The following were proposed by the Council as Associates of the Society:—

Frederick Hanley Seares, Solar Observatory, Mount Wilson, California, U.S.A.;
Harlow Shapley, Solar Observatory, Mount Wilson, California, U.S.A.; and
Joel Stebbins, Professor of Astronomy, University of Illinois, Urbana, U.S.A.

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Fifty presents were announced as having been received since the last meeting, and the thanks of the Society were returned to the respective donors.

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## *The Secular Acceleration of the Sun as determined from Hipparchus' Equinox Observations; with a Note on Ptolemy's False Equinox.*

By J. K. Fotheringham, M.A., D.Litt.

It is well known that the ancient eclipses of the Sun and the magnitudes of the ancient eclipses of the Moon yield a positive correction to the longitude of the Sun measured from the Moon's node as determined from modern observations and theory. While most investigators have corrected the motion of the Moon's node to account for these phenomena, Dr. Cowell has since 1905 * preferred to assume an acceleration of the Sun's motion, and Newcomb has taken the extreme course of setting aside the evidence as too untrustworthy to furnish corrections to the motion either of the Sun or of the node, though he has regarded the recorded times of the ancient lunar eclipses as good evidence on which to base corrections to the Moon's longitude. In 1915 † Miss Longbottom and I determined the secular acceleration of the Moon's motion in longitude from the ancient occultations; we found a value substantially larger than the acceleration resulting from the times of the ancient lunar eclipses and inferred that the only possible explanation of the discrepancy was a secular acceleration of the Sun's motion which affected the times of the eclipses, but which had no influence on the times of the occultations. On comparing the solar acceleration thus obtained with the acceleration as determined from the magnitudes of lunar eclipses, we found a small correction to the acceleration of the node, but it was accompanied by so large a probable error that we expressed a doubt whether any correction to the theoretical acceleration of the node was necessary.

In the present paper I attempt to determine the Sun's acceleration independently of possible corrections to the lunar theory.

* *Monthly Notices,* **66**, 3–5. † *Ibid.*, **75**, 377–396.

It may be well to avoid possible misunderstandings by explaining at the outset of this inquiry that by the phrase "acceleration of the Sun" I refer merely to the acceleration of the Earth's motion of revolution as compared with its motion of rotation, which we adopt as the standard of time, and do not wish to commit myself to any view as to the extent to which the phenomenon under discussion is due to a physical acceleration of the one motion or a physical retardation of the other motion. I also wish it to be understood that I am dealing only with the mean acceleration between the time of Hipparchus' observations and the time of the observations by which the present mean motion of the Sun is determined. I do not express any opinion on the question whether this acceleration has been uniform or subject to fluctuations. For the purpose of this paper it will be assumed that Newcomb's mean longitude and mean motion of the Sun are correct for the epoch +1800 Jan. 0·0. Any inexactness in this assumption can hardly affect the acceleration resulting from this discussion by more than a tenth of a second of arc per century, a small quantity compared with the probable error attaching to the quantity which I seek to determine.

### *Hipparchus' Equinox Observations.*

In the first chapter of the third book of the μαθηματικὴ σύνταξις* or *Almagest*, Ptolemy cites from Hipparchus' treatise on the Displacement of the Solstitial and Equinoctial Signs a series of twenty equinox observations, presumably made by himself, which Hipparchus had used in the discussion of the question whether the length of the year is uniform or whether it is subject to an inequality such as would be produced by an oscillation of the equinoctial points. I insert a translation of the passage.

"Then he sets forth first the times of autumn equinoxes observed with the greatest possible accuracy, (1) in the 17th year of the third Callippic cycle on the 30th of Mesore about sunset. (2) Three years later in the 20th year on the first of the supplementary days in early morning, though it should have been at midday, so that there was a discordance of a fourth part of a day. (3) A year later in the 21st year at the sixth hour, which was in agreement with the preceding observation. (4) And after eleven years in the 32nd year between the third and the fourth supplementary days at midnight, though it should have been in early morning, so that again there was the discordance of a fourth. (5) And after one year in the 32nd year on the fourth of the supplementary days in early morning, which was in agreement with the preceding observation. (6) And after three years in the 36th year on the fourth of the supplementary days at evening, though it should have been at midnight, so that again there was a discordance of the fourth [of a day] only.

* Ed. Heiberg, i. (1898) pp. 195, 196.

"And after this he sets forth also the spring equinoxes observed with similar accuracy: (7) in the 32nd year of the third Callippic cycle, on the 27th of Mechir in early morning: the *κρίκος* at Alexandria, however, he says, was equally illuminated on both sides about the fifth hour; so that here two observations of the same equinox differed by five hours approximately. (8) to (12) And the following equinoxes, he says, down to the 37th year are in accordance with the excess of one-fourth [*i.e.* with a value of the solar year exceeding the Egyptian year of 365 days by one-fourth of a day]. (13) to (19) And after eleven years in the 43rd year on the 29th of Mechir after the midnight at the beginning of the 30th the spring equinox took place, which was also in agreement with the observation in the 32nd year and accords also, says he, with the observations in the following years down to the 50th year; (20) for [in that year] it took place on the 1st of Phamenoth about sunset one day and three-fourths approximately after the equinox in the 43rd year, an interval which is divided over the seven intervening years."

References to the Callippic cycle are too numerous to leave any doubt as to the interpretation of these dates,* and it is remarkable that the whole series of observations should frequently have been placed one year too late. A careful reduction of these, as of all other dates in the *μαθηματικὴ σύνταξις*, will be found in Manitius's translation.† Manitius reckons his days from midnight and adopts the historical method of reckoning years before Christ. I have substituted astronomical days reckoned from Greenwich mean noon and negative numerals for the years, so that $-161$ is equivalent to 162 B.C.

It will be observed that the whole list of observations is impersonal. That is probably only a matter of grammatical style. The fourth and seventh of the series are stated later in the same chapter ‡ to be specially indicated by Hipparchus as having been most reliably observed by him, though elsewhere the reference is once more impersonal. § There seems, therefore, to be no reason for questioning, as is sometimes done, the right of Hipparchus to be regarded as the observer of the first three equinoxes in the series. Of which of his predecessors would so severe a critic as Hipparchus have said that his observations were made with the greatest possible accuracy? An opposite tendency may be traced in those who infer from Hipparchus' citation of the observation made with the *κρίκος* at Alexandria that he must have been on a visit to Alexandria at that time. His express statement that the rival observation was "most reliably observed by him," and his

* Ptolemy actually gives the interval in years and days between two of these observations and two dated observations of his own; *ubi supra*, pp. 204, 205.

† 1912, 1913, 2 vols. It is curious that Manitius himself should have given the false dates in his edition of Hipparchus' *Commentary on Aratus* (1894, p. 282).

‡ Ed. Heiberg, p. 203. § *Ibid.*, p. 204.

use of it in deducing an oscillation of the equinox, are clear evidence that the observation on the κρίκος was derived from some source which he regarded as less trustworthy. There is one instance preserved to us where Hipparchus cites the magnitudes of a solar eclipse as observed both at the Hellespont and at Alexandria; he cannot have observed the eclipse at both places, and it is most probable that he observed it at neither.* The astronomers of his time must have been in the habit of communicating their observations when occasion demanded.†

It will be observed that Hipparchus is content with recording the observed times of the equinoxes to the nearest quarter of a day, and it is noteworthy that Ptolemy in commenting on the observations regards an error of this magnitude as possible. It is not merely possible, but certain that some of the observations are affected by errors of this magnitude; but in view of modern methods of dealing with errors, we may perhaps regret the scientific caution which deterred Hipparchus from recording the time more exactly. Ptolemy, though he knew himself to be an inferior observer, does not shrink from giving the results of similar observations of his own to the nearest hour.

In the passage cited above no mention is made of the method by which the observations were made apart from that made with the κρίκος at Alexandria, and I observe that Newcomb ‡ seems to have supposed that Hipparchus determined the times of the equinoxes by observing the dates when sunset was exactly opposite to sunrise. As some of the equinoxes are stated to have happened at midnight, it is clear that the results must have been obtained by interpolation, and Ptolemy, § who had the whole discussion before him, refers to the error in the observed declination of the Sun which would arise from a hypothetical error of $6'$ on the declination circle due to an error in the setting or graduation of the instrument. This leaves no doubt that the time of each equinox was obtained by interpolation between two observations, presumably meridian observations, of the Sun's declination.|| I regret that in an earlier paper ¶ I endorsed an erroneous opinion that these observations were vitiated by refraction.

Postponing consideration of the observation made with the

* See Hultsch, *Hipparchos über die Grösse und Entfrenung der Sonne*, *Berichte der sächsischen Gesellschaft*, 52 (1900), *Phil.-hist. Classe*, pp. 169–200, or my paper in *Monthly Notices*, 69, 204.

† The views combated in this and the previous paragraph have been copied from book to book. I have thought it unnecessary to trace them to their source.

‡ *Compendium of Spherical Astronomy* (1906), p. 253. His words do not expressly assert this view, but they at least suggest it.

§ *Ubi supra*, p. 197.

|| Ptolemy's own equinox observations were made with an instrument which he had specially devised for meridian observations; *ubi supra*, p. 203, and compare pp. 64–67. We may assume that Hipparchus was equally alive to the advantage of making observations on the meridian. The point is not absolutely essential to my discussion.

¶ *Monthly Notices*, 75, 378.

κρίκος at Alexandria, and confining myself to the twenty observations of the Sun's declination, I form the following table, where the first column gives the reference number of each observation, the second the observed date of each equinox reduced to Greenwich mean time, the third the apparent longitude of the Sun at that date, computed from Newcomb's *Tables of the Sun*, and the fourth the correction to Newcomb's apparent longitude resulting from the observation. In reducing the observed times I have assumed that all the observations were made at Rhodes (east longitude 28° 14′; the difference between Rhodes and Alexandria is unimportant for the present purpose) and that the observed times are local solar time. Ptolemy only applies the equation of time when dealing with observations of the Moon; there is no reason to suppose that Hipparchus would apply it when he did not attempt to define the time more exactly than to the nearest quarter of a day. I have not, however, introduced an artificial difference into the observed times by computing the small variations in the equation of time at different autumn or different spring equinoxes. The Sun's true longitude was computed by the method of which an example is given on p. 32 of Newcomb's *Tables of the Sun*, in which planetary terms and nutation are ignored. The apparent longitude was found by applying a constant of −21″ at each autumn equinox and of −20″ at each spring equinox.

TABLE I.

| No. | Date. d h m | Tabular Longitude. ° ′ ″ | Observed − Tabular Longitude. ′ ″ |
|---|---|---|---|
| 1 | −161 Sept. 27 4 0 | 180 30 52 | −30 52 |
| 2 | −158 Sept. 26 16 0 | 180 17 14 | −17 14 |
| 3 | −157 Sept. 26 22 0 | 180 17 41 | −17 41 |
| 4 | −146 Sept. 26 10 0 | 180 7 35 | − 7 35 |
| 5 | −145 Sept. 26 16 0 | 180 8 2 | − 8 2 |
| 6 | −142 Sept. 26 4 0 | 179 54 21 | + 5 39 |
| 7 | −145 Mar. 23 16 15 | 359 30 55 | +29 5 |
| 8 | −144 Mar. 22 22 15 | 359 31 23 | +28 37 |
| 9 | −143 Mar. 23 4 15 | 359 31 50 | +28 10 |
| 10 | −142 Mar. 23 10 15 | 359 32 17 | +27 43 |
| 11 | −141 Mar. 23 16 15 | 359 32 46 | +27 14 |
| 12 | −140 Mar. 22 22 15 | 359 33 14 | +26 46 |
| 13 | −134 Mar. 23 10 15 | 359 35 58 | +24 2 |
| 14 | −133 Mar. 23 16 15 | 359 36 26 | +23 34 |
| 15 | −132 Mar. 22 22 15 | 359 36 53 | +23 7 |
| 16 | −131 Mar. 23 4 15 | 359 37 21 | +22 39 |
| 17 | −130 Mar. 23 10 15 | 359 37 49 | +22 11 |
| 18 | −129 Mar. 23 16 15 | 359 38 16 | +21 44 |
| 19 | −128 Mar. 22 22 15 | 359 38 44 | +21 16 |
| 20 | −127 Mar. 23 4 15 | 359 39 11 | +20 49 |

The only error in the modern tables sufficiently large to be detected by an analysis of these observations is in the secular acceleration of the Sun's mean motion. The ancient observations must be regarded as subject to a constant error in declination which would include the effect of any constant error in the setting or graduation of the instruments and the effect of the neglect of refraction. We shall have to consider later the effect of expressing the result to the nearest quarter of a day only; this indefiniteness of expression will affect the probable error of the result, but does not provide any additional term which can be determined by an analysis of the observations. There appears to be no other error which would have a systematic effect on the result. I have, therefore, analysed the observations for two unknown quantities, (1) the secular acceleration of the Sun, (2) the constant correction to be applied to Hipparchus' declinations.

If $s$ be the secular acceleration expressed in seconds of arc, $\epsilon$ the obliquity of the ecliptic, $g$ the Sun's mean anomaly, T the time measured in Julian centuries from 1800 Jan. 0·0, and $\odot$ the longitude of the Sun, the correction to the Sun's declination due to secular acceleration will be $s\mathrm{T}^2 \sin \epsilon \cos \odot (1 + 0{\cdot}0351 \cos g + 0{\cdot}0004 \cos 2g)$, where $\cos \odot = -1$ at autumn equinoxes and $+1$ at spring equinoxes. I have computed the obliquity of the ecliptic from the formula

$$\epsilon = 23^\circ\ 27'\ 31''{\cdot}85 - 47''{\cdot}090\mathrm{T} - 0''{\cdot}009\mathrm{T}^2 + 0''{\cdot}0018\mathrm{T}^3,$$

where T is the time measured in solar centuries from 1850·0. In this formula the obliquity at epoch and centennial motion are taken from Dr. Brown's paper in *Monthly Notices*, **75**, 507, 508 (1915). The same value for the centennial motion may be obtained by multiplying Boss's value for planetary precession in longitude as given in his *Preliminary General Catalogue* (1910) by $\cot N \tan \epsilon$, where $N$ is the angular distance of the instantaneous axis of rotation of the ecliptic from the equinox. The coefficients of $\mathrm{T}^2$ and $\mathrm{T}^3$ were obtained by computing the centennial motion with Dr. Brown's correction to obliquity and Boss's corrections to precession for successive epochs separated by half-centuries from 1750 to 2100, and performing a least squares analysis for two unknown quantities varying respectively with the square and cube of the time. The formula was obtained with a view to all work on ancient observations; a less exact value would have been sufficient for the present purpose.

The value of $g$ was taken from the quantity M in Newcomb's *Tables of the Sun* by the formula, $g = 0^\circ{\cdot}9856003\,(\mathrm{M} - 5{\cdot}370)$. The differences between observed and tabular longitude were converted into declination by the factor $\sin \epsilon \cos \odot$. All work except the computation of $g$ was done with four-figure tables. That is the reason why the last figures in the next table do not range. Putting $\delta$ for the constant error in Hipparchus' declinations, I thus obtained the following equations of condition:—

TABLE II.

| | | | |
|---|---|---|---|
| 1 | $-157{\cdot}0s$ | $-\delta$ | $=+745''{\cdot}0$ |
| 2 | $-156{\cdot}4s$ | $-\delta$ | $=+415{\cdot}9$ |
| 3 | $-156{\cdot}3s$ | $-\delta$ | $=+426{\cdot}8$ |
| 4 | $-154{\cdot}5s$ | $-\delta$ | $=+183{\cdot}0$ |
| 5 | $-154{\cdot}4s$ | $-\delta$ | $=+193{\cdot}8$ |
| 6 | $-153{\cdot}8s$ | $-\delta$ | $=-136{\cdot}4$ |
| 7 | $+150{\cdot}2s$ | $-\delta$ | $=+701{\cdot}8$ |
| 8 | $+150{\cdot}0s$ | $-\delta$ | $=+690{\cdot}5$ |
| 9 | $+149{\cdot}8s$ | $-\delta$ | $=+679{\cdot}8$ |
| 10 | $+149{\cdot}6s$ | $-\delta$ | $=+668{\cdot}9$ |
| 11 | $+149{\cdot}5s$ | $-\delta$ | $=+657{\cdot}3$ |
| 12 | $+149{\cdot}4s$ | $-\delta$ | $=+646{\cdot}0$ |
| 13 | $+148{\cdot}5s$ | $-\delta$ | $=+579{\cdot}9$ |
| 14 | $+148{\cdot}3s$ | $-\delta$ | $=+568{\cdot}5$ |
| 15 | $+148{\cdot}1s$ | $-\delta$ | $=+557{\cdot}8$ |
| 16 | $+148{\cdot}0s$ | $-\delta$ | $=+546{\cdot}6$ |
| 17 | $+147{\cdot}8s$ | $-\delta$ | $=+535{\cdot}3$ |
| 18 | $+147{\cdot}7s$ | $-\delta$ | $=+524{\cdot}4$ |
| 19 | $+147{\cdot}5s$ | $-\delta$ | $=+513{\cdot}3$ |
| 20 | $+147{\cdot}3s$ | $-\delta$ | $=+502{\cdot}3$ |

I have treated all the equations as of equal weight in spite of the statement that the 4th and 7th were made with special accuracy, because it appeared that the practice of expressing the observed time in quarters of a day would smother the superior accuracy of any particular observations.

### *First Analysis of Observations.*

A least squares solution gives the results—

$$s=+1''{\cdot}0\pm0''{\cdot}18, \quad \delta=-7'{\cdot}6\pm0'{\cdot}46 \quad . \quad . \quad (1)$$

The probable error in each observed declination after correcting for the constant error in declination amounts to $\pm1'{\cdot}9$. Of the constant error in declination, which it may be simpler to describe as the equator error, $-0'{\cdot}7$ is explained by Hipparchus' ignorance of refraction.

But this result requires considerable modification. The effect of giving the observed times to the nearest quarter of a day is to divide the observations into groups, producing an artificial concord between observations belonging to the same group and an artificial discord between observations of the same equinox belonging to different groups. The reason for this is that the unit of time, a quarter of a day, is nearly an exact factor of the tropical year, and in consequence the interval between two successive observations of the same equinox appears either as an exact multiple of 365 days 6 hours, or as 6 hours less than such a multiple. While the spring observations show the uniform interval of 365 days 6 hours and thus possess an artificial concord, the autumn observations disclose three leaps of 6 hours, exaggerating the discord where the leap takes place, *i.e.* between the first and second, third and fourth, fifth and sixth observations, but minimising it between the second and third, fourth and fifth observations. A least squares solution can take proper account of the discordances of the six autumn observations, but not of the artificial character of the concord of the fourteen spring observations. If there were no errors of observation, the maximum inaccuracy in each longitude of the Sun resulting from the expression of the observed time to the nearest fourth of a day would be one-eighth of the daily motion of the Sun, which at the dates of these observations would be about $437''\cdot 7$. The semi-range of the tabular longitudes of the fourteen observations is $248''$. It therefore follows that a systematic error of $189''\cdot 7$ may be lurking in these longitudes. As it is probable that Hipparchus did not always determine the time exactly to the nearest quarter of a day, a larger error is possible; but it is equally possible, I might say probable, that if some of the observed times had lain almost exactly three hours distant from noon or midnight, they would have been attributed to the wrong quarter and have disturbed the apparent concord. A separate analysis of the residuals obtained from these fourteen observations, if conducted by the ordinary method, gives $\pm 13''$ as the probable error attaching to the mean correction of declination, whereas the probable error really arising from the inexactness of the statements of the observed times should be $\pm \frac{1}{2} \sin \epsilon \; 189''\cdot 7$ or $\pm 38''$. It will be seen, then, that the error attaching to these observations is only very imperfectly represented by a least squares solution, and we shall not greatly exaggerate the probable error attaching to the result if we treat the probable error arising from the inexactness of the statements of time as independent of the probable error arising from the discordances. I have therefore tried the effect of introducing a constant correction of $+189''\cdot 7$ into the observed $-$ tabular longitudes for the fourteen spring equinoxes. Solving the twenty equations of condition with this correction, I get an addition of $+0''\cdot 25$ to the value of $s$ determined above, and of $-0'\cdot 65$ to the value of $\delta$ determined above. The probable error arising from the inexactness of stated times should be half these quantities, *i.e.* $\pm 0''\cdot 13$ in $s$ and $\pm 0'\cdot 33$ in $\delta$. Combining these probable errors with those deduced from the discordances, I get

$$s = +1''\cdot 0 \pm 0''\cdot 22, \; \delta = -7'\cdot 6 \pm 0'\cdot 56 \quad . \quad . \quad (2)$$

*Comparison with Stellar Observations.*

We are happily able to test this conclusion, so far as the declinations are concerned, by Hipparchus' observations of star declinations. A large number of declinations are given in an early work of Hipparchus, his *Commentary on Aratus*, while eighteen are cited by Ptolemy,* apparently from the treatise on the "Displacement of the Solstitial and Equinoctial Signs." Ptolemy treats these observations as having been made 265 years before his own, which, as he tells us elsewhere,† were mostly made in the first year of Antonine (+137 July 20 to +138 July 19). The beginning of that year was, as is well known, the epoch of Ptolemy's star catalogue.‡ I have, therefore, taken from Dr. Neugebauer's *Sterntafeln*,§ which are sufficiently accurate for our purpose, the declinations of these eighteen stars for the epoch −127·28, which is 265 Egyptian years before +137·55, the epoch of Ptolemy's catalogue. The eighteen observations show that Hipparchus' declinations require a mean correction of $+0'\cdot5$; but, as we are only concerned in the present investigation with the error of his equator, I have made a separate analysis of observations within 10° of the equator. I tabulate accordingly:

TABLE III.

| | Hipparchus' Declination. | Declination according to Neugebauer. | Hipparchus' Error. (Tabular − Observed). |
|---|---|---|---|
| | ° | ° | ° |
| Altair . . . | +5·80 | +5·68 | −0·12 |
| Aldebaran . . | +9·75 | +9·70 | −0·05 |
| Bellatrix . . | +1·80 | +1·79 | −0·01 |
| Betelguese . . | +4·33 | +4·25 | −0·08 |
| Spica . . . | +0·60 | +0·54 | −0·06 |
| α Libræ . . . | −5·60 | −5·64 | −0·04 |
| β Libræ . . | +0·40 | +0·25 | −0·15 |

It will be observed that in each instance Hipparchus places his equator too far to the south. Using the same nomenclature as before, I find

$$\delta = -4'\cdot4 \pm 0'\cdot74 \quad . \quad . \quad . \quad . \quad (3)$$

where, as before, $-0'\cdot7$ is the error due to refraction, while the probable error of a single observation after correction of the equator is $\pm 2'\cdot0$, or almost the same as that found from the equinox observations.

The large discrepancy between the false equators found from the two series of observations is most probably explained by the supposition that Hipparchus revised his equator. As the equinox observations are spread over 33½ years, while the star declinations

* *Ubi supra*, ii. (1903), pp. 19–23. † *Ibid.*, p. 15. ‡ *Ibid.*, p. 36.
§ *Tafeln zur astronomischen Chronologie*, i. (1912).

were observed at the very end of that range of dates, Hipparchus would not be the man we conceive him to have been, if his equator at the latter date had not been considerably better than his mean equator for the whole range of dates. With a view to obtaining light on this revision of the equator, I examined the four declinations of stars within 10° of the equator given in the *Commentary on Aratus*, computing the declinations where they were not given by Neugebauer. I found, however, that the discordances were too great to give any reliable indication of the equator to which they were referred.

### *Discussion of Assorted Observations.*

Had the spring and autumn observations been equally distributed over the whole interval under discussion, the revision of the equator would have been unimportant. But, as it is, the mean epoch of the autumn observations is much earlier than that of the spring observations, and there is therefore every reason to suppose that there was an appreciable difference between the mean equators of the two series. I have, therefore, thought it best to make a separate analysis of observations 4–11, containing three autumn and five spring equinoxes, the two series being not unevenly distributed. The analysis yielded the results, $s = +2''\cdot 0$, $\delta = -6'\cdot 4$. The probable errors found in the usual way were $\pm 0''\cdot 18$ for $s$ and $\pm 0'\cdot 45$ for $\delta$.

I then tried the effect of adding to the spring declinations a constant of $154''\cdot 5$, the largest addition consistent with a uniform statement of time to the nearest fourth of a day. This added $+0''\cdot 51$ to the value of $s$ and $-1'\cdot 31$ to the value of $\delta$. I therefore took as probable errors due to the method of stating time half these quantities, viz. $\pm 0''\cdot 25$ and $\pm 0'\cdot 65$ for $s$ and $\delta$ respectively. The combination of the probable errors from the two sources gives

$$s = +2''\cdot 0 \pm 0''\cdot 31, \quad \delta = -6'\cdot 4 \pm 0'\cdot 79 \qquad . \qquad . \qquad (4)$$

The error in Hipparchus' equator as determined from the star declinations places us in a position to determine the secular acceleration from the spring equinoxes alone for the years in the neighbourhood of the star observations. It will be observed that there is a gap of six years between observations 12 and 13, after which we have eight observations made in consecutive years, the epoch of the star observations being near the close of the series. I have, therefore, made a separate analysis of these eight observations, taking the equator correction and also its probable error from the analysis of star declinations. In this series the discordances of the original observations are smothered by the method of stating the time, and I have, therefore, not attempted to find the probable error to be deduced from the discordances. I have assigned to each observation a weight proportional to the square of the coefficient of $s$ in the equation of condition and determined the weighted mean value of $s$, which comes out as $+1''\cdot 9 \pm 0''\cdot 30$. The probable error thus obtained must be modified by the probable error arising from the method of stating the time. This time I find that the maximum constant correction to the declinations is $\pm 137''\cdot 25$. I accordingly treat half this quantity as a probable error attaching to each observed declination and find a probable error from this source of $\pm 0''\cdot 46$. The combination of the two probable errors gives as the secular acceleration resulting from observations 13–20:

$$s = +1''\cdot 9 \pm 0''\cdot 55 \quad . \quad . \quad . \quad . \qquad (5)$$

This agrees very satisfactorily with the value independently determined from observations 4–11. A combination of the two series gives the final value:—

Correction to Newcomb's secular acceleration of the Sun

$$= +1''\cdot 95 \pm 0''\cdot 27 \quad . \quad . \quad . \quad . \qquad (6)$$

As Newcomb's value for the acceleration is only $-0''\cdot 02$, it is unimportant whether we call the value just determined the acceleration or the correction to the acceleration.

I append a separate determination from observations 4 and 7 for what it may be worth, though I have questioned above whether, in view of the method of stating the time, their superior accuracy is of any importance. These two observations give $s = +1''\cdot 7$, $\delta = -7'\cdot 4$. As the probable error of the mean of two observations should be twice that given in (4) as the mean of eight observations, it will be seen that these results are well within the probable error attaching to the series to which they belong.

### *Observation with the κρίκος.*

The single observation made with the *κρίκος* at Alexandria is of interest because of the value attached to this instrument by Hipparchus.* The instrument was fixed in the plane of the equator with a slit so placed that the illumination from the Sun's rays would pass from one side to the other when he crossed the equator. Ptolemy † raises the objection that, if permanently fixed, it is liable to be affected by a settlement in the ground, as was proved, in his opinion, by the fact that it often showed the Sun's passage across the equator twice in one day. This phenomenon, as has been frequently remarked, may easily be explained by refraction.

Taking 29° 51′ East as the longitude of Alexandria, I compute as follows:—

| Date. | Tabular Longitude. | Observed – Tabular Longitude. |
|---|---|---|
| $-145$ Mar. $23^{d}$ $21^{h}$ $9^{m}$ | 359° 42′ 50″ | +17′ 10″ |

With terrestrial latitude 31° 11′, hour-angle 345°, and apparent declination 0, we get 34° 17′ for the Sun's apparent zenith distance

* Ptolemy, *Math. Syn.*, i. p. 195. † *Ibid.*, p. 197.

and a refraction of $39''\cdot7$. The error in declination due to the failure to correct for refraction will be given by the formula

$$\delta = -R \sin\phi \operatorname{cosec} z,$$

where $\delta$ is the correction to be applied to the observed declination, $R$ is refraction, $\phi$ is the terrestrial latitude, and $z$ is the apparent zenith distance. In the present instance $\delta$ will be $= -36''\cdot5$. Assuming that there is no error in the observed declination apart from refraction, taking the coefficient of $s$ from No. 7 in Table II. above, and reducing the difference between observed and tabular longitudes to declination by the factor $\sin\epsilon \cos \odot$ as above, I get the equation

$$+150\cdot2s + 36''\cdot5 = 414''\cdot3$$

from which it follows that $s = +2''\cdot5$.

This would be the correct value, if we could be sure that the κρίκος was set in the true plane of the equator. But, since Hipparchus at Rhodes and Ptolemy at Alexandria both placed the equator too far to the south, there is a presumption that the κρίκος was misplaced in the same direction, and the result probably requires a negative correction which we have no means of determining. For this reason the observation made with the κρίκος cannot be used to determine the secular acceleration, but only to confirm a result obtained by other instruments whose errors can be determined.

### *Comparisons of Eclipsed Moon with Spica.*

In addition to the series of twenty observations and the one observation made with the κρίκος, Hipparchus recorded a number of observations in which he had compared the position of the eclipsed Moon with Spica. Eclipses were selected near the spring equinox. The motion of the Sun between the equinox and the middle of the eclipse was taken from his theory of the Sun; the Moon was assumed to be opposite the Sun, and parallax was applied in order to obtain her apparent position.* The observed difference in longitude was supposed to give the longitude of Spica, and the fluctuations in this longitude as resulting from different observations were supposed to be a measure of the oscillation of the equinoctial point. As having led the greatest of ancient astronomers to favour a false hypothesis, these observations will always be of historical interest. Unfortunately for our present purpose, Ptolemy has cited no figures except two of the inferred longitudes of Spica and the dates of the equinoxes from which these were computed, and the two longitudes that he has selected are admitted to be the most discrepant of the series, † and are, therefore, in

* Ptolemy makes no reference to parallax in his summary of Hipparchus' method, *ubi supra*, p. 199, but he makes the suggestion a little lower that Hipparchus' parallax may have been erroneous, *ibid.*, p. 200.

† *Ibid.*, p. 198.

all probability the least accurately observed. Happily we are sufficiently acquainted with Hipparchus' theory to compute the motion of the Sun between the equinox and the middle of the eclipse, and to make an approximation to the necessary correction for parallax; combining these with the inferred longitude of Spica, we can estimate to within a few minutes the observed difference in longitude between the Moon and Spica. I have taken the times of the eclipses from Oppolzer's "Syzygientafeln,"* and have computed the Sun's longitude and Moon's parallax both from Hipparchus' theory and from Newcomb's tables combined with modern theory. The exact time is unimportant, because a small error would affect equally the lunar place as determined by Hipparchus and by modern theory. I have thought it unnecessary to print any details of the computation, because the result would hardly justify such a trespass on the Society's space. But the following results may be of interest:—

TABLE IV.

| | | Modern Theory. | | |
|---|---|---|---|---|
| | Greenwich Mean Time. | Apparent Longitude of Spica, as affected by Refraction. | Apparent Longitude of Moon, as affected by Parallax and Refraction. | Difference in Apparent Longitude. |
| | d h m | ° ′ ″ | ° ′ ″ | ° ′ ″ |
| 1 | −145 Apr. 21 9 26 | 174 3 26 | 207 36 36 | +33 33 10 |
| 2 | −134 Mar. 20 12 47 | 174 12 47 | 176 34 42 | + 2 21 55 |

| | Hipparchus' Theory. | | | |
|---|---|---|---|---|
| | Inferred Longitude of Spica. | Apparent Longitude of Moon. | Observed Difference in Longitude. | Correction to Tabular Longitude. |
| | ° ′ | ° ′ ″ | ° ′ ″ | ′ ″ |
| 1 | 173 30 | 207 55 32 | 34 25 32 | +52 22 |
| 2 | 174 45 | 176 58 36 | 2 13 36 | − 8 19 |

| | Secular Acceleration of Sun. |
|---|---|
| | ″ |
| 1 | +9·0 |
| 2 | −1·3 |

The weighted mean of the two results for the secular acceleration of the Sun is $+3''\cdot7$, and, if it is permissible to derive a probable error from the discordance of only two observations, the probable error according to the ordinary theory should be $\pm 3''\cdot5$. This, however, does not take account of the fact that the two observations are the two extremes of a series; in the present case the mean ought to possess a higher value than the mean of two observations selected at haphazard, but, even so, the observations can scarcely do more than support the likelihood of the existence

* *Publication der astronomischen Gesellschaft*, 16 (1881).

of a secular acceleration of the Sun. They do not help us to determine its value.*

*Ptolemy's Equinox Observations.*

This completes the equinox observations cited by Ptolemy from Hipparchus. For the sake of completeness I proceed to the discussion of those of his own equinox observations which Ptolemy has recorded. One of these is recorded in the seventh chapter of the third book and two others in the first chapter of the third book of the *μαθηματικὴ σύνταξις*, from which Hipparchus' observations were cited. I quote the actual records of all three observations, though I recommend my readers to study the whole of the third book if they wish to grasp the conditions of ancient observations and their relation to the theory of the Sun's motion. The first of these observations possesses a special interest, because it was the means of that false determination of the equinox which gave rise to the curious allegation that Ptolemy's star catalogue was not authentic.

"One of the most accurate of the first equinoxes observed by us was an autumn equinox which fell in the 17th year of Hadrian on the Egyptian 7th of Athyr two equinoctial hours approximately after noon." †

"And after 285 years, in the 3rd year of Antonine, which is the 463rd after the death of Alexander, we observed again most reliably the autumn equinox which fell on the 9th of Athyr one hour approximately after sunrise." ‡

"And we find that the spring equinox, which was similarly 285 years later in the 463rd year from the death of Alexander, fell on the 7th of Pachon one hour approximately after noon." §

Computing as in Table I. above, except that I have on the present occasion introduced the largest term in nutation into the tabular longitude, I find :—

TABLE V.

| | Date. | Tabular Longitude. | Observed – Tabular Longitude. |
|---|---|---|---|
| | d h m | ° ′ ″ | ° ′ ″ |
| 1 | + 132 Sept. 24 23 53 | 181 17 25 | − 1 17 25 |
| 2 | + 139 Sept. 25 16 50 | 181 17 26 | − 1 17 26 |
| 3 | + 140 Mar. 21 23 9 | 0 45 14 | − 0 45 14 |

* An opinion has recently become current that, because Hipparchus found that Spica had a longitude of 174° in his time, compared with one of 172° in the time of Timocharis (Ptolemy, *ubi supra*, ii. pp. 12, 13), he must have adopted a value of precession doing justice to this change of longitude. But in view of the discordances in his own observations of Spica, it is unlikely that Hipparchus drew any such conclusion, and there is no evidence that he ever attempted anything more precise than his minimum value of 36″ per annum.

† *Math. Syn.*, ed. Heiberg, i. p. 256.

‡ *Ibid.*, p. 204. § *Ibid.*, pp. 204, 205.

I then proceed as in the case of Hipparchus' observations, substituting $+0{\cdot}0349$ for $+0{\cdot}0351$ as the coefficient of cos $g$ in the formula for the correction to the Sun's declination due to secular acceleration. This substitution is necessitated by the diminution in the eccentricity of the Earth's orbit between the epochs of Hipparchus' and Ptolemy's observations. As I have chosen to introduce nutation, the formula for obliquity has received the addition of the term $+9''$ cos $\Omega$, where $\Omega$ is the longitude of the Moon's ascending node. I thus form the following table:—

TABLE VI.

| | | | |
|---|---|---|---|
| 1 | $-113{\cdot}0s$ | $-\delta$ | $=+1865''$ |
| 2 | $-112{\cdot}1s$ | $-\delta$ | $=+1865$ |
| 3 | $+109{\cdot}4s$ | $-\delta$ | $=-1090$ |

These give the result

$$s=-13''{\cdot}3\pm0''{\cdot}035,\quad \delta=-6'\ 7''\pm3''{\cdot}9 \quad . \quad (7)$$

The smallness of the probable error attaching to a result so greatly at variance with those derived from other observations is doubtless due to two causes. (1) The observations are so few in number that an accidental concord can easily affect the apparent probable error. (2) The observations are selected for their accuracy, *i.e.* in all probability for their agreement with Hipparchus' tabular places, and hence show a surprising agreement in favour of a false result.

*Comparison with Stellar Observations.*

This will become more apparent when we compare the declination error with that found from the star declinations. I have compared Ptolemy's star declinations as given in the third chapter of the seventh book of the *μαθηματικὴ σύνταξις*,* with the declinations in Neugebauer's tables for $+137{\cdot}55$, the epoch of Ptolemy's catalogue. I find that Ptolemy's eighteen declinations require a mean correction of $-2'{\cdot}4$. As before, I tabulate separately the observations within $10°$ of the equator.

TABLE VII.

| | Ptolemy's Declination. | Declination according to Neugebauer. | Ptolemy's Error. (Tabular – Observed.) |
|---|---|---|---|
| Altair . . | $+5{\overset{\circ}{\cdot}}83$ | $+5{\overset{\circ}{\cdot}}79$ | $-0{\overset{\circ}{\cdot}}04$ |
| Bellatrix . . | $+2{\cdot}50$ | $+2{\cdot}65$ | $+0{\cdot}15$ |
| Betelguese . . | $+5{\cdot}25$ | $+4{\cdot}95$ | $-0{\cdot}30$ |
| Spica . . | $-0{\cdot}50$ | $-0{\cdot}94$ | $-0{\cdot}44$ |
| α Libræ . . | $-7{\cdot}17$ | $-7{\cdot}07$ | $+0{\cdot}10$ |
| β Libræ . . | $-1{\cdot}00$ | $-1{\cdot}11$ | $-0{\cdot}11$ |

* *Ubi supra*, ii. pp. 19–23.

These give the result $\delta = -6'\cdot5 \pm 3'\cdot8$, while the probable error of a single observation after correction of the equator comes out as $\pm 9'\cdot3$, or nearly five times as much as that of a similar observation by Hipparchus. The discordances in the star declinations are clear proof that the agreement of the equinox observations is either fortuitous or artificial. I have thought it worth while to work up the equinox and stellar declinations as a single series of observations, allowing double weight to the equinox observations as being the mean of two observations in each case. The result then becomes

$$s = -13''\cdot3 \pm 1''\cdot8, \quad \delta = -6'\cdot3 \pm 2'\cdot3 \qquad . \qquad . \qquad (8)$$

From a comparison of the probable error attaching to $s$ in this equation with that found in (6), it would appear that even if Ptolemy's observations had not been selected on the ground of their accord with Hipparchus' tables, the secular acceleration deduced from Hipparchus' observations would be entitled to a weight forty-three times as great as that deduced from Ptolemy's observations. The latter can hardly be used to modify the value given in (6).

### *The Equinox of Ptolemy's Star Catalogue.*

Of the three equinox observations just discussed the first is that from which Ptolemy deduced the epoch of the Sun's longitude. From Hipparchus' theory of the Sun he finds that at the autumn equinox the equation of the centre should be $-2° \ 10'$, and therefore makes the mean longitude of the Sun at the date of this observation $182° \ 10'$. Newcomb's tables give for the mean longitude unaffected by aberration $183° \ 9' \ 31''$. The correction for secular acceleration found in (6) is $+9' \ 2''$, so that the corrected mean longitude should be $183° \ 18' \ 33''$, and the correction required to reduce Ptolemy's mean longitude to the true mean longitude is $+1° \ 8'\cdot5$. Hipparchus' mean motion of the Sun in a Julian year is $360° \ 0' \ 12''$. At the epoch in question the true value was $360° \ 0' \ 26''$. Hipparchus' error is therefore $+14''$ per annum. In 4·82 years, the interval between the observation and the epoch of the star catalogue, the error in the mean motion would amount to $+1'\cdot2$, so that the error in the deduced mean longitude at the epoch of the star catalogue would be $+1° \ 9'\cdot7$. Now Mr. Knobel has found * that the mean error of Ptolemy's longitudes of zodiacal stars, when compared with the true longitudes for $+100$, is $+34'\cdot9$. 37·55 years precession at $49''\cdot9$ per annum amounts to $+31'\cdot2$, so that the mean error in the star longitudes for $+137\cdot55$, the epoch of the star catalogue, is $+1° \ 6'\cdot1$. It is clear, therefore, that the false equinox in the catalogue reproduces to within a few minutes the false equinox as obtained five years

* C. H. F. Peters and Knobel, *Ptolemy's Catalogue of Stars: a Revision of the Almagest*, Carnegie Institution of Washington, **86**, 15, 17 (1915).