initial values of the co-ordinates, on which the mean motions $m_1, \ldots m_n$ depend; for $m_1, \ldots m_n$ are discontinuous functions of these initial values, and since $w_1, \ldots w_n$ are used as the arguments in the series f_v, g_v, the coefficients in these series must also be discontinuous functions. Indeed, in Lindstedt's series, as in the series (4) above, there occur divisors which depend on the constants of integration, and since some of the divisors vanish in any range of these constants, the series are not even formally valid over any finite range of these constants : it may plausibly be suggested that this peculiar complication arises from the use of angular variables.

The equations of Celestial Mechanics initially present themselves in a form in which none of the co-ordinates are angular, and the object of so transforming them that half the co-ordinates become angular is presumably to throw them into a form as nearly as possible allied to the desired trigonometric form of the solution. The above note has shown that in general this procedure must defeat its own object.

The " Arcus Visionis " of the Planets in the Babylonian Observations.
By C. Schoch.

(*Communicated by J. K. Fotheringham, D.Litt.*)

In his recently published book* Father Kugler has stated that the Babylonian astronomers of the last four centuries before Christ always put the heliacal setting of Sirius on May 14 Julian and the heliacal rising on July 19, if we reckon the day from midnight. The *arcus visionis* (in this paper called γ) of a planet or of Sirius is the depression of the Sun below the horizon, measured in the vertical circle for the moment when the star sets on the last evening when it is visible or rises on the first morning when it is visible, refraction being disregarded in the case of both bodies. In the case of the outer planets and fixed stars the heliacal setting will be called elast (evening last) and the heliacal rising mfirst (morning first). Father Kugler has computed that the mfirst of Sirius at Babylon (latitude $= 32°\cdot5$) occurred on July 19 with $\gamma = 7°\cdot4$. I have computed several mfirsts of Sirius and have found for γ a mean value of $7°\cdot8$. This value for mfirst is in striking contradiction to the value used by Ptolemy ($\gamma = 11°$), and adopted by all astronomers and chronologists (Ideler, *Handbuch der Chronologie*; Ginzel, *Handbuch der Chronologie*; Neugebauer, *Tafeln zur astron. Chronologie*). This difference of $3°\cdot2$ in γ is increased if we compute γ for elast of Sirius. I find a mean value of $6°\cdot5$, which is smaller than Ptolemy's value ($11°$) by $4°\cdot5$. The Babylonians could therefore observe the elasts of Sirius five days later than Ptolemy, the mfirsts four days earlier. The cause of this enormous difference can be found only in the circumstance that at Alexandria there

* *Sternkunde und Sterndienst in Babel*, II. Buch, II. Teil, 2 Heft, 1924, p. 523.

was always a layer of mist covering the lower part of the horizon, even when the rest of the sky was quite clear.   On the other hand, at Babylon (and Athens) the lower part of the horizon was free and the brighter stars were visible till rising or setting.   It is quite impossible to assume that the Babylonians were in error, because their observations of Mercury and Jupiter are in full agreement with those of Sirius.

I use the following abbreviations for the phenomena of the inner planets, Mercury and Venus.   After superior conjunction the planet becomes visible in the evening (efirst), disappears in the evening before inferior conjunction (elast), then reappears after inferior conjunction in the morning (mfirst), and disappears in the morning before superior conjunction (mlast).   Now, for Mercury so many observations have been published by Father Kugler (*Sternkunde und Sterndienst in Babel*, I. Buch, 1907) and still more by Epping (*Zeitschrift für Assyriologie*, vols. v.–viii., 1890–93), that I could compute the angle γ for 30 Babylonian dates.   The result is that for all four phenomena of Mercury the angle γ was different, agreeing well with the stellar magnitude and the conditions of observation ("lasts" or "firsts").   While in our latitudes it is not easy to see Mercury with the naked eye, the Babylonians as far back as the year -567 noted on Airu 10 an efirst, and the value γ of this observation corresponds accurately with that of the later observations of efirst.

All my computations are based on observations, not on Babylonian prediction-tablets.   I have used : for Mercury 30 observations, for Venus 15, for Jupiter 12, for Saturn 9, for Mars only 4.   For this planet therefore the result is uncertain.   As the computations to obtain the angle γ for about 70 observations are very intricate, I have used my MS. tables for geocentric places of the planets, constructed several years ago.   In these tables the following sidereal accelerations are assumed : Earth $+ 0''\cdot89s^2$, Mercury $+ 4''\cdot11s^2$, Venus $+ 1''\cdot65s^2$, Mars $+ 0''\cdot45s^2$, where s signifies the number of centuries elapsed since 1900.   The total corrections applied to Newcomb's mean longitudes are :

$$\text{Earth} \quad + 0''\cdot51 + 1''\cdot08s + 0''\cdot907s^2$$
$$\text{Mercury} + 1''\cdot62 + 3''\cdot60s + 4''\cdot133s^2$$
$$\text{Venus} \quad + 0''\cdot61 + 1''\cdot12s + 1''\cdot645s^2.$$

The resultant corrections to Newcomb's mean heliocentric longitudes for 1925 are : Earth $+ 1''$, Mercury $+ 3''$, Venus $+ 1''$.   The effect of these accelerations is to place ancient conjunctions of Mercury and Venus with the Sun earlier than the times that Newcomb's tables would give.   The value of γ is dependent : (1) on the right ascensions and declinations of the Sun and the star ; (2) on the stellar magnitude (m) of the star ; (3) on the difference in azimuth (A) of the Sun and star, all computed for the moment of efirst, mlast, etc.*   The greater the magnitude is, the smaller γ is, and the greater A is, the smaller γ is. A is always small for the planets, but in the case of Sirius it is very great, 51°.   So γ for Sirius must be smaller than for a planet with the same value of m.   But the

* See Vogt, *Sitzungsberichte der Heidelberger Akademie, philosoph.-hist. Classe*, 1920, Abhandl. 15, p. 8.

Babylonian observations prove also that all "lasts" had a smaller γ than the "firsts." The Babylonians were persistent observers, and it was comparatively easy for them to find the place of a star, known by the observation of the previous day (the case of the "lasts"). For a re-appearance (the case of the "firsts") the place of a star was unknown and not given by previous observations.

Mercury.—m is computed approximately. All 30 observations show that γ was smaller for superior conjunction (mlast, efirst) than for inferior conjunction (elast, mfirst), in full agreement with the value of m. All "lasts" have a smaller γ than the "firsts". The minimum of mlast is $\gamma = 9°\cdot0$.

Venus.—m is equal for superior and inferior conjunction, but the observations seem to give a slightly smaller γ for elast and mfirst than for mlast and efirst. The mean value is $5°\cdot2$ to $5°\cdot8$; the minimum $4°\cdot9$ elast, the sky probably being very clear during this observation. The value given by Dr. Neugebauer,[*] $7°$, is far too high. Of the greatest importance is the interval of invisibility at inferior conjunction, between elast and mfirst. It varies between 2 and 19 days for Babylon and depends only on the geocentric latitude of Venus. The smaller intervals of 2 or 3 days can recur on the same Babylonian date only after periods of 56, 64, or 120 years, and they provide a very good means of identifying the Babylonian year where such observations are recorded. The first observations of Venus were probably made as far back as 3000 years before Christ.

Jupiter.—For elast γ is much smaller than for mfirst. Jupiter has at conjunction the same m as Sirius, but it is easier to observe Sirius, because the difference of azimuth Sun − Sirius is $51°$; hence γ is smaller for Sirius so as to affect the time of mfirst and elast by nearly 2 days. Dr. Neugebauer's value, $7°$, is too small. It is strange that he attributes to Venus and Jupiter the same value of $7°$ although their brilliance differs by nearly two stellar magnitudes.

Sirius has been already discussed. Elast in the time from −300 to +100 for Babylon fell on May 14 (with Ptolemy's false value of γ on May 10). Mfirst fell on July 19 (with Ptolemy's value on July 23), for Memphis (latitude = 30°) with the value of γ as determined for Babylon on July 17 (with Ptolemy's value on July 21).

Saturn.—The value of m depends on whether the planet's ring is open or not, and may vary as much as half a magnitude from the mean value which I tabulate. γ for elast is much smaller than for mfirst.

Mars.—m is right, for the stellar magnitude at conjunction is subject to little variation; at opposition, however, m varies between $-2\cdot7$ and $-0\cdot9$. (At the opposition of this year, 1924 August, m is $-2\cdot6$; that is the most favourable opposition within the next hundred years.) To determine γ I had only 4 observations, which are quite insufficient.

The following table gives the approximate stellar magnitude m and the *arcus visionis* γ, arranged according to the value of m. All values of m and γ are mean values; m is computed for the time of elast, etc., not for the time of conjunction.[†]

[*] *Tafeln zur astron. Chronologie*, Heft 3, 1922, p. 60.
[†] See the formula given in Müller, *Photometrie der Gestirne*.

Babylon. Observ.	m	γ		Babylon. Observ.	m	γ
Venus, inf. conj. :		°		*Mercury*, sup. conj. :		°
elast	−3·2	5·2		mlast	−0·5	9·5
mfirst	−3·2	5·7		efirst	−0·5	10·5
sup. conj. :				inf. conj. :		
mlast	−3·3	5·8		elast	+1·2	11·1
efirst	−3·3	5·8		mfirst	+1·2	13·2
Jupiter :				*Saturn* :		
elast	−1·6	7·4		elast	+0·9	10·5
mfirst	−1·6	9·3		mfirst	+0·9	13·0
Sirius :				*Mars* :		
elast	−1·5	6·5		elast	+1·8	14·2
mfirst	−1·5	7·8		mfirst	+1·8	15·5

For places with a very clear sky (such as Babylon and Athens) I recommend the use of the following values of γ for stars near the ecliptic (A not greater than $25°$) :—

	γ	
m.	Lasts.	Firsts.
	°	°
−3·0	5·8	6·5
−2·0	7·0	8·0
−1·0	8·5	9·5
0·0	9·5	10·5
+1·0	10·5	11·5
+2·0	12·5	13·5
+3·0	15·0	16·0

From the constants given above I have computed the following dates for the phases of Sirius and summer solstice at Babylon, time being reckoned from Babylon midnight :—

Year.	Elast.	Mfirst.	Solstice.
0	May 14	July 19	June 24·8
−1000	,, 11	,, 19	July 2·9
−2000	,, 8	,, 18	,, 10·7
−3000	,, 5	,, 18	,, 18·3

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The Stellar Magnitude Scales of the Astrographic Catalogue. Eighteenth Note: The San Fernando Magnitudes for − 4° *and* − 6°, *and the Algiers Magnitudes for* − 1°. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. Sr. Herrero has kindly furnished counts for zones − 4° and − 6°, following on those previously sent for − 3° and − 5° and published in the seventeenth note (*M.N.*, **83**, 385). These are given in Tables I. and II. below. Cumulative counts were formed from them up to and

TABLE I.

Counts of Zone − 4° (*San Fernando*) *for each hour.*
Numbers of plates, 8, 7, 8, 7, etc.

h	]8·5.	8·5.	9·0.	9·5.	10·0.	10·5.	11·0	11·5.	[11·5.	All.
0	48	34	74	142	182	307	228	134	8	1157
1	35	37	54	108	199	241	205	91	6	976
2	43	33	85	139	210	307	253	166	6	1242
3	47	39	72	119	201	283	226	193	2	1182
4	65	82	112	181	306	456	310	188	36	1736
5	63	62	114	185	250	366	333	429	201	2003
6	81	119	224	352	671	1020	936	672	100	4175
7	73	113	253	505	1036	1363	1000	489	54	4886
8	73	104	217	416	582	690	478	220	19	2799
9	38	91	138	248	336	243	120	48	2	1264
10	36	54	121	229	318	264	165	74	5	1266
11	33	43	84	192	263	262	148	56	8	1089
12	41	45	106	203	319	326	146	78	10	1274
13	32	43	102	216	339	404	279	153	26	1594
14	31	57	95	204	259	307	192	57	12	1214
15	33	35	98	179	252	219	122	38	6	982
16	47	54	118	280	340	288	144	34	29	1334
17	55	28	86	176	294	245	142	74	11	1111
18	64	70	134	227	331	299	145	94	14	1378
19	59	69	131	278	376	372	198	142	9	1634
20	55	80	149	412	770	799	579	237	26	3107
21	45	50	95	222	406	385	228	119	10	1560
22	41	51	106	197	320	266	210	70	25	1286
23	32	40	64	129	260	312	212	123	20	1192
Sum	1170	1433	2832	5539	8820	10024	6999	3979	645	41441