

## THREE DOUBLE-LINED CLASS B SPECTROSCOPIC BINARY SYSTEMS

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### *Introduction*

IN accordance with the principles of celestial mechanics the two components of a binary system revolve in a common plane about their centre of gravity, in the same time and describe similar ellipses. The centre of gravity moves in a straight line with a constant velocity, which is readily determined from the spectroscopic observations, being designated by  $V_0$ , or by the greek letter  $\gamma$ . Ordinarily, only the brighter star records on the photographic plate, so that the observations of velocity when plotted against the time give rise to one velocity curve, generally resembling a distorted sine curve, which may be solved by one of several methods and the elements of the orbit found. When the two

components differ little in mass, when  $\frac{m_1}{m_2} \cong 0.7$ , the spectral lines of the less luminous star appear as faint companions to those of the brighter; and if their intensity, quality and separation permit their measurement, each observation furnishes the individual velocities of the two stars.

The measurement of double-lined spectra, therefore, gives rise to two velocity curves as shown in Fig. 1, having the same period  $P$ , eccentricity  $e$ , and whose  $\omega$ 's differ by  $180^\circ$  ( $\omega$  is the angle defining the orientation of the ellipse). The curves are not solved separately, but the common elements are found from the combined data, the number of observations being thus virtually doubled.

Double-lined spectra provide additional information of the system, for the ratio of the semi-amplitudes of the curves is equal to the ratio of the masses, *i.e.*,  $\frac{k_1}{k_2} = \frac{m_1}{m_2}$ . It is, therefore, highly desirable when the secondary spectrum can be moderately well measured to find the amplitude and thus determine the ratio of

the masses, which cannot be determined from observations of the brighter star alone. The three double-lined binaries discussed in this article were discovered here, the orbits of the first two being completed by the Director, Dr. J. S. Plaskett, and that of the last star by the writer.

$\zeta^2$  COR. BOR.—H.D. 139892

1900 R.A. 15h 35.6m, Dec.  $+36^{\circ}58'$ , vis. mag. 5.07, spectral type B8n.

The binary character of this star was discovered by W. H. Christie from a double-lined plate showing a relative separation of 220 km. The spectrum is of type B8n with broad, strong and ill-defined lines of hydrogen, together with weak diffuse lines of helium. The difficulties of measuring such nebulous lines were increased by their double character, the resulting probable errors per plate being  $\pm 11.8$  km. per sec. for the brighter, and  $\pm 15.1$  km. per sec. for the fainter component. The lines measured are given in Table I, the average number of lines being five and the maximum number nine.

The I-prism spectrograph with the medium-focus camera resulting in a linear dispersion of 29A per mm. at  $H\gamma$  was used, although two of the plates were obtained with the short-focus camera which gives a linear dispersion of 49A per mm. at  $H\gamma$ .

Thirty-two of the thirty-six plates secured were used in the determination of the orbit. Four plates were omitted, for, on account of their proximity to the crossing points, the double lines were blended and even approximate measures were impossible.

The preliminary elements determined by a trial and error method of a series of velocity curves for different eccentricities and longitudes of the line of apsides, were corrected by a least-squares solution after the method of Lehmann-Filhés. Sixty observation equations involving six unknowns were solved and corrections applied to all the elements except  $\omega$ . This solution was performed by F. G. Young, an undergraduate in Mathematics at the University of Alberta, Edmonton. The resulting elements together with their probable errors are given in Table II.

The projected masses and separations of the components are

given in Table III. The masses are unusually high for a star of this type, and are certainly not exceeded by any other B8 stars with the possible exception of  $\mu$  Sagittarii whose mass function is  $3.63 \odot$ . But the spectrum of  $\mu$  Sagittarii, and also of  $\nu$  Sagittarii and  $\beta$  Lyrae with masses of the same order, can hardly be called of normal B8 type.

The absolute magnitude  $-3.2$  was accepted, being the mean of the two methods later described. This value combined with the apparent magnitude of 5.8 gives a parallax of  $0''.0016$  or an approximate distance of 2,000 light years.

TABLE I—LINES MEASURED IN THE SPECTRA

$\zeta^2$ Cor. Bor. -B8n	H.D. 25833 - B3		H.D. 216014 - B0n	
$\lambda$ Atom	$\lambda$ Atom	$\lambda$ Atom	$\lambda$ Atom	$\lambda$ Atom
4922 He	4922 He	4144 He	*4922 He	4267 C+
4861 H $\beta$	4861 H $\beta$	4121 He	*4861 H $\beta$	*4144 He
4481 Mg+	4713 He	4102 H $\delta$	4713 He	*4121 He
4471 He	4481 Mg+	4026 He	4686 He+	4116 SiIV
4388 He	4471 He	4009 He	4651 C++	*4102 H $\delta$
4340 H $\gamma$	4388 He	3970 He	4575 SiIII	4089 SiIV
4144 He	4340 H $\gamma$	3934 Ca+(K)	4568 SiIII	4076 OII
4101 H $\delta$	4267 C+		4553 SiIII	*4026 He
4026 He			4481 Mg+	4009 He
3970 He			*4471 He	*3970 He
			*4388 He	3968 Ca+(H)
			*4340 H $\gamma$	3934 Ca+(K)

\*The amplitude of H.D. 216014 is exceeded only by that of V Puppis, for which  $K_1 + K_2 = 604$  km.<sup>14</sup>

The proper motion of this star according to Boss is  $0''.0135$  corresponding to a linear tangential velocity of 40 km. per sec. Combining this with the radial velocity of 30 km. per sec. gives a space velocity of 50 km. per sec.

This star is the brighter of the visual double Bu 7352, the 6th magnitude companion being distant  $6''.15$  in position angle  $304^\circ.3$ . These values of separation and position angle have remained unchanged for over 50 years according to Burnham, the stars possessing a common proper motion. The fainter star is of type B5 with moderately sharp lines and constant radial velocity, from

seven plates of  $-20$  km. per sec. as compared with  $-30$  km. per sec. for the brighter star. From the fixity of position, from the difference in radial velocity and from the fact that, with a parallax of  $0''.0016$  the minimum separation is 4,000 astronomical units, it seems unlikely that there can be any orbital connection between the visual pair.

#### H.D. 25833

1900 R.A. 4h 0.6m, Dec.  $+33^\circ 11'$ , vis. mag. 6.61, spectral type B3.

The first plate of this star, obtained by the Director, showed a double-lined spectrum with a relative separation of 314 km. The spectrum is of type B3 with rather ill-defined lines, given in Table I. The maximum number of lines measured on a plate was eleven, and the average number eight. Although they are of somewhat better quality than those of  $\zeta^2$  Cor. Bor. the measures were difficult for the reasons previously given, the probable error of a single plate being  $\pm 13.3$  km. per sec.

The 1-prism spectroscope with the short-focus camera giving a linear dispersion of 49A per mm. at  $H\gamma$  was used, though two of the plates were taken with the medium-focus camera. Three of the twenty-seven plates secured showed the lines blended which rendered the measures uncertain and hence they were not used in the determination of the orbit.

Owing to the uncertainty of the measures of the secondary, the elements were determined from the primary alone, and after these were obtained, the amplitude of the secondary determined, the other elements being considered as fixed. The preliminary elements showed that the eccentricity was small and therefore both  $\omega$  and  $T$  could not be separately defined. As a correction to the period was desired the observations were necessarily treated separately each being assigned a weight in conformity with the general quality of the plate, the number of lines measured and the inter-agreement of the measures. Schlesinger's equation of condition was employed, and a least-squares solution of the resulting twenty-four observation equations involving five unknowns was performed. The corrected elements and their probable errors are given in Table II.

Assigning the probable values of a temperature of  $15,000^\circ$  K,

a density of 0.1 times the sun, and a surface brightness of  $-3.3$  magnitudes with respect to the sun, to this B3 star, we derive an absolute magnitude of  $-1.4$ . Eddington's method, hereafter described, gives  $-1.6$ . The mean value  $-1.5$  combined with its apparent magnitude of  $7.4$  gives the hypothetical parallax of  $0''.0016$  equivalent to a distance of 600 parsecs or nearly 2,000 light years; practically identical with  $\zeta^2$  Cor. Bor.

TABLE II—ELEMENTS OF THE SYSTEMS

	$\zeta^2$ Cor. Bor.	H.D. 25833	H.D. 216014
P	$12.58485 \pm 0.1578$ days	$2.02858 \pm 0.00118$ dys.	$2.28754 \pm 0.00047$ days
$e$	$0.030 \pm 0.0183$	$0.0512 \pm 0.0102$	$0.086 \pm 0.007$
$\omega_1$	$90^\circ$	$30^\circ.103 \pm 1^\circ.00$	$289^\circ.31 \pm 4^\circ.80$
$\omega_2$	$270^\circ$	$210^\circ.103 \pm 1^\circ.00$	$109^\circ.31 \pm 4^\circ.80$
$K_1$	$134.82 \pm 3.19$ km.	$164.97 \pm 1.89$ km.	$225.0 \pm 2.66$ km.
$K_2$	$137.71 \pm 3.20$ km.	$187.32 \pm 6.1$ km.	$258.8 \pm 3.33$ km.
T	$3855.681 \pm 0.158$ days	$4039.335$ days	$4077.0010 \pm 0.0325$ days
$\gamma$	$-29.61 \pm 1.80$ km/sec.	$+15.84$ km/sec.	$-23.10 \pm 1.70$ km/sec.

- P Period
- $e$  Eccentricity
- $\omega_1$  Periastron minus node, primary
- $\omega_2$  Periastron minus node, secondary
- $K_1$  Semi-amplitude, primary
- $K_2$  Semi-amplitude, secondary
- T Time of periastron passage
- $\gamma$  Velocity of the system

TABLE III—DIMENSIONS OF THE SYSTEM

	$\zeta^2$ Cor. Bor.	H.D. 25833	H.D. 216014
$a_1 \sin i$	23,310,000 km.	4,549,000 km.	7,051,000 km.
$a_2 \sin i$	23,820,000 km.	5,165,000 km.	8,111,000 km.
$m_1 \sin^3 i$	13.35 $\odot$	4.86 $\odot$	14.23 $\odot$
$m_2 \sin^3 i$	13.06 $\odot$	4.29 $\odot$	12.37 $\odot$

The K line of ionized calcium was measured on 18 plates of this star. The line is very weak and generally difficult of measurement. Although there is a total range of some 35 km. in the velocity with some indication of orbital effect, the latter is decidedly

uncertain; and the mean calcium velocity  $+10.5$  km. does not differ much from the solar component of  $+7.1$  km., nor indeed from the velocity of the system  $+15.8$  km. Type B3 appears to be about the lower limit of temperature which will produce excitation in inter-stellar calcium, and the effect hence appears to be present in this example with a possible influence due to stellar calcium.

#### H.D. 216014

1900 R.A. 22h 44.2m, Dec.  $+64^{\circ} 32'$ , vis. mag. 6.83, spectral type B0n.

The first plate of this star showed a double-lined spectrum of unusually wide separation, the relative velocity being approximately 400 km. The binary nature was discovered by Dr. J. S. Plaskett.

The spectrum is classified in the Henry Draper catalogue as B3 with the following remark: "The line  $H\beta$  is not seen as a dark line, and is suspected to be bright." The type, however, is really E0n. The lines measured for determining the radial velocity are given in Table I. Both components appear to have identical spectra, the asterisks in the table denoting those lines measured as double in an average plate. The single-lined plates show many oxygen lines but these are very weak and not suitable for measurement. The diffuse character of the lines is shown in the rather high probable error of an average plate  $\pm 7.34$  km. The lines of calcium, however, are strong and, compared with the nebulous nature of the other lines, are narrow, admitting of accurate measurement, as is shown by the probable error of their mean velocity  $\pm 0.31$  km. The suspected emission of  $H\beta$  referred to in the quotation above is not confirmed, it appeared as a good absorption line on all the plates.

Twenty-one I-prism plates were secured principally with the short-focus camera. These were measured, as were those of the two stars previously described, on a Toepfer micrometer engine, having a binocular eyepiece which is more restful on the eyes than the ordinary single-ocular machine. The secondary spectrum was measured on all but three of the plates, so that the elements of the orbit were determined from thirty-nine observations.

In the least-squares correction to the elements, the equation of condition of Lehmann-Filhés was used. A correction to the period was included, as the observations only extended over thirty-eight cycles. Weights were assigned to the measured velocities in accordance with the quality of the plate, the number of lines measured and degree of precision of the fainter measures with respect to those of the brighter component. The thirty-nine observation equations involving seven unknowns thus formed resulted in the elements given in Table I.

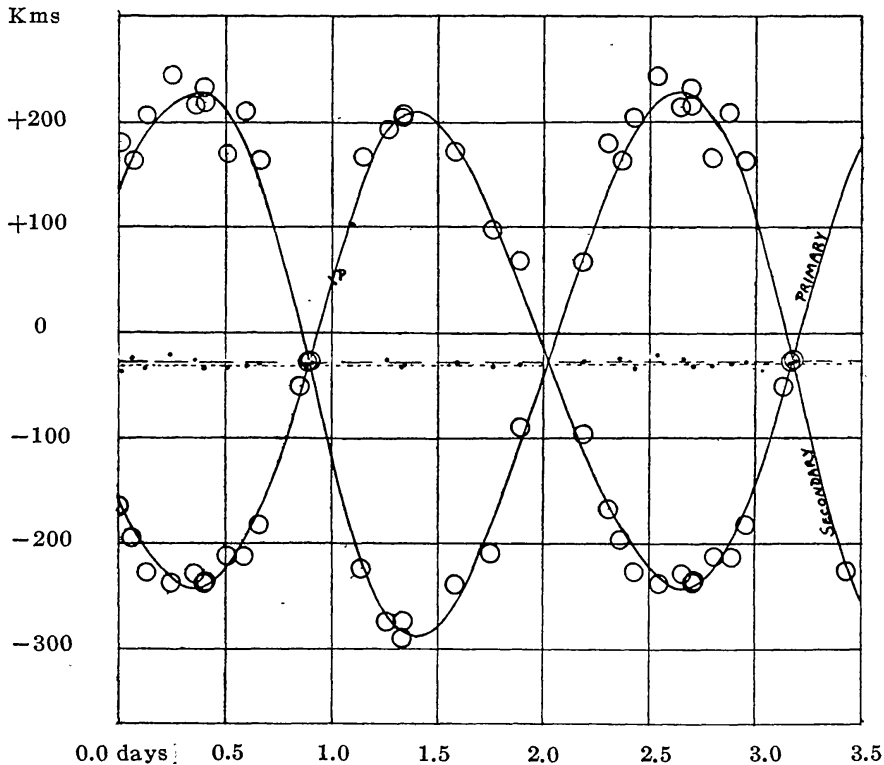


Fig. 1. Velocity Curves of H.D. 216014

In order to see whether the inclusion of the less certain measures of the fainter component materially contributed in defining the elements, or whether they produced a detrimental effect, a second solution was carried out for the brighter component only. The resulting elements differed very little from the first set, although their probable errors in all cases except that of  $K_1$  were higher, thus showing that the inclusion of the secondary measures was justifiable, as we would expect from theory. We are here, how-



ever, dealing with diffuse lines, and not with well-defined ones, and it is questionable whether with very faint secondary lines of early type spectra the gain in accuracy compensates for the additional labour involved. Experience has led some investigators of double-lined B-type binaries to base the elements upon the measures of the primary, and the test made in the case of this star would support that contention in those cases where the quality of the lines is such as to make the measures of the secondary component untrustworthy. For a comparison of the two sets of elements and their probable errors the reader is referred to Pub. D.A.O., III, No. 6.

The elements lead to the values of the projected dimensions and masses of the stars given in Table III. These minimum masses are of the same order as those for  $\zeta^2$  Cor. Bor., being well above the average for stars of this spectral class.

Assigning the conservative values of a temperature of  $20,000^\circ$  K, surface brightness of  $-3.75$  and a density of  $0.08$  to this B0 star, we find by two methods an absolute magnitude of  $-2.58$  corresponding to a parallax of  $0''.00095$ ,  $1050$  parsecs, or more than  $3,400$  light years.

The stationary nature of the calcium lines in the spectrum is important. In contradistinction to the diffuse hydrogen and helium lines, the H and K lines of ionized calcium are strong and sharp, giving a constant velocity of  $-26.28 \pm 0.31$  km. per sec. This value is close to that of  $-23.10 \pm 1.70$  km. per sec. found for the velocity of the system. If these lines are produced by an interstellar cloud of calcium vapour quite apart from the spectroscopic binary, the observations indicate that the cloud has a velocity of approach of  $15$  km. per sec., as the component of the solar motion in this direction is  $-11.40$  km. only. Differences of this order and even greater, between the velocity of calcium and the component of the solar motion, do not appear to be unusual among the O- and early B-type stars, as has been shown by Dr. Plaskett,<sup>1</sup> Young<sup>2</sup> and others.

An alternative explanation is to place the calcium cloud within the star's gravitational domain, at a distance of  $40$  to  $50$  astronomical units. It would then hardly be expected to show the motions of the rotating binary, and the resulting lines would be



stationary, possessing approximately the velocity of the system. It is reasonable to suppose that the ionized calcium has been expelled from the star through ionization impact as Saha<sup>3</sup> has pointed out, and at this far distance settled into a state of equilibrium between the gravitational pull of the star and the outward pressure also acting radially. This alternative location for the ionized calcium does not appear to be inconsistent with the well-established hypothesis of Dr. Plaskett in regard to stationary clouds of calcium and sodium vapour.

### *Explanation of the methods employed*

The readers of the Journal may be interested in following in greater detail the methods used in assigning values of the absolute magnitudes to the stars previously described, and thus in finding their distances which are so enormously great, that the question may well be asked, "How are these distances determined?" As an illustrative example we will take H.D. 216014 and outline the various steps involved.

The fundamental equation is the relation between the absolute and apparent magnitude of the star, *i.e.*,

$$M = m + 5 + 5 \log \pi \dots \dots \dots (1)$$

where  $M$  = abs. mag.

$m$  = app. mag.

$\pi$  = parallax

the unit of distance being a parallax of  $0''.1$ , 10 parsecs or 32.6 light years. If the  $M$  can be found, the distance is readily computed by the above formula since the  $m$  is known. There are two ways of determining  $M$ : (1) By assigning probable values of the surface brightness and surface area, and thus determine the luminosity, (2) By the theoretical considerations of Eddington.

(1) *Luminosity Method*

This method is based upon the two equations:

$$M' = \frac{4}{3}\pi\rho b^3 \dots\dots\dots (2)$$

$$L = \pi b^2 j \dots\dots\dots (3)$$

where  $M'$  = mass

$\rho$  = density

$b$  = radius

$L$  = luminosity

$j$  = surface temperature

Since the sun is taken as the standard, these equations become:

$$M' = \rho b^3 \odot \dots\dots\dots (4)$$

$$L = b^2 j \odot \dots\dots\dots (5)$$

*The Density*

Some uncertainty exists in our estimation of the density. We know from the early class B eclipsing variables that the average density of these stars is 0.05 with a range from 0.0004 to 0.18. Also, Seares<sup>4</sup> in his discussion on the masses and densities of the stars, gives the density of a B0 star of mass  $M' = 10\odot$ , as 0.04<sup>5</sup> for the dwarfs, and 0.08 for the giants. Adopting  $\rho = 0.08$  for the brighter component of the star under discussion, we have its volume is 178, linear diameter is 5.62 and surface area 31.6 times the sun.

*The Surface Brightness*

The surface brightness,  $j$ , may be computed by Hertzsprung's<sup>5</sup> formula based on Planck's radiation law:

$$J_H = +2.3 \left( \frac{14300}{T} \right)^{0.93} + \text{constant} \dots\dots\dots (6)$$

For a temperature of 20,000° K, corresponding to B0,  $J_H$  is found to be -3.47 mag.

Again the surface brightness for this type of star may be found in another way. It has been found that the difference in surface brightness of two stars expressed in stellar magnitudes is proportional to the difference in their colour indices. That is, this equation holds:

$$j_2 - j_1 = K(i_2 - i_1) \dots\dots\dots (7)$$

The constant  $K$  is the same for all stars, but depends on the wavelength of the light used in measuring the colour indices. Russell<sup>6</sup> has obtained as a mean  $K=4.0$  and he showed how the apparent diameter of a star can be determined from its surface brightness. Later, it was found from the interferometer measures of stellar diameters, made at Mt. Wilson, that a better value for  $K$  is 4.8. Hence, again adopting the sun as standard, we have:

$$\begin{aligned} \Delta j &= -4.8 (\Delta i) \dots\dots\dots (8) \\ &= -4.03 \text{ mag. where } \Delta i = i_{G_0} - i_{B_0} = 0.84 \end{aligned}$$

Giving equal weight to these estimates of  $j$ , we take the mean  $j = -3.75$  mag. That is, a B0 star of temperature  $20,000^\circ$  is 3.75, mag. or 32 times more brilliant than the sun, per unit area.

The luminosity now readily follows:

$$\begin{aligned} L &= (2.512)^{3.75} X (5.62)^2 \dots\dots\dots (9) \\ &= [3.00008] \end{aligned}$$

over 1,000 times as luminous as the sun, or expressed in magnitudes, 7.50. The absolute magnitude of the sun being  $+4.83$ , we have the absolute magnitude of H.D. 216014 as  $-2.67$ .

## (2) Eddington's Method

In his articles on the radiative equilibrium of the stars, Eddington<sup>7</sup> develops the formula for computing the total radiation of a giant star, which is found to be directly proportional to the mass and inversely proportional to the opacity of the star. The absolute magnitude of the giant star is found by converting the total radiation into magnitudes, and may be tabulated as a function of the mass. Eddington's table<sup>8</sup> for masses up to thirteen times the sun, has been extended by Plaskett<sup>9</sup> in his memoir on the O-type stars to cover a range in from ten to eighty times the sun. Interpolating this table we have for our star a bolometric absolute magnitude of  $-3.98$ . For types F and G (temperature  $6000^\circ$ ) the bolometric absolute magnitude deduced from the total radiation, agrees with the visual absolute magnitude, but for the earlier or later types a correction is necessary. This is given by Eddington<sup>10</sup> for the temperatures up to  $12,000^\circ$ , and for higher temperatures, may be calculated by the formula given by Seares.<sup>11</sup>

$$M - M_B = j + 10 \log T - \text{constant} \dots\dots\dots (10)$$

In this case the correction is +1.48, the resulting visual absolute magnitude being  $-2.50$ . Mean  $M = -2.58$ .

### *The Visual Magnitude and Distance*

The apparent visual magnitude is 6.83, and it is necessary to find the magnitude of the separate components. Since the spectra are identical their temperatures will be of the same order. The difference in mass indicates a difference in magnitude of 0.10, so that the magnitude of the brighter star is found to be 7.53 from the relation

$$m_1 = m_{(1+2)} + 2.5 \log (2.5)^{m_1 - m_2} + 1 \dots \dots \dots (11)$$

Hence with  $M = -2.58$ ,  $m = 7.53$ , the parallax is found by (1) to be  $0''.00095$ , over 1050 parsecs or more than 3,400 light years.

### *Corrected Linear Diameter.*

With the adopted  $M = -2.58$  we find a corrected value for the linear diameter,  $D = 5.40 \odot$ , 4,667,000 miles, and this value is used in the subsequent computations.

### *The Apparent Diameter*

It may be interesting to compute the apparent diameter according to Russell,<sup>6</sup> and find the distance at which the adopted linear diameter of  $5.40 \odot$  or 4,667,000 miles would subtend this small angle. By the formula:

$$d = 0''.0087 (0.631)^{m-j} \dots \dots \dots (12)$$

For  $m-j = 11.28$ , we have  $d = 0''.000048$ . Hence the distance in light years would be

$$\pi_{L.Y.} = \frac{206264.8 \cdot D}{d \times [12.770]} \dots \dots \dots (13)$$

$$[12.770] = 1 \text{ L.Y.}$$

$$= [3.532] > 3400 \text{ light years.}$$

### *Check Formulae*

In order to check the computations we may use the following formulae expressing relations between the various physical constants of a star. They are given by Seares;<sup>12</sup> Nos. (14) and (15)

are equivalent to Russell's<sup>6</sup> formulae, and No. (18) is identical with No. (14).

$$\log d = 0.2 (j - m) - 2.061 \dots \dots \dots (14)$$

$$\log D = 0.2 (j - m) - \log \pi - 0.030 \dots \dots \dots (15)$$

$$\log D = 0.2 (j - M) + 0.970 \dots \dots \dots (16)$$

$$D = \frac{107.4d}{\pi} \dots \dots \dots (17)$$

$$j = 5 \log d + m + 10.30 \dots \dots \dots (18)$$

For convenience we collect the adopted values of these constants for our star.

$m = 7.53$	By (14) we have $d = 0''.00048$	
$M = -2.58$	(15)	$\log D = 0.737$
$D = 5.40 \odot [0.732]$	(16)	$\log D = 0.736$
$d = 0''.000048$	(17)	$\pi = 0.00095$
$j = -3.75$	(18)	$j = -3.75$
$\pi = 0''.00095$		

All in complete agreement.

### *The Inclination of the Orbital Plane*

It is well to remember that the computations have been based upon the minimum value of the mass ( $m \sin^3 i$ ) and therefore the distance deduced is the minimum value for the system. The actual dimensions of the binary cannot in general be found from spectroscopic observations, since the inclination of the orbital plane remains indeterminate. Only in the case of the eclipsing binaries where  $i$  approaches  $90^\circ$  and  $\sin^3 i$  is taken as unity do we know the actual masses. The average inclination is taken in statistical discussions on the masses of stars of different spectral types, and it is found that the average class B star is ten times, and class O star fifty times as massive as the sun. Adopting the average inclination for H.D. 216014, for which  $\sin^3 i = 0.65$ ,<sup>13</sup> the resulting masses would be 21.90 and 19.05 times the sun, which for the conditions of density, temperature and surface brightness assumed above give a parallax of  $0''.00079$  or 4100 light years.

Recognizing that large amplitudes\* point to large values for  $i$ ,

\*The amplitude of H.D. 216014 is exceeded only by that of V Puppis, for which  $K_1 + K_2 = 604 \text{ km}^{14}$ .

it is interesting to assign values which would cause the brighter star to partially eclipse the fainter and find the variation in magnitude on the assumption of the star being an eclipsing variable. If the star were examined with a photometer and no change in brightness detected we could thus find a limiting value for the inclination, and hence a closer approximation to the real mass. Drawing the system to scale and using a planimeter to find the areas eclipsed we have the results given in Table IV.

For an inclination of  $70^\circ$  the mass of the system would be increased by 12% being  $32.06 \odot$  and the resulting  $M = -2.80$ , corresponding to a distance of 3800 light years. Variations in magnitude of 0.15 can be readily detected with a photometer, but until the observations are made we shall accept the minimum masses.

The above considerations on the plane of the system are added merely to emphasize the fact that the distance determined from the minimum mass and the conservative estimates of the physical properties is the smallest value that the spectroscopic observations allow us to adduce, and that the farther values of 3800 to 4100 light years are not only possible but quite probable.

TABLE IV—ECLIPSE DATA

Inclination	Area of Secondary eclipsed	App. Mag. of Secondary at eclipse	Combined Magnitude
$90^\circ$	100%	....	7.53
73	40	8.18	7.06
70	27	7.97	6.98
66	16	7.82	6.92
57	0	7.63	6.83

### *Conclusion*

These three systems well exemplify the chief characteristics of this interesting class of very hot and very massive stars. In  $\zeta^2$  Cor. Bor. we have a B8 giant binary of low density, great volume and high luminosity. On the up-grade of its evolutionary course it has yet to pass through the early sub-divisions of the class and reach its maximum temperature. H.D. 216014, B0, is at

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TABLE V—SUMMARY OF SYSTEMS

	Sun	$\zeta^2$ Cor. Bor.	H.D. 25833	H.D. 216014
Spectral class.....	G0	B8n	B3	B0n
Temperature K.....	5,800°	12,000°	15,000°	20,000°
Temperature F.....	10,400°	21,600°	27,000°	36,000°
Mass, brighter component.....	1.0	13.35	4.86	14.23
Mass, fainter component.....		13.06	4.29	12.37
Density, brighter component.....	1.0	0.015	0.10	0.08
Volume.....	1.0	890	49	178
Surface brightness.....	1.0	16	21	32
Luminosity.....	1.0	1580	330	1000
Absolute magnitude (mean).....	+4.83	-3.2	-1.5	-2.58
Velocity in orbit, brighter star... (Earth = 18.4 M.P.S.)		217.0 M.P.S.	265.5 M.P.S.	362.1 M.P.S.
Velocity in orbit, fainter star....		221.6 M.P.S.	301.5 M.P.S.	416.5 M.P.S.
Distance, parallax (minimum)....	sun = 8".80	0".0016	0".0016	0".00095
Distance, miles (minimum).....	$93 \times 10^6$	$12 \times 10^{15}$	$12 \times 10^{15}$	$20 \times 10^{15}$
Time by aeroplane 120 M.P.H....	86.1 years	$11 \times 10^9$ years	$11 \times 10^9$ years	$19 \times 10^9$ years
Time by light 186,400 M.P.S.	498 seconds	2000 years	2000 years	3400 years

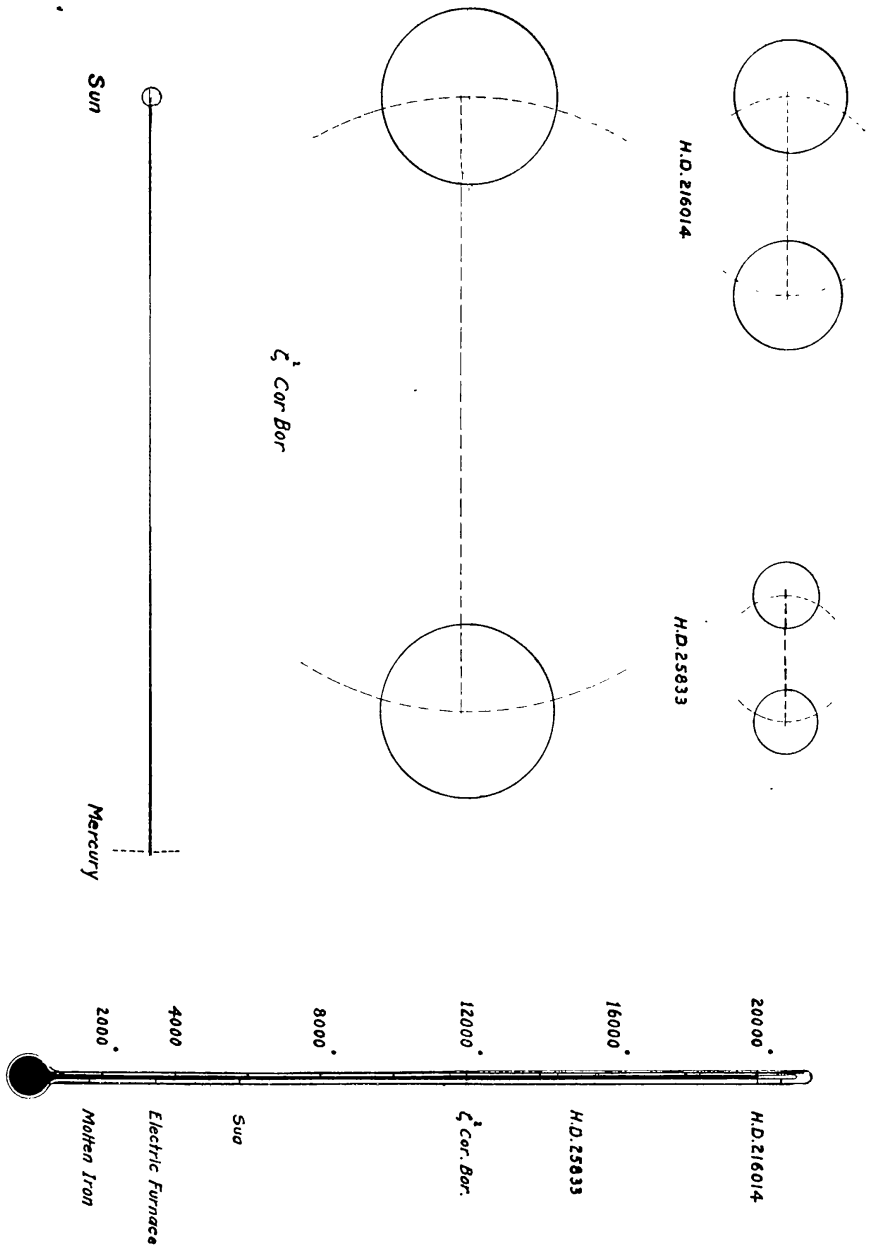


or near the culmination of its career, its mass may have carried it into the later subdivisions of the O's,  $\lambda 4686$  He+ being the last of the lines characteristic of this type to vanish from the spectrum. H.D. 25833, B3, the least massive and the densest of the systems, precedes the other two, having passed through its apex and started downward on the dwarf branch of decreasing temperature. The minimum sizes and separations are contrasted with the sun and Mercury in Fig. II. On the same scale the earth is a dot 1/100 of the sun's diameter at two and one-half times the distance of Mercury. The first two systems are approximately twice as remote as the average class B star,<sup>15</sup> while the distance of H.D. 216014 is of the order of the average O<sup>16</sup> and N-type,<sup>17</sup> the most distant of our ordinary stars. There seems to be no reasonable doubt that the light of this giant binary started on its journey one thousand years before the founding of the city of Rome, at about the time when the nomad Abraham commenced his wanderings, and after these thirty-four centuries, one-half the span of authentic history, reached our spectroscope, with the information outlined in the above article.

Victoria, B.C.,  
May 1925.

#### REFERENCES

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- <sup>3</sup>Nature, 107, p. 488, 1921
- <sup>4</sup>Mt. W. Cont. 10, p. 404
- <sup>5</sup>Ibid, 10, p. 399
- <sup>6</sup>Pub. A.S.P., 32, p. 307
- <sup>7</sup>M.N., 77, 16, 596; Ap. J., 48, p. 205
- <sup>8</sup>Ap. J., 48, p. 211
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- <sup>10</sup>M.N., 77, p. 605
- <sup>11</sup>Mt. W. Cont., 10, p. 398
- <sup>12</sup>Ibid, 10, p. 402
- <sup>13</sup>Campbell, Stellar Motions, p. 263
- <sup>14</sup>L.O.B., XI, p. 173, 1924
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*Sizes and Separations compared with the Sun and Mercury*  
 Figure II.