for a possible correlation between the spectral type of the nucleus and of the nebulosity. There are, apparently, two classes of nuclear spectrum—the one with a continuous background and the other with additional bright lines. The correlation, if present at all, is very slight and in the sense that the stars with continuous spectra are found in the nebulae whose condition is more highly excited. It is to be hoped that future research will reveal such a sequence in the spectra of the nuclei that further subdivision will be possible.

Future investigation will solve many of the difficulties which the planetaries now appear to present. These objects are well adapted to astrophysical investigations and it is certain

[40] *Op. Cit.*, p. 260.
[41] *Op. Cit.*, p. 195.

that applications of radiation and quantum theories, atomic structure, Saha theory and the other well-established scientific laws will go far to solve the jig-saw puzzle. Here we have some bits of evidence pieced together, there some more, and it is but a matter of time until various investigators will have reached the solution and will present to us a completed picture of the constitution and nature of the planetary nebulae and their nuclei.

Lick Observatory, September, 1926.

LENSES AND THEIR FOCAL ADJUSTMENT IN RELATION TO PHOTOMETRY

By Frank E. Ross

At the Yerkes Observatory there are four high-speed lenses in operation, namely, a 10-inch Petzval, a 6-inch Voigtlander refigured by Brashear, a 6-inch Zeiss U. V. doublet, and a 3-inch Ross wide-angle doublet. In the case of each of these lenses we find the apparently paradoxical fact that the best focal setting is dependent on the magnitude of the star. Thus the Ross doublet, of focal length 53 cm, for a two-minute exposure on the pole, has a focal setting for *Polaris* 1.2 mm longer than for a neighboring star of magnitude 7.5. Dr. R. Trumpler has noted the phenomenon in determining the focal point of the short focus lenses used by Dr. Campbell in measuring the Einstein shift at the Australian eclipse of 1922[1]. The late Mr. J. A. Parkhurst was familiar with the phenomenon in the case of the Zeiss doublet.

If one examines the image of an artificial star formed by a lens, using a high-power microscope, the cause of the phenomenon is seen. The examination is most conveniently made on a lens-bench, in which the examining microscope, sliding in and out, along the axis of the lens, will disclose the form of the image produced by the lens at any point along its axis. It is found that in one position, some of the light from a point source, transmitted by the lens, is concentrated in a very fine point, which is seen to be surrounded by a diffused halo or disk. If, accordingly, the photographic plate is placed in this position,

[1] *Publ. A. S. P.*, **35**, 153.