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## ON THE APPLICATION OF INTERFERENCE METHODS TO ASTRONOMICAL MEASUREMENTS[1]

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ABSTRACT

*An interference method of measuring extremely small angles and changes of angle.*—Thirty years ago the author called attention to the possibility of measuring such minute angles as the *diameters of planetoids and satellites* and the *distance between the components of double stars* by observing the interference fringes produced at the focus of a telescope when only two portions of the objective, located on the same diameter, are used; for it was shown that as the distance apart of the apertures is increased the visibility of the fringes reaches a minimum for a distance equal to $1.22\ \lambda/\alpha$, or $0.5\ \lambda/\alpha$, for a disk or double star respectively, when $\lambda$ is the effective wave-length and $\alpha$ is the desired angle. This beautiful and simple method was applied successfully by the author in 1891 to the accurate measurement of the size of Jupiter's satellites but was not tested on stellar objects, probably because it was supposed to require ideal seeing conditions. Last year, however, the author discovered by tests at Yerkes Observatory and at Mount Wilson that even when the "seeing" was bad, clear and relatively steady fringes could be obtained; and Anderson, using the 100-inch reflector, has recently measured the separation of the components of Capella, 0″0545, within less than 1 per cent by the application of this method. In fact the 100-inch could measure with accuracy separations as small as 0″025. But to determine the *diameter of a fixed star*, a distance between apertures of at least 10 meters would be required; so that an interferometer arrangement would have to be substituted for the telescope. The author also suggests that by the use of prisms to superpose the fringe systems of two stars, small *relative motions* and *parallaxes* could be measured.

*Relative brightness of components of double stars* may also be determined from the relative visibility of the focal fringes at minimum and maximum visibility.

[1] *Contributions from the Mount Wilson Observatory*, No. 184.

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In the number of the *Philosophical Magazine* for July 1890 (**30**, 1) a method was described for the measurement of the angular magnitude of astronomical objects such as the diameter of planetoids and satellites and the distance between double stars, when these are beyond the powers of the largest telescopes, and the hope was there expressed that it might not be impossible thus to measure the diameter of the fixed stars.

Briefly, the process consists in utilizing only the two portions of a large objective at opposite ends of a diameter. The interference fringes at the focus under these conditions will be a series of equidistant interference bands which are most distinct with a source subtending an infinitesimal angle. For an object presenting an appreciable angle the visibility is less and may become zero—the exact relation being readily expressed for any given distribution of light in the source. Thus if $\phi(\alpha)d\alpha$ represent the intensity of a strip of the source of angular width $d\alpha$, and $s$ the distance between the apertures (supposed small compared with $s$), and if $P=\int\phi(\alpha)d\alpha$, $C=\int\phi(\alpha)d\alpha\ \cos 2\pi\frac{s}{\lambda}\alpha$, and $S=\int\phi(\alpha)d\alpha\ \sin 2\pi\frac{s}{\lambda}\alpha$, then the visibility of the interference bands is

$$V=\frac{\sqrt{C^2+S^2}}{P}\ .$$

Thus for a double star, the brightness of whose components is in the ratio $1:r$ and whose angular distance $=\alpha$,

$$V=\frac{\sqrt{1+r^2+2r\cos 2\pi\frac{s}{\lambda}\alpha}}{1+r}\ .$$

For equal components this reduces to $\cos\frac{\pi s\alpha}{\lambda}$, which vanishes for $\alpha=\frac{1}{2}\frac{\lambda}{s}$. Accordingly this angle can be accurately measured when it is only half of the limit of resolution of the full-apertured telescope.

Again, by comparing the visibility at maximum and at minimum, the ratio of the brightness of the component stars may be found by

$$r=\frac{V_1-V_2}{V_1+V_2}\ .$$

For a uniformly illuminated disk

$$V=\int_0^1 \sqrt{1+\omega^2}\cos \omega n d\omega$$

where $n=\pi\frac{s\alpha}{\lambda}$, $\alpha$ being the angular diameter. For such an object the fringes vanish for an angular diameter $\alpha=1.22\frac{\lambda}{s}$.

A series of observations was taken on the satellites of Jupiter at the Lick Observatory the following year with results which amply confirmed the practicability and accuracy of the method.

It is clear, however, that as in all probability the stars present an angular diameter less than one-hundredth of a second, it would be almost hopeless to make such measurements, utilizing the largest telescope in existence; for it would require a distance between the apertures of at least 10 meters to observe the vanishing of the fringes.[1] While such a large telescope would be entirely out of question, the interferometer arrangements figured in the article referred to may serve the purpose, as there is theoretically no limit to the effective base line and practically only that which depends on the atmospheric disturbances.

With a view to testing the effect of these, a trial was made (August 25, 1919) with the 40-inch refractor at Yerkes Observatory, using two apertures 4 inches by 5 inches at opposite ends of a diameter. The result was very encouraging, the interference bands being remarkably steady, notwithstanding the relatively poor "seeing"—2 to 3 on a scale of 5.

On invitation from Dr. George E. Hale the test was applied (September 18, 1919) to the 60-inch reflector of the Mount Wilson Observatory and then to the 100-inch reflector, and in both cases the experience at the Yerkes Observatory was confirmed.

In the case of the 60-inch telescope the apertures were applied, as in the case of the 40-inch, to the objective; while for the 100-inch, it was found quite as effective and far more convenient to use a small screen with two apertures near the eyepiece, the distance and orientation being thus much more readily controlled while the effective size and separation of the two interfering pencils remain the same.

[1] A diminution in visibility, however, might be observed with a diameter of 2 meters.

The interference fringes in both observations remained remarkably clear and steady, notwithstanding the excessive "boiling" of the highly magnified images corresponding to "seeing" 2 on a scale of 10.

Subsequent observations showed that the interference bands remain visible even when the seeing is so poor that the usual type of observation is impracticable.

A statement of results obtained by this method by Dr. J. A. Anderson of the staff of the Mount Wilson Observatory from observations of the spectroscopic binary Capella is herewith appended.

STATEMENT OF RESULTS FROM OBSERVATIONS OF CAPELLA WITH THE MICHELSON INTERFEROMETER

| Date of Observation | | | Distance | Position Angle | Remarks |
|---|---|---|---|---|---|
| December | 30.6 | 1919 | 0″.0418 | (153°.9) | P.A. calculated. Only the distance observed accurately* |
| February | 13.6 | 1920 | 0.0458 | 5.0 | |
| February | 14.6 | 1920 | 0.0451 | 1.0 | |
| February | 15.6 | 1920 | 0.0443 | 356.4 | |
| March | 15.6 | 1920 | 0.0505 | 242.0 | |
| April | 23.6 | 1920 | (0.0439) | 107.0 | Distance calculated. Only the position angle observed accurately† |

* The rough observation of position angle was 148°±10°.

† Observation made in strong daylight. The blue background shifted the effective wave-length to the red, causing the observed distance of 0″.0402 to be much too low. The background could not affect the position angle.

Using the following elements[1] for the orbit, these observations are satisfied as indicated:

$$a_1+a_2 = 0''.05249 \qquad (a_1+a_2)\sin i = 83{,}277{,}900 \text{ km}$$
$$e = 0.016$$
$$\omega = 117^\circ.3$$
$$i = 140^\circ 30'$$
Position angle of $\Omega = 45^\circ 55'$
$$T = \text{J.D. } 2{,}422{,}387.9 \qquad \text{or} \qquad P = 104.006 \text{ days}$$

[1] See orbit by H. M. Reese, *Astrophysical Journal*, **14**, 263, 1901, for values of $(a_1+a_2)\sin i$, of $e$, and of $\omega$. $a_1$ and $a_2$ are the semi-major axes; $e$, the eccentricity (same for each orbit); omega is the distance of periastron from the ascending node in the plane of the orbit; $i$ is the inclination of the orbit to the line of sight; $\Omega$ is the symbol for the ascending node and is here equivalent to the position angle on the tangential plane.

| Date | $t-T$ | $a$ | $i$ | $r$ | Position Angle |
|---|---|---|---|---|---|
| 1. December 30.6 1919 | 39$^d$.7 | 1 and 5 0″.05249 | 180°−39°31′ | 0″.04180 | 153°.9 |
| 2. February 13.6 1920 | 84.7 | 2 and 5 0.05250 | −39 36 | 0.04583 | 4.6 |
| 3. February 14.6 1920 | 85.7 | 3 and 5 0.05248 | −39 22 | 0.04506 | 1.0 |
| 4. February 15.6 1920 | 86.7 | 4 and 5 0.05249 | −39 31 | 0.04430 | 357.3 |
| 5. March 15.6 1920 | 11.7 | ............... | −39 30 | 0.05050 | 242.4 |
| 6. April 23.6 1920 | 50.7 | | | 0.04391 | 107.2 |
| | | Means 0.05249 | 180°−39 30 | | |

Parallax of Capella 0″.0600.

It appears from a comparison of these results with the orbit that an order of accuracy of one ten-thousandth of a second of arc is attainable in the case of Capella with a base line of 100 inches.

In these observations the distance between the apertures was fixed, and the vanishing of the fringes was observed at the appropriate position angle.

The complete disappearance of the interference bands showed that the component stars are of equal brightness; otherwise the visibility would have a minimum from which the ratio of the brightness may be obtained as indicated above.

For this, however, it would be necessary to measure the visibility, or at least to calibrate the eye estimates.

This may be effected by adjusting the relative width of two auxiliary apertures so that the resulting visibility of a single comparison star (or of the double star in a direction perpendicular to the plane of the components) is the same as that which is actually observed.

If the ratio of the width of the apertures is $\rho$, then when the visibilities are equal in the two systems of fringes

$$V=\frac{2}{\rho+\frac{1}{\rho}}\,.$$

Other problems which involve the comparison of two stars, such as the measurement of stellar parallax, proper motion, variation of latitude, etc., may also be undertaken by a modification of the interference method. For this two prisms or prism couples are similarly placed in front of the apertures, with the plane of refraction parallel with that passing through the two apertures and the axis. This plane is rotated until the direction coincides with that of the stars to be compared. The refraction by the prisms is then altered (by rotating the single prisms in the same plane, or by equal and opposite rotations of the elements of the prism couples) until the two systems of fringes are superposed. Any change in the relative position of the stars is accompanied by a corresponding alteration in the appearance of the fringes, which is then compensated by rotation of the prisms, from which the change in position may be determined.

Preparations are now under way at Mount Wilson for testing the possibilities of the interferometer method with a base line of 18 to 20 feet.

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