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THE ECLIPSING VARIABLE ι H. CASSIOPEIAE, WITH EVIDENCE ON THE DARKENING AT THE LIMB OF A STELLAR DISK

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ABSTRACT

Variable star ι H. Cassiopeiae; photometric study.—Observations during 1917–1921 with a photo-electric photometer show that this star is an *eclipsing variable*. The primary and secondary minima have ranges of $0^m.132$ and $0^m.032$ respectively, each with an interval of constant light showing that the eclipses are annular or total. From the *light-curve* are derived the *elements of the binary system* (Table IV, Fig. 3), which are in good agreement with the Allegheny spectroscopic results. There is, however, a discrepancy in the phase and duration of the secondary minimum which points to a *motion of the line of apsides*, and which in view of the *high eccentricity*, $e=0.25$, could be readily verified by a new determination of the spectroscopic elements. The *density* of the smaller and fainter body is probably seven times that of the primary.

Darkening at the limb of a star of spectrum B₃.—This star furnishes an unusually favorable test for darkening at the limb, as there is an annular eclipse by a small companion; but the results indicate that there can at most be only a small fraction of the amount of darkening which is present in the case of the sun.

Comparison stars, ι and σ Cassiopeiae.—These have been found to be constant in light, within perhaps $0^m.01$.

Photo-electric photometer for stars.—The *precision* which may be reached is indicated by the residuals for seventy-five normal magnitudes, which give the probable error of one normal = $\pm 0^m.0036$.

The variation of ι H. Cassiopeiae (B.D. $57^{\circ}.2748$, H.R. 8926, magnitude 4.89, spectrum B₃), was detected with the photo-electric photometer in 1918.¹ The spectroscopic orbit by Baker² depends upon plates taken in 1908 and 1909, and the period,

¹ *Publications of the American Astronomical Society*, 4, 115, 1919.

² *Publications of the Allegheny Observatory*, 2, 28, 1910.

6.067 days, was derived by comparison with several Yerkes observations in 1903. In our searching for an eclipse some ten years after the spectroscopic measures it was not surprising to find the star faint at a phase nearly half a day before the prediction, which difference after more than six hundred revolutions could be due to a small error in the assumed period. The photometric observations of this star have been continued at times during four seasons, and it is found that there are primary and secondary eclipses of σ^M_{13} and σ^M_{03} respectively, also that the eclipses are annular and total, which facilitate the derivation of the elements even though the light-range is so small.

ι H. Cassiopeiae, No. 12405 in Burnham's *General Catalogue*, is listed as a wide multiple star with no fewer than eight components, but of these the combination AB is all that is measured through a diaphragm of the photometer which limits the field to $72''$ diameter. As B is more than four magnitudes fainter than A, the effect of the former is almost negligible.

There are two good comparison stars available:

H.R. 8797, ι Cassiopeiae, magnitude 4.93, spectrum B₀
 H.R. 9071, σ Cassiopeiae, magnitude 4.93, spectrum B₂

As these are on either side of the variable, it is found that the differential visual extinction between ι H. and the mean of the two stars is less than 0^M_{004} from 20^h_{0} to 4^h_{0} sidereal time. Thus for eight hours the stars are in good observing position, and there has been no suspicion of variability of either ι or σ . From favorable measures of the comparison stars it is found that the photo-electric difference, ι brighter than σ , is $0^M_{010} \pm 0^M_{002}$.

In Table I are given the photo-electric observations of ι H. Cassiopeiae. The first date is September 10, 1917. The times were reduced to the sun and the phases computed from the elements:

$$\text{Minimum} = \text{J.D. } 2422586.492 + 6^d 06630 \cdot E.$$

The difference of magnitude is in the sense $\iota - \iota$ H., a positive sign meaning that ι H. was brighter than ι . Unless otherwise noted, each difference is the mean of three sets of four readings each on the variable and comparison star, the latter being ι Cassiopeiae unless σ is in the last column. The designation (ι , σ) is given to the few sets where the mean of ι and σ was used.

TABLE I
OBSERVATIONS OF 1 H. CASSIOPEIAE

Date, G.M.T.	Phase	i-i H.	Remarks	Date, G.M.T.	Phase	i-i H.	Remarks
2421482.812...	0 ^d .390	+0 ^M .029	2422525.888...	0 ^d .059	-0 ^M .096
1483.805...	1.383	.036	2528.859...	3.030	+ .028	2 sets
1837.733...	3.464	.003	} Re- jected	2530.828...	4.999	.030	(1, σ)
1845.683...	5.349	.006		2531.710...	-0.185	.028
1924.657...	5.461	.010		725...	-0.170	.018
1927.631...	2.368	.036		739...	-0.156	- .009
1928.567...	3.304	.020		749...	-0.146	- .020
1932.615...	1.285	.034		766...	-0.129	- .042
1939.538...	2.142	.036		776...	-0.119	- .070
1943.512...	0.050	- .106		792...	-0.103	- .086
535...	0.073	- .095		803...	-0.092	- .095
549...	0.087	- .089		829...	-0.066	- .100	(1, σ)
573...	0.111	- .072	842...	-0.053	- .102	1 set	
587...	0.125	- .048	2533.765...	+1.870	+ .043	
601...	0.139	- .026	2534.774...	2.880	.037	
615...	0.153	- .020	2539.778...	1.817	.031	
629...	0.167	- .003	2540.768...	2.807	.037	4 sets	
642...	0.180	+ .023	2553.799...	3.707	.011	
659...	0.197	.010	809...	3.717	.000	
674...	0.212	.024	849...	3.757	- .003	
1967.498...	-0.230	.026	861...	3.769	+ .005	
522...	-0.206	.005	886...	3.794	- .014	(1, σ)	
537...	-0.191	.013	2554.733...	4.641	+ .030	
551...	-0.177	.013	743...	4.651	.030	
588...	-0.140	- .028	2558.810...	2.652	.021	
604...	-0.124	- .058	2561.756...	5.598	.022	
1976.572...	+2.776	+ .016	767...	5.609	.036	
1979.556...	5.760	.010	2566.766...	4.541	.039	
1983.517...	3.655	.009	781...	4.556	.036	
530...	3.668	- .003	2567.772...	5.547	.056	
1988.553...	2.624	+ .025	2573.749...	5.458	.032	2 sets	
1990.537...	4.608	.032	2580.640...	0.217	.030	
568...	4.639	.035	651...	0.228	.034	
2307.493...	0.053	- .100	671...	0.248	.038	
511...	0.071	- .095	2583.754...	3.331	.035	
535...	0.095	- .081	769...	3.346	.033	
547...	0.107	- .070	2584.719...	4.206	.038	
586...	0.146	- .023	735...	4.312	.021	
599...	0.159	+ .013	2585.766...	5.343	.025	
621...	0.181	.012	785...	5.362	.023	
637...	0.197	.032	2586.578...	0.089	- .105	
2313.494...	-0.012	- .088	622...	0.133	- .016	
501...	-0.005	- .083	662...	0.173	+ .014	
542...	+0.036	- .083	710...	0.221	.040	
551...	0.045	- .095	764...	0.275	.030	
2517.792...	4.096	+ .015	798...	0.309	.021	
2524.828...	5.065	.036	2589.659...	3.170	.033	
2525.730...	-0.099	- .091	2592.578...	0.023	- .076	
744...	-0.085	- .105	593...	0.038	- .092	
785...	-0.044	- .107	636...	0.081	- .097	
797...	-0.032	- .103	706...	0.151	- .032	
814...	-0.015	- .102	728...	0.173	+ .023	
825...	-0.004	- .116	739...	0.184	.023	
836...	+0.007	- .085	2593.692...	1.137	.057	
853...	0.024	- .094	708...	1.153	.037	
876...	0.047	- .100	2595.685...	3.130	.032	

TABLE I—Continued

Date, G.M.T.	Phase	r-r H.	Remarks	Date, G.M.T.	Phase	r-r H.	Remarks
2422595.722...	3 ^d 167	+0 ^M 040	2422613.737...	2 ^d 983	+0 ^M 027
2596.646...	4.091	.053740...	2.986	.043	σ
.657...	4.102	.039	2614.572	3.818	.034
2598.672...	0.050	— .114577...	3.823	.000	σ
.700...	0.078	— .084	2616.578...	—0.243	.029
2599.683...	1.061	+ .028582...	—0.239	.030	σ
.694...	1.072	.036601...	—0.220	.028
2601.685...	3.063	.034604...	—0.217	.031	σ
.698...	3.076	.031624...	—0.197	.025
2603.634...	5.012	.038626...	—0.195	.038	σ
.644...	5.022	.036647...	—0.174	.013
2604.537...	—0.151	.002650...	—0.171	.006	σ
.551...	—0.137	— .022685...	—0.136	— .029
.564...	—0.124	— .052687...	—0.134	— .031	σ
.576...	—0.112	— .070704...	—0.117	— .051
.596...	—0.092	— .074708...	—0.113	— .073	σ
.608...	—0.080	— .096725...	—0.096	— .084
.658...	—0.030	— .085729...	—0.092	— .088	σ
.669...	—0.019	— .088753...	—0.068	— .092
.691...	+0.003	— .119758...	—0.063	— .093	σ
.701...	0.013	— .103776...	—0.045	— .085
.722...	0.034	— .105781...	—0.040	— .100	σ
.733...	0.045	— .088	2618.640...	+1.819	+ .013
.753...	0.065	— .120644...	1.823	.030	σ
.766...	0.078	— .111	2619.578...	2.757	.051
2605.621...	0.933	+ .028582...	2.761	.047	σ
.630...	0.942	.040	2626.579...	3.692	.020	2 sets
2606.709...	2.021	.039583...	3.696	— .007	σ, 2 sets
.723...	2.035	.036599...	3.712	+ .001	2 sets
2607.628...	2.940	.043606...	3.719	— .005	σ, 2 sets
.644...	2.956	.039	2627.586...	4.699	+ .052
2608.524...	3.836	.001590...	4.703	.045	σ
.563...	3.875	— .008	2628.592...	5.705	.032	2 sets
.572...	3.884	— .028596...	5.709	.029	σ, 2 sets
.610...	3.922	.000	2632.592...	3.639	.032
.619...	3.931	— .013598...	3.645	.013	σ
.645...	3.957	+ .008	2633.600...	4.647	.062
.654...	3.966	.019603...	4.650	.047	σ
.667...	3.979	.013	2640.693...	5.674	.019
.678...	3.990	.018698...	5.679	.045	σ
.694...	4.006	.023	(1, σ)	2641.563...	0.477	.054
.708...	4.020	.032567...	0.481	.042	σ
.718...	4.030	.021	2642.558...	1.472	.047
.727...	4.039	.039562...	1.476	.051	σ
.751...	4.063	.027	2647.541...	0.389	.058
.753...	4.065	.045	σ	.547...	0.395	.031	σ
.770...	4.082	.032	2648.615...	1.463	.019
.773...	4.085	.042	σ	.620...	1.468	.063	σ
.795...	4.107	.042	2656.637...	3.417	.009
.799...	4.111	.048	σ	.642...	3.422	.047	σ
2609.608...	4.920	.030	2699.521...	3.836	.012
.612...	4.924	.048	σ	.524...	3.839	.000	σ
2610.723...	—0.031	— .092551...	3.866	— .005
.730...	—0.024	— .118	σ	.556...	3.871	+ .015	σ
.754...	0.000	— .097578...	3.893	— .006
.765...	+0.011	— .103	σ	.582...	3.897	+ .037	σ
.781...	0.027	— .091601...	3.916	— .001
.785...	0.031	— .092	σ	.604...	3.919	+ .027	σ

The normal magnitudes in Table II were formed in the usual way, each normal being ordinarily the mean of three observations from Table I. The residuals give a comparison with the computed

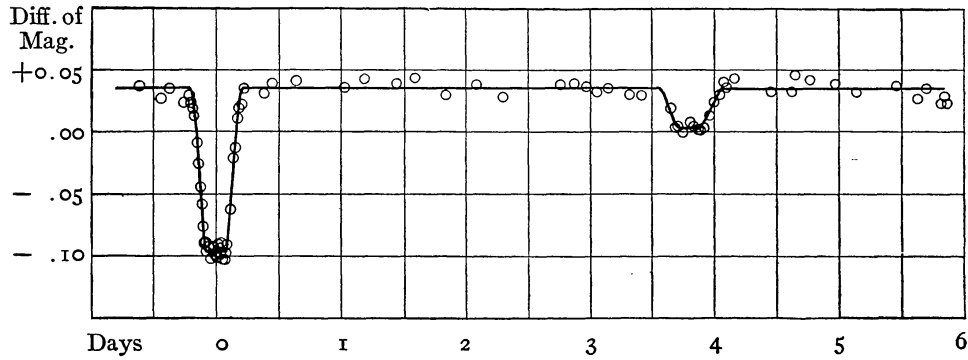


FIG. 1.—The light-curve of 1 H. Cassiopeiae

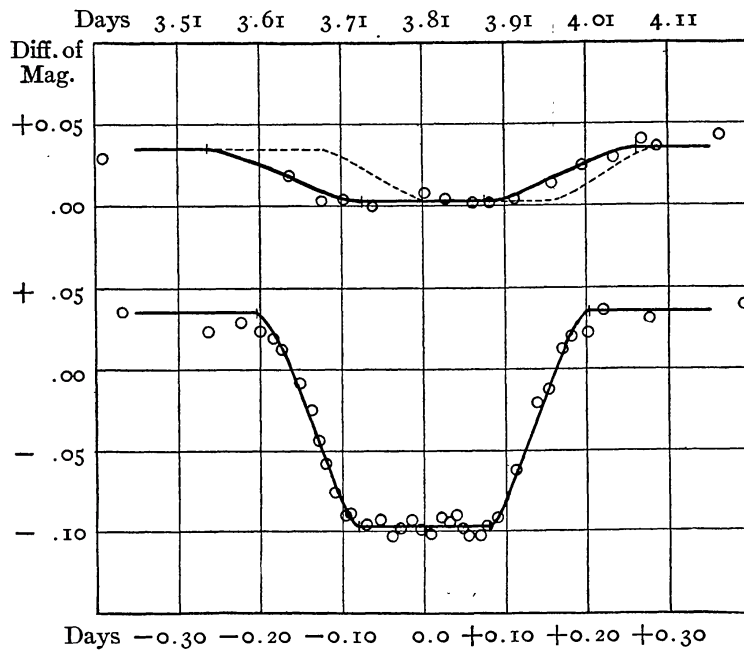


FIG. 2.—Primary and secondary minima of 1 H. Cassiopeiae. Broken line for secondary is computed from spectroscopic elements.

light-curve which is shown in Figures 1 and 2. From all the residuals is derived

$$\text{Probable error of one normal magnitude} = \pm 0.0036.$$

The constant light at each minimum shows that each eclipse must be either annular or total. As there is little variation between minima the bodies are presumably too far apart to show the effects of radiation and ellipticity of figure which are often observed in

TABLE II
NORMAL MAGNITUDES

Phase	$r-i$ H.	Residual	Phase	$r-i$ H.	Residual
-0.263.....	+0 ^M .023	-0 ^M .012	1.192.....	+0 ^M .043	+0 ^M .008
-0.222.....	.028	- .007	1.438.....	.039	+ .004
-0.199.....	.023	- .010	1.588.....	.043	+ .008
-0.184.....	.018	- .005	1.837.....	.029	- .006
-0.172.....	.012	+ .001	2.066.....	.037	+ .002
-0.151.....	- .009	+ .005	2.548.....	.027	- .008
-0.138.....	- .026	+ .005	2.765.....	.038	+ .003
-0.129.....	- .044	.000	2.876.....	.039	+ .004
-0.120.....	- .058	- .002	2.975.....	.036	+ .001
-0.109.....	- .076	- .005	3.056.....	.031	- .004
-0.096.....	- .090	- .005	3.156.....	.035	.000
-0.090.....	- .089	+ .001	3.327.....	.029	- .006
-0.071.....	- .096	+ .001	3.420.....	.028	- .007
-0.054.....	- .093	+ .004	3.646.....	.018	+ .001
-0.039.....	- .103	- .006	3.685.....	.003	- .005
-0.028.....	- .098	- .001	3.712.....	.004	+ .001
-0.015.....	- .093	+ .004	3.748.....	- .001	- .004
-0.003.....	- .099	- .002	3.812.....	+ .007	+ .004
+0.007.....	- .102	- .005	3.837.....	.004	+ .001
0.020.....	- .091	+ .006	3.871.....	.001	- .002
0.031.....	- .094	+ .003	3.891.....	.001	- .002
0.040.....	- .090	+ .007	3.922.....	.003	- .003
0.047.....	- .098	- .001	3.967.....	.013	- .002
0.054.....	- .103	- .006	4.005.....	.024	.000
0.070.....	- .103	- .006	4.044.....	.029	- .003
0.079.....	- .097	.000	4.078.....	.040	+ .005
0.090.....	- .092	- .002	4.096.....	.036	+ .001
0.114.....	- .063	+ .002	4.171.....	.043	+ .008
0.139.....	- .022	+ .008	4.470.....	.032	- .003
0.154.....	- .013	- .003	4.629.....	.032	- .003
0.171.....	+ .011	+ .001	4.649.....	.046	+ .011
0.182.....	.019	- .003	4.774.....	.042	+ .007
0.202.....	.022	- .013	4.978.....	.039	+ .004
0.222.....	.035	.000	5.143.....	.032	- .003
0.277.....	.030	- .005	5.456.....	.037	+ .002
0.391.....	.039	+ .004	5.627.....	.026	- .009
0.630.....	.041	+ .006	5.698.....	.035	.000
1.025.....	.035	.000			

close binaries. From the observations between minima we have the constants in Table III. It is to be remembered that these constants are the actual observed quantities, and include the light of the visual component B, which is assumed to be 4.4 magnitudes

fainter than A, and hence gives 1/60 of the light of the eclipsing system. In the further work this extra light has been duly allowed for.

Preliminary circular elements were found by Russell's method, and then the transition to an elliptical orbit was accomplished in

TABLE III

	Magnitude	Range	Light	Loss
Maximum.....	+0 ^M .035	1.000
Primary minimum.....	-0.097	0 ^M .132	0.886	0.114
Secondary minimum.....	+0.003	0.032	0.971	0.029

TABLE IV

ELEMENTS OF 1 H. CASSIOPEIAE

		Photometric	Spectroscopic
Minimum, J.D. 242.....	8224.822	8224.798
Period.....	<i>P</i>	6 ^d 06630	6 ^d 067±0 ^d 0005
Phase of secondary minimum.....	3 ^d 81	3 ^d 89
Time of periastron.....	<i>T</i>	-0 ^d 5454	-1 ^d 036
Longitude of periastron.....	<i>w</i>	37°25	3°35±8°17
Eccentricity.....	<i>e</i>	0.25	0.224±0.025
Compound of eccentricity.....	<i>e cos w</i>	0.199	0.224
Ratio of radii of bodies.....	<i>k</i>	0.346
Light of first body.....	<i>L</i> ₁	0.970
Light of second body.....	<i>L</i> ₂	0.030
Ratio of surface brightnesses.....	<i>J</i> ₁ / <i>J</i> ₂	3.9
Radius of first body.....	<i>a</i> ₁	0.192
Radius of second body.....	<i>a</i> ₂	0.066
Cosine of inclination.....	cos <i>i</i>	0.096
Mean density of system.....	<i>ρ</i> ₀	0.049

Assumption, $m_1 + m_2 = 10 \odot$

		Sun=1	
Mass of first body.....	<i>m</i> ₁	7.7
Mass of second body.....	<i>m</i> ₂	2.3
Radius of first body.....	<i>a</i> ₁	5.8
Radius of second body.....	<i>a</i> ₂	2.0
Density of first body.....	<i>ρ</i> ₁	0.039
Density of second body.....	<i>ρ</i> ₂	0.28

the manner indicated by him, except that the square of the eccentricity, $e=0.25$, may not be neglected. However, only a couple of trials were necessary to secure a satisfactory approximation. Complete elements were derived from the light-curve so as to

have an independent set for comparison with the spectroscopic results.

From the light measures alone there are two solutions, one with a small faint star, $k=0.346$, in front at primary minimum, and the other with much greater difference in size, $k=0.18$, and with total eclipse of the smaller and brighter star at primary. The greatest difference between the corresponding light-curves is only $0^m.003$, and we must depend upon the spectroscopic evidence for a decision. This is quite definite, for there is only one spectrum visible on the plates, and it is that of the component which is eclipsed at primary minimum, when only one-ninth of the combined light is lost. The second solution is therefore impossible, and we have a system in which the large star is the more intense,

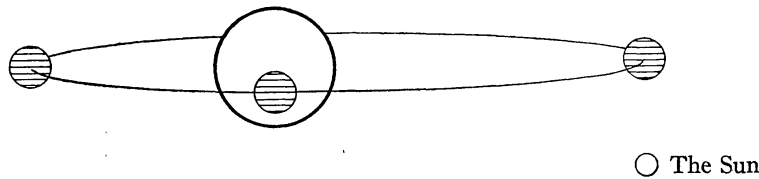


FIG. 3.—The system of 1 H. Cassiopeiae

while the companion has about one-third the diameter and one-fourth the surface-brightness of the main body. The photometric elements, together with a comparison with the corresponding spectroscopic results, are in Table IV.

The light elements in Table IV are represented by the system in Figure 3, and the final light-curve was computed rigorously from the positions in the elliptical orbit and the geometrical relations of overlapping disks.

In comparing the photometric and spectroscopic results we must remember that in the nature of the case the two methods are of different relative advantage, depending upon which element is concerned. Taking up first the time of minimum, the difference of 0.024 day corresponds to less than 2 km/sec. in the velocity-curve, a very good agreement. The period is much the better determined by the light measures, as the change during increasing or decreasing light at primary is quite rapid. In the phase of secondary minimum there is a large discrepancy, shown in Figure 3, and it is quite impossible to bring the two results into agreement.

The time of periastron T depends upon the adopted value of ω , and need not be discussed further.

The elements e and ω are fixed by the phase and duration of the secondary minimum; in particular $e \cos \omega$ is very accurately determined by the light-curve, and the photometric result is here of much greater weight, though it differs from the spectroscopic value by just the probable error of the latter. For the other component, $e \sin \omega$, we should expect the spectroscopic measure

TABLE V
LIGHT-CURVE

Phase	I-I H.	Phase	I-I H.
± 0.00	-0^M097	3^d62	$+0^M023$
± 0.080	$- .097$	3.66	$.014$
± 0.10	$- .082$	3.70	$.006$
± 0.12	$- .056$	3.734	$.003$
± 0.14	$- .029$	3.884	$.003$
± 0.16	$- .003$	3.92	$.006$
± 0.18	$+ .020$	3.96	$.014$
± 0.20	$.034$	4.00	$.023$
± 0.204	$.035$	4.04	$.031$
$+3.546$	$.035$	4.073	$.035$
3.58	$.031$	5.986	$.035$

to be superior, but for $\omega = 3^\circ 35$ the secondary minimum should be almost of the same duration as the primary, whereas this seems quite impossible in the light-curve. I have computed a velocity-curve using the photometric values $e = 0.25$, $\omega = 37^\circ$, together with the semi-amplitude $K = 58.5$ km/sec. found by Baker. This curve differs on the average from his by about 5 km/sec. and increases the sum of the squares of the residuals to about threefold the value in the original. Consequently, unless there is some unknown source of error in the velocity measures this alternative orbit must be ruled out.

We therefore seem forced to the conclusion which perhaps we should have assumed as true in the first place, namely, that the line of apsides is rotating. This explanation has been overworked to account for changes in the periods of other variable stars like Algol, but, as has been said, it would be more surprising if the major axis of the orbit were stationary than if it were in motion. Fortunately there is no particular difficulty in the case of this star.

The eccentricity of 0.25 is the largest of any eclipsing system that has come to my attention, and a new series of spectrograms should easily determine whether or not there is any change in the orbit.

The remaining elements of Table IV down to the masses were found in the usual way. The spectroscopic mass-function,

$$\frac{m_2^3 \sin^3 i}{(m_1 + m_2)^2} = 0.12,$$

combined with a reasonable assumption for the total mass, $m_1 + m_2$, gives the actual masses and dimensions of the system. It is well known that the average star of spectrum B₃ is more massive than the sun. Russell¹ found for the total mass of systems where the brighter component is of spectrum B₀–B₅ the following averages:

	Number of Stars	Mass
Spectroscopic binaries.....	13	17.5☉
Physical pairs.....	8	10.4☉
From parallactic motions.....	36	7.1☉

He made an allowance for the probable average inclination of the orbits. In Ludendorff's list² of spectroscopic binaries with two measurable components, nine stars give

$$(m_1 + m_2) \sin^3 i = 10.5 \text{ } \odot .$$

It therefore does not seem much of an exaggeration to assume that the system of γ H. Cassiopeiae has the round figure of ten times the mass of the sun. If it be objected that there is an observational preference for spectroscopic binaries of large velocity-variation and hence of large mass, it is still true that γ H. Cassiopeiae is simply one of these stars for which there is such preference. In any event, since the hypothetical dimensions vary as the cube root of the assumed mass, we have a good approximation as to the scale of this system.

It is to be noted that the companion is much more dense than the primary; in fact, the formerly used assumption of equal density for the components of a close binary would give with the spectroscopic mass-function a total for the system of nearly two thousand times the mass of the sun.

¹ *Publications of the American Astronomical Society*, 3, 327, 1917.

² *Astronomische Nachrichten*, 211, 105, 1920.

DARKENING AT THE LIMB

The observations furnish a very good test of the darkening at the limb of the brighter star. It is evident from Figure 2 that there is no measurable variation during the annular eclipse at primary minimum, whereas by close approximation it is found that for a completely darkened star the difference would be $0^m.033$ between internal contact and conjunction. For the sun Abbot gives the intensity at the edge about one-fourth that at the center, measured in light of 4560 Å, which is close to the maximum sensitivity of the potassium cell. For a star of this degree of darkening the variation during the annular eclipse would be $0^m.026$. Either of these amounts of variation should be easily detected, as there are fourteen normal magnitudes during the constant light, with a probable error of one normal equal to $\pm 0^m.0023$, or more than ten times smaller than the quantity to be observed.

It seems therefore that there is no evidence whatever of darkening at the limb in this star of spectrum class B₃. Since the darkening effect is conspicuous for the sun, it seems plausible from this result and from other considerations to expect that there is progressive darkening in the spectral types, ranging from almost nothing in the B stars to extreme effects in classes K and M. For a number of years I have been trying to find an eclipsing red star, but as is well known there are very few spectroscopic binaries of short period and "late" types of spectrum. Somewhere in space there must be such a star which as viewed from the earth has a companion which will conveniently move across the disk of the primary, but the probability of the early discovery of such a case by present methods seems small indeed.

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