Conclusion.

The most remarkable feature is the rapid forward movement in longitude of a "leader" spot at its formation, accompanied by its increase in area. For the first two days the diurnal motion is $+1^\circ{\cdot}0$ greater than the average movement of a spot in the same latitude. At 15° from the Equator this amounts to $15^\circ{\cdot}2$. Comparing this result with sidereal diurnal movements found in other ways, we get

At Latitude 15°

	°
Average spot *	14·2
Reversing layer †	14·3
Faculæ (Greenwich) ‡	14·4
λ4227 †	14·8
H_a †	14·9
Spot at commencement	15·2

If these differences may be correlated with variation in level, a "leader" spot at its formation is high and rapidly descends. At the same time it increases in area.

It may be worth while to examine a "leader" spot when a suitable one appears for velocity in the line of sight, to see what vertical motion accompanies the horizontal motion. Possibly an examination on spectroheliograms of the flocculi surrounding a "leader" spot for a few days after its formation might also throw some light on the matter.

Royal Observatory, Greenwich :
1925 *April* 3.

On the Cause of Anomalous Determinations of Time.
By Professor R. A. Sampson, F.R.S. (Plates 14, 15.)

With a view to locating the source of the discrepancies in time determination which I pointed out, as between the observatories of Paris and Edinburgh, in 1920,§ and in 1922 showed to be inherent, with varying character, in the time determinations of six of the principal observatories,|| attention has been given at Edinburgh to improving time determinations in any way that offered, and especially in the clock and chronograph system. Thus the clocks are compared daily by the

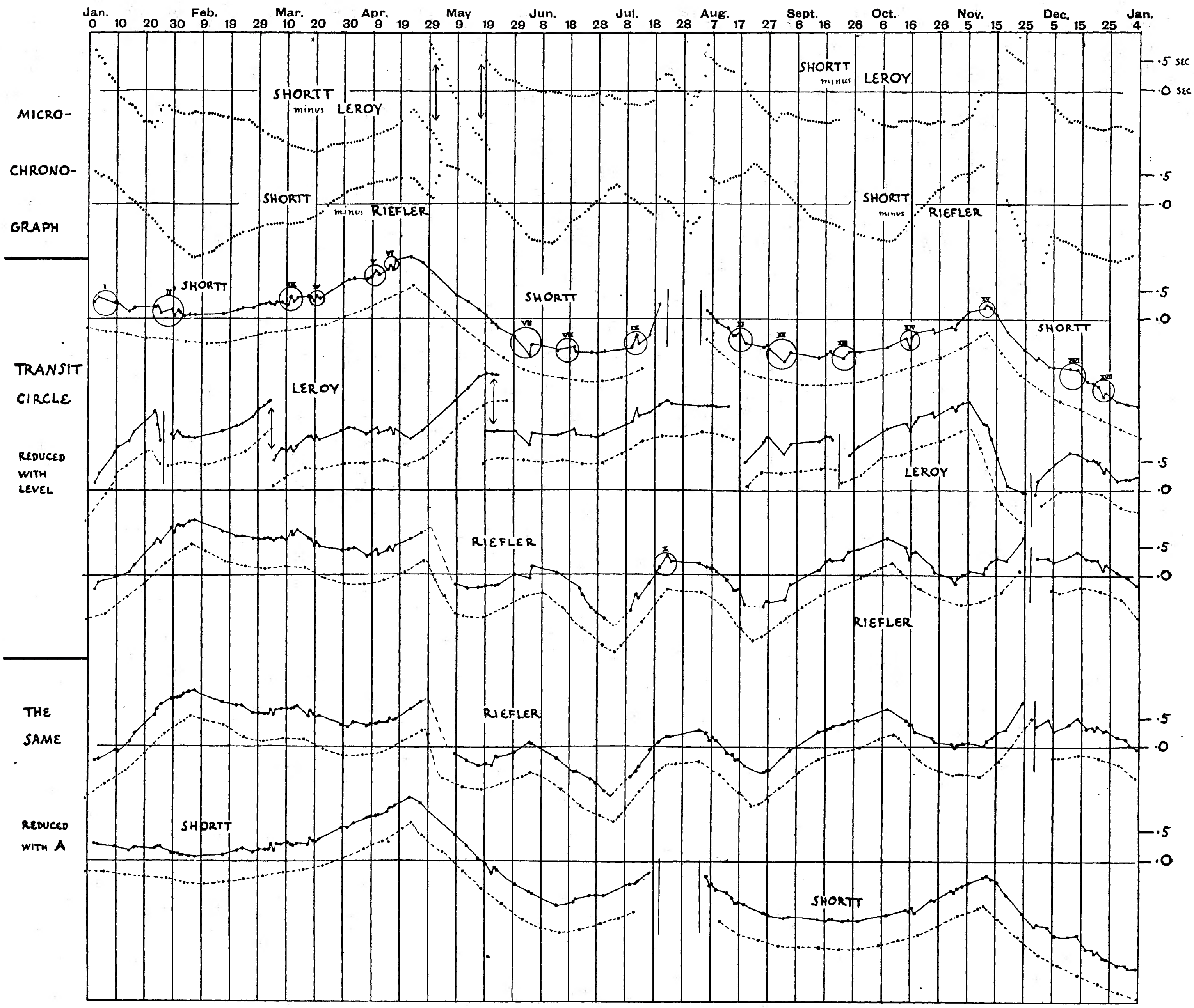
* "The Sun's Rotation Period from Long-Lived Spots," preceding paper, p. 551.

† "Investigation of Rotation Period of the Sun," by W. S. Adams and J. B. Lasby, p. 118.

‡ *M.N.R.A.S.*, **84**, 441.

§ "On Clock Errors and Wireless Time Signals," *M.N.*, **71**, 90.

|| "On the Determination of Time at Different Observatories," *M.N.*, **72**, 215.



Jan. 0 10 20 30 Feb. 9 19 29 Mar. 10 20 30 Apr. 9 19 29 May 9 19 29 Jun. 8 18 28 Jul. 8 18 28 Aug. 7 17 27 Sept. 6 16 26 Oct. 6 16 26 Nov. 5 15 25 Dec. 5 15 25 Jan. 4
MICRO-
CHRONO-
GRAPH
SHORTT minus LEROY
SHORTT minus RIEFLER
TRANSIT CIRCLE
REDUCED WITH LEVEL
THE SAME
REDUCED WITH A
SHORTT
LEROY
RIEFLER
·5 sec
·0 sec
·5
·0

microchronograph,* which excludes all errors in registration, and in the year 1924, to which this paper refers, there were three clocks of the first grade in use—Riefler No. 258, Leroy No. 1230, and the original model of Mr. Shortt's clock. To these has been added in the present year another Shortt clock of a slightly improved pattern—Shortt No. 4. This equipment is not redundant. Certainty over prolonged periods to the hundredth of a second is the requirement. The observed corrections of these clocks are shown on the accompanying plate (Plate 14). The scale of this chart is such that a gradient of 45° implies a rate of ·05 second per day. It is therefore intended to show errors up. Accordingly it will be remarked how many accidents have beset the clocks, of leaks in the cases, failure in temperature control, stoppages, changes of rate, either by calculated barometric change or without assignable cause. When any such accident supervenes upon a clock, that clock is for the time being out of count for discriminating between different classes of discrepancies shown in the transit-circle time observations. It is only by possessing a battery of evidence in reserve that certainty can be reached on this point.

It happens that at this observatory, in the year 1924, the cases of anomaly in clock error were both numerous and well marked. The evidence assembled in Plate 14 illustrates the method by which time work is here discussed.

The two upper traces show the relative errors of the three clocks, recorded by the microchronograph. Their general smoothness will be appreciated at once. At the same time they are far from being straight for prolonged periods. It is not necessary to particularise the causes of all the breaks that are noted. The abrupt changes of gradient are, as a rule, due to barometric changes, deliberately introduced in order to alter the rate. For determination of the absolute errors of the clocks the relative errors here recorded are taken as fiducial data. Even on the evidence of these charts, in which the scale of representation is reduced and the actual measures are rounded off from the thousandths to the hundredths of a second, it will be readily conceded that where the trace of two clocks appears smooth, anything like an abrupt change of error in either one or the other of them, to the amount of a few hundredths of a second, is excluded, and must be attributed to the source that avers it.

Now consider the transit-circle observations for clock correction. The usual observation made here consists of a measurement of collimation error, upon the north and south collimators, an observation of level, with the mercury bath and Bohnenberger eyepiece before and again at the conclusion of the series, two polar stars, and ten time stars. The tabular places of the stars are uniformly referred to Boss's P.G.C. The observations are made with the clock Leroy marking seconds on a Fuess chronograph. The corrections of Shortt and Riefler are then deduced from the relative data supplied by the microchronograph. From the clocks that keep sidereal time the nutation is removed. The results for the year are then those shown on Plate 14.

* Cf. *M.N.*, **78**, 592.

It will be noted, as has often been remarked before, that this imposes numerous marked cases of simultaneous deviations upon all the clocks. It now remains to assign what is true clock correction and what is transit-circle error. For this purpose, the most reliable clock is taken as "director," in this case the clock Shortt, and an error adopted for it which shall keep as close as may be to the transit-circle record, while taking due note of the microchronograph relative record. The record for the clock Shortt shows hardly any feature except a curvature due to a minute leak which has not been located. The other clocks, though almost equally steady for short periods, are more liable to fluctuations and breaks. The results are shown, as adopted, below the observed corrections of each clock. These adopted traces are therefore a single coherent system, as are also the three observed traces. The values "*Observed—Adopted*" are referred to as the erratics.

There will be noted at once several conspicuous erratics that no one would ascribe to the clocks. Proceeding from these to the less conspicuous cases, many more will be conceded. Without by any means exhausting the evidence, I have marked by a ring seventeen clear cases for examination, with the intention of using them to determine what feature in the observations has introduced them.

In the process of measuring collimation error, the movable wire of the N. collimator is set upon the wire of the S. collimator, which is untouched. We denote this setting by N/S. The collimation line of the transit circle is then set in turn upon the wire of the N. collimator and that of the S. collimator, the measures being denoted by TC/N and TC/S respectively. Further, L being the setting of the collimation line upon its own image in the mercury bath, as habitually, we have

$$\begin{aligned}\text{collimation} &= \textit{constant} - \tfrac{1}{2}[\text{TC/S}+\text{TC/N}] = c\\ \text{level} &= \text{L} - \tfrac{1}{2}[\text{TC/S}+\text{TC/N}] = l,\end{aligned}$$

and besides this we may add

$$\begin{aligned}&\text{azimuth of transit circle east of S. collimator}\\ &\qquad = \textit{constant} + \tfrac{1}{2}[\text{TC/N} - \tfrac{1}{2}\text{TC/S}] = \text{A}\\ &\text{azimuth of N. collimator east of S. collimator}\\ &\qquad = \textit{constant} - \text{N/S} = [a].\end{aligned}$$

Since 1913 the third and fourth data have been recorded here, with the view that they might serve as a check on azimuths derived from polars.* These were supposed to be the weak point of the transit observations. There is a proportion of cases when the polars disagree, and, following the formula $m+n \tan \delta$, they might introduce an uncertainty which on occasion could amount to several hundredths of a second in the clock correction. On the other hand, level seemed particularly well determined. The mercury bath is carried between the piers. It is used immediately before and immediately after the observations, but requires to be removed while they are in progress. Hence the two measures are wholly independent. Yet their average

* *Cf.* "The Temperature Coefficients of the Edinburgh Transit Circle," *M.N.*, **75**, 69

range from their mean is only ·01 second, which has to include all progressive changes due to temperature or other causes, and the cases of erratics examined below in Table I offer no suggestion of an unusual feature. Accordingly I have experimented from time to time by employing A to replace the stellar azimuth derived from the formula $a=l \tan \phi - n \sec \phi$ in the reductions, so as to eliminate n, but always with inconclusive results.

In considering the erratics presented by the year 1924, it occurred to me to retain n and use A in order to eliminate l. One thing that suggested this course was the following. In each time set a certain number of high stars are included. If in the formula of correction, $m+n \tan \delta$, the quantity n was at fault, these should stand out. Hardly anything of the kind was shown even when they were grouped for the purpose.

The complete elimination of l from the observations is effected very simply. Deriving n as usual from the polars and time stars, compute

$$a=l \tan \phi - n \sec \phi,$$

and then, noting that

$$m=a \operatorname{cosec} \phi + n \cot \phi,$$

apply to m the correction

$$+(\text{A}-a) \operatorname{cosec} \phi,$$

and the same quantity with the reversed sign to the clock correction. This was done to all the observations; they were marked on a chart as before, using, of course, the same microchronograph differences, and a fresh set of adopted clock errors derived, without any other variation of method.

If now the seventeen dates of occurrence of striking erratics are looked for, fifteen will be found to have disappeared totally, one (No. IV) partially, while one (No. XVI) cannot be said to be improved. At the same time the new reduction introduces some faults of its own at other places. These are of less amount. The mean erratic for the whole year is reduced from ±·04 second to ±·03 second. This is a very large reduction, as the latter has to cover every possible residual source of error. But it is no part of the present argument to prove that the S. collimator supplies a permanent fiducial mark. It was not designed for any such purpose. It appears, as will be shown further below, to have behaved with extraordinary steadiness in the year under review. It does not follow that it would do so another year. If on any occasion the spider thread in its focal plane had moved east or west relative to its object-glass by as much as ·001 inch, it would introduce an erratic in the clock correction of ±0·2 second. It is astonishing that, as will be seen below, it does not appear to have moved so much in the course of the temperature changes of the whole year. But I would repeat that the thesis is, that the erratics originally conspicuous disappear when the level measures are discarded.

For closer examination I append a table giving numerical details regarding these seventeen erratic cases along with their regular neighbours.

Table I.

Details of Selected Cases.

Column 1. Number and date.
,, 2. Erratic = TC − Adopted, reduced with *l*.
,, 3. ,, ,, ,, reduced with A.
,, 4, 5, 6. Instrumental corrections to TC.
,, 7. A−*a*.
,, 8. Range in clock correction permitted by extreme measures of *l*.
,, 9. Temperature.

1.		2. Erratics. With *l*. sec.	3. Erratics. With A. sec.	4. *l*. sec.	5. *n*. sec.	6. *a*. sec.	7. A−*a*. sec.	8. *l* Range from Mean. ±sec.	9. Temp. ° F.
(I) Jan.	2	*·02*	*·01*	·42	*·03*	·67	·00	·006	44
	3	·09	·00	·39	·00	·58	·08	·015	40
	9	·02	*·01*	·39	*·03*	·62	·03	·012	28
(II) Jan.	24	·07	·07	·43	*·03*	·70	*·03*	·017	..
	25	*·04*	·07	·47	*·06*	·80	*·13*	·017	45
	29	·04	*·01*	·48	·05 *	·61	·03	..	46
	30	*·06*	*·01*	·46	*·03*	·73	*·06*	·003	44
	31	·04	*·01*	·42	*·01*	·64	·04	·003	44
Feb.	1	·01	*·02*	·41	*·02*	·64	·02	·000	41
(III) Mar.	8	·02	·01	·43	*·01*	·64	·02	·012	41
	10	*·05*	·03	·46	*·02*	·70	*·05*	·003	43
	11	·08	*·01*	·43	·05	·55	·09	·003	41
	12	*·02*	*·03*	·44	·03	·61	·03	·020	45
	13	·05	*·01*	·41	·02	·57	·08	·017	45
(IV) Mar.	18	·02	·05	·40	·02	·57	*·01*	·003	39
	19	*·06*	·00	·39	*·03*	·63	*·03*	·003	38
	20	·03	*·04*	·40	·06	·49	·07	·017	39
	21	*·03*	·01	·41	·00	·60	*·01*	·006	38
(V) Apr.	8	·01	·03	·39	*·03*	·62	*·03*	·029	44
	10	·08	·03	·36	·02	·50	·05	·008	37
	11	*·02*	·02	·38	*·02*	·59	*·03*	·003	38
(VI) Apr.	14	·01	·00	·36	*·01*	·54	·02	·017	39
	15	*·06*	·04	·38	·00	·57	·01	·026	42
	16	*·01*	·01	·39	*·01*	·59	*·02*	·008	43
	17	*·09*	·02	·38	·02	·52	·04	·038	45
(VII) May	29	·00	*·01*	·34	·05	·41	·11	·006	53
June	3	*·23*	·04	·33	*·07*	·62	*·09*	·006	52
	4	*·03*	·03	·31	·00	·46	·10	·017	53

* One polar only.

TABLE I—*continued.*

1.		2. Erratics. With *l*. sec.	3. Erratics. With A. sec.	4. *l*. sec.	5. *n*. sec.	6. *a*. sec.	7. A–*a*. sec.	8. *l* Range from Mean. ±sec.	9. Temp. ° F.
(VIII)	June 13	*·04*	*·03*	·28	·02	·38	·20	·003	52
	19	·07	·02	·25	*·01*	·38	·20	·026	61
	20	*·02*	·07	·32	·00	·48	·08	·003	61
(IX)	July 9	*·04*	·01	·28	*·07*	·55	*·07*	·000	63
	11	·12	*·03*	·23	*·04*	·41	·11	·012	..
	12	*·03*	*·03*	·27	*·09*	·57	*·04*	·006	66
(X)	July 19	·09	·02	·19	*·06*	·38	·06	·017	58
	22	·10	*·02*	·20	*·04*	·35	·14	·012	59
	23	·00	*·03*	·21	*·05*	·41	·07	·000	57
(XI)	Aug. 15	·02	*·02*	·18	*·07*	·39	*·02*	·012	57
	16	·11	·03	·14	*·09*	·37	·00	·012	55
	18	·00	·04	·16	*·13*	·48	*·12*	·008	55
(XII)	Aug. 27	·01	*·02*	·18	*·13*	·48	*·04*	·003	56
	Sept. 1	*·17*	*·01*	·18	*·21*	·65	*·20*	·012	58
	3	·03	*·01*	·20	*·11*	·48	*·07*	·003	59
(XIII)	Sept. 16	·04	*·04*	·17	*·11*	·45	*·04*	·008	56
	17	·09	·01	·17	*·12*	·45	*·03*	·006	56
	18	·02	·02	·16	*·13*	·47	*·08*	·012	53
(XIV)	Oct. 14	·02	·00	·15	*·11*	·42	·02	·000	49
	15	*·14*	*·02*	·20	*·14*	·54	*·09*	·015	51
	16	·03	·02	·17	*·11*	·42	·01	·008	50
(XV)	Nov. 10	·04	·00	·19	*·09*	·44	·01	·008	44
	11	·00	·02	·20	*·07*	·43	·02	·026	45
	13	·09	·06	·16	*·09*	·40	·03	..	42
(XVI)	Dec. 5	·03	*·02*	·24	*·10*	·52	*·04*	·000	46
	10	·11	·08	·21	*·11*	·51	*·04*	·012	49
	13	·15	·18	·22	*·13*	·55	*·08*	·006	45
	16	·02	*·01*	·22	*·09*	·45	*·02*	·017	42
(XVII)	Dec. 20	·02	·05	·24	*·12*	·58	*·07*	..	51
	22	*·10*	*·02*	·25	*·15*	·63	*·13*	·000	46
	23	*·01*	·00	·23	*·14*	·59	*·07*	·008	48

We have so far dealt directly with the transit-circle observations and their visibly erratic features. These are removed in the smooth adopted curve of clock correction. It may be noted, however, that the two adopted corrections, derived respectively with l and A, are not the same, but present features of fluctuation and semi-systematic difference throughout the year. I turn now to this point.

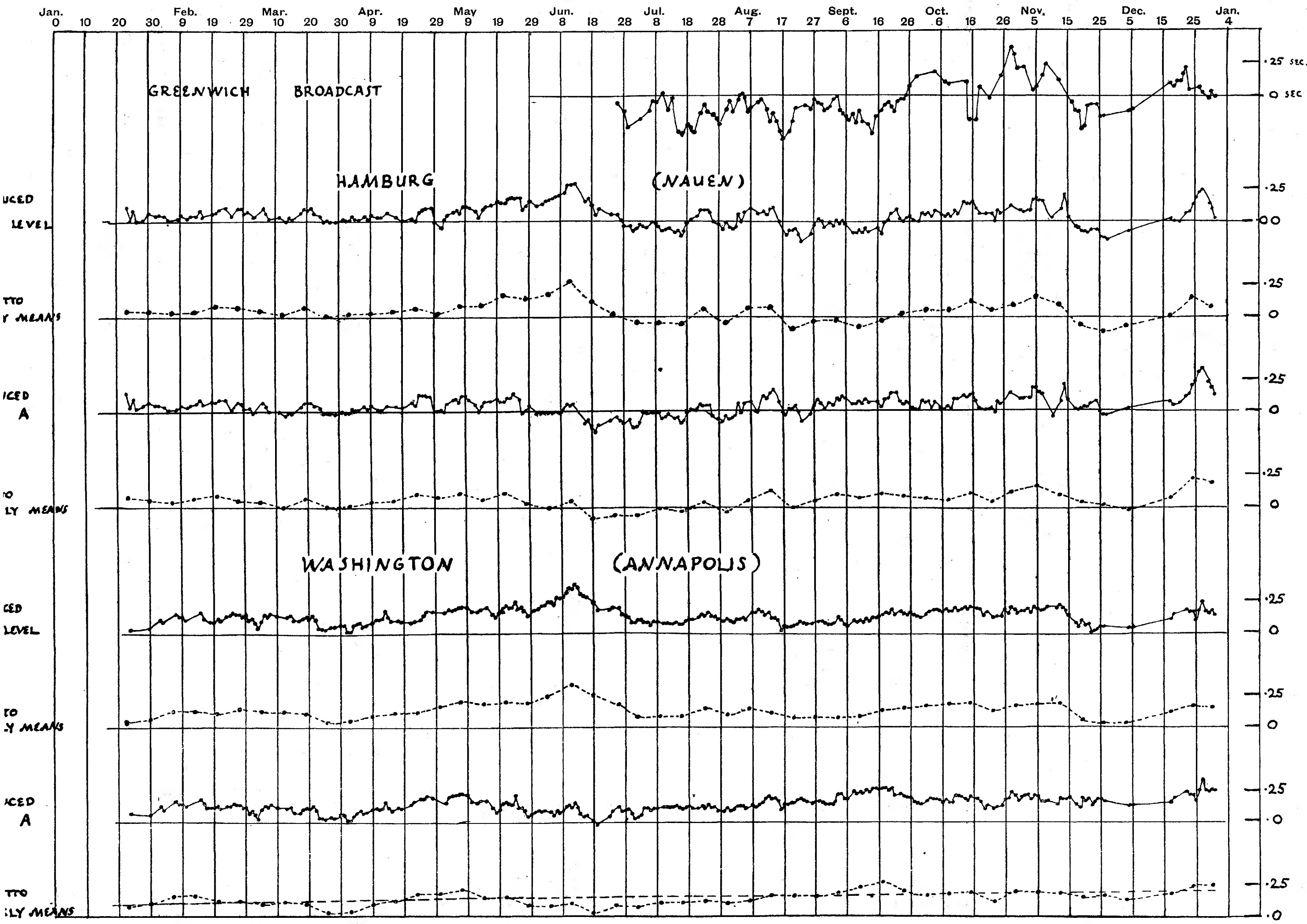
In W.T. signals, each issuing observatory emits its signals at a time provisionally determined, afterwards publishing its definitive time when its clock correction has been adopted. An exception may be found in Greenwich, where the signal relayed by the British Broadcasting Company is not followed by any published corrections. Apart from this case, the final outcome of the receipt of W.T. signals is a comparison of the definitive times derived from the adopted clock corrections of each observatory. One might expect a close argeement, constant corrections of longitude or lag being set aside. But, as I have already remarked, this is not the case. The comparisons made in *M.N.*, **82**, 1922, depended upon uncorrected times, which were the only times available at the moment. The discrepancies may be changed, but are little diminished, when definitive times are employed.*

Since about the end of January 1924, W.T. signals have been regularly recorded automatically here, including those from Washington (Annapolis). I give on Plate 15 the charts for definitive times of receipt of the Annapolis (17^h) and the Nauen (12^h) signals ("Washington slow" and "Hamburg slow"), the final corrections to these signals being available. I give also since June 26 the Greenwich Broadcast Signal. For the Washington and Hamburg times I add the means for each week, without any other smoothing. In the upper curves for Hamburg and Washington, the Edinburgh adopted times have been derived with level, in the old way. It will be noticed that besides resemblances that may or may not be significant, both curves show one very strong deviation which must certainly be due to a peculiarity of the Edinburgh adopted time. But, referring to the adoption in question and the T.C. observations from which it comes, it cannot be admitted that any adjustment is allowable that would remove this deviation.

Next below the comparisons with adopted times derived from T.C. observations reduced into level, I give those where the level is discarded and the T.C. is reduced with A and n. It will be seen that the deviation in question has disappeared. At the same time the mean deviation for Nauen is reduced from $\pm$·058 sec. to $\pm$·045 sec. This is with regard to a medial constant value. For Annapolis the mean deviation is not reduced; for both curves it remains at $\pm$·047 second. The reason of this appears to be that the true medial line is not level throughout the year, but slightly tilted, as shown by the dotted trace on the diagram. A similar tilt, running over the same months, was noted in the Washington times in 1920.† With respect to such a tilted

* It has been pointed out by B. Wanach (*Ast. Nach.*, No. 5199) that in the paper just referred to the times ascribed to Berlin are really due to the Hamburg Seewarte.

† Cf. *M.N.*, **82**, 219.


Jan. 0 10 20 30 Feb. 9 19 29 Mar. 10 20 30 Apr. 9 19 29 May 9 19 29 Jun. 8 18 28 Jul. 8 18 28 Aug. 7 17 27 Sept. 6 16 26 Oct. 6 16 26 Nov. 5 15 25 Dec. 5 15 25 Jan. 4
GREENWICH BROADCAST
HAMBURG (NAUEN)
WASHINGTON (ANNAPOLIS)
·25 sec.
0 sec
·25
0

medial the average discrepancy is ±·037 second. In any case the resultant comparison curve could hardly be more satisfactory, considering that it contains all errors incident to the Washington adoption as well as those of Edinburgh. But a general reduction is not the point. The matter for remark is that the level alone is responsible for a large semi-systematic and persistent discrepancy.

I do not at present enter upon any discussion of the way in which the use of the level introduces these anomalies. I am well aware of the difficulties. There is no method more firmly built into fundamental astronomy than the use of the normal to the geoid, found with the spirit-level or the mercury bath. Besides the subject of the present paper, it enters all the most refined observations of latitude and declination. None the less, I consider that there is no escape from the conclusion that the level as determined here has introduced these rather gross faults, both erratic and semi-systematic, without bearing any evidence upon its face. It may be that something will be found in the method by which level is habitually found, as it is found here, that will explain the anomaly. I cannot say that I see any probability of that, and if not, the subject becomes even more interesting.

I would add some remarks which might seem superfluous but for the position which may be taken by those astronomers who regard time questions as having no bearing of importance upon astronomy proper, inasmuch as time merely measures the phase of the Earth's rotation, and this is eliminated from permanent records. This is a complete misapprehension. The hour hands of the clocks may be regarded as mechanical devices for reproducing for continuous reference the positions of the First Point of Aries (without the nutation) or the Mean Sun, as the case may be. The clocks for right ascensions correspond to the vertical circle of the instrument for declinations. Precisely the same part which the declination circle plays in determining one co-ordinate of a star is played by the clock in determining the other. The only difference is that on the divided circle the division errors are treated as fixed and the nadir point approximately so, while the corresponding quantities of clock rate and clock error require continual redetermination. In either case, if the index error is incorrectly assigned, the deficiency is debited to the co-ordinate of the star, and appears tacitly and automatically in the star ledger. If the fault consisted entirely of what I have called erratics, and if the observations are merely differential, as is commonly assumed to be the case, this would have little effect upon the outcome, except a general deterioration of the material, in respect to its internal consistency. The case is far otherwise when there is a systematic deviation and when we are dealing with fundamental right ascensions. Especially if it appears that such a deviation may be of seasonal character, no single observatory can effectively clear the error from its own observations, and it will remain as a systematic difference between the catalogues it prepares and those prepared elsewhere. If these systematic differences in catalogues are changed from one epoch to another, their differences are transferred to the proper motions of the stars and may distort some of the most comprehensive and delicate conclusions drawn by astronomy. Such was the position between different observatories until recently, and so it remains in regard to declinations. But now in respect to right ascensions, wireless time signals permit a continuous and searching comparison of the datum adopted every day, with that adopted at other observatories. The fact that this comparison deals with the general adopted index errors for whole groups of fundamental stars, which may be in widely different regions of the sky, and not with the co-ordinates of individual stars, should surely be held to enhance its significance.

Method of Variation of Axes, Time, and Scale.

By T. C. Hudson.

In a previous paper (*M.N.R.A.S.*, **84**, 589) the directed differential equation of the second order

$$^2\mathrm{R}R^2 = \mathring{\rho} = -r$$

was exhibited as a statement of Newton's law of gravitation.

It may be written in a binary form :

$$\mathrm{F}=0, \quad \left(1+R^3\left(\frac{d}{dt}\right)^2\right)\mathrm{R}=\mathrm{F}.$$

The right member of F=0 may be regarded as a *negative* idealisation ; as to *non*-departure from the simple law.

Whatever directed value F may possess, if we denote $\mathrm{VR}^1\mathrm{R}$ by H and $\mathrm{V}^1\mathrm{RH}-r$ by E, then

$$\mathrm{EH}=\mathrm{HV}^1\mathrm{RH}-r\mathrm{VR}^1\mathrm{R}=0-0=0,$$

the reason, of course, being that the relation between H and E is that of perpendicularity.

Now, so long as F=0, we find ΔH=0 and ΔE=0 ; and conversely, if ΔH=0 and ΔE=0, we have F=0. Thus the negation as to F may be translated into negation as to both H and E, without gain and without loss of import. The import consists in the *invariableness* of

$\begin{cases} h \text{ planarity ;} \\ \tfrac{1}{2}H \text{ temporal variation of area (angular momentum) ;} \end{cases}$

$\begin{cases} e \text{ perihelion ;} \\ E \text{ eccentricity.} \end{cases}$

H and E contain three numbers each as defining-attributes possessing a quantitative nature ; but there is a relation between them (EH=0). Thus there are $2\times3-1$ numerical independencies, whatever system of reference is adopted. Of these 5, the lengths of H and E require 2. The other 3 belong to h and e. It takes two numbers to define h ; the remaining number defines the half-plane which contains e.