THE COLOR-CHANGES OF CERTAIN VARIABLE STARS OF SHORT PERIOD[1]

By F. C. JORDAN

The determination of the colors of stars has occupied an increasingly important place in astronomical investigations of recent years. They have been studied partly for the mere purpose of ascribing to each star its exact position in the color-scale; but the investigation becomes much more important when we consider the intimate connection between color and spectral type, color and temperature, and the place which colored stars occupy in the scheme of stellar evolution. Slow changes in the colors of some stars have been suspected, but not thoroughly proved.

It is well known that the long-period variable stars, all of which are reddish in color, become more strongly tinted as they decrease in brightness. As the mere decrease in brightness would make the tint appear less intense, the cause must lie in the star itself, and indicate a change in the absorption, and hence in the distribution, of energy in the spectrum. The cause of this is entirely unknown. The same phenomenon is found in short-period variables of certain types; but though there is a clue here to the cause, many points remain obscure, and much further investigation will be necessary in order to arrive at a satisfactory explanation.

By the study of star-colors combined with spectroscopic investigation we shall ultimately increase our knowledge of stellar evolution, and consequently of the development of the universe.

COLOR-DETERMINATIONS AND COLOR-SCALES

In the determination of the exact grade of color in a star various methods have been suggested and used, the most obvious one being that of eye-estimates. Various more or less fantastic names have been given to stellar tints in a general description of them for the

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use of the amateur observer. A few of these are: "red lilac," "pale gray," "flushed purple." It is probable that all star-colors are comprehended within the limits white to red, through the various shades of yellow and orange, with a possible trace of blue in some; though the latter color may usually be explained as the effect of contrast.

In indicating star-colors various scales have been suggested and used, such as the well-known numerical one ranging from 0, pure white, to 10, pure red. Müller and Kempf use the very simple method of naming the colors: white, yellowish white, yellow, and so on. Whatever the nomenclature may be, the object of all is the same, namely, that of locating the star as accurately as possible on the color-scale. None of these give place for any colors other than the various shades of yellow, orange, and red; and no others are needed.

DIFFICULTIES OF EYE-ESTIMATES

In nearly all the work which has been done in this line of research the eye has been the sole determining factor. It is true that various kinds of colorimeters have played some part in these investigations, but here also the eye is the final resort, and the result depends upon what the individual eye sees and records. In studying the results given by various observers curious anomalies are occasionally found, and such can hardly fail to be the case. Different observers have different color-perceptions; this perception may change as the observer becomes older; the same observer with larger or smaller aperture obtains varying results; stars of different brightness have a different physiological color-effect on the eye. These and other causes conspire to make the eye a rather faulty instrument in the determination of colors, and also in estimating the magnitudes of other than white stars.

It is well known that, in general, color increases with advance in spectral type; indeed, it is possible that there is no exception to this rule. But in catalogues of colored stars the order will sometimes be reversed, a star of more advanced type being credited with less color than one preceding it in the spectral scale. These anomalies are undoubtedly due to the effect of personal equation in determining the colors.

THE PHOTOGRAPHIC METHOD OF DETERMINING COLORS

Since the eye, because of its limitations as above mentioned, is unsatisfactory in this line of research, other methods have been proposed which seek to eliminate as far as possible any dependence upon the eye. It was Schwarzschild who first suggested the photographic method of determining colors: that is, the difference between the photographic and visual magnitudes of a star may be taken as the indication of its color. This effect is called by him *Farbentönung*. The method is becoming more and more extensively used in one form or another. As originally proposed, it meant merely the substitution of determinations of stellar magnitudes for the estimates of color, the visual magnitudes being found as usual by the eye, the others from the photographic plate. Thus were eliminated only partially the difficulties before mentioned in regard to colored stars, for there still remains the problem of comparing, for example, a deep yellow star with a neighboring white one, or with the artificial one of the photometer. This is always a difficult task and gives rise to decided differences in determinations of magnitude by different observers.

METHOD USED IN THIS WORK

In the present paper the Schwarzschild definition of color is used with the designation "Color-Index"; but the method of obtaining it is entirely photographic. The process is fully described in a paper by Professor J. A. Parkhurst and the writer, "The Photographic Determination of Star-Colors and Their Relation to Spectral Type."[1]

I shall give only an outline of the parts necessary for the work of this paper.

The instrument.—The telescope used was the two-foot reflector of the Yerkes Observatory. Since with the full aperture the field of good definition is very limited, it was always stopped down to an aperture of twelve inches, or a ratio of 1 to 7.8, which makes the effect of curvature of the field very much less. Even with this aperture it is necessary to make a correction for magnitude, depend-

[1] *Astrophysical Journal*, **27**, 169, 1908.

ing upon the distance of the star's image from the optical axis, as described later.

The plates.—All plates were taken in the primary focus of the instrument. For the photographic magnitudes Seed 27 plates were employed; for the photo-visual, Cramer Trichromatic and Wallace "Pan-Iso"[1] with a special color-filter constructed for this work by Mr. R. J. Wallace.[2] The Trichromatic and Pan-Iso plates with the color-filter have the same effect on colored stars and can be used interchangeably. The spectral luminosity-curves of the two, though somewhat different in shape, and also in the position of the maxima, give practically the same integrated effect, as can be seen in Figs. 1 and 2 (reprinted from this *Journal*, **27,** 171, 173, 1908). The actual working out of the results with the filter and Trichromatic plates is shown (Plate XI, *Astrophysical Journal*, **27** [opposite p. 170], 1908) in the reproduction of the photographs of the region of the intensely red star U Cygni, when its visual magnitude, as far as could be judged by the eye, was practically the same as that of its white companion. On the Seed 27 plate the difference of magnitude, or color-index, is 5.6, while on the Trichromatic plate the two images are equal. This is perhaps as severe a test as could be applied to the red-sensitive plate and filter, but the combination fully stands the test.

It is well known that Müller and Kempf are probably as accurate in their color-estimates as any other observers; therefore a comparison of their results with those obtained by the use of the visual-luminosity filter and color-sensitive plates will furnish further evidence as to the validity of the photographic method of determining star-colors. In Table IV of the paper "The Photographic Determination of Star-Colors and Their Relation to Spectral Type" will be found the comparison.[3] While in most of the individual visual groups there is a considerable range in the photographic color, they agree in general. In the comparison of spectral type and photographic color, the probable error of the mean color-index is but ±0.05 magnitude, part of this being due to determinations of color, part to errors in estimating spectral type. This shows that

[1] *Astrophysical Journal*, **26,** 299, 1907.

[2] *Ibid.*, **24,** 268, 1906.

[3] *Ibid.*, **27,** 169, 1908.

the visual-luminosity filter with properly sensitized plates is true to its name and really gives visual magnitudes.

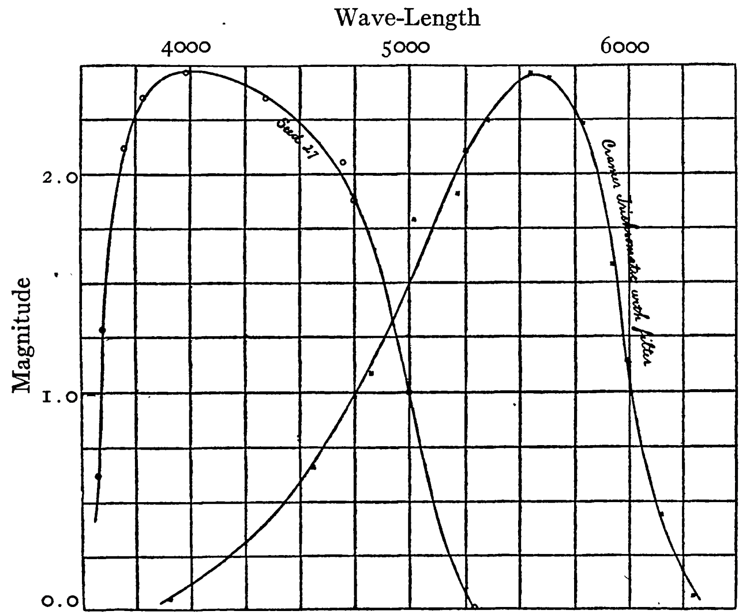

FIG. 1.—Spectral intensity-curves. Seed, and Trichromatic with filter

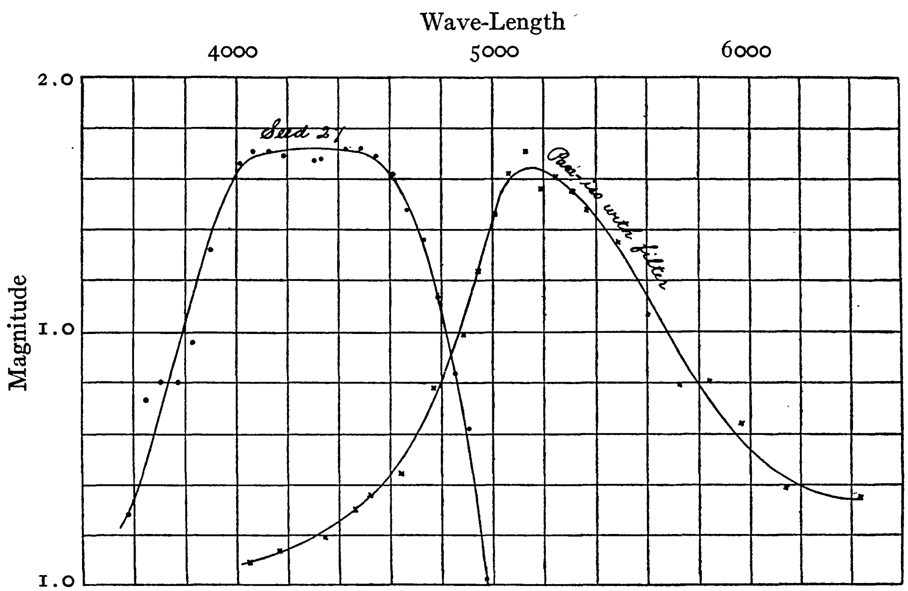

FIG. 2.—Spectral intensity-curves. Seed, and Pan-Iso with filter

Thus in the determination of color the eye is entirely eliminated as far as actual work on the stars is concerned, and in its place is substituted the measuring engine and star images, in the measurement of whose diameters the only possible source of error is that of setting the measuring wires on their more or less diffuse edges. Since for the results here given all the measuring was done by the writer, and since the images of stars of all colors are exactly similar, they would all be affected alike. Hence this would introduce no error in the derived magnitudes. The errors remaining, therefore, are the accidental ones of the plate itself, which cannot be eliminated, but which can be reduced by taking a sufficient number of plates.

THE MAGNITUDE-FORMULA

I have hitherto tacitly assumed that we have a satisfactory formula for translating star diameters into stellar magnitudes. Before going further it will be necessary to prove that this is the case. Various formulae have been suggested and used, all of them empirical, though some of them employ in one form or another the light-ratio for one magnitude, a number whose logarithm is 0.4. However, the action of light on the sensitive film and the cause of the growth of a star image with increase of exposure are so imperfectly known that it will suffice to select that formula which most nearly satisfies the results obtained with the particular instrument and plates with which the observations are made.

In the earlier work of photographic photometry with the two-foot reflector, Charlier's well-known formula, $m=a-b \log D$, was used,[1] but it was found on further investigation that this did not exactly suit, and the formula $m=a-b\sqrt{D}$ was substituted. A graphical representation of the two formulae is given by Mr. J. A. Parkhurst in the "Yerkes Actinometry,"[2] and is here reproduced (Fig. 3) by permission of Mr. Parkhurst. Although it was drawn from data furnished by the six-inch Zeiss doublet of the Yerkes Observatory the results are of the same character for the two-foot reflector.

APPLICABILITY OF THE FORMULA

I now proceed to show that the formula satisfies the observations. Suppose a group of white stars be photographed. Then plot the stars with magnitudes as abscissae and square roots of

[1] *Astrophysical Journal*, **23**, 79, 1906. [2] *Ibid.*, **36**, 185, 1912.

diameters of the images as ordinates. If these plotted points lie on a straight line the square-root formula applies. The Pleiades is a group eminently suited for this because of the spectral type of its stars and the careful determinations which have been made of their magnitudes. In pursuance of this plan a number of photographs of the group were taken with the two-foot reflector diaphragmed

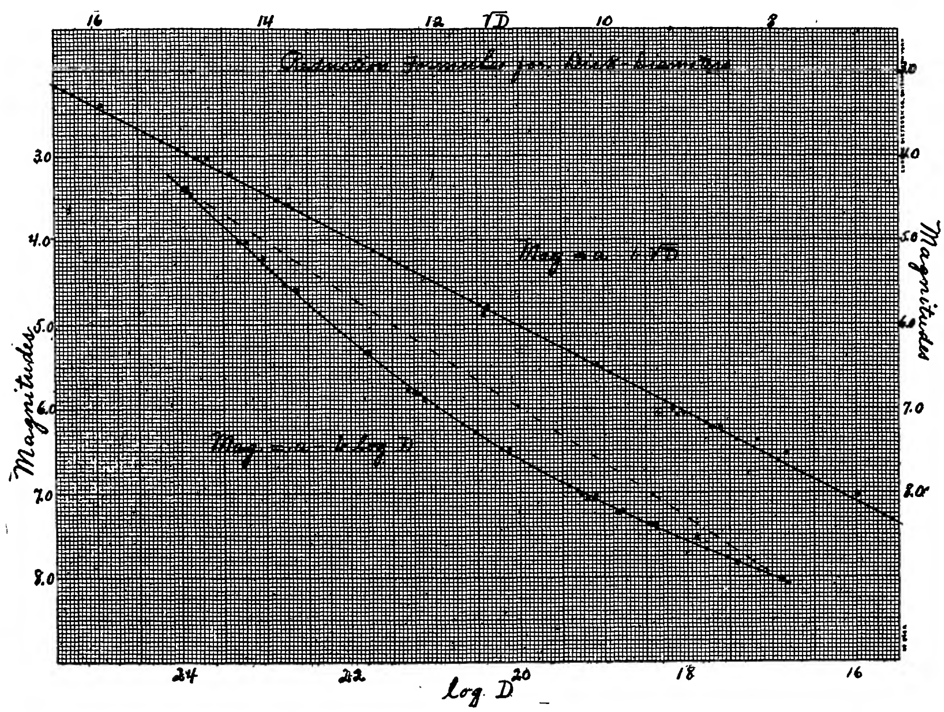

FIG. 3.—Reduction formulae for focal plates (by J. A. Parkhurst)

to twelve inches. The plates used were Seed 27, Cramer Trichromatic, and Wallace Pan-Iso. The diameters of the images were measured on a Gaertner measuring engine to 0.001 mm. The magnitudes of the stars were determined by Mr. Parkhurst by the extra-focal method, and are certainly as accurate as, or even a little better than, other determinations of magnitude in this much-studied group. The basis of the extra-focal determinations is given in "An Absolute Scale of Photographic Magnitudes of Stars."[1]

[1] *Astrophysical Journal*, **26**, 244, 1907.

To illustrate, I have selected one each of the different kinds of plates for the construction of the diagrams Figs. 4, 5, and 6, and give the data for them in Table I.

TABLE I

DATA FOR CONSTRUCTION OF MAGNITUDE-CURVES

Star	Mag.	Plate 532, Seed 27			Plate 613, Trich.			Plate 1501, Pan-Iso		
		Exp.	D	$\sqrt{D}$	Exp.	D	$\sqrt{D}$	Exp.	D	$\sqrt{D}$
		Sec.			Min.			Min.		
η	3.32	...	....	....	1	204.5	14.30	1	202.0	14.21
		...	....	....	2	229.5	15.15	2	228.5	15.12
		...	....	....	4	264.0	16.25	4	265.5	16.29
b	4.03	8	183.0	13.53	1	171.5	13.10	1	172.0	13.11
		15	200.0	14.14	2	196.5	14.02	2	197.0	14.04
		...	....	....	4	224.0	14.97	4	226.0	15.03
c	4.20	8	173.5	13.17	1	163.0	12.77	1	170.5	13.06
		15	194.5	13.95	2	188.5	13.73	2	191.5	13.84
		30	212.5	14.58	4	215.0	14.66	5	221.5	14.88
d	4.52	8	168.0	12.96	1	159.0	12.61	1	156.0	12.49
		15	189.0	13.75	2	180.0	13.42	2	179.0	13.40
		30	204.5	14.30	4	207.5	14.40	5	206.5	14.37
e	4.57	8	168.5	12.98	1	153.0	12.37	1	159.0	12.61
		15	189.0	13.75	2	181.0	13.45	2	183.5	13.55
		30	203.0	14.25	4	205.0	14.32	5	207.5	14.40
g	5.77	8	134.0	11.58	1	117.5	10.84	1	125.5	11.20
		15	150.5	12.27	2	142.5	11.94	2	150.0	12.25
		30	162.5	12.75	4	161.5	12.71	5	170.5	13.06
24	7.07	8	104.0	10.20	1	83.5	9.14	1	91.5	9.57
		15	123.0	11.09	2	107.0	10.34	2	107.0	10.34
		30	135.0	11.62	4	124.5	11.16	5	131.0	11.45
19	7.21	...	....	....	.....	....	....	1	82.5	9.08
		...	....	....	.....	....	....	2	101.0	10.05
		...	....	....	.....	....	....	5	122.5	11.07
10	7.54	8	92.5	9.62	.....	....	....	1	76.5	8.75
		15	107.0	10.34	.....	....	....	2	91.0	9.54
		30	116.0	10.77	4	112.0	10.59	5	113.5	10.65

The positions of points in Figs. 4, 5, and 6 are affected by accidental errors in the plates, errors in the assumed magnitudes, and in the measured diameters. Within the consequent allowable limits of errors the points in every case lie on a straight line, the magnitude-curve of the plate. This shows that the formula is applicable for white stars without any reference to the values of a and b, since it depends solely upon the measured diameters of the star images. The value of a varies from plate to plate and depends upon the effective exposure. In the work of this paper the value of b has been determined once for all for each kind of plate, and the fact that in each diagram the curves for the different exposures are practically parallel shows that this value does not depend upon the time of exposure. I have tested this matter for the three kinds of plates with a range of exposure-time from one to forty on any one plate,

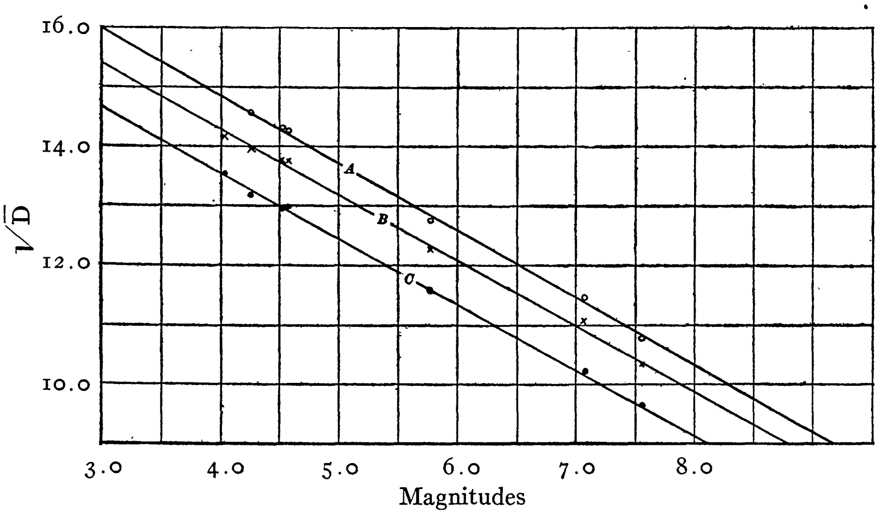

FIG. 4.—Magnitude-curve for Seed 27 plate
A, 30-second exposure
B, 15-second exposure
C, 8-second exposure

and find that even this extreme difference does not necessitate any change in the value of b. It probably varies somewhat from plate to plate because of changes in the seeing, but unless a standard field be photographed on the plate, in addition to the field containing the stars whose magnitudes are to be obtained, this change cannot be determined. This was not done for the plates whose results are to be given later; hence the value of b is considered as fixed.

METHODS OF DETERMINING THE VALUE OF b

From known magnitudes of certain Pleiades stars.—The data for the first method are given in the table and the lines drawn in the diagrams through the plotted points. Each line furnishes two equations of the form $m=a-b\sqrt{D}$. For example, in Fig. 4, for plate 532, Seed 27, the line representing the exposure of eight seconds intersects the line for magnitude 3.0 at ordinate 14.66, and for 8.0 magnitude at ordinate 9.11; hence the two equations are:

$$3.0=a-b\ (14.66)$$
$$8.0=a-b\ (9.11)$$

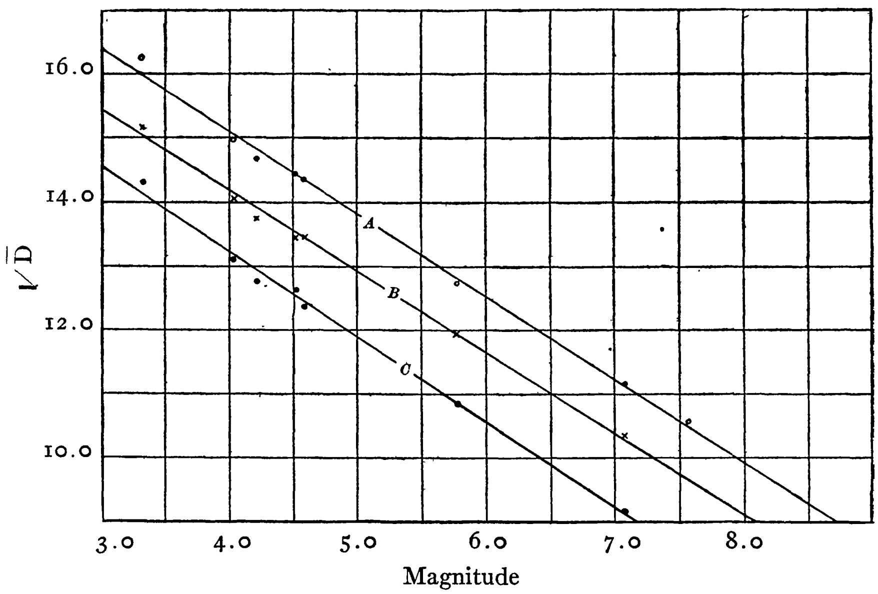

FIG. 5.—Magnitude-curve for Trichromatic plate
A, 4-minute exposure
B, 2-minute exposure
C, 1-minute exposure

The solution of these gives $b=0.901$. Similarly for the exposures of fifteen seconds and of thirty seconds the values of b are respectively 0.904 and 0.882. A number of other plates were reduced in the same way, and from the results 0.90 was adopted as the value of b for the Seed 27 plates. The probable error of this determination is ± 0.01.

The same method applied to the Trichromatic and Pan-Iso plates yields, for the former, the values 0.755, 0.790, and 0.777; and for the latter, 0.789, 0.773, and 0.794. From these and other plates the value of b adopted for both the Trichromatic and Pan-Iso plates is 0.77 ± 0.01.

A least-squares solution is applied to the observations in the following manner. Each measured diameter of an image together with the magnitude of the star gives an observation equation. From these are formed normal equations in the usual way. Below is given in detail the solution for the exposure of plate 532 of eight seconds.

$$a-b(13.53)=4.03$$
$$a-b(13.17)=4.20$$
$$a-b(12.96)=4.52$$
$$a-b(12.98)=4.57$$
$$a-b(11.58)=5.77$$
$$a-b(10.20)=7.07$$
$$a-b(9.62)=7.54$$

Normal Equations

$$7a-84.04b=+37.70$$
$$-84.04a+1023.63b=-439.20$$
$$b=0.914$$

The exposures of fifteen and of thirty seconds give the values respectively 0.881 and 0.886. Observation equations for the Trichromatic plate 613 give the values 0.743, 0.802, and 0.764 for the respective exposures. Pan-Iso plate 1501 yields in the same manner the values 0.789, 0.781, and 0.787.

The grating method.—A second and entirely independent way of determining the value of b is offered by the use of the so-called *Halb-Gitter*. In his "Plan of Selected Areas" Kapteyn suggested the use of an absorbing plate over half of the field of the camera, by which the magnitudes of the corresponding stars could be diminished by a known amount, and hence compared with the same images obtained without the absorbing medium. Schwarzschild used in place of the absorbing plate a "*gitter*" of fine wire, as described in *Astronomische Nachrichten* (**183**, 297, 1910).

In pursuance of this plan a grating was used in connection with the reflector. The data for the grating are as follows: Mean mesh, center to center, 0.125 mm; diameter of wire (b) 0.0433 mm; free mesh (a) 0.0817 mm. Placed a short distance (about 75 mm) in front of the sensitive plate, this forms a central image surrounded by four sets of diffraction images arranged at intervals of 90°. The central image is exactly similar in appearance to the ordinary star image, and can be measured with equal facility.

The reduction in magnitude with this grating is 1.878 magnitudes as determined by the photometer. Theoretically the fraction of the incident light thrown into the central image is given by the formula $\left(\frac{a}{a+b}\right)^4$. This gives a reduction in magnitude of 1.628. One advantage of this method is the fact that we need know nothing

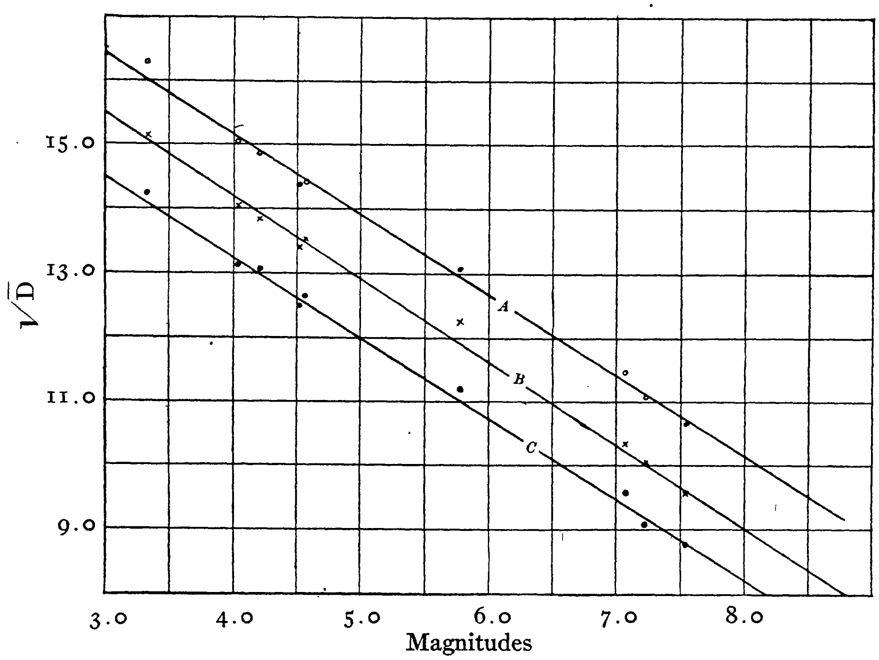

FIG. 6.—Magnitude-curve for Pan-Iso plate

A, 4-minute exposure
B, 2-minute exposure
C, 1-minute exposure

about the magnitudes of the stars used, either absolute or relative. We are concerned solely with the square roots of the measured diameters.

EXPOSURES WITH THE GRATING

Two methods of exposure were used: (1) A plate is taken of a region, say the Pleiades, without the grating; then on the same or a different plate another exposure is made with half the plate covered by the grating. The effective exposures need not be the same, and if different plates are used it is not necessary that they be obtained at the same time or even under the same conditions. (2) An exposure is made with half the plate covered by the grating. The grating is then reversed so as to cover the other half of the plate and another exposure made.

In the first method, suppose that one plate is used: we then have two groups of stars, one of them normal on both exposures, the other normal on one and through the grating on the other. Let d^1, d^2, d^3, and d^4 be the mean square roots of the measured diameters in the respective cases. Let $d^2-d^1=\Delta^1$, $d^4-d^3=\Delta^2$, then $\Delta^1-\Delta^2$ is the change produced by the grating in the square-root factor; therefore $\frac{1.878}{\Delta^1-\Delta^2}$ is the value of b.

Using the same notation in the second method, $\Delta^1+\Delta^2$ is double the absorption effect of the grating. Table II gives the details of the measurement of two plates.

Other plates taken with the grating lead to an average value for b of about 0.90. This constant has also been determined in both ways by Dr. C. H. Gingrich, who finds the values 0.90 and 0.91 from given magnitudes and from exposures with grating, respectively.[1] In the work which follows I have therefore assumed 0.90 as the definitive value of b for Seed 27 plates. Trichromatic plates and filter were also tested with the grating, giving results such that 0.77 is adopted as the definitive value of b for these plates.

APPLICATION TO SHORT-PERIOD VARIABLE STARS

The magnitude-formula having been established and the values of b in this formula having been found for the various kinds of plates, it remains now to give the methods of work and results for individual stars. All those selected for investigation are stars of the type of variation of δ Cephei, or at least of a similar spectral type. The instrument and plates used were as described in the earlier part of this paper.

Each complete observation consists of two exposures, one on a Seed 27 plate, the other on a Trichromatic or Pan-Iso plate with visual-luminosity filter, which was placed in contact with the sensitive film of the plate. Usually the Seed 27 plate was taken first, then the other with an interval of only about a minute necessary

[1] *Astrophysical Journal*, **36**, 171, 1912.

for the changing of plate-holders. The Trichromatic plates with filter required nine times the exposure of the Seed 27 plates to give

TABLE II

GRATING MEASUREMENTS

First Method (Pleiades Plate 2511, Seed 27)

First Exposure		Second Exposure
Stars	$\sqrt{D}$ Normal	$\sqrt{D}$ Normal
e	15.83	16.93
g	14.27	15.73
k	13.89	15.28
1	13.32	14.19
4	12.24	13.46
72	10.42	11.99
101	10.39	11.77
Means,	12.91 $=d^1$	14.19 $=d^2$
	Normal	Grating
f	16.37	15.42
h	15.07	14.00
32	13.41	12.51
s	13.20	12.57
p	12.93	12.07
29	12.59	11.81
19	12.56	11.72
24	12.53	11.79
31	12.32	11.23
22	12.29	11.63
17	12.17	11.42
33	11.78	10.84
23	11.75	10.96
27	11.03	10.08
18	10.63	10.01
13	10.63	9.89
15	9.75	8.88
Means,	12.41 $=d^3$	11.58 $=d^4$

$d^2-d^1=+1.28=\Delta^1$

$d^4-d^3=-0.83=\Delta^2$

$\Delta^1-\Delta^2=+2.11$

$1.878\div 2.11=0.89=b$

Second Method (Pleiades Plate 2520, Seed 27)

First Exposure		Second Exposure
Stars	$\sqrt{D}$ Normal	$\sqrt{D}$ Grating
31	14.42	11.98
32	15.43	12.93
s	15.30	13.00
p	14.95	12.68
19	14.76	12.40
29	14.66	12.29
24	14.63	12.16
22	14.44	12.08
31	14.42	11.98
17	14.32	11.98
23	14.03	11.37
33	13.82	11.47
18	12.92	10.42
27	12.89	10.46
13	12.72	10.28
Means,	14.23 $=d^1$	11.82 $=d^2$
	Grating	Normal
4	12.25	13.81
7	11.39	13.14
1	11.15	12.94
51	11.11	12.84
101	10.49	12.24
72	10.37	12.45
Means,	11.13 $=d^3$	12.90 $=d^4$

$d^1-d^2=\Delta^1=2.41$

$d^4-d^3=\Delta^2=1.77$

$$\frac{\Delta^1+\Delta^2}{2}=2.09$$

$1.878\div 2.09=0.90=b$

images of about the same size; with the Pan-Iso plates used in the later observations, this ratio was only five to one, thus affecting a considerable saving of time at the telescope. The maximum exposure for any field with the Seed 27 plates was three minutes and increased correspondingly for the other plates. All were developed with hydroquinone for just ten minutes. It is not necessary that the same care should be taken to have a constant temperature of developer as in the case of extra-focal plates, but the temperature was usually between 15° and 20° C.

Position of the stars on the plate.—The variable and comparison stars were located as symmetrically as possible with reference to the optical center of the plate, and a suitable star was selected for guiding. The double-slide plate-holder is furnished with three scales: right ascension, declination, and guiding eyepiece. The latter can be moved through a range of about two inches, the others considerably less. Care was taken in each exposure on a given field to set all these scales at the readings noted in the first exposure; hence the stars have always the same position on the plate, and consequently the matter of magnitude-corrections for distance from the center is much simplified.

I am indebted to Mr. Parkhurst for a table of corrections, the necessary portion of which is given in Table III with his explanation of its use.

TABLE III

REDUCTION TO THE CENTER FOR REFLECTOR PLATES (APERTURE, TWELVE INCHES)

Distance from Center	Correction		Distance from Center	Correction	
	Seed 27	Trich.		Seed 27	Trich.
7′	−0.03	−0.03	19′	−0.24	−0.18
8	− .04	− .04	20	− .26	− .20
9	− .06	− .05	21	− .28	− .22
10	− .07	− .06	22	− .30	− .24
11	− .09	− .07	23	− .32	− .28
12	− .10	− .08	24	− .35	− .30
13	− .11	− .09	25	− .37	− .32
14	− .13	− .10	26	− .40	− .34
15	− .15	− .12	27	− .43	− .37
16	− .17	− .14	28	− .46	− .40
17	− .19	− .16	29	− .49	− .42
18	−0.21	−0.17	30	−0.53	−0.45

"The size of the image and the corrections are expressed in terms of the square root of *D*, in thousandths of a millimeter. The corrections are proportional to the values of the square root of *D*. Therefore, since the reductions are tabulated for a value of 10.0, to find the correction for an image of any size multiply the tabular correction by one-tenth of the square root of *D*."

The corrections are a function both of the distance from the center and the square root of the measured diameters of the images. The distance from the center is always the same because of the arrangement of the field on the plate, but the size of the image of any one star varies somewhat from plate to plate because of variations in the effective exposures. However, it seemed sufficiently accurate to make the correction once for all for the comparison stars by taking the mean square roots of the diameters from all the plates. The corrections applied were made, not to the center, but to the star which was used on each plate as the standard of magnitude. As the stars are so arranged on the plate that differences are in general only a few minutes of arc, the corrections are but a few hundredths of a magnitude. One extreme case requires a change of 0.26 magnitude, while the variation in square roots of diameters of the images is 2.1. Therefore the extreme error caused by using the mean is only 0.027 magnitude, a quantity which is completely masked by the accidental errors of the plate itself.

DERIVATION OF THE MAGNITUDES OF THE COMPARISON STARS

One star of the first-type spectrum was selected in each field, its magnitude assumed, and then from it were derived the magnitudes of the comparison stars on all plates of that field. These varied considerably from plate to plate because of accidental errors, but the means of all were used in obtaining the magnitude of the variable. Using the mean from each plate would be equivalent to rejecting all comparison stars but the one selected as a standard.

The method of using the comparison stars in deriving the magnitude of the variable may be best explained by giving the details for a single plate. All plates have been reduced by this method.

X CYGNI, PLATE 1311, SEED 27

Stars	D	$\sqrt{D}$	$b\sqrt{D}$
X	152.0	12.33	11.10
1	162.5	12.75	11.48
2	127.5	11.29	10.16
3	108.5	10.42	9.38
4	127.5	11.29	10.16

Mean $b\sqrt{D}$ of comparison stars = 10.30.
Mean of magnitudes from all plates = 8.12.
Δ Mag. (X−comparison stars) = +0.80.
8.12−0.80−0.01 (correction) = 7.31 = magnitude of X.

ABSOLUTE AND RELATIVE VALUES

Since the principal object of this research is the determination of the difference between the visual and photographic magnitudes of the stars at various phases of the light-curves, no special effort has been made to have the magnitudes conform to those of any light-curves which may have been derived by other observers. From the fact that the magnitude-formula and the values of *b* have been definitively determined, the range of magnitude will also be correct. The assumption of a different magnitude for the standard star would have no effect upon the shape or relative position of the two curves, visual and photographic. Furthermore, the limited number of observations of each star does not justify a statement that the derived curves are definitive either as to accurate shape or the exact phase of maximum light.

Some of these Cepheid variables have been investigated spectroscopically,[1] and it has been shown that they have all the characteristics of spectroscopic binaries whose orbital period is the same as that of the variation in light, and whose maximum and minimum light correspond in time nearly to maximum velocity of approach and recession, respectively. The same is undoubtedly true of all other variables of the same class. Perhaps the best suggestion of the cause as well as the working out of the details of this relation of light and orbital velocity has been given by Dr. F. H. Loud.[2]

An alternative theory has been advanced, however, which seeks to explain their real variation in light and apparent variation in radial velocity as due to the pulsations of a single body.[3] This theory has many points in its favor, but it is not the purpose here to discuss the relative merits of the two hypotheses.

The tables.—In the tables of observations for individual stars, in the second column, *S* indicates Seed 27, *T*, Trichromatic, *P*, Pan-Iso plates. All other columns are self-explanatory. The residuals in the sixth and eighth columns are to be added algebraically to the observed magnitudes to produce the magnitudes of the smooth curve.

X CYGNI

This star, B.D.$+35°4234$ ($\alpha=20^h39^m$, $\delta=+35°\ 13'.6$), was discovered to be a variable star by Chandler in 1886.[4] From observa-

[1] *Lick Observatory Bulletin*, **4**, 130, 1907.

[2] *Astrophysical Journal*, **26**, 369, 1907.

[3] *Ibid.*, **40**, 448, 1914.

[4] *Astronomical Journal*, **7**, 32, 1886.

tions made at the Yerkes Observatory, it has been found to have a variable radial velocity, with a range of at least 50 km.[1]

In deriving the phases the following elements are used:

$$\text{J.D. } 2410190.684+16^{d}.38543 \text{ E}$$
$$=1886, \text{ Oct. } 10, 16^{h}25^{m}+16^{d}9^{h}15^{m}1^{s}.2 \text{ E (Luizet).}^{2}$$

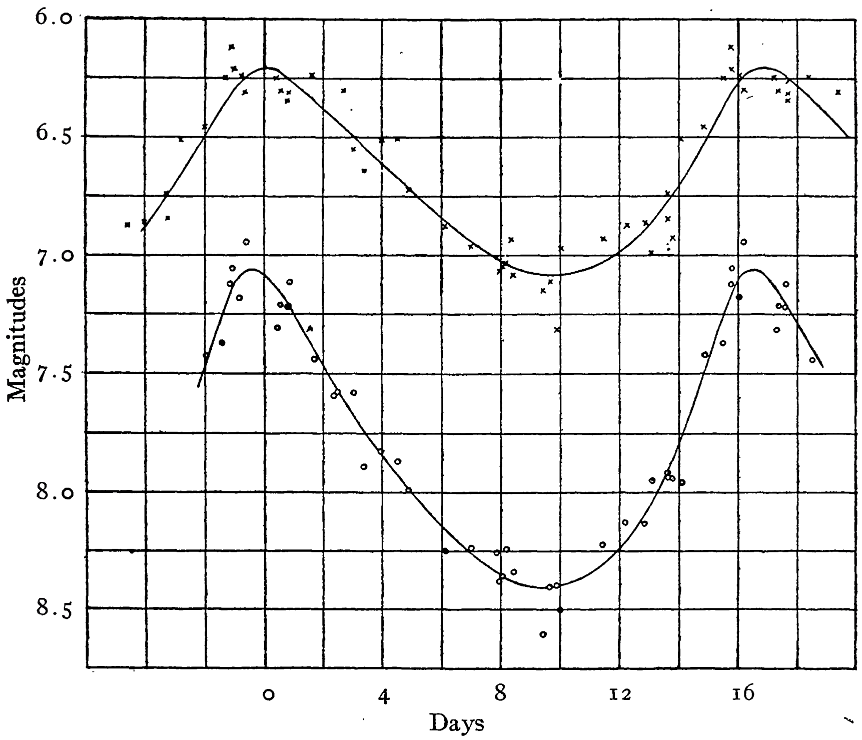

FIG. 7.—Light-curves of X Cygni

Data for the variable and comparison stars are:

Stars	B.D.	B.D. Mag.	Adopted Magnitudes		Distance from Center
			Photographic	Photo-visual	
X.........	+35°4234	Var.			11′
1.........	34 4127	7.5	6.96	6.96	11
2.........	35 4219	7.5	8.20	7.56	12
3.........	35 4221	8.3	9.12	8.51	14
4.........	35 4232	8.5	8.20	8.12	10

From the smooth curves drawn through the individual observations, it can easily be seen (Fig. 7) that the maximum is less

[1] *Astrophysical Journal*, **25**, 60, 1907.

[2] *Bulletin de la Société Belge d'Astronomie*, No. 12, 1913.

TABLE IV

X CYGNI: OBSERVATIONS

No. of Plate	Kind of Plate	Date, G.M.T.	Phase	Mags. and Residuals				Color-Index
				Photographic		Photo-visual		
				Mag.	Res.	Mag.	Res.	
		1906	Days					
714.....	S	Oct. 27d 15h 00m	13.04	7.91	+0.08			
715.....	T	27 15 13	13.05			6.98	−0.13	0.93
718.....	S	28 15 18	14.05	7.96	−0.20			
719.....	T	28 15 26	14.06			6.82	+0.17	1.14
726.....	S	30 14 38	16.02	7.17	−0.07			
727.....	T	30 14 49	16.03			6.23	−0.03	0.94
747.....	S	31 13 29	0.59	7.21	−0.05			
748.....	T	31 13 43	0.60			6.30	−0.06	0.91
775.....	S	Nov. 1 13 39	1.60	7.44	−0.03			
776.....	T	1 13 50	1.61			6.22	+0.09	1.22
796.....	S	9 14 13	9.62	8.40	0.00			
797.....	T	9 14 35	9.64			7.11	−0.03	1.29
809.....	S	13 14 36	13.64	7.92	0.00			
810.....	T	13 14 47	13.65			6.74	+0.03	1.18
818.....	S	22 12 40	6.17	8.24	−0.09			
819.....	T	22 12 51	6.18			6.88	−0.03	1.36
845.....	S	23 14 17	7.24	8.23	+0.01			
846.....	T	23 14 28	7.25			6.96	−0.02	1.27
857.....	S	24 13 28	8.20	8.26	+0.12			
858.....	T	24 13 39	8.21			7.03	0.00	1.23
868.....	S	28 13 26	12.20	8.12	+0.09			
869.....	T	28 13 37	12.21			6.87	+0.08	1.25
874.....	S	Dec. 3 13 32	0.82	7.22	0.00			
875.....	T	3 13 43	0.83			6.33	−0.09	0.89
898.....	S	18 12 22	15.77	7.13	+0.04			
899.....	T	18 12 35	15.78			6.11	+0.19	1.02
		1907						
952.....	T	Jan. 8 11 37	3.97			6.59	+0.09	
953.....	S	8 11 54	3.98	7.82	+0.01			1.23
1049.....	S	May 8 21 17	7.99	8.40	−0.03			
1050.....	T	8 21 29	8.00			7.07	−0.05	1.33
1099.....	S	12 21 10	13.67	7.94	−0.01			
1100.....	T	12 21 21	13.68			6.84	−0.07	1.10

TABLE IV—*Continued*

No. of Plate	Kind of Plate	Date, G.M.T.	Phase	Mags. and Residuals: Photographic Mag.	Photographic Res.	Photo-visual Mag.	Photo-visual Res.	Color-Index
		1907	Days					
1111.....	S	May 17d 19h 49m	2.23	7.58	0.00			
1112.....	T	17 20 00	2.24			6.30	+0.15	1.28
1126.....	S	19 20 42	4.27	7.87	+0.06			
1127.....	T	19 20 53	4.28			6.52	+0.15	1.35
1176.....	S	31 19 07	16.20	6.94	+0.13			
1177.....	T	31 19 22	16.21			6.30	−0.06	0.64
1189.....	S	June 1 20 41	0.88	7.11	+0.10			
1190.....	T	1 20 52	0.89			6.31	−0.05	0.80
1209.....	S	5 20 24	4.87	7.98	0.00			
1210.....	T	5 20 35	4.88			6.72	−0.01	1.26
1226.....	S	8 20 05	7.85	8.26	+0.08			
1227.....	T	8 20 16	7.86			7.01	0.00	1.25
1236.....	S	10 20 15	9.86	8.39	+0.01			
1237.....	T	10 20 36	9.88			7.31	−0.23	1.08
1252.....	S	13 19 28	12.83	8.13	−0.04			
1253.....	T	13 19 40	12.84			6.85	+0.02	1.28
1262.....	S	14 18 19	13.78	7.94	−0.06			
1263.....	T	14 18 30	13.79			6.92	−0.17	1.02
1283.....	S	15 19 54	14.85	7.43	+0.05			
1284.....	T	15 20 05	14.85			6.45	+0.07	0.98
1298.....	S	16 18 43	15.80	7.05	+0.11			
1299.....	T	16 18 54	15.81			6.21	+0.10	0.84
1311.....	S	17 20 13	0.48	7.31	−0.17			
1312.....	T	17 20 24	0.48			6.25	−0.02	1.06
1325.....	S	19 18 38	2.41	7.59	−0.05			
1326.....	T	19 18 49	2.42			6.58	−0.16	1.01
1335.....	S	20 15 58	3.30	7.89	−0.13			
1346.....	T	20 16 09	3.31			6.64	−0.09	1.25
1356.....	S	25 18 49	8.42	8.33	+0.04			
1357.....	T	25 19 00	8.43			7.08	−0.04	1.25
1358.....	P	25 19 24	8.44			6.93	+0.11	1.40
1367.....	S	26 18 42	9.41	8.50	−0.20			
1368.....	P	26 18 53	9.42			7.14	−0.06	1.46

TABLE IV—*Continued*

No. of Plate	Kind of Plate	Date, G.M.T.	Phase	Mags. and Residuals				Color-Index
				Photographic		Photo-visual		
				Mag.	Res.	Mag.	Res.	
		1907	Days					
1387.....	S	June 28^d 19^h 34^m	11.45	8.22	+0.08			
1388.....	T	28 19 46	11.46			6.92	+0.10	1.30
1401.....	S	July 2 20 21	15.48	7.37	−0.10			
1402.....	P	2 20 31	15.49			6.25	+0.12	1.12
1411.....	S	6 18 48	3.03	7.58	+0.09			
1412.....	P	6 18 58	3.04			6.55	−0.04	1.03
1425.....	S	11 19 08	8.04	8.35	0.00			
1426.....	P	11 18 18	8.05			7.04	−0.02	1.31
1433.....	S	13 18 19	10.01	8.50	−0.10			
1434.....	P	13 18 29	10.02			6.97	+0.11	1.53

accurately determined than the minimum. As the curves are drawn, the photo-visual maximum and minimum are respectively 6.21 and 7.08, giving a range of 0.87 magnitude; the photographic are 7.06 and 8.40, or a range of 1.34 magnitudes. The color-index at maximum is 0.85, at minimum 1.32. The photographic range is therefore 1.54 times the photo-visual.

These results differ quite decidedly from those given by Wilkens in his "Photographische-Photometrische Untersuchungen," where he gives the visual range as 1.0 and the photographic as 1.80 magnitudes.[1] These, however, are the extremes from individual plates, and cannot be interpreted as meaning that the actual range of a smooth light-curve would be as much. Interpreted in the same way, my observations would give as the visual and photographic ranges 1.20 and 1.64 magnitudes, respectively.

S SAGITTAE

This variable, B.D. $+16°4067$ ($\alpha=19^h52^m$, $\delta=+16°22'$), was discovered by Gore in 1885.[2] It was found by R. H. Curtiss, from the measure of plates obtained at the Lick Observatory, to have a

[1] *Astronomische Nachrichten*, **172**, 305, 1906.

[2] *Monthly Notices*, **46**, 106, 1886.

variable radial velocity with a range of at least 36 km.[1] Phases are derived from the formula

J.D. 2409860.36+8^{d}.38209 E.=1886, Nov. 14, 8^{h} 38^{m}+8^{d} 9^{h} 10^{m} 9^{s} E.

Data for the comparison stars are as follows:

Stars	B.D.	B.D. Mag.	Adopted Magnitudes		Distance from Center
			Photographic	Photo-visual	
S.........	+16° 4067	Var.			15′
1.........	16 4081	5.8	5.67	5.67	17
2.........	16 4085	7.0	6.77	6.70	18
3.........	16 4073	8.4	8.11	8.29	11

The light-curve of this star is peculiar, inasmuch as all published ones show a slight depression where maximum light would be expected. This peculiarity is also made manifest by the results of

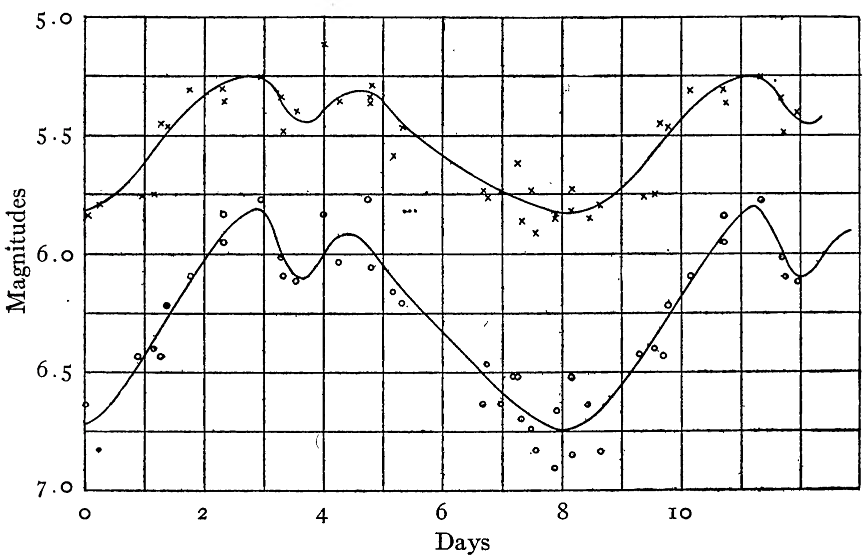

FIG. 8.—Light-curves of S Sagittae

the observations here given, though the curve as drawn at that phase could be considerably varied in shape and still satisfy the observed magnitudes equally well. The photo-visual curve has a range from 5.25 to 5.83 magnitudes; the photographic from 5.81

[1] *Lick Observatory Bulletins*, 3, 40, 1904.

TABLE V

S SAGITTAE: OBSERVATIONS

No. of Plate	Kind of Plate	Date, G.M.T.	Phase	Mags. and Residuals: Photographic Mag.	Photographic Res.	Photo-visual Mag.	Photo-visual Res.	Color-Index
		1906	Days					
710.....	S	Oct. 27d 13h 37m	6.74	6.47	+0.06			
711.....	T	27 13 50	6.75			5.73	−0.06	0.70
724.....	S	30 13 56	1.37	6.22	+0.05			
725.....	T	30 14 09	1.38			5.46	+0.02	0.76
743.....	S	31 12 13	2.30	5.83	+0.07			
744.....	T	31 12 31	2.31			5.31	−0.02	0.52
773.....	S	Nov. 1 13 11	3.34	6.09	−0.08			
774.....	T	1 13 20	3.35			5.48	−0.13	0.61
783.....	S	5 12 29	7.31	6.70	−0.05			
784.....	T	5 12 40	7.32			5.86	−0.08	0.84
792.....	S	9 12 48	2.94	5.77	+0.05			
793.....	T	9 12 55	2.95			5.25	0.00	0.52
805.....	S	13 13 13	6.96	6.64	−0.06			
806.....	T	13 13 22	6.97			5.73	0.00	0.91
816.....	S	22 12 10	7.54	6.83	−0.13			
817.....	T	22 12 18	7.54			5.86	−0.11	0.97
843.....	S	23 13 49	0.22	7.83	−0.15			
844.....	T	23 13 56	0.23			5.84	0.00	0.99
853.....	S	24 12 07	1.15	6.40	−0.03			
854.....	T	24 12 15	1.16			5.75	−0.20	0.65
864.....	S	28 12 09	5.15	6.16	−0.06			
865.....	T	28 12 17	5.16			5.28	−0.17	0.88
896.....	S	Dec. 18 12 35	0.02	6.63	+0.07			
897.....	T	18 12 44	0.03			5.84	−0.03	0.67
916.....	S	22 12 01	4.00	5.88	+0.16			
917.....	T	22 12 10	4.01			5.11	+0.28	0.77
		1907						
1047.....	S	May 8 20 49	7.25	6.52	+0.12			
1048.....	T	8 20 58	7.26			5.62	+0.15	0.90
1058.....	S	9 18 58	8.18	6.85	−0.12			
1059.....	T	9 19 10	8.19			5.82	+0.02	1.03
1082.....	S	10 21 11	0.89	6.44	+0.02			
1083.....	T	10 21 20	0.89			5.76	−0.13	0.68
1115.....	S	17 21 04	7.88	6.91	−0.17			
1116.....	T	17 21 13	7.89			5.85	−0.03	1.06

TABLE V—*Continued*

No. of Plate	Kind of Plate	Date, G.M.T.	Phase	Mags. and Residuals				Color-Index
				Photographic		Photo-visual		
				Mag.	Res.	Mag.	Res.	
		1907	Days					
1211.....	S	June $5^d 20^h 57^m$	1.73	6.09	+0.01			
1212.....	T	5 21 06	1.74			5.31	+0.08	0.78
1228.....	S	8 20 38	4.72	5.77	+0.20			
1229.....	T	8 20 47	4.72			5.37	−0.05	0.40
1234.....	S	10 19 46	6.68	6.63	−0.12			
1235.....	T	10 19 55	6.69			5.74	−0.05	0.89
1250.....	S	13 19 00	1.27	6.43	−0.13			
1251.....	T	13 19 10	1.27			5.45	+0.07	0.98
1268.....	S	14 20 00	2.31	5.95	−0.04			
1269.....	T	14 20 09	2.31			5.36	−0.07	0.59
1281.....	S	15 19 24	3.28	6.02	−0.03			
1282.....	T	15 19 33	3.29			5.29	+0.02	0.73
1296.....	S	16 18 19	4.24	6.04	−0.11			
1297.....	T	16 18 26	4.24			5.36	−0.03	0.68
1309.....	S	17 19 44	5.30	6.21	−0.06			
1310.....	T	17 19 53	5.30			5.47	−0.02	0.74
1319.....	S	19 16 55	7.18	6.52	+0.11			
1337.....	S	20 16 31	8.16	6.52	+0.22			
1338.....	T	20 16 40	8.17			5.73	+0.11	0.79
1348.....	S	25 16 27	4.78	6.06	−0.07			
1349.....	T	25 16 36	4.79			5.29	+0.03	0.77
1350.....	P	25 16 55	4.81			5.34	−0.02	0.72
1385.....	S	28 19 06	7.89	6.66	+0.08			
1386.....	P	28 19 14	7.90			5.83	−0.01	0.83
1399.....	S	July 2 19 36	3.53	6.12	−0.02			
1400.....	P	2 20 03	3.55			5.45	+0.04	0.67
1409.....	S	6 18 23	7.48	6.74	−0.06			
1410.....	P	6 18 30	7.48			5.73	+0.05	1.01

to 6.75, or a loss at minimum of respectively 0.58 and 0.94 magnitude. The color-indices at maximum and minimum phases are respectively 0.56 and 0.92. The ratio of photographic to photo-visual range is 1.62.

TT AQUILAE

This star, B.D. $+1°3899$ ($\alpha = 19^h03^m$, $\delta = +1° 09'$), was discovered to be a variable by Miss Annie J. Cannon from photographs taken at the Harvard College Observatory.[1] The statement is made that the spectrum seems also to be variable, classified as K at minimum and G at maximum. This variation in spectrum has since been found to be true in a number of Cepheid variables which have been observed spectroscopically, and is probably true in all variables of this type. Such a variation would be expected from the known change in color of these stars at the two epochs.

The elements used in determining the phases are those given by Ichinohe from early observations at the Yerkes Observatory, and from later ones made at Tokyo.[2] The formula is:

$$\text{J.D. } 2411873.865 + 13^d.753 \text{ E, or, 1891, April 21, } 20^h46^m + 13^d18^h4^m19^s.2 \text{ E.}$$

The data for the comparison stars follow:

Stars	B.D.	B.D. Mag.	Adopted Magnitudes		Distance from Center
			Photographic	Photo-visual	
TT	+1°3899	7.5	....	....	13′
1.........	1 3905	8.0	7.44	7.44	20
2.........	1 3896	9.0	8.76	8.72	15
3.........	1 3889	7.7	8.46	8.16	17
4.........	1 3898	8.1	7.75	7.90	18

The photo-visual range is from 6.70 to 7.53, or 0.83 magnitude; the photographic range is from 7.0 to 8.37, or 1.37 magnitudes. The latter is almost exactly the range given by Miss Cannon, 1.40 magnitudes, although the actual magnitudes differ by 0.6, the Harvard values being numerically the greater. The color-index at maximum is 0.30, at minimum, 0.84, a ratio of 2.8; greater than that of any of the other stars whose results are given here. These values are only approximate because of the comparatively few plates from which the curves are drawn.

[1] *Harvard College Observatory Circular*, No. 129.

[2] *Astronomische Nachrichten*, **187**, 299, 1911.

TABLE VI

TT AQUILAE: OBSERVATIONS

No. of Plate	Kind of Plate	Date, G.M.T.	Phase	Mags. and Residuals: Photographic Mag.	Photographic Res.	Photo-visual Mag.	Photo-visual Res.	Color-Index
		1907	Days					
1182.....	S	June $1^d18^h31^m$	9.88	8.04	+0.04			
1183.....	T	1 18 43	9.89			7.50	−0.07	0.54
1203.....	S	5 18 36	0.13	7.07	−0.03			
1204.....	T	5 18 47	0.14			6.76	−0.02	0.31
1220.....	S	8 18 29	3.13	7.56	+0.01			
1221.....	T	8 18 47	3.14			7.04	+0.04	0.52
1230.....	S	10 18 35	5.13	7.89	+0.11			
1231.....	T	10 18 51	5.14			7.23	+0.03	0.66
1246.....	S	13 17 47	8.10	8.30	+0.05			
1247.....	T	13 18 03	8.11			7.53	−0.02	0.77
1264.....	S	14 18 54	9.15	8.14	+0.07			
1265.....	T	14 19 10	9.16			7.55	−0.03	0.59
1277.....	S	15 18 16	10.12	7.89	+0.07			
1278.....	T	15 18 32	10.13			7.35	+0.05	0.54
1292.....	S	16 17 20	11.08	8.03	−0.25			
1293.....	T	16 17 31	11.09			7.30	−0.11	0.73
1305.....	S	17 18 40	12.14	7.52	+0.05			
1306.....	T	17 18 56	12.15			7.12	−0.13	0.30
1321.....	S	19 17 29	0.33	7.07	0.00			
1322.....	T	19 17 46	0.34			6.82	−0.05	0.25
1339.....	S	20 17 05	1.32	7.36	−0.12			
1340.....	T	20 17 22	1.33			6.90	−0.02	0.46
1351.....	S	25 17 19	6.33	8.33	−0.12			
1352.....	T	25 17 36	6.34			7.41	−0.02	0.92
1374.....	S	27 17 10	8.32	8.30	+0.05			
1375.....	P	27 17 27	8.33			7.54	−0.02	0.76
1381.....	S	28 18 04	9.36	8.22	−0.05			
1382.....	P	28 18 18	9.37			7.48	+0.02	0.74
1392.....	S	29 15 25	10.25	7.90	+0.07			
1393.....	P	29 15 35	10.25			7.23	+0.14	0.67
1396.....	S	July 2 17 27	13.33	6.97	+0.13			
1397.....	P	2 17 41	13.34			6.64	+0.08	0.33
1421.....	S	11 17 37	8.58	8.38	−0.07			
1422.....	P	11 17 50	8.59			7.48	+0.04	0.90
1439.....	S	13 17 40	10.59	7.88	+0.10			
1440.....	P	13 17 54	10.60			7.25	+0.05	0.63
1466.....	S	19 17 37	2.83	7.39	+0.12			
1467.....	P	19 17 58	2.85			7.10	−0.05	0.29

δ CEPHEI

This well-known variable, B.D. $+57°2548$ ($\alpha = 22^h\ 24^m$, $\delta = +57°\ 40'$), was discovered by Goodricke in 1784. In 1894 Belopolsky found it to be a spectrocsopic binary with the orbital period the same as the light-period.[1]

The latest elements of the light-variation as given in the *Vierteljahrsschrift* are:

Maximum, 1840, Sept. 26, $9^h 57^m.8 + 5^d 8^h 47^m 45^s.00\ E - 0^s.00075\ E^2 - 0^s.00000062\ E^3$.

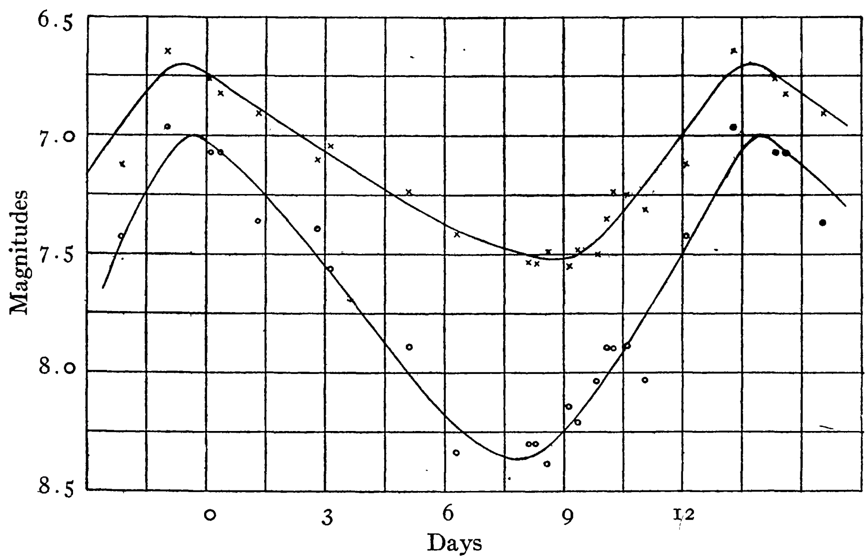

FIG. 9.—Light-curves of TT Aquilae

Because of the brightness of the star and the very limited field of the reflector, only one comparison star is available, the fainter, white component of the double, whose magnitude is here assumed to be 6.61.

Usually four exposures were made on the Seed 27 plates, ranging from ten to fifty seconds. The adopted magnitude for each plate is the mean of those from the different exposures. With the filter and the other two kinds of plates three exposures were made, ranging from one to five minutes.

[1] *Astronomische Nachrichten*, **136**, 281, 1894.

There is unfortunately a lack of exposures just at the maximum, making the curves somewhat uncertain at this phase, though the probable error in the location of the curve at this point can scarcely be as much as 0.05 magnitude.

The range obtained for the photo-visual curve as drawn is from 4.02 to 4.73, or 0.73 magnitude; that of the photographic curve

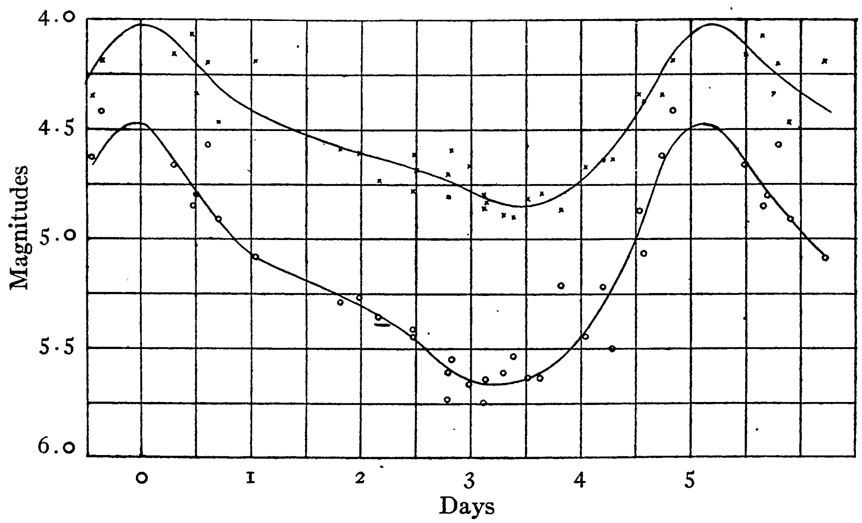

FIG. 10.—Light-curves of δ Cephei

from 4.47 to 5.67, or 1.20 magnitudes. The ratio of photographic to photo-visual range is 1.64. The color-indices at maximum and minimum are respectively 0.45 and 0.92.

SUMMARY

The object of this paper, the determination of stellar magnitudes, both photo-visual and photographic, by means of the photographic plate alone, and the application of this to short-period variables has necessitated the following investigations:

1. It has been shown that a properly sensitized photographic plate used with a suitable color-filter does actually give magnitudes of stars of all colors on a visual scale, and hence the expressions "photo-visual magnitudes" and "photo-visual light-curve" can be properly applied to such results.

TABLE VII

δ CEPHEI: OBSERVATIONS

No. of Plate	Kind of Plate	Date, G.M.T.	Phase	Mags. and Residuals				Color-Index
				Photographic		Photo-visual		
				Mag.	Res.	Mag.	Res.	
		1906	Days					
693.....	S	Oct. 21d 16h 34m	2.78	5.73	−0.14			
694.....	T	21 16 40	2.79			4.70	+0.03	1.03
730.....	S	30 15 56	1.02	5.08	0.00			
731.....	T	30 16 00	1.03			4.19	+0.22	0.89
751.....	S	31 14 50	1.98	5.23	+0.03			
752.....	T	31 14 56	1.98			4.61	0.00	0.66
779.....	S	Nov. 1 15 05	2.99	5.67	−0.02			
780.....	T	1 15 10	2.99			4.67	+0.11	1.00
788.....	S	8 13 56	4.58	5.07	−0.16			
789.....	T	8 14 03	4.58			4.37	+0.01	0.70
800.....	S	9 15 50	0.29	4.67	−0.02			
801.....	T	9 15 55	0.29			4.15	−0.05	0.52
813.....	S	13 15 51	4.29	5.50	−0.27			
814.....	T	13 15 58	4.29			4.63	−0.05	0.87
822.....	S	22 13 47	2.47	5.45	0.00			
823.....	T	22 14 00	2.48			4.78	−0.10	0.67
847.....	S	23 14 51	3.51	5.63	+0.01	4.82	+0.03	0.81
848.....	T	23 15 01	3.52					
872.....	S	28 14 46	3.14	5.64	+0.03			
873.....	T	28 14 54	3.15			4.83	−0.02	0.81
886.....	S	Dec. 17 13 25	0.62	4.57	+0.28			
887.....	T	17 13 34	0.63			4.20	+0.05	0.37
		1907						
938.....	T	Jan. 4 12 59	2.50			4.68	0.00	
1123.....	S	May 19 19 55	3.63	5.63	−0.03			
1124.....	T	19 20 00	3.64			4.79	+0.04	0.84
1186.....	S	June 1 19 56	0.53	4.80	−0.01			
1187.....	T	1 20 00	0.54			4.34	−0.13	0.46
1207.....	S	5 19 58	4.54	4.87	+0.08			
1208.....	T	5 20 05	4 54			4.34	+0.07	0.53
1224.....	S	8 19 46	2.16	5.36	0.00			
1225.....	T	8 19 52	2.17			4.73	−0.11	0.63

TABLE VII—*Continued*

No. of Plate	Kind of Plate	Date G.M.T.	Phase	Mags. and Residuals: Photographic Mag.	Photographic Res.	Photo-visual Mag.	Photo-visual Res.	Color-Index
		1907	Days					
1238.....	S	June 10^d 20^h 49^m	4.20	5.22	+0.08			
1239.....	T	10 20 54	4.21			4.63	−0.01	0.59
1254.....	S	13 20 01	1.80	5.29	−0.03			
1255.....	T	13 20 06	1.81			4.64	−0.01	0.65
1270.....	S	14 20 27	2.82	5.55	+0.06			
1271.....	T	14 20 32	2.83			4.59	+0.15	0.96
1285.....	S	15 20 25	3.82	5.21	+0.33			
1286.....	T	15 20 32	3.82			4.86	−0.07	0.35
1301.....	S	16 20 43	4.83	4.41	+0.16			
1302.....	T	16 20 48	4.84			4.18	−0.03	0.23
1313.....	S	17 20 44	0.47	4.85	−0.08			
1314.....	T	17 20 49	0.47			4.06	+0.11	0.79
1331.....	S	19 20 44	2.47	5.42	+0.04			
1332.....	T	19 20 50	2.47			4.61	+0.06	0.81
1341.....	S	20 18 43	3.38	5.53	+0.12			
1342.....	T	20 18 48	3.39			4.90	−0.05	0.63
1359.....	S	25 19 44	3.06	5.75	−0.08			
1360.....	T	25 19 50	3.07			4.80	+0.01	0.95
1361.....	P	25 20 06	3.08			4.86	−0.05	0.89
1369.....	S	26 19 14	4.04	5.45	−0.02			
1370.....	P	26 19 20	4.04			4.67	+0.04	0.78
1387.....	S	28 20 03	0.71	4.91	−0.01			
1388.....	T	28 20 08	0.71			4.47	−0.16	0.44
1401.....	S	July 2 20 42	4.73	4.62	+0.05			
1402.....	P	2 20 52	4.74			4.34	−0.11	0.28
1411.....	S	6 19 16	3.31	5.61	+0.05			
1412.....	P	6 19 22	3.31			4.89	−0.05	0.72
1519.....	S	Aug. 12 20 24	2.79	5.61	−0.02			
1520.....	T	12 20 28	2.79			4.80	−0.07	0.81

This is shown by a comparison of these results with those of competent visual observers, by the curves of spectral intensity from the plates, and by an actual photograph of a region containing a strongly colored star.

2. It has been shown that the magnitude formula $m=a-b\sqrt{D}$ represents within the limits of error the results with the reflector and the three kinds of plates used; also that the value of b in the formula, or the slope of the magnitude-curve, does not change perceptibly with the varying length of exposure used.

3. The value of b has been determined in two entirely independent ways: first, from the photographs of the Pleiades reduced with known magnitudes derived from extra-focal images, and an "absolute scale" of magnitudes; second, from grating exposures on the Pleiades, in which it is not necessary to assume any magnitudes of the stars used, since the reductions and the final value depend only upon the measured diameters of the images and the known magnitude-absorption of the grating.

TABLE VIII

		X Cygni	S Sagittae	TT Aquilae	δ Cephei
α		$20^h 39^m$	$19^h 52^m$	$19^h 03^m$	$22^h 24^m$
δ		+35° 14′	+16° 22′	+1° 09′	+57° 40′
Galactic latitude		−5°	−7°	−4°	−1°
Spectrum		F8 to G	G	G to K	G
No. of Plates	Photographic	37	31	19	31
	Photo-visual	38	31	19	33
Magnitudes at Max.	Photographic	7.06	5.81	7.00	4.47
	Photo-visual	6.21	5.25	6.70	4.02
Magnitudes at Min.	Photographic	8.40	6.75	8.37	5.67
	Photo-visual	7.08	5.83	7.53	4.73
Color-Index at	Max.	0.85	0.56	0.30	0.45
	Min.	1.32	0.92	0.84	0.92
Ratio $\frac{\text{Color at Min.}}{\text{Color at Max.}}$		1.55	1.64	2.80	2.04

The adopted values of b are: 0.90 for Seed 27; 0.77 for Trichromatic and Pan-Iso plates.

4. The selected fields, variable and comparison stars, have been arranged as symmetrically as possible with reference to the optical center of the plate, and care has been taken to have them

always in the same position on the plate. Corrections have then been applied to reduce their magnitudes to the center, or rather to one star as a basis. This fundamental star is of the first spectral type, and upon its assumed magnitude depends the position of the light-curves in the scale of magnitudes.

5. It has been shown that in each case the photographic range is greater than the photo-visual; or, in other words, that the star becomes redder as it becomes fainter, indicating some change in the spectrum.

The color of the star, or color-index, is expressed in magnitudes, and is a perfectly definite quantity, depending in no way upon the personal equation of the observer.

The results may be duplicated by any person using similar instruments and plates.

ACKNOWLEDGMENTS

My thanks are due to Director E. B. Frost, who placed at my disposal the two-foot reflector and all other instruments and supplies necessary for carrying out this investigation; to Mr. R. J. Wallace, whose skill and success in constructing a "visual-luminosity" filter made possible this method of carrying on the work; and especially to Professor J. A. Parkhurst, to whom is due the inception of the work, and by whose general oversight and helpful suggestions its completion was made possible.

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