in twenty-three years, for 3745 −·002 +·001, and for 3746 +·001 +·002 (unit = 0″·3).

In $+26^\circ\ 7^{\mathrm{h}}\ 4^{\mathrm{m}}$, Oxford 20286 should be omitted; it is the same as 20285.

TABLE II.

List of P.Ms.

Plate Centre.			Oxford.		*A.G.C.* No.	Δ*x*.	Δ*y*.	Cent. P.M. in Arc.	Interval in Years.
	h	m	No.	φ Mag.		(·001 = 0″·3)		″	
+26°	7	4	20123*	8·9	3831	−·032	−·004	42·0	22·98
+26	2	56	7181	10·8	...	+·017	−·017	31·1	23·15
+27	11	48	28359	11·7	...	+·008	·000	26·1	9·22
+26	2	56	7233*	7·6	1553	+·017	−·010	25·6	23·15
+26	2	56	7247*	9·2	1530	−·001	+·014	18·2	23·15
+26	7	4	20192*	7·5	3774	−·007	−·013	14·8	22·98
+26	2	56	7201	11·7	...	−·003	−·007	9·9	23·15
+26	7	4	20176*	9·4	3814	−·005	−·005	9·2	22·98
+26	2	56	7179	10·1	...	−·007	−·001	9·2	23·15
+26	2	56	7172	12·1	...	+·005	−·004	8·3	23·15
+26	4	16	10336	11·2	...	+·003	−·005	7·6	23·15
+26	2	56	7116	10·3	...	−·002	−·005	7·0	23·15
+26	2	56	7318*	9·3	1534	−·005	+·002	7·0	23·15
+26	7	4	20172	11·5	...	+·005	+·001	6·7	22·98
+26	7	4	20162*	10·9	3768	−·003	−·004	6·5	22·98
+26	2	56	7167	11·9	...	+·003	−·004	6·5	23·15
+26	2	56	7175	11·7	...	+·005	·000	6·5	23·15
+26	2	56	7218*	8·8	1522	+·004	+·003	6·5	23·15
+26	4	16	10380	11·3	...	+·003	−·004	6·5	23·15
+26	4	16	10413	10·9	...	+·004	−·003	6·5	23·15

University Observatory, Oxford:
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On the Measurement of Time to the Thousandth of a Second.

By Professor R. A. Sampson, F.R.S., Astronomer Royal for Scotland.

All time errors are relative quantities, for they arise from the comparison of two standards, whether these are merely two clocks, or a clock and a stellar transit, or otherwise. Hence when an irregularity appears the question arises which is technically known as "bringing home the guilt." When the matter is looked into more closely the problem becomes more intricate, because in the best-established methods the actual determination from any timekeeper is highly indirect. To take for definiteness the method employed at Edinburgh for registering time from the standard mean time clock, Riefler No. 258. The pendulum is the ultimate standard; this releases the scape wheel; the scape wheel carries on the same arbor an auxiliary wheel which throws a loose pivoted arm into or out of contact with a stop, closing or opening an electric circuit; the current in this circuit being too feeble to operate the chronograph directly, operates instead a relay; the relay current goes through one of the electromagnets of the Fuess chronograph, throwing down a somewhat massive arm: this is furnished with a pricker which marks a paper tape; the tape is moved forward by clockwork, which is driven by a weight and controlled by a fan governor. If an anomaly appears in a determined error, any one of these elements may be, and from time to time is, presumably at fault, either accidentally or systematically; for instance, it will appear below that the determinations are systematically different by an amount ranging up to and beyond a hundredth of a second, or, as it will be convenient to name it, to 10 *mil-seconds* (ms.), according to which of the thirty teeth of the scape wheel it is taken from, while three or four times that amount may be introduced by varying the adjustment of the relay.

Attention has of late been directed at the Royal Observatory, Edinburgh, to the improvement of time-keeping, and consideration of the position indicated above showed that little convincing advance could be made without a more direct as well as a more refined method of registration. Chronographs are in existence which are said to record the time to 1 ms., but so far as I know without any sufficient guarantee of their performance. One naturally turns to photography. I believe that the late Professor F. L. O. Wadsworth had a project of taking time by photographing the position of a pendulum at the moment; whether any steps were taken to carry it into practice I do not know, but it would evidently involve difficulties of its own, the chief being a true theory of the motion of the pendulum, and the design of a camera which would operate without introducing a variable lag. A more direct method appeared to offer in the registration of the signal current sent out by the clock itself by means of an oscillograph or kindred instrument. It is true that in only one of the clocks referred to below—Cottingham (C.), with a version of Gill's escapement—is the signal given directly by the pendulum; but in Riefler (R.) and in the synchronome clock (Σ.) the signal is the most direct external message that the clock issues, and the remaining sources of irregularity which they contain are so few in number—being irregularity of the scape wheel in R., the pendulum off beat in Σ., besides accidental variations due to set of the moving parts—that they can be studied without obscurity and their causes and amounts assigned. When this is done the signal current becomes a true and absolute signal from the pendulum. That such a current can be registered by an oscillograph without introducing any variable extraneous lag whatsoever is quite well known, and it is only necessary to adapt the method to the present case. The oscillograph, in essential, is an Einthoven galvanometer, in which the wire which carries the current to be recorded passes in a loop up and down between the poles of an external electromagnet. When the current passes, the magnetic field causes the two branches of the loop to deviate in opposite directions, and if they are spanned by a mirror, this mirror turns and moves a spot of light which can be used for registration. The fact that the magnetic field is external gives its power to the device; for, no matter how feeble the signal current may be, it can be made to yield a sensible mechanical effect by increasing the strength of the magnetic field, and the same resource permits the leads of the current between the poles of the magnet to be shortened and tightened until the time taken for the wire to assume a new position is reduced to any assigned smallness. Under these circumstances its movement is quite dead-beat under air resistance, and requires no further damping; and, moreover, the time taken for a movement of any amplitude is the same. But in any case the amount of this time is eliminated by the method of measurement, as will be seen below.

We possess at the Royal Observatory a remarkable electromagnet built by Lord Crawford about 1870; no doubt its design would not be repeated now, but it is still a powerful and convenient instrument; the poles are square flats, 5 inches a side (pierced by a central circular hole about 1 inch diameter). These poles can be made to approach or recede from one another, and a field of upwards of 12,000 gausses per sq. cm. has been measured between them. This instrument forms the basis for the microchronograph which I shall now describe. The loop which carries the signal current is made of platinoid wire, the two sides being $\frac{1}{8}$ inch apart, and under a joint tension of 8 lbs., which is sufficiently short of the point at which the elasticity of the wire gives way; the length of span of the loop is $1\frac{1}{2}$ inches; as regards the mirror, I have had to be content with what could be obtained at the present time, and it might be improved upon; the time taken by the system to assume a new position is about 3 mil-seconds, but, as remarked above, this does not come into the measures.

The general arrangement of the microchronograph is shown in plan in the figure. M, M are the poles of the great electromagnet; m is a small flat mirror mounted across the up-and-down wires, l, that lead the signal current. A is an arc lantern, focussed upon a slit S. IJ is an interruptor, to be described more particularly below. L is a lens, which with the intervention of the mirror, m, throws an image of S, magnified about 4 times, upon the camera, C, where it is received by a cylindrical lens, c, which focuses it upon the film, f. f is a cinema film, perforated and wound off a drum by means of a sprocket wheel, actuated by the gear G, which is driven by a band and small electric motor. A variety of speeds can be given to the film, those most commonly employed being from 3 to 4 cm. per second. So long as no change of current passes in *l*, the spot of light traces a straight line on the film. When the clock or other signal makes a change, the spot is deviated. In order then to measure correctly the time of any given signal in terms of the clock signal, it is requisite to interpolate correctly between two successive marks made by the clock. For this the motor cannot be trusted; its audible note rises and falls sensibly within the second. It is therefore necessary to mark out the second into correct subdivisions which shall be short enough to make any acceleration of the motor within them negligible; that is to say, incapable of introducing an error which shall not be well below 1 ms. This is the purpose of the interruptor, IJ. It is a straight steel tongue, securely clamped at J and loaded at I till its semi-period is near $0^s \cdot 1$. It is below the level of the slit, but carries a steel wire, *i*, projecting from it, almost in contact with the slit, of about the same breadth with it, and occulting it at each semi-oscillation. The tongue IJ is not actuated by any maintaining device. It is simply held aside to a measured degree, and released 20 seconds before the film is exposed. Its oscillations die down naturally in amplitude, the occultations grow broader and become slightly accelerated if the exposure is continued for, say, a minute, but there is nothing of the kind to be noticed in them for an ordinary exposure which lasts for about 15 seconds. The loading of the interruptor is such that the divisions are intermediate in length between the tenths of solar and sidereal seconds, or thereabout. The period is very nearly but not quite the same from one occasion to another; it is convenient to have a name for it that does not suggest that it is an exact subdivision of the second or constant in value, neither of which is requisite for its correct use, and I call it a *decim.*

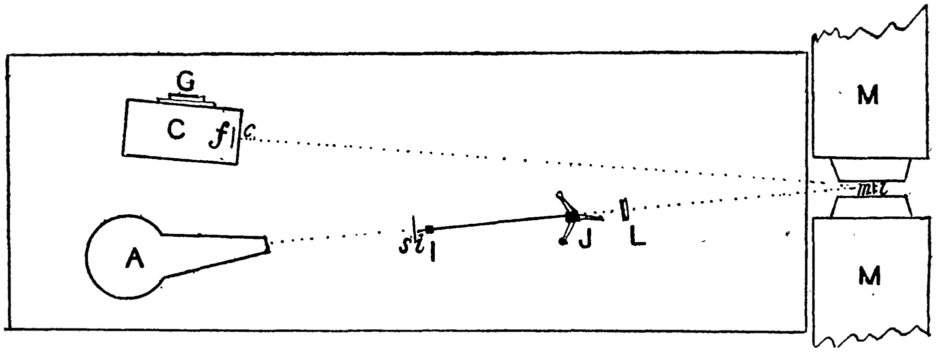

FIG. 1.

The film is measured under a travelling micrometer microscope, in terms of the decim and interpolated fractions of the same; backlash is eliminated as usual, by setting on each mark first for the right and then for the left and taking the mean; at the same time the period taken by the mirror to assume a fresh position is eliminated by making the first setting on the end and the second setting on the beginning of its motion. With any except a very defective trace the work is easy and certain to ·01 mm. Provided the decim is constant in time for this experiment, and the run of the motor is sufficiently steady over this one decim, this is equivalent to a measurement to 0·3 ms. or less. The factors that enter then into consideration are the constancy of the period of the interruptor for each occasion, and the uniformity of run of the motor for any single decim. It requires then to be established that the faults of these have been reduced to a point where they cannot introduce an error amounting to 1 ms. To this I now proceed, the standard for comparison being the clock Cottingham, which makes and breaks its current with the pendulum itself. Presumably then its 2-second signals are equal much within 1 ms., and if, as will be shown, in each experiment, the measure of Cottingham's seconds in decims is completely steady, I take it as a demonstration both that Cottingham's seconds *are* so equal (which cannot be said of the other clocks) and that the decims are equal too. It will then remain to examine the linear measures of the decims upon the trace, in order to study the acceleration of the motor and show that it is cut up into sufficiently small pieces to make its noxious effects negligible; or, in other words, to prove that a linear interpolation is admissible within a single decim. This is due and proper as a direct test, but in substance it is effectively anticipated by the comparison of Cottingham's seconds with the decim, which is made through the linear measure of the trace, and which would therefore exhibit any faults arising from interpolation from the latter.

In Table I. are shown for the first twenty traces which have Cottingham's seconds upon them the measures of these in terms of the decim. One faint trace that could not be measured accurately has been omitted. Beneath each set are shown numbers completely smoothed, as if calculated from a formula. The total discrepancies are given on the right; for 120 comparisons they amount to 26 units. That is to say, including all errors of record, of measurement, as well as irregularity in the interruptor which marks the decim or of Cottingham's second, for a single record on the trace the mean discrepancy from complete smoothness is ±0·22 ms. As the method professes to measure to 1 ms., this is well below what we need consider. The conclusion is that, in spite of any faults in the motor or the sprocket drive of the film, the method of measuring from the decim-marks and interpolating between them gives a completely reliable result to the thousandth of a second, so that any irregularities that are found when other clocks or signals record themselves must be attributed to these signals themselves, and are not introduced by the microchronograph. The majority of the measures given below were made by Miss K. A. Williams.

TABLE I.

Cottingham's Two-second Intervals in Terms of the Decim.

Trace.	1^{s}.	3^{s}.	5^{s}.	7^{s}.	9^{s}.	11^{s}.	Total Discrepancy.	Measurer.
	0^{d}·+	20^{d}·+	40^{d}·+	60^{d}·+	80^{d}·+	100^{d}·+		
No. 24	·11	·14	·17	·21	·25	·28	...	R.A.S.
Cf.	[·11	·14	·18	·21	·25	·28]	1	
25	·11	·14	·17	·19	·23	·26	...	R.A.S.
Cf.	[·11	·14	·17	·20	·23	·26]	1	
26	·88	·91	·94	·97	1·00	1·03	...	R.A.S.
Cf.	[·88	·91	·94	·97	1·00	1·03]	0	
27	·30	·34	·36	·39	·43	·45	...	R.A.S.
Cf.	[·30	·33	·36	·39	·42	·45]	2	
28	·15	·19	·22	·24	·27	...	...	K.A.W.
Cf.	[·16	·19	·22	·24	·27	]	1	
31	·61	·64	·69	·71	·76	·77	...	K.A.W.
Cf.	[·61	·64	·68	·71	·74	·77]	3	
32	·64	·68	·71	·74	·77	·80	...	K.A.W.
Cf.	[·64	·68	·71	·74	·77	·80]	0	
33	·12	·15	·19	·21	·25	·27	...	K.A.W.
Cf.	[·12	·15	·18	·21	·24	·27]	2	
34	·92	·96	1·00	1·00	1·04	1·07	...	K.A.W.
Cf.	[·92	·95	·98	1·01	1·04	1·07]	4	
35	·82	·85	·87	·90	·92	·95	...	K.A.W.
Cf.	[·82	·85	·87	·90	·92	·95]	0	
36	·45	·48	·51	·53	·56	·58	...	K.A.W.
Cf.	[·45	·48	·51	·53	·56	·58]	0	
38	1·73	1·74	1·76	1·79	1·81	1·84	...	R.A.S.
Cf.	[·72	·74	·76	·79	·81	·84]	1	
41	2·71	2·73	2·75	2·78	2·80	2·81	...	K.A.W.
Cf.	[·71	·73	·75	·78	·80	·82]	1	
42	3·13	3·15	3·18	3·21	3·23	3·25	...	K.A.W.
Cf.	[·13	·15	·18	·21	·23	·26]	1	
43	2·84	2·86	2·88	2·91	2·91	2·94	...	K.A.W.
Cf.	[·84	·86	·88	·90	·92	·94]		
44	3·09	3·11	3·13	3·15	3·18	3·21	...	K.A.W.
Cf.	[·09	·11	·13	·15	·18	·20]	1	
45	4·28	4·28	4·30	4·33	4·36	4·37	...	R.A.S.
Cf.	[·28	·29	·31	·33	·35	·37]	3	
46	4·16	4·16	4·19	4·21	4·22	4·24	...	K.A.W.
Cf.	[·16	·17	·19	·21	·22	·24]	1	
47	4·33	4·37	4·39	4·41	4·44	4·46	...	K.A.W.
Cf.	[·34	·37	·39	·41	·44	·46]	1	
48	4·48	4·50	4·53	4·54	4·56	4·58	...	K.A.W.
Cf.	[·48	·50	·52	·54	·56	·58]	1	

Consider now the absolute value of the decim in any operation. Taking the span of 10 seconds of mean time, its value appears as slightly over 100 decims. We get from the successive traces the values recorded in Table II. Between 69 and 71 there was a break of about 0·6 ms., for which I have no recorded reason to give; presumably there was some disturbance. Thereafter the decim remains very constantly near the same value, but with a slight though distinct tendency to increase in duration. The smaller variations, from one experiment to the next, run to a thirtieth of a mil-second or less. These must be regarded at present as inevitably incident, and are due perhaps to slight changes in the rigidity of the support.

TABLE II.

Value of Decim in Seconds of Mean Time.

Trace No.	Decim $=0^{\mathrm{s}}{\cdot}1-$	Trace No.	Decim $=0^{\mathrm{s}}{\cdot}1-$	Trace No.	Decim $=0^{\mathrm{s}}{\cdot}1-$	Trace No.	Decim $=0^{\mathrm{s}}{\cdot}1-$
	s		s		s		s
24	·0017	69 *a*	·0010	81 *b*	·0062	90 *d*	·0058
25	15	*b*	10	*c*	61	*e*	60
26	15	*c*	8	*d*	60	91 *a*	57
27	15	71 *a*	64	*e*	62	*b*	57
31	16	*b*	65	82 *a*	61	*c*	60
32	16	*c*	67	*b*	60	*d*	57
33	15	72 *a*	68	*c*	59	92 *a*	56
34	15	*b*	67	*e*	60	*b*	57
35	13	*c*	70	83 *a*	61	*c*	58
36	13	74 *a*	70	*b*	59	*d*	58
38	11	*b*	70	*c*	60	*e*	56
41	10	*c*	69	*d*	60	93 *a*	55
42	12	75 *c*	66	*e*	58	*b*	55
43	10	76 *b*	70	84 *e*	62	*c*	55
44	12	*c*	69	85 *a*	54	*d*	53
45	9	77 *a*	67	86 *a*	56	*e*	54
46	8	*b*	67	87 *a*	57	94 *a*	55
47	13	*c*	67	*b*	56	*b*	52
48	10	*g*	64	*c*	58	95 *b*	52
50	9	78 *a*	66	*d*	56	*c*	52
51	8	*b*	69	*e*	57	*e*	54
52	8	*c*	70	88 *a*	54	96 *a*	52
53	12	*d*	66	*b*	56	*e*	51
54	12	*e*	66	*c*	56	97 *a*	50
55	11	*f*	66	*d*	57	*e*	50
56	10	*g*	67	*e*	59	98 *a*	53
57	11	79 *b*	63	89 *a*	59	*b*	50
58	11	*c*	66	*b*	58	*c*	50
59	11	*d*	66	*c*	58	*d*	50
60	10	80 *a*	63	*d*	59	99 *b*	45
61	12	*b*	64	*e*	59	*d*	47
62	14	*c*	57	90 *a*	60	*e*	45
63	13	*e*	64	*b*	57	100 *a*	46
64	12	81 *a*	61	*c*	59	*f*	48

Note.—From 69 onwards several traces, distinguished as *a*, *b*, . . ., were taken in succession on the same film.

As the oscillations of the interruptor die down and its arc diminishes, its period may shorten. An analysis of the traces upon which Table II. is based shows a sharp separation between Nos. 69*c* and 71*a*. In the first part no trace of an acceleration is shown; the mean value of the intervals 1^s–3^s and 11^s–13^s, measured in decims, is exactly the same, viz. $20^d{\cdot}025$. In the second part there is a smooth and well-marked progression.

Table III.

Values in Decims of Cottingham's Successive Two-second Interval, derived from Traces 71a to 80e.

s s	d	d
1–3	20·126	*Cf.* Calc. 20·125
3–5	·126	·127
5–7	·130	·130
7–9	·133	·134
9–11	·139	·137
11–13	·141	·141
13–15	·147	·144

The column added for comparison is calculated with a factor for progression of length of the decim 1 − ·00075 per 10 sec. Both the change in length of the decim after Trace No. 69 and the sudden appearance of this acceleration are obscure in origin, but are probably due to changes in the fixture of the interruptor. In some earlier work an acceleration had made itself apparent, but of less amount, giving a factor 1 − ·0003 per 10 sec. In later traces again, from No. 86*a* to No. 93*e*, it is much reduced, showing a factor 1 − ·00015; the mean values for the intervals 1^s–3^s and 11^s–13^s being $20^d{\cdot}113$ and $20^d{\cdot}116$ respectively. In the latest traces it seems to have disappeared completely.

It is of interest now to examine the actual running of the film lineally, taking it as established by the foregoing comparisons that the decims represent known time intervals. We may anticipate variations from three causes: (1) if the vibrating tongue which causes the interruptions is out of centre, the vibrations will be "off beat," and alternate ones will be of different lengths, a difference that will grow more pronounced as the vibration dies away; (2) the lineal measure of the decim on the film will fluctuate with the speed of the motor; and (3) it may be supposed that the sprocket drive on the perforations of the film will move it forward with a certain proportion of jumps and hesitations. These faults may be examined by measuring in millimetres all the decims on the trace. This is somewhat laborious; it was done at the beginning in a considerable number of cases until the character of the motion became clear. Since it is only necessary to show that the faults in question are not present in a degree that will prejudice the accuracy aimed at, once that was made clear, complete measurement was abandoned, and the regular measure now consists of each signal, and the two extremities of the decim that contains it, with perhaps a few others as specimens. As an average illustration I have copied out below the complete measure of Trace No. 29. The measures of the decims are given in millimetres alternately, under the columns A, B. In perfect adjustment A and B should be equal. They are set as nearly equal as can be judged, by bringing the interruptor to rest, and seeing that its shadow, which just fills the object glass, falls correctly upon it. This is easily secured, and, as it appears, is sufficient. It is assumed that in calculating we need not distinguish the series A from series B in length. For this to be justified, consider the effect of a residual error such that the ratio $A : B = 1 + a : 1 - a$.

Measuring from middle to middle we should have equal spaces, and if we calculate as if the decims were equal, the actual interruption itself would be out of truth by $\pm \frac{1}{2}a$, according as we reckon it as belonging to series A or series B. Provided therefore a does not exceed ·01, an error of 1 ms. can only be introduced by this calculation if another accidental error happens to conspire with it. To examine the value of a the column C has been formed, being equal to $\frac{1}{2}(A_{-1} + A_{+1}) - B_0$. In this column positive values are set to the left and negative values to the right. The breaks in this column show to what degree the film is moved in jerks Fluctuations in the drive may be seen by taking the average value of the decims as we proceed down the series A or B; they are, however, of no significance apart from their effect in column C for they will not otherwise disturb linear proportionality.

The mean values of the column C run

	mm	
1st quarter	+ ·054	$\frac{1}{2}a =$ ·003
2nd ,,	+ ·045	·003
3rd ,,	+ ·068	·004
4th ,,	+ ·108	·007

the increase being due to damping of the vibration. The corresponding values of $\frac{1}{2}a$ are given alongside. From these it is apparent that, systematically, a is well below the prescribed limit (·01). As regards individual values for a, there are 9 cases out of 86 in which a reaches or exceeds the value ·01, the largest being $a = +·015$. It is apparent therefore that the detailed measurement of this trace would lead us to expect fully the prescribed degree of accuracy in time measures which has already been established by comparison of C and the decim. Other traces, of which a large number were measured in detail, led to exactly the same conclusion. It is unnecessary to give the actual figures for them.

Table IV.

Trace No. 29. Linear Measures of Decim.

A.	B.	C.	A.	B.	C.	A.	B.	C.
mm	mm		mm	mm		mm	mm	
3·40	3·35	+·07	3·73	3·77	−7	3·78	3·68	+ 8
·43	·42	·00	·67	·67	+ 5	·73	·68	6
·41	·53	−·05	·76	·60	19	·75	·74	5
·55	·48	12	·81	·72	2	·82	·76	5
·65	·53	12	·66	·77	−12	·80	·63	16
·64	·63	1	·63	·62	9	·77	·82	−8
·63	·62	3	·79	·62	11	·70	·66	12
·67	·70	−2	·67	·69	−4	·85	·63	21
·69	·61	11	·63	·59	12	·82	·68	12
·74	·57	20	·79	·60	10	·78	·60	18
·80	·62	12	·61	·67	−·01	·77	·67	8
·68	·70	−2	·71	·63	10	·73	·72	−5
·68	·74	0	·75	·63	11	·61	·73	−2
·80	·67	14	·73	·64	10	·80	·60	22
·82	·67	10	·75	·68	5	·83	·73	6
·72	·74	−1	·70	·65	6	·75	·69	12
·73	·68	4	·72	·68	9	·87	·64	19
·70	·73	0	·82	·66	15	·79	·70	5
·76	·73	6	·80	·71	2	·70	·76	−2
·81	·77	4	·66	·75	−·04	·77	·70	11
·80	·73	7	·76	·74	6	·84	·63	23
·80	·70	6	·84	·66	12	·88	·71	11
·71	·80	−8	·72	·79	−6	·75	·69	10
·73	·79	−2	·73	73	5	·82	·71	10
·81	·74	10	·83	·68	16	·79	·73	5
·86	·70	7	·85	·76	10	·76	·74	5
·68	·82	−10	·87	·73	11	·81	·70	11
·76	·71	11	·81	·75	4	·82	·68	+12
·87	·70	10	·77	·72	6	3·77	3·73	
3·73	3·77	−7	3·78	3·68	+ 8			

Having completed these preliminaries, we can now examine the anomalies shown in the signals sent out by the clocks Σ. and R., in which the signal current is not taken directly from the pendulum. Take first the clock Σ. This clock consists of a pendulum and maintenance merely — the counting train being external and subsidiary, and operated by the signal current. The maintenance is a stroke alternately on one side and the other of a crutch that is linked to the pendulum. The pendulum unlocks the striking piece, and when the stroke is made the striking piece continues to fall and releases a winding piece which resets the striking piece in preparation for the return of the pendulum; the winding piece moves on and makes a contact which serves as signal current and also resets the winding piece.

It may be expected that the several operations separating the signal from the pendulum itself will appear in the form of an increased accidental error of the signals recorded. Besides this error of a purely accidental character, there may be a systematic difference between alternate seconds owing to the pendulum being off beat. Table V. shows the result of measuring five traces. The numbers shown are *Obs.-Calc.*, expressed in mil-seconds. The calculation was made by comparing the observed measures in decims with a series proceeding by constant differences. The even seconds and odd seconds were then compared for indications of systematic difference. For Traces Nos. 29 and 31 there was a difference of 2 ms., for the rest zero. This difference was removed, and the residuals shown in the table are the accidental effect of the method of sending the signal. The general mean for 82 measures is ± 1·1 ms., which may be compared with ± 0·2 ms. for C. The greatest individual discrepancy is 3 ms. As the discrepancies are accidental, they must be removed by taking the mean of a sufficient number of comparisons.

As Cottingham is a mean time clock and synchronome a sidereal one, it is of interest to compare the ratios of their seconds to one another as taken from these traces over a ten-second run in each case. We find the following values, with which may be compared the true ratio 1·00273.

Trace No.	Σ.	C.	Ratio.
	d	d	
31	99·91	100·16	1·0025
32	90	·17	27
33	·84	·14	30
34	·87	13	26
		Mean	1·0027

TABLE V.

Anomalies in Seconds of Clock Σ.

Trace No. sec	29. ms	31. ms	32. ms	33. ms	34. ms
1	+2	+2	−2	0	0
2	+1	+2	+2	−1	0
3	−1	0	+2	+3	0
4	+1	−1	−1	−2	+1
5	−1	0	*	+2	0
6	0	−1	0	0	−2
7	−1	−1	0	0	0
8	−2	0	+1	0	+1
9	+3	0	−1	0	+3
10	−2	0	+1	+1	−1
11	+1	−1	−1	+3	+1
12	0	+3	−1	0	0
13	−1	0	+3	+2	0
14	+2	0	−1	0	+1
15	−1	−1	−3	−1	0
16	−1	−1	−2	+3	
17	−1	...	−2		
18	0	...	+3		
Means	$\pm 1{\cdot}2_{18}$	$\pm 0{\cdot}8_{16}$	$\pm 1{\cdot}5_{17}$	$\pm 1{\cdot}1_{16}$	$\pm 0{\cdot}7_{15}$

General mean $_{82}$ ±1·1 ms.

I take next the signals of the clock Riefler. The signals here also are indirect. The clock possesses a dial and wheel train. The driving weight actuates the "third wheel" which engages with the scape wheel. The scape wheel engages the anchor which maintains the pendulum in motion by bending its suspension spring. The scape wheel has 30 teeth, as usual. On the scape wheel arbor another wheel is mounted, possessing also 30 sloped teeth—less one, cut away to mark the beginning of the minute. These teeth alternately lift and let fall a pivoted arm, and thereby break and make the contact for the signal. We may expect, then, two sources of irregularity in the signals: first, a systematic one due to differences in the teeth of the scape wheel, and the second, more or less accidental, due to "set" of these various pieces in slightly different places allowed them by the "shake" they have to possess. The question of beat does not enter, because only the break is measured, and, as in Cottingham, this occurs only every two seconds.

The positions of the teeth of the scape wheel, with respect to one another, are of course very important, for, as it will appear, some of them are out of relative place by more than 10 ms., and in consequence no consistent micro-measures can be made unless the positions are found and the second with which any other signal is compared is known and its error applied. The determination of their positions was made twice over, first, in a preliminary way with paper traces, and secondly, for use, with films. I shall deal with the latter first.

The determination is made by reading Riefler and Cottingham simultaneously upon the trace; then Riefler's two-second intervals are measured in terms of the decim and the decim converted by reference to Cottingham's two-second intervals. It is proposed, in use, generally speaking, in comparing any set of signals with Riefler, to employ a run of, say, 10 or 12 secs. of Riefler, beginning from 1^s and going to 11^s or 13^s, and therefore this section has been done with more fulness than the rest. The results are shown in Table VI. When the mean of each column is taken and the residues derived, the mean of 244 residues is ±1·1 ms., exactly equal to the mean accidental discrepancy of a signal of Σ. The outcome of the table shows that if Riefler's signal at 1 sec. be taken as the standard, a comparison with the other seconds may be reduced to it by applying to the measures: for

s	ms	s	ms
3	− 7	9	− 12
5	− 11	11	− 11
7	− 9	13	− 3

where a negative sign indicates that the interval from 1 sec. to the second in question is too short.

Table VI.

Clock Riefler. Value of Successive Two-second Intervals.

Trace.	$1^s - 3^s$. s	$1^s - 3^s$. ms	$3^s - 5^s$. ms	$5^s - 7^s$. ms	$7^s - 9^s$. ms	$9^s - 11^s$. ms	$11^s - 13^s$. ms
No. 35	2	− 1	− 7	+ 4	− 2	+ 1	*
36		− 8	− 5	+ 3	− 2	+ 1	+ 8
37		− 5	− 8	+ 1	0	+ 1	*
38		− 4	− 5	+ 2	− 1	+ 2	+ 7
41		− 7	− 6	+ 2	− 1	+ 1	+ 7
42		− 5	− 3	+ 1	− 2	+ 2	+ 8
43		− 4	− 5	+ 1	0	+ 2	+ 6
44		− 5	− 4	+ 2	− 2	+ 1	+ 5
45		− 6	− 6	+ 2	− 1	+ 3	+ 5
46		− 7	− 4	+ 2	− 4	0	+ 10
47		− 4	− 5	+ 3	− 4	+ 2	+ 5
48		− 7	− 3	+ 2	− 3	+ 1	+ 5

TABLE VI.—*continued.*

Trace.	1^s-3^s.	3^s-5^s.	5^s-7^s.	7^s-9^s.	9^s-11^s.	11^s-13^s.
	s ms	ms	ms	ms	ms	ms
No. 69 A	−7	−4	+2	−2	0	+7
B	−7	−5	+2	−2	−1	+6
C	−7	−4	+2	−4	+2	+6
71 A	−5	−7	+3	−3	+2	+9
B	−8	−5	+3	−4	+1	+8
C	−7	−4	+2	−2	0	+10
72 B	−7	−4	+3	−2	+1	+11
C	−7	−5	+2	−2	0	+10
74 A	−7	−5	+3	−5	0	+9
B	−6	−6	+4	−7	+2	+8
C	−6	−5	0	−3	+2	+8
76 B	−8	−6	0	0	−2	+10
C	−10	−2	−2	−3	+3	+5
77 A	−5	−4	+1	−3	+1	+8
C	*	−4	+1	*	*	+7
G	−9	−2	+1	−4	+1	+9
78 A	−6	−6	+2	−2	+1	+9
B	−5	−6	+1	−3	+1	+8
C	−7	−4	+4	−6	+4	+6
D	−4	−6	0	−1	+2	+8
E	−8	−5	+3	−3	+3	+7
F	−8	−3	+2	−3	+2	+8
G	−8	−1	0	−1	+2	+8
79 B	*	*	+2	−5	+1	+9
C	−5	−5	+2	−5	+1	+8
D	*	−5	+2	−4	+1	+9
80 A	−9	−5	+1	−3	0	+8
B	−9	−3	+1	−3	+1	+9
C	−4	−4	0	−3	+2	+9
E	−11	−4	0	−2	+2	+5
Means	$-6, 5_{39}$	$-4{\cdot}6_{41}$	$+1, 7_{42}$	$-2, 7_{41}$	$+1, 3_{41}$	$+7, 7_{40}$

Table VI. is continued by Table VII., which gives the corresponding intervals as determined for the remainder of the wheel. For this purpose the 16 traces Nos. 49–64 were taken expressly and the results collected in the same way. The excesses or defects of the two-second intervals in mil-seconds are given under the column *mean.*

TABLE VII.

Riefler's Escape Wheel. Value of Successive Two-second Intervals—continued.

	No. 49.		No. 53.		No. 57.		No. 61.		Mean.
s s	s ms		s ms		s ms		s ms		ms
11–13	2 –		2 +9		2		2 +7		+8
13–15	−2		0		+2		+4		+1
15–17	0		+2		+2		+1		+1
17–19	−6		−6		−7		−9		−7
19–21	−1		1		0		+2		0
21–23			0		+1		−1	No. 62. s ms	0
23–25		No. 50. s ms	+6	No. 54. ms	+2	No. 58. s ms		2 +2	+3
25–27		2 −4	−2	−6	0	2 −1		0	−2
27–29		+4		+3		+4		+2	+3
29–31		−7		−8		−5		−7	−7
31–33		+8		+11		+8		+7	+8
33–35		−2		−2		−7	No. 63.	−2	−3
35–37	No. 51.	+8	No. 55.	+7	No. 59.	+10	+8	+9	+8
37–39	−2	−1	−2	0	−3	−4	−1		−2
39–41	+7		+8		+3		+5		+6
41–43	−10		−4		−3		−6		−6
43–45	−1		−7		−4		−3		−4
45–47	+6		+6	No. 56.	+5	No. 60.	+5	No. 64.	+5
47–49	−5	No. 52.	−1	−4	−2	−1	−6	−5	−4
49–51	0	−2	−4	−2	−5	−3		−3	−3
51–53		+4		+4		+3		+5	+4
53–55		−3		+1		−1		−3	−2
55–57		2 −1		+3		+1		+4	+2
57–1		4 +3		+2		0		+3	+2

A preliminary determination of these intervals had previously been made upon paper traces, and the final determination just recorded was intended to confirm it; but an accident occurred between the taking of the two sets, by which Riefler's current was short circuited for an instant, and the fine helical wire leading the current to the insulated pivoted arm was burnt out. The clock was opened and the wire was replaced by one that may have been slightly stronger, no other change being made within the clock. When the new determination was made, it appeared on comparison that the adjustment had undergone a systematic change. The values of the preliminary and final corrections from any given second to 1 sec. as standard are shown in Table VIII. The preliminary determination need not be given in detail; it is little inferior to the final one, and the changes are unquestionably real. The mean change has been reduced to zero by applying +2, 5 ms. to each; that is to say, by considering the one-second point to be altered by that amount.

TABLE VIII.

Riefler's Wheel. Reductions to 1 *Second and Change consequent on Adjustment.*

	Final.	Prelim.	Change.		Final.	Prelim.	Change.
s	ms	ms	ms		ms	ms	ms
3	− 7	− 2	− 2	33^s	− 3	0	− 1
5	− 11	− 9	0	35	− 6	− 1	− 2
7	− 9	− 4	− 2	37	+ 2	+ 4	0
9	− 12	− 7	− 3	39	0	+ 3	0
11	− 11	− 8	0	41	+ 6	+ 5	+ 3
13	− 3	− 2	+ 1	43	0	+ 2	+ 1
15	− 2	− 3	+ 4	45	− 4	+ 1	− 3
17	− 1	− 3	+ 4	47	+ 1	+ 3	+ 1
19	− 8	− 7	+ 2	49	− 3	0	− 1
21	− 8	− 6	0	51	− 6	− 3	0
23	− 8	− 8	+ 3	53	− 2	+ 1	− 1
25	− 5	− 6	+ 3	55	− 4	− 3	+ 2
27	− 7	− 2	− 2	57	− 2	+ 1	− 1
29	− 4	+ 1	− 3	1	0	...	+ 3
31	− 11	− 5	− 3				

The change is presumably due to some difference in set of the pivoted arm with respect to the toothed wheel that actuates it. The example may be taken as demonstrating how undesirable is a train of toothed wheels in a precision clock, particularly in connection with sending out signals. We very soon revert to the difficulty remarked at the beginning of this paper, in which it would be impossible to fix the responsibility for the occurrence of sensibly erratic signals, since they are dependent upon a number of adjustments and upon accidental set.

I shall next give some examples of the use of the microchronograph, and first for determining certain lags between signals originating from the same source.

Lag of the Fuess Chronograph.—This is the chronograph we use for general purposes. It is sufficient here to say that it carries three prickers, mounted side by side, and actuated by separate electromagnets. These make imprints upon a tape of paper which is carried past them by clockwork. In normal working the clock R. operates the first pricker, the sidereal clock Σ. operates the middle one, and the mean time clock C., or the transit circle signal, operates the third. A current of about 0·3 amp. is required to operate these prickers, so that the clocks have to work them by means of relays. It is required to find the amount of lag between the true clock current and the perforation of the paper tape.

The clocks and the chronograph are on different floors, and are separated by the whole length of the buildings, but it is a simple matter to arrange that the pricker shall close a circuit when it makes its perforation and to lead the resulting current to the microchronograph, where it records itself alongside the original clock signal.

The results show that the lag is remarkably consistent, not only throughout a single experiment, but from one experiment to another, provided no readjustment of the pens or relays is made; but the lags are liable to differ for the different pens, and are dependent upon adjustment, presumably also upon the current through the electromagnets, the range of alteration amounting to several hundredths of a second. The accidental variations of the clocks, and for the clock R. the inequalities of the scape wheel, are reproduced with fidelity by the chronograph.

Take, for example, the traces 75 *a*, *b*, *c*. Denote the Fuess chronograph signal by X, and its lag upon the clock by X_R, X_C, or X_Σ.

TABLE IX.

Lag of Fuess Chronograph (FX) in Successive Seconds.

No. 75*a*.			No. 75*b*.			No. 75*c*.		
R.	FX.	$X_{R.}$	Σ.	FX.	$X_{\Sigma.}$	C.	FX.	$X_{C.}$
d	d	ms	d	d	ms	d	d	ms
0·61	1·14	53	0·40	1·34	94	0·81	1·38	57
20·66	21·19	53	10·47	11·40	93	20·94	21·50	56
40·73	41·27	54	20·48	21·42	94	41·06	41·60	54
60·89	61·44	55	30·54	31·47	93	61·20	61·75	55
81·00	81·55	55	40·57	41·50	93	81·33	81·87	54
101·15	101·70	55	50·61	51·56	95	101·45	102·02	57
121·37	121·92	55	60·65	61·57	92	121·60	122·15	55
	Mean	54	70·71	71·64	93	141·75	142·31	56
			80·74	81·65	91		Mean	56
			90·80	91·73	93			
			100·84	101·76	92			
				Mean	93			

It will be remarked that the seconds of R. are noticeably unequal and that Σ. is slightly off beat, and that the chronograph, as we might expect, reproduces these inequalities.

The following are records of the mean lag for different experiments determined as above:—

TABLE X.

Lag of Fuess Chronograph on Primary Signals.

No. of Trace.	$X_{R.}$	No. of Trace.	$X_{\Sigma.}$	No. of Trace.	$X_{C.}$
	ms		ms		ms
75*a*	54	75*b*	93	75*c*	56
78*a*	63	96*c*	125	95*e*	46
82*c*	62	97*b*	125	96*a*	47
94*c*	52	98*e*	125	97*e*	46
94*d*	52	100*b*	126	98*b*	46
95*a*	57	100*d*	121	98*c*	46
95*d*	56	100*e*	117	100*a*	45
96*b*	58	103*b*	119	100*f*	46
97*d*	56	103*d*	145		
98*d*	61	104*e*	62		
99*c*	52				

A bar marks a change of adjustment of the Fuess system.

Slight adjustments of the relay operated by R. are made from day to day. This no doubt is the reason why the lag X_R is less consistent than C_R. Where no adjustment is made, the lag is surprisingly constant, but it is dependent upon adjustment to the amount of certainly $0^{s}\cdot03$. Moreover, while the clocks R. and C., which have a similar system and hold the current on and off for whole seconds alternately, do not show much difference in lag, the clock Σ., which gives a short signal lasting about $0^{s}\cdot05$ out of each second, differs from them sometimes by as much as $0^{s}\cdot08$.

Besides the Fuess chronograph signal, a signal is taken from the clock R. for comparison with the wireless signals sent from Paris. This is done by means of a controlled clock. The direct current from R. energises in its circuit an electromagnet. A clock, standing in the observatory lobby, carries on its pendulum an armature which swings over and close to this magnet at its extreme excursion on one side, and the clock is thus controlled. For the purpose of sending a signal to the wireless apparatus which shall be as clean and neat as possible, the pendulum in passing its lowest position lifts a short spring which throws a small hammer into momentary contact with a plate. On each occasion the contact is adjusted until it sounds absolutely sharp in the telephone.

As we might expect, this ticker does not reproduce the inequalities of R.'s wheel, and R.'s signals require to be reduced to standard before a smooth comparison can be made; for illustration, consider the trace No. 74*c*.

Table XI.

Trace No. 74*c*. *Lag of the Ticker* (T_R) *on Riefler—Individual Seconds—before and after Correction for Riefler's Wheel.*

	T_R.			T_R.	
	Uncorrected.	Corrected.		Uncorrected.	Corrected.
	s	s	s	s	s
1	+·147	+·147	9	+·158	+·146
3	·153	·146	11	·157	·146
5	·156	·145	13	·150	·147
7	·154	·145	15	·149	·147

As the contact is out of operation when not in use and is reset on each occasion, the lag T_R must be expected to differ in some degree from one experiment to another. The amount of this variation appears to lie within $\pm 0^{s}\cdot01$. But in addition, as is well known, the lag of the controlled clock will be dependent upon the controlling current. In normal working this current amounts to 10 milliamps. To examine this point, the current was reduced from 10 milliamps to 6 milliamps, and then raised again, leaving in each case intervals of about an hour for the clock to pick up its new position. The following table shows the collected results:—

TABLE XII.

Lag of Ticker (T_R) on Riefler; Successive Experiments.

No. of Trace.	T_R.	No. of Trace.	T_R.	No. of Trace.	T_R.
	s		s		s
68	+·168	71*b*	+·160	74*a*	+·155
69*a*	·164	71*c*	·154	*74*b*	·172
69*b*	·160	72*a*	·159	74*c*	·146
69*c*	·160	*72*b*	·193	99*e*	·145
71*a*	·157	72*c*	·159		

* Controlling current reduced to 6 milliamps.

As the indications of the Fuess chronograph have been adopted hitherto as the time given by Riefler, when its error is found with the transit circle, we see by comparison with Table X., column X_R, that the ticker system for comparison with the wireless signals is slow by $0^s\cdot 10$. This lag had previously been determined by ear by the help of an intermediate free pendulum which gained about one beat per minute upon the seconds ticks. The value adopted was $0^s\cdot 08$; but from a variety of circumstances it was rather troublesome to find, and the determination might well be $0^s\cdot 02$ in error.

I shall give next some examples of determination of the relative error of two clocks. Three points of interest present themselves: (1) the consistency of independent experiments taken so close together that rate hardly enters; (2) allied to the former, definite indication of irregularities of going which cannot be ascribed to the method of registration; and (3) the value of indications taken with the Fuess chronograph.

It was seen above that the mean of the error of a registered signal of R. or Σ. was ±1·1 ms., while that of C. was 0·2 ms. Hence the probable error of a single comparison of R. or Σ. with C. is ±0·95 ms., and of R. with Σ., ±1·31 ms. Six or seven comparisons are measured upon the trace, and hence the probable error of a relative time determination from a single trace is not more than 0·4 ms. for C. with R. or Σ., and 0·5 ms. for R. with Σ. Hence we may expect comparisons made in this way to be rarely ±1 ms. in error, although naturally one cannot insist on the last unit, for a variety of reasons.

The following examples show the comparison of this expectation with actual results.

Between Nov. 23 and Dec. 2 sets of traces were taken with different parts of R.'s scape wheel in order, primarily, to determine the inequalities of the wheel. They are, however, also available as relative time determinations, if the reduction is applied to bring them to a comparable basis at 1 sec. from Table VIII. Each set consisted of four traces, and there were four sets in all. Three further sets were taken between Dec. 7 and 9, in the standard way between 1 sec. and 13 sec. of R.'s wheel. The results are as follows, no allowance being made for rate except for the last, where the interval between two traces is upwards of an hour.

TABLE XIII.

Determination of Cottingham's Correction relative to Riefler (C_R) at Short Intervals.

No. of Trace.	Time.			C_R.	No. of Trace.	Time.			C_R.
	d	h	m	s		d	h	m	s
49	Nov. 23	22	17	+ ·492	61	Dec. 2	21	49	+ ·395
50		22	27	·494	62		21	59	·395
51		22	31	·493	63		22	6	·394
52		22	36	·491	64		22	11	·392
53	Nov. 25	21	45	+ ·630	69*a*	Dec. 7	5	31	− ·376
54		21	49	·628	*b*		5	38	·376
55		21	58	·627	*c*		5	43	·374
56		22	3	·630	71*a*	Dec. 8	2	42	− ·513
57	Nov. 26	21	53	+ ·720	*b*		2	46	·515
58		21	58	·718	*c*		2	51	·514
59		22	4	·719	72*a*	Dec. 8	22	24	− ·653
60		22	9	·718	*b*		23	41	·653
					c	9	1	0	·654

If we compare the determinations for one day with the next, it is obvious that the general relative rate of the clocks has been by no means constant. If we consider then that out of the discrepancies of the determinations above we have to provide for actual changes of error as well as of faults of the microchronograph, the numbers seem fairly to confirm the claim that they are a true record to 1 ms., and wherever we find inequalities beyond that order they must be ascribed to actual changes in the clocks.

I now come to the question of the comparative value of times taken by the Fuess chronographs (FX). As mentioned at the beginning, FX is operated with the aid of relays. The great influence of these upon the time recorded was not realised when the experiments began, and their arrangement had been settled more or less arbitrarily. R. operated its relay upon the make of the current, and Σ. and C. upon the break. After Dec. 9, with the intention of putting R. and C. upon the same footing, C.'s relay was made to operate also upon the make. The make for R. is fairly sharp, as may be seen by inspecting the traces, but that of C. is liable to some obscure variation when the loose crutch is picked up by the moving pendulum. The consequence of the change is very conspicuous, both total amount and variation of the equation FX − MX being much increased. When this was seen, both R. and C. were set to operate upon the break, and the discrepancy disappeared. The conclusion is that the indications of FX are now reliable to $\pm 0^s{\cdot}01$, provided always no change is made in the adjustment of the relays or the prickers. But in this connection Table X. should be looked at, and the remarks upon it. By changes of adjustment the readings of FX can be altered by several hundredths of a second, nor can one tell by how much it is altered without the aid of the microchronograph.

Table XIV.

Lag of Fuess Chronograph on Microchronograph in comparison of Relative Errors of Clocks C., R.

(1)			(2)			(3)		
Date. d	h	FX − MX. s	Date. d	h	FX − MX. s	Date. d	h	FX − MX. s
Nov. 16	9	·00	Dec. 10	9	− ·05	Jan. 24	9	·00
	21	·00		21	− ·04		21	·00
17	9	− ·01	11	9	− ·03	25	9	·00
	21	·00		21	+ ·07		21	·00
18	9	− ·01	12	9	+ ·06	26	9	+ ·01
	21	+ ·01		21	+ ·07	27	9	·00
19	9	− ·01	...				21	·00
	21	− ·02	18	9	+ ·05	28	9	− ·01
20	9	− ·01		21	+ ·05		21	·00
	21	− ·01	19	9	+ ·07	29	9	+ ·01
21	9	− ·01		21	+ ·07		21	− ·01
	21	− ·01	21	9	+ ·10	30	9	·00
22	9	+ ·01	22	9	+ ·07	31	21	·00
	21	·00		21	+ ·10	Feb. 1	9	·00
23	9	− ·02	26	9	+ ·11		21	+ ·01
	21	− ·01	31	9	+ ·07	3	21	− ·01
24	9	·00		21	+ ·09	5	9	− ·01
	21	− ·01	Jan. 1	9	+ ·10		21	·00
25	9	+ ·02		12	+ ·12	6	9	− ·02
	21	·00	2	9	+ ·10	7	9	·00
26	9	+ ·01		12	+ ·12		21	+ ·01
	21	·00	3	9	+ ·06	8	9	·00
Dec. 3	9	·00		12	+ ·08			
	21	·00	4	9	+ ·07			
4	9	− ·01		12	+ ·07			
	21	− ·02	5	9	+ ·06			
5	9	− ·03		12	+ ·06			
	21	− ·01	6	9	+ ·06			
6	9	− ·03		12	+ ·08			
	21	− ·02	7	9	+ ·08			
7	9	− ·02		21	+ ·09			
	21	− ·01	...					
8	9	− ·02	21	21	− ·01			
	21	+ ·02	22	9	+ ·06			
9	9	− ·02		21	+ ·06			
	21	− ·01	23	9	+ ·06			
R. operating on make C. ,, on break			R. and C. operating on make.			R. and C. operating on break.		

This table requires no further comment.

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The present communication is confined to a description of the microchronograph and to establishing its performance and capa-

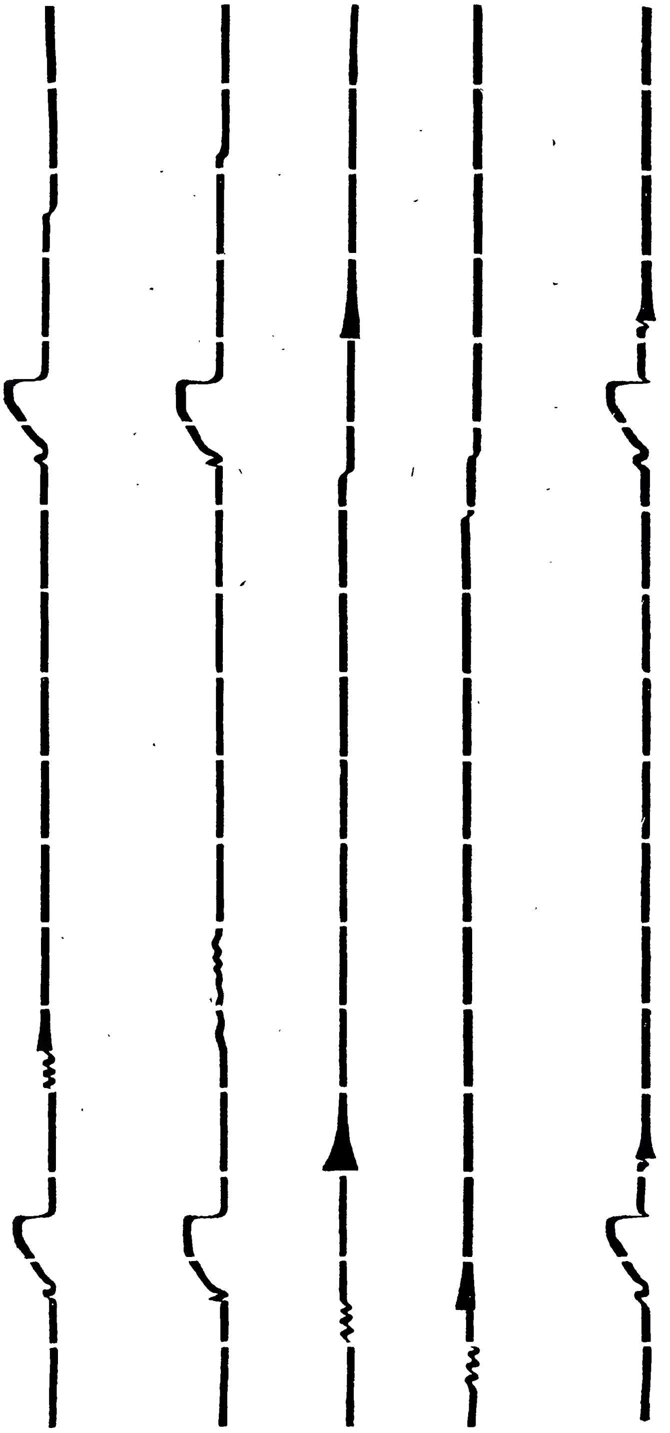

FIG. 2.—Drawings from actual Traces. Enlarged about three times.

bilities; I shall not discuss at all the going of the clocks themselves, but shall proceed to describe the illustrative traces shown in fig. 2. These five traces are respectively: (1) clocks Riefler and synchronome I.; (2) clocks Cottingham and synchronome II.; (3) Riefler with controlled ticker; (4) Riefler with Fuess chronograph; (5) synchronome I. with Fuess chronograph. The method of sending the signal recorded differs in each case. To take the synchronome clocks first, the signal is made every second by an arm which is thrown into contact by gravity by the release of a click, and is broken again after about $0^s{\cdot}05$ by a spring worked by an electromagnet which the signal itself actuates. The indentation of the line of the trace therefore gives on small scale a graph of the growth of the current. It may be compared with the full set of oscillograms of this current under various circumstances shown by Mr. Hope Jones in his paper "Modern Electric Time Service," *Inst. Elec. Eng. Jour.*, vol. xxix., 1900, p. 119. The essential features are here readily recognisable. After the first contact the pieces rebound mechanically and the current breaks, showing a zigzag in the line. They then lie in contact and the current grows, but not smoothly to its maximum, owing to the entry of self-induction. In Σ. I. the contact pieces are of carbon, in Σ. II. they are of platinum; a difference in character between the two can be quite well recognised in the traces.

In the clocks Riefler and Cottingham the contacts are made and broken on alternate seconds. Riefler, by a wheel upon the scape wheel arbor, at the odd second, lifts out of contact a freely pivoted arm, and at the even second allows it to fall into contact again. Hence the current is on for a whole second and off for the same time. The mechanical rebound when the arm falls into contact may be seen very clearly and regularly at the make. In Cottingham the make of the current is effected by the pendulum itself striking and carrying with it the crutch. As might be expected, the make is less decided and sharp than in Riefler, but the construction of the clock makes the break very neat and clean.

Little need be said of the three traces showing records by controlled systems. The first shows, along with Riefler's seconds, the tick taken from the pendulum of the controlled lobby clock which is taken to the telephone at the wireless table. As shown above, this is free from the inequalities of Riefler's scape wheel. Its lag on R. can be varied by varying the strength of R's. current. It comes on every second, but its mark is somewhat different according to whether Riefler's own current is off or on. The signals in the last two traces are produced by the pricker arms of the Fuess chronograph, actuated respectively by Riefler and synchronome I. They reproduce any original inequalities of the signals sent by the respective clocks, and, as shown above, their lags, which are here evidently different, can be varied by large amounts by changing the connections of the relays or other details.

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On the Literal Development of the Motion of the Lunar Perigee. By R. Moritz.

(*Communicated by P. H. Cowell, D.Sc., F.R.S.*)

Some years ago, at the suggestion of Dr. Cowell, Engineer-Commander Godbeer, R.N., repeated Hill's well-known development of the variational terms in the motion of the moon.* The very surprising fact was then discovered that these famous series are vitiated by no less than three errors, which have apparently remained undetected for forty years. Dr. Cowell and I both repeated the work, and satisfied ourselves that the facts are as stated. The terms in error are those in m^7 in a_{-3}, in m^8 in a_{-2}, and in m^8 in a_{-3} respectively, and their correct values are given below. Throughout this paper the symbol m refers to Hill's parameter, being $\frac{m}{1-m}$ in terms of that used by the older lunar theorists.

Partly as a result of this work, Dr. Cowell and I embarked upon an investigation of the literal series of characteristic e in the lunar theory, following a method which he had propounded for obtaining the power series expressing that portion of the motion of the perigee which is dependent on the ratio of the mean motions only. The work was afterwards extended to further classes of terms, *e.g.* those depending upon e^2, e^3, g, and $\frac{a}{a'}$ and its powers; but the pressure of other duties and interests precluded any idea of the completion of an algebraic lunar theory. It is sufficient to say that all these results, when transformed into M. Andoyer's notation, agree without exception with the full developments which he has given.†

Since the publication of Professor E. W. Brown's Lunar Theory,‡ containing a complete numerical expression for the motion of the perigee carried to a degree of accuracy comparable with modern observations and, indeed, agreeing with the latter to a surprising extent, the question of the development of this quantity in power series has in a sense lost its interest. But though in the case of our own moon no algebraic series can compare in accuracy with this numerical theory, nevertheless the historical interest of the subject, and the possibility of a need for such developments in the case of other satellites, still invest the matter with sufficient importance to warrant this investigation. I ought to state that the inception of the plan and the details of its working are entirely due to Dr. Cowell, who kindly permits me to publish the results.

The history of the expansion in power series of the motion of the perigee is a matter of some interest. Following upon the earlier investigations of Newton and Clairaut, the famous and unexpectedly

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* *Collected Works*, vol. i. p. 317.

† *Bulletin Astronomique*, **18**, 177 (May 1901) *et seq.*

‡ *Mem. R.A.S.*, **53**, 39, 163; **54**, 1; **57**, 51; **59**, 1.