APPLICATION OF MICHELSON'S INTERFEROMETER METHOD TO THE MEASUREMENT OF CLOSE DOUBLE STARS[1]

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ABSTRACT

Michelson interference method of measuring close double stars.—It is surprising that this method of accurately measuring angles as minute even as half the limit of resolution of the telescope used has heretofore been applied only to Jupiter's satellites. But after Michelson found by tests last year at the Yerkes Observatory and at Mount Wilson that clear and relatively steady fringes can be obtained even when the "seeing" is bad, steps were taken to apply the method to the measurement of the separation of the components of Capella. The *arrangement of apparatus* adopted is described in full. The apertures were placed near the focus instead of at the objective and set at a fixed distance apart, somewhat greater than the distance for minimum visibility, and were then rotated and the four position angles which gave minimum visibility were determined. After the *effective wave-length* for a G-type star had been found by laboratory experiments with sunlight to be about 0.550 μ, the position readings gave both the angular separation of the components and the direction of the line joining them. The *effect of atmospheric dispersion* is in general to shift the center of the system. Two methods of compensating for this effect are suggested. A method of determining the *relative brightness of components* when the ratio is not more than 1.5 is described and illustrated.

Accuracy.—In the case of Capella, the separation, about 0."05, was determined within 1 per cent. As to the *limits of applicability of the 100-inch reflector* in such measurements, the theoretical resolution limit with the interferometer is 0."025, and double stars down to about the eleventh magnitude should be measurable under ordinary observing conditions.

Elements for Capella as determined by the Michelson interference method.—Observations made on six dates, December 1919 to April 1920, give the following results: a=0."05249; T=J.D. 2422387.9; i=140°30′. These combined with spectroscopic elements give: P=104.006 days; a_1+a_2=130,924,000 km; $m_1=4.62\odot$; $m_2=3.65\odot$.

Interference method of measuring atmospheric dispersion, by its effect on the fringe systems of stars at various zenith distances, is suggested by the author as likely to prove very sensitive.

The astronomical applications of the interferometer have been discussed by Professor Michelson in a number of papers.[2] He

[1] *Contributions from the Mount Wilson Observatory*, No. 185.

[2] *Philosophical Magazine* (5), **30**, 1, 1890; (5), **31**, 338, 1891; (5), **34**, 280, 1892; *American Journal of Science* (3), **39**, 115, 1890.

explains how the diameters of stars can be measured, considered as uniformly luminous disks; and, in case there is darkening toward the limb, how the amount of darkening can be determined. Further, if a celestial object is not circular in apparent shape, he explains how its exact shape may be found. Special cases, such as double stars, are fully treated.

In view of the great beauty and simplicity of the method, it is rather surprising to find that the only application it has had up to the present time is to the determination of the diameters of Jupiter's satellites, and this was done by Professor Michelson himself. It is possible that astronomers who in general are so much troubled by the phenomena of "bad seeing" have had a feeling that an instrument so extraordinarily sensitive as the interferometer is known to be could hardly, if ever, be used, especially with the larger telescopes.

On September 18, 1919, Professor Michelson made a final test of this point by applying the interferometer to the 60-inch and 100-inch telescopes of the Mount Wilson Observatory, and found that the interference fringes were easily observed, although the seeing at the time was rated about 2 on a scale of 10. He had on August 25 found similar results with the 40-inch refractor of the Yerkes Observatory. Accordingly, it was decided to give the method a trial with the 100-inch reflector, and Mr. Hale requested the writer to undertake the experiments described below.

As it appeared probable that an aperture much larger than 100 inches would be required for effective work in measuring stellar diameters, it was decided to apply the method to the measurement of some double star, if possible, to one too close to be measured with the usual method. Capella was selected, because an estimate of the separation of its components, based on knowledge of its spectroscopic orbit and parallax, places this around $\frac{1}{20}$ second of arc, which should be easy to measure with the interferometer applied to the 100-inch telescope.

A preliminary observation (by Mr. Pease and the writer) was made on December 30, 1919, the chief object of which was to learn just how the interferometer should be constructed in order that it might be suited to the measurement of double stars. This observation gave the distance between the components of Capella with a probable error of about 1 per cent; a reading of the position angle was also made, but it was so very rough that it might easily be in error by 10° either way. Regular observations with the improved form of apparatus were made on February 13, 14, and 15, March 15, and April 23, 1920.

As shown in the diagram (Fig. 1) the interferometer consists simply of a plate (A) having two apertures in it, placed in the

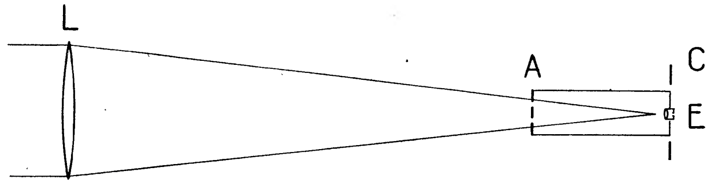

FIG. 1

converging beam of light coming from the telescope objective or mirror. The interference fringes formed in the focal plane are viewed with a high-power eyepiece E. The entire interferometer can be rotated about the telescope axis so as to vary the position of the line joining the centers of the two apertures; and the apertures themselves are so arranged that their distance apart can be varied, the actual separation being read on a scale at the eye end of the instrument, where the circle is also located from which the position angle of the apertures is read. The plate carrying the apertures can also be moved entirely out of the beam of light to facilitate the accurate centering of the star to be observed.

The method of making the measurements differs slightly from that described by Professor Michelson,[1] and will therefore be described in sufficient detail to be clear even without reference to any previous work.

Let the light from a star fall upon two apertures placed (for simplicity) in front of the telescope objective or mirror. Let the width of each aperture measured along the line joining their centers be d; let the distance between their centers be D. The

[1] *Philosophical Magazine* (5), **30**, 1, 1890.

shape of the diffraction pattern seen in the focal plane of the telescope will depend upon that of the apertures; but we are here concerned only with its dimension in the direction of the line joining the two apertures, and this, in angular measure, as seen from a distance equal to the equivalent focal length of the telescope, is $\alpha = 2C\lambda/d$. The intensity, being, say, unity at the center of the pattern, falls to the first zero value at an angle $\alpha/2$ from this point. C is a factor depending on the shape of the apertures. For rectangular slits, $C=1$; for circular apertures, $C=1.22$ nearly. Upon the diffraction pattern will appear the interference fringes, these being at right angles to the line joining the two apertures. The angular distance between two bright fringes is λ/D. Hence the number of fringes which can be seen on the central diffraction disk depends on the ratio D/d, and is equal to $2CD/d$. If this number is greater than 10, the fringes farthest from the center will, in general, be invisible in white light, because of the overlapping of the different colors.

Now assume that this arrangement is pointed at a double star, the angular separation of its components being β. If β is larger than $2C\lambda/d$, two separate diffraction patterns will be seen, each with its own system of interference fringes. When β is less than $2C\lambda/d$ the patterns will overlap more or less; and if β is just equal to $\lambda/2D$ and the position angle of the double star is the same as that of the line joining the two apertures, the conditions are such that a bright fringe due to one component falls on a dark fringe due to the other component, or we may say that the two fringe systems are out of step by just one-half a fringe; and hence, if the two component stars are of the same intensity the visibility of the fringes near the center of the pattern will be zero. (It is, of course, clear that minima of visibility will occur for $\beta = N\lambda/2D$, where N is any odd integer.) Hence we may say that the interferometer "resolves" two stars whose angular separation is $\lambda/2D$, just as a circular telescope objective of diameter D will resolve two stars whose separation is $1.22\lambda/D$; that is, the resolving power of the interferometer is somewhat more than twice that of a telescope of the same aperture. It should also be borne in mind that useful measurements may be made with the interferometer even when the angular separation is much less than $\lambda/2D$, as will be discussed more fully below.

Let D_0 denote the smallest value of D which will cause the fringes to disappear for a double star having equal components. If we choose D a little larger than D_0, so that $D_0 = D \cos \theta$, it is evident that the fringes will be visible when the position angle of the apertures is the same as that of the double star. If now the interferometer be rotated through an angle $\pm\theta$, the fringes will just disappear; and the same thing will happen when the instrument is rotated through an angle $180° \pm \theta$. For any value of D greater than D_0 there are, therefore, four values of the position angle, for each of which the fringes disappear, or have minimum visibility, according as the two components of the double star are of equal or of unequal intensity. From these four position angles and the known value D one can obviously find the value of D_0 and the position angle of the double star. There will, of course, be two possible position angles differing by 180°, but, as will be explained presently, unless the two components have exactly the same intensity and the same color, even this uncertainty may be removed.

The method just described was employed in the present work on Capella. As a rule, a complete observation included three complete rotations of the interferometer for each of three values of D, making a total of thirty-six readings of position angle. The values of D were so chosen that θ, as defined above, should lie between 30° and 50°. Under these conditions and with reasonably good seeing the probable error of a single reading should not exceed 3°. It is the author's opinion that with a little practice a good observer will be able to reduce the probable error of a single setting to about 1°. The corresponding error in the distance is about 1.8 per cent. The time required for a complete observation was about one hour, but it is reasonable to expect that when one becomes accustomed to observations of this kind no more than fifteen minutes will be required.

Given a suitable arrangement for measuring the "visibility" of the fringes, the following method of observation may be used. Choose a value of D smaller than D_0. Determine the visibility at position angles differing from each other by, say, 15° all the way around the circle. If the object is a double star, the visibility will show two maxima and two minima in a revolution. (This will also be true if the object has the form of a luminous surface longer in one dimension than at right angles thereto. Further measures will, however, readily distinguish between this case and that of a double star.) Repeat twice, using two other values of D. The data thus obtained should be sufficient to determine both position angle and distance of the double star, with a high degree of accuracy, and without requiring a value of D as large as D_0.

Having found D_0 as explained above, we need to know only λ in order to compute β, since $\beta = \lambda/2\, D_0$. The value of λ for the sun was found from laboratory measures, using as an artificial double star two small round holes illuminated by sunlight reflected from freshly silvered mirrors. The constants of the apparatus were determined by direct measurements, and also by observations on the artificial double star illuminated by very nearly monochromatic light of known wave-length. The results from two series of observations with sunlight were 5498 A and 5500 A. It seems safe, therefore, to use for a G-type star $\lambda = 0.0000550$ cm, and this value was employed in reducing the observations of Capella.

In this connection it is important to bear in mind the rôle played by the background on which the interference pattern is observed. On April 23 an observation of Capella was made in full daylight. The observation was very easy to make, but on being reduced, using the value of λ given above, the distance between the components came out approximately 10 per cent too small. A little consideration shows that this might have been predicted, for the skylight, being relatively very rich in blue light, would reduce the visibility of the blue fringes much more than that of the yellow or red fringes, thus resulting in a considerable increase in the value of the effective wave-length.

EFFECTS OF ATMOSPHERIC DISPERSION

When observations are made at some distance from the zenith it is found that for certain position angles of the interferometer the center of the fringe system does not fall on the center of the diffraction pattern and may even be displaced to such an extent that the fringes cannot be seen at all. This happens when the line joining the apertures is perpendicular to the horizon. The cause of the phenomenon is atmospheric dispersion, as became evident from a series of observations on a star at zenith distances between 50° and 20°. The phenomenon is well known to physicists who have used interferometers. For completeness, however, the explanation will be reproduced here.

To fix ideas, let us consider a star at zenith distance 45°. As seen on the sky it is really a short spectrum, the violet end above and the red end pointing down toward the horizon. Set the interferometer at right angles to this spectrum, that is, with the line joining the apertures parallel to the horizon. The fringe system will be seen central on the diffraction pattern, but the fringes will be closer together at the upper edge than at the lower, for the former are in violet light, the latter in red light. Hence the fringe system is fan-shaped, very much as indicated in the diagram (Fig. 2). Now, if we rotate the interferometer through an angle of 90°, the central fringe will no longer be white, but colored—a spectrum whose width is equal to the length of the little stellar spectrum seen on the sky. The order of colors in this fringe is violet above, red below. The angular width of this fringe for the case we are considering ($z=45°$) is about 0.″8, and is therefore equal to about ten times the distance between two bright fringes when $D=113$ cm. Consequently, in this part of the system the fringes overlap to such an extent that the result is perfectly uniform illumination. Let us for a moment imagine the atmospheric dispersion removed, all other conditions being the same. The diffraction pattern will then show a perfectly normal fringe system, the central fringe being white, the others being spectra whose dispersion increases linearly with the distance from the central fringe. For convenience let us number the fringes, calling the central fringe 0; then +1, +2, +3, etc., upward, and −1, −2, −3, etc., downward. The order of colors in the + fringes is red above, violet below, this order being just the reverse for the − fringes. Since the dispersion

FIG. 2

increases with the order, it is evident that we can find some fringe which will have a given width, say 0″.8, and hence will be an inverted duplicate of the central fringe as seen with atmospheric dispersion. This will evidently be one of the + fringes, say the $+p$th. If we now introduce atmospheric dispersion, the $+p$th fringe will be white or very approximately so, and will accordingly be regarded as the center of the system. Indeed, the fringes $p-1$, $p-2$, etc., will be violet above, red below, while the fringes $p+1$, $p+2$, etc., will be violet below, red above, as they should be. Now if p is greater than CD/d, the new central fringe will be off the diffraction pattern, in the direction away from the horizon.

We may calculate p as a function of D and the zenith distance as follows: Let the limits of the spectrum be taken as λ 4500 and λ 6500. Let the width of the +1st fringe be w, and its distance from the 0 fringe be W. The width of the pth fringe will be pw, or since $w/W=4/11$, the width of the pth fringe is $4pW/11$. In angular measure $W=\lambda/D$ or, in seconds of arc, $W=11''.3/D$. Hence the width of the pth fringe in seconds of arc is $45''.2\, p/11\, D$ where D is, of course, in centimeters.

The mean atmospheric refraction in seconds of arc for zenith distances less than about 70° is given to a sufficient approximation by $r''=60 \tan z$. The atmospheric dispersion between the wave-length limits chosen is 0.0138 times this, according to the values given in *Bulletin of the Bureau of Standards*, **14**, No. 4, 697–740, 1919. Hence the dispersion will be given by $\delta''=0.83 \tan z =45''.2\, p/11\, D$, whence $p=0.22\, D \tan z$. Taking $D=150$ cm, $z=45°$, we have $p=33$, which indicates that with good conditions of seeing this should be a very sensitive method for the actual determination of atmospheric dispersion.

In order to make observations at all zenith distances, it is necessary to be able to compensate for this effect of atmospheric dispersion. The method thus far used is not entirely satisfactory. Two plates of plane parallel glass of exactly the same thickness are mounted, one in front of each of the apertures of the interferometer. The normals to the plane faces of these plates make an angle of about 5° with the axis of the telescope, and hence, by tilting one of the plates slightly so as to vary the angle, one of the two beams can be retarded or accelerated with respect to the other sufficiently to bring the fringe system back to the center of the diffraction disk. Unfortunately, as the interferometer is rotated, one must constantly vary the tilt of the movable plate, and this is annoying, especially with bad seeing, when the fringes are none too easy to observe anyway. A much better way, which, however, has not been tried yet, would be to place a prism with a variable angle in front of the interferometer, but mounted so as not to rotate with the latter. By maintaining its refracting edge always downward it would then only be necessary to vary the refracting angle with the zenith distance. It should be easy to arrange the mounting so that the adjustments would be made automatically by the action of gravity.

SIZE OF APERTURES AND BRIGHTNESS

The apertures which were used with the 100-inch reflector measure 18.3×27.5 cm as seen in projection on the telescope mirror. Their distance apart was varied from about 120 to 200 cm. On the average, then, $D=160$ cm, and $d=0.12\,D$. With d as small as this, the diffraction disk is, of course, quite large compared to the size of the fringe system. A relatively small amount of light is admitted by the small apertures, and this is spread over a rather large area in the focal plane; hence the intensity is low. A little consideration will show that doubling the size of the apertures will result in a 16-fold increase in the intensity of the image; consequently, if we are interested in faint stars, we must use apertures as large as possible. In the present work on Capella this question was of no importance, for with such a bright star there is plenty of light for observations with small apertures even in daytime. Sufficient data were gathered, however, both with the large telescope and from laboratory experiments, to enable us to see pretty clearly just what can be done.

With $d=0.12\,D$ it was found possible with the 100-inch reflector to observe stars down to the seventh magnitude, the seeing being usually 1 on a scale of 10. With good seeing perhaps one might reach a magnitude fainter. Laboratory experiments showed that there is practically no loss in the distinctness of the fringes when d

is as large as 0.5 D, which would increase the intensity two hundred and fifty-six times, or about six magnitudes. Hence with an instrument as large as the 100-inch, one might be able to observe stars down to the thirteenth magnitude, and there should be no difficulty in reaching the eleventh magnitude under ordinary conditions of observing. It is, of course, important to bear in mind that, as d is increased, the size of the diffraction disk decreases according to theory only if the seeing is very good. A small

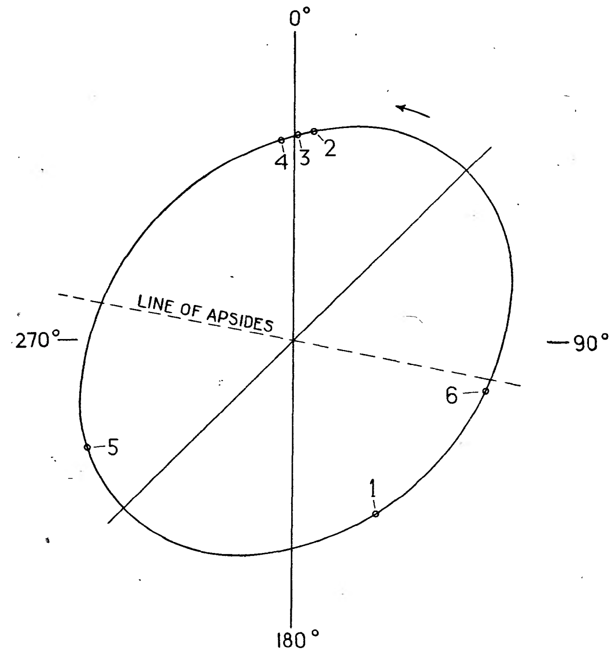

FIG. 3.—Apparent orbit of Capella

instrument will, therefore, be much more likely to reach the theoretical limiting magnitude than one as large as the 100-inch.

When d is greater than 0.3 D, so that the fringe system covers all of the diffraction disk, the normal phase-changes over the latter begin to produce an appreciable effect. If, for example, two stars of equal intensity are observed, d being greater than 0.3 D, and D being so chosen that the fringes should just disappear, one finds that although they do disappear at the center of the disk, they may still be seen at the ends of a diameter parallel to the line

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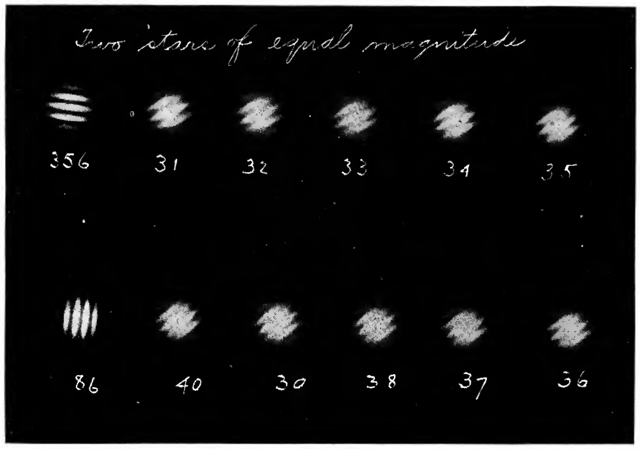
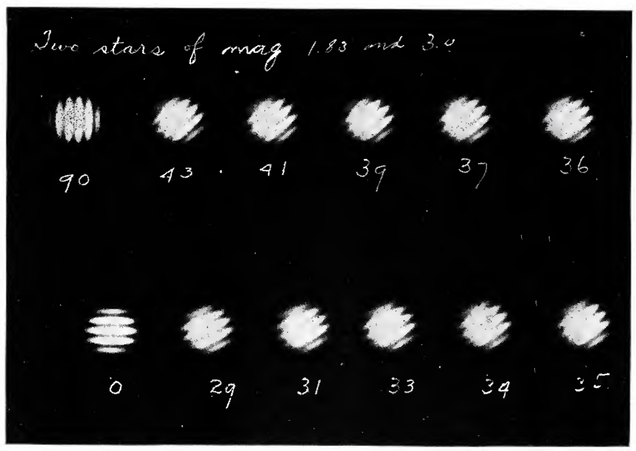
PLATE XIV

a



Position Angle of Stars 266°

b



Position Angle of Stars 270°

APPEARANCE OF INTERFERENCE SYSTEM FROM ARTIFICIAL STARS IN THE LABORATORY joining the two stars. If, say, the star on the right side is brighter than that on the left, the fringes will be visible at the center, but will disappear at some point between the center and the left side. These phenomena are beautifully shown if D is so chosen that θ is around 45°, the fringes being inclined at 45° to the line joining the two stars. Plate XIV is reproduced from two series of photographs made to illustrate this point. The position angle of the artificial double star used in Plate XIV *a* was 266°, and the two stars had the same magnitude. The number under each image shows the position angle of the interferometer when the photograph was made. Plate XIV *b* shows a similar set for two stars in position angle 270°, their magnitudes differing by 1.17. It will be observed that in each set the image made at 35° shows the fringes on the right side out of step by half a fringe with the fringes on the left side. In *a*, however, the two systems meet in a vertical line across the center of the image, while in *b* this line is near the left edge. From this it is clear that if one uses a relatively large value of d, and the seeing is good enough to enable one to see the phenomena just described, there should be no difficulty in estimating the difference in brightness of the components, provided this does not exceed say 1.5 mag., or in removing completely the uncertainty of 180° in the value of the position angle. Experience also shows that with a pattern such as shown in Plate XIV the probable error of a single setting for θ is not greater than 1°; in fact, the probable error of the mean of 10 settings seldom exceeded 0°.15. The observed co-ordinates of Capella are as follows:

RESULTS FOR CAPELLA

Observation	Date G.M.T.	Distance	Position Angle	Remarks
1.......	1919 Dec. 30.6	0″.0418	148° ±10°	Position angle only roughly estimated
2.......	1920 Feb. 13.6	0.0458	5.0	
3.......	Feb. 14.6	0.0451	1.0	
4.......	Feb. 15.6	0.0443	356.4	
5.......	Mar. 15.6	0.0505	242.0	
6.......	Apr. 23.6	....	107.0	Observed distance (0″.0402) greatly in error on account of daylight

The following spectroscopic elements for Capella are taken from Campbell's *Second Catalogue of Spectroscopic Binaries:*[1]

P	T	ω	e	$a \sin i$	$m \sin^3 i$
104.022	J.D. 2414899.5	117.3	0.016	36,847,900 km	0.19
....	....	(297.3)	....	46,430,000 km	0.94

Adding 72 P to T, we have T' = J.D. 2422389.1 as a convenient time of periastron for our calculations. The data are not well distributed and indeed are insufficient for a complete revision of the elements; but a slight change in P is, of course, allowable, and for purposes of fitting the observed and calculated positions we may regard T' as variable with limits of ± 2 days.

Let ρ and v be the radius vector and true anomaly in the orbit plane. Let r and ϕ be the radius and angle from the node in the apparent orbit. Then

$$\rho \cos (v+\omega) = r \cos \phi$$
$$\rho \sin (v+\omega) \cos i = r \sin \phi$$

Since the eccentricity is small, we may set $\rho = a(1-e \cos v)$ so that

$$r^2 = a^2(1-e \cos v)^2[\cos^2(v+\omega)+\sin^2(v+\omega) \cos^2 i]$$
$$= a^2(1-e \cos v)^2[1-\sin^2 (v+\omega) \sin^2 i]$$

Combining the fifth observation with each of the first four we can derive four values of a. With the mean value of a we can then find five values of i. The values of a and i thus obtained should agree except for errors in the observations or in the given elements. We have assumed that the given elements are all accurate except P, as indicated above. Using T' = 2422389.1 we derive:

Observation	$t-T'$	M	v	a	i
1...........	38ᵈ5	133°25	134°59	0″05334	137° 51′
2...........	85.5	289.00	287.27	.05394	132 49
3...........	84.5	292.45	290.75	.05377	134 2
4...........	85.5	295.92	294.27	.05368	134 42
5...........	10.5	36.34	37.43	....	134 42
6...........	49.5	171.32	171.60	....	....
			Mean	0.05368	

[1] *Lick Observatory Bulletin*, No. 181.

Reducing T' by 0.9 and then by 1.2 days we have:

Observation	$t-T'$	a	i	$t-T'$	a	i
1..........	39ᵈ4	0″05270	139°56′	39ᵈ7	0″05249	140°29′
2..........	84.4	.05280	138 46	84.7	.05250	140 24
3..........	85.4	.05276	139 11	85.7	.05248	140 38
4..........	86.4	.05276	139 6	86.7	.05249	140 29
5..........	11.4	....	139 14	11.7	....	140 30
	Mean	0.05275	....	Mean	0.05249	140 30

By subtracting 1.2 days from T, we arrive at a remarkable constancy in the values of a and i, and conclude that the observations to date are best represented by

$$T' = \text{J.D. } 2422387.9, \qquad a = 0''05249, \qquad i = 140°30'$$

Computing the distance and position angle with these values we have:

Observation	r (Observed)	Pos. Ang. (Observed)	r (Calc.)	Pos. Ang. (Calc.)	O−C	
					r	Pos. Ang.
1.........	0″0418	148° ±10°	0″04180	153°54′	0″00000	....
2.........	.0458	5.0	.04583	4 34	−0.00003	+0°4
3.........	.0451	1.0	.04506	1 0	+0.00004	0.0
4.........	.0443	356.4	.04430	357 18	0.00000	−0.9
5.........	.0505	242.0	.05050	242 26	0.00000	−0.4
6.........	....	107.0	0.04391	107 12	....	−0.2

The agreement is, perhaps, better than we have any reason to expect, due possibly to the fact that the number of observations is small. However, taking the results as they stand, we can substitute in the elements as given by Campbell and derive:

$$P = 104.006 \text{ days.} \quad a_1 + a_2 = 130{,}924{,}000 \text{ km.}$$
$$\pi = 0''0600; \quad m_1 = 4.62\odot; \quad m_2 = 3.65\odot.$$

It remains to be seen whether recent spectroscopic observations are in agreement with this new value of the period.

Mount Wilson Observatory
June 1920