

NOTES ON THE COLOR-CURVE AND LIGHT-ELEMENTS OF W URSAE MAJORIS¹

BY HARLOW SHAPLEY AND J. VAN DER BILT

The variable W Ursae Majoris² is of particular interest because of its exceptionally short period (four hours between minima) and because when interpreted as an eclipsing binary its mean density is more than twice that of the sun. Discussions of the orbit have been published by H. N. Russell³ and H. Shapley.⁴ A further study of the light-variations and orbit, on the basis of simultaneous photographic and photo-visual observations, was begun with the 60-inch reflector in 1916. The work was discontinued, however, upon learning that a similar investigation, based on Potsdam visual observations and unpublished photographic measures from Harvard, was already under way at Princeton.⁵ The following pages contain a brief discussion of the color-curve obtained from the preliminary Mount Wilson observations, and an investigation of the light-elements based upon a considerable number of published and unpublished observations of minima during the last thirteen years.

OBSERVATIONS AT MOUNT WILSON AND UTRECHT

The magnitudes of the comparison stars used on the photographs with the 60-inch reflector were determined in the usual manner from ten polar-comparison plates (Table I). Magnitudes of the variable, derived from a series of Seed 27 and Cramer isochromatic plates made on April 26, 1916, are given in Table II. Six, seven, or eight exposures were made on each plate with exposure-

¹ *Contributions from the Mount Wilson Solar Observatory*, No. 140.

² W Ursae Majoris = B.D. + 56° 14 00; $\alpha = 9^{\text{h}} 36^{\text{m}} 43^{\text{s}}$, $\delta = +56^{\circ} 24' 5$ (1900.0); spectrum G.

³ *Astrophysical Journal*, 36, 139, 1912.

⁴ *Contributions from the Princeton University Observatory*, No. 3, p. 82, 1915.

⁵ Since this paper was written the Princeton results have appeared in *Astrophysical Journal*, 45, 306, 1917.

times of one-half, one, or two minutes. The phases were computed with the formula given below. A plot of these results shows that the color-index on the Mount Wilson system is about $+0.85$, with little evidence of change during the cycle of light-variation.¹

TABLE I
MAGNITUDES OF COMPARISON STARS

STAR	B.D.	H.A., 63 VISUAL	MOUNT WILSON		COLOR-INDEX
			Photo-visual	Photographic	
a.....	$+56^{\circ}1397$		6.64	7.65	$+1.01$
b.....	$+56^{\circ}1399$	8.74	8.76	9.90	$+1.14$
c.....	$+56^{\circ}1398$	8.73	8.80	10.08	$+1.28$

TABLE II
MOUNT WILSON OBSERVATIONS OF W URSAE MAJORIS
April 26, 1916; J.D. 2420980

PHOTOGRAPHIC					PHOTO-VISUAL				
Plate	Gr. H.M.T.	Apertures	Phase	Mag.	Plate	Gr. H.M.T.	Apertures	Phase	Mag.
3034P	d672	14, 9, 6	$-0^{\circ}142$	8.76	3035P	d676	14, 9, 6	$-0^{\circ}138$	7.91
3036...	.702	14, 9	$-0^{\circ}112$	8.70	3037...	.706	14	$-0^{\circ}108$	
3038...	.720	14, 9	$-0^{\circ}094$	8.64	3039...	.724	14, 9	$-0^{\circ}090$	7.88
3040...	.737	14, 9	$-0^{\circ}077$	8.81	3041...	.740	14, 9	$-0^{\circ}074$	7.96
3042...	.754	14, 6	$-0^{\circ}060$	8.84	3043...	.758	14, 9	$-0^{\circ}056$	8.07
3044...	.771	14, 9	$-0^{\circ}043$	8.96	3045...	.774	14, 9	$-0^{\circ}040$	8.03
3046...	.789	14	$-0^{\circ}025$	9.06	3047...	.792	14, 9	$-0^{\circ}022$	8.18
3048...	.802	14, 9, 6	$-0^{\circ}012$	9.20	3049...	.805	14, 9	$-0^{\circ}009$	8.41
3050...	.814	14, 9	$-0^{\circ}000$	9.27	3051...	.817	14, 9	$+0^{\circ}003$	8.29
3052...	.829	14, 9, 6	$+0^{\circ}015$	8.79	3053...	.832	14, 9	$+0^{\circ}018$	8.15
3054...	.841	14, 9	$+0^{\circ}027$	8.67	3055...	.844	14, 9	$+0^{\circ}030$	7.98
3056...	.856	14, 9, 6	$+0^{\circ}042$	8.52	3057...	.860	14, 9, 6	$+0^{\circ}046$	7.81

Though the light-curves of this variable often show an asymmetry which, if real, could not be accounted for completely by the simple eclipse theory, the star evidently is not a Cepheid variable, as is sometimes supposed, since no conspicuous change of color is found. Moreover, Russell has no difficulty in interpreting the light-curve, or in explaining the differences between visual and photographic curves through differences in darkening at the limb.²

¹ The last two plates are uncertain because of the low altitude of the field.

² *Popular Astronomy*, 25, 30, 1917.

At the Utrecht Observatory 105 observations of *W Ursae Majoris* were obtained with the polarizing photometer attached to the $4\frac{1}{2}$ -inch refractor.¹ At maximum the variable is a little too bright for this instrument, but the minima can be determined very sharply. The following comparison stars were used:

$$\begin{array}{ll} a = \text{B.D.} + 54^{\circ}1329 = \text{Potsdam Gen. Cat. 5731; Mag.} = 7.73 \\ b = \quad + 56^{\circ}1412 = a + 0^{\text{M}}_{33} & 8.06 \\ c = \quad + 56^{\circ}1399 = a + 1 \ 38 & 9.11 \end{array}$$

The brightness of both *b* and *c* was derived from the measured differences with *a*.

The method of observation differed somewhat from the usual one inasmuch as more settings were made on the rapidly changing variable than on the comparison stars. In the interval covered by the observations on each evening three to five measures of each of the comparison stars were secured, and these were plotted against the time on squared paper. After allowance for extinction, the settings were usually remarkably constant and could be read accurately for the times corresponding to the measures on the variable. In this way the latter could be closely followed through the steep ascents and descents of its light-curve.

The Utrecht observations are given in detail in Table III, the fifth column containing the differences in hundredths of a magnitude between the adopted value and that given by each of the comparison stars.

THE LIGHT-ELEMENTS

Since about 6000 epochs have elapsed since the latest discussion, a revision of the light-elements can now be made. In view of the possibility that the light-curve may be changeable in form, as many individual minima are included in the computation as can be derived with sufficient accuracy from the published series of observations.

The list in Table IV contains 67 minima, extending over an interval of 29,273 epochs. That numbered 61 has not been used in the derivation of elements. Nos. 59 and 60 were received from Professor Jordan after the completion of the solution and have been

¹ The instrument has been described in *Recherches astronomique de l'Observatoire d'Utrecht*, 6, 10, 1916.

TABLE III
UTRECHT OBSERVATIONS OF W URSAE MAJORIS

Date	J.D. and Gr.M.T. 2420000+	Phase	Mag.	Residuals*		
				a	b	c
1916 April 28.....	{ 0982.461 .505 .548	0.071 .116 .158	8.42	+ 5, ...	- 5	
			8.16	- 2, ...	+ 3	
			7.72	+ 1, ...	0	
April 29.....	{ 0983.408 .420 .429 .440 .452 .503 .515 .524 .535 .544	.018 .029 .039 .049 .062 .113 .125 .134 .144 .153	8.07	- 9, ...	+ 9	
			8.11	- 7, ...	+ 7	
			8.06	- 5, ...	+ 5	
			8.14	- 1, ...	+ 2	
			8.24	0, ...	0	
			8.30	+ 1, ...	0	
			8.23	+ 2, ...	- 2	
			7.97	+ 3, ...	- 2	
			7.90	+ 4, ...	- 3	
			7.87	+ 5, ...	- 5	
May 11.....	{ 0995.408 .421 .428 .433 .438 .444 .450 .459 .468 .473 .480 .489 .495 .501 .508 .514 .525 .547 .556	.008 .021 .028 .033 .038 .044 .050 .059 .068 .073 .080 .089 .095 .101 .108 .114 .125 .147 .156	7.80	- 11, ...	+ 12	
			7.94	- 9, ...	+ 10	
			7.87	- 8, ...	+ 8	
			7.92	- 7, ...	+ 7	
			8.02	- 6, ...	+ 7	
			8.06	- 5, ...	+ 6	
			8.15	- 5, ...	+ 5	
			8.24	- 4, ...	+ 5	
			8.48	- 4, ...	+ 5	
			8.43	- 4, ...	+ 4	
			8.65	- 4, ...	+ 5	
			9.09	- 3, ...	+ 3	
			8.71	- 1, ...	+ 2	
			8.76	0, ...	+ 1	
			8.54	+ 1, ...	0	
			8.30	+ 1, ...	0	
			8.16	+ 1, ...	0	
			7.99	+ 2, ...	- 1	
			7.99	+ 1, ...	0	
May 17.....	{ 1001.410 .423 .428 .434 .438 .448 .461 .465 .469 .474 .483 .486 .491 .495 .500 .505 .517 .527 .536	.005 .018 .023 .029 .033 .043 .056 .060 .065 .069 .078 .082 .086 .090 .095 .101 .112 .122 0.131	7.92	, + 1, - 1		
			7.92	, + 1, 0		
			7.88	, 0, 0		
			7.97	, 0, 0		
			7.99	, 0, 0		
			7.97	, - 2, + 3		
			8.14	, - 3, + 3		
			8.19	, - 3, + 3		
			8.28	, - 3, + 4		
			8.39	, - 4, + 5		
			8.48	, - 5, + 5		
			8.60	, - 5, + 5		
			8.69	, - 5, + 5		
			8.70	, - 5, + 5		
			8.55	, - 5, + 6		
			8.44	, - 5, + 5		
			8.32	, - 5, + 6		
			8.02	, - 6, + 6		
			7.89	, - 5, + 6		

* A positive residual means that the adopted magnitude is brighter than that derived from the comparison star.

TABLE III—Continued

Date	J.D. and Gr.M.T. 2420000+	Phase	Mag.	Residuals		
				<i>a</i>	<i>b</i>	<i>c</i>
1916	1003.408	0 ^d 001	7.95	+ 8,	- 4,	- 3
	.415	.009	7.88	+ 5,	- 3,	- 3
	.424	.019	7.98	+ 3,	- 2,	- 2
	.429	.024	7.96	+ 1,	- 1,	- 1
	.434	.027	8.00	0,	+ 1,	- 1
	.439	.034	7.99	- 2,	+ 2,	0
	.448	.043	8.01	- 4,	+ 3,	0
	.456	.051	7.99	- 4,	+ 4,	+ 1
	.465	.060	8.18	- 4,	+ 3,	+ 2
	.469	.064	8.26	- 4,	+ 3,	+ 2
	.473	.068	8.50	- 4,	+ 2,	+ 3
	.478	.072	8.53	- 4,	+ 2,	+ 3
	.482	.076	8.41	- 5,	+ 1,	+ 3
	.487	.081	8.47	- 5,	0,	+ 4
May 19.....	.496	.090	8.70	- 3,	- 1,	+ 4
	.500	.094	8.71	- 3,	- 1,	+ 3
	.505	.098	8.65	- 2,	- 1,	+ 3
	.510	.104	8.41	- 2,	- 1,	+ 2
	.520	.113	8.26	0,	0,	+ 1
	.530	.123	8.05	- 1,	+ 2,	- 2
	.541	.135	7.93	- 3,	+ 4,	0
	.554	.148	7.89	- 6,	+ 5,	+ 1
	1005.415	.007	8.18	,	+ 2,	- 2
	.421	.013	8.03	,	+ 3,	- 3
	.433	.025	8.10	,	+ 6,	- 6
	.443	.035	7.90	+ 9,	+ 1,	- 10
	.451	.043	8.01	+ 7,	+ 3,	- 9
	.459	.051	7.97	+ 6,	+ 2,	- 7
May 21.....	.466	.058	8.14	+ 5,	0,	- 6
	.472	.064	8.30	+ 4,	0,	- 5
	.475	.067	8.35	+ 5,	0,	- 5
	.479	.071	8.44	+ 5,	- 1,	- 4
	.483	.075	8.60	+ 5,	- 1,	- 4
	.487	.079	8.63	+ 5,	- 2,	- 3
	.490	.082	8.54	+ 5,	- 2,	- 2
	.496	.088	8.84	+ 5,	- 2,	- 4
	.500	.092	8.86	+ 6,	- 1,	- 5
	.505	.097	8.70	+ 7,	0,	- 6
	.509	.101	8.62	+ 8,	0,	- 8
	.516	.108	8.51	+ 10,	+ 1,	- 11
	.526	.118	8.28	+ 10,	+ 2,	- 13
	.533	.125	8.09	+ 11,	+ 2,	- 14
May 28.....	1012.420	.007	8.08	,	+ 3,	- 2
	.432	.018	8.01	,	0,	0
	.437	.023	7.96	,	- 1,	+ 2
	.442	.028	7.93	,	- 3,	+ 3
	.450	.036	8.07	,	- 6,	+ 6
	.459	.045	8.26	,	- 6,	+ 6
	.480	.067	8.52	,	- 2,	+ 2
	.486	.073	8.44	,	0,	0
	.499	.085	8.74	,	0,	+ 1
	.509	.095	8.57	,	- 2,	+ 2
	.517	.104	8.54	,	- 1,	+ 2
	.525	0.112	8.34	,	+ 1,	0

TABLE IV
LIST OF MINIMA

No.	OBSERVED MINIMA DATE	RED. TO SUN	EPOCH	OBSD. J.D. (HELIOC.)	O.-C.		OBSERVER	REMARKS	REFERENCE
					Days	Min.			
1.....	1903 Jan. 14	+6 ^m	0	6129.194	-0.006	8.0	MK		Ap. J., 17, 205
2.....	17	+6	18	6132.199	-	5.2	"		
3.....	17	+6	19	6132.359	-	11	"		
4.....	18	+6	25	6133.361	-	10	"		
5.....	Feb. 20	+6	222	6166.230	-	5.2	"		A.N., 4005
6.....	Apr. 24	0	601	6229.452	-	9.5	"		
7.....	1904 Jan. 10	+6	2165	6490.372	+	10.7	F		
8.....	17	+6	2207	6497.378	+	10.7	"		
9.....	27	+6	2266	6507.224	+	11	"	Uncertain	A.N., 3963
10.....	27	+6	2267	6507.387	+	10.7	"	Good	
11.....	Feb. 13	+6	2368	6524.222	-	9.5	MK		
12.....	17	+6	2393	6528.407	+	12.1	F	Fair	
13.....	19	+6	2404	6530.242	+	12.1	"	Fair	A.N., 3963
14.....	Mar. 14	+4	2548	6554.274	+	18	"	Uncertain	
15.....	Jan. 17	+6	4401	6803.367	-	8.0	MK		
16.....	Nov. 9	+1	6178	7159.803	+	11.8	PJ	Fair	
17.....	Dec. 7	+4	6346	7187.841	+	5.9	"	Poor	Ap. J., 23, 84
18.....	July 26	-6	7728	7418.376	-	8.0	MK	Good	
19.....	Mar. 4	+5	9052	7639.253	+	3.3	T	Orange filter*	
20.....	5	+5	9059	7640.419	+	0.9	"	Green-blue†	
21.....	7	+5	9071	7642.417	-	4.9	"	Orange†	Pulkova Mitt., 2, No. 21
22.....	24	+3	9172	7659.269	-	0.1	"	Orange*	
23.....	29	+3	9202	7664.277	+	4.9	"	Ultra-violet*	
24.....	30	+3	9208	7665.280	+	4.9	"	Ultra-violet*	
25.....	Apr. 1	+2	9220	7667.281	+	6.6	"	Ultra-violet†	
26.....	10	+2	9274	7676.289	+	6.2	"	Ultra-violet*	
27.....	12	+1	9286	7678.288	+	2.2	"	Orange§	
28.....	13	+1	9292	7679.292	+	6.6	"	Green-blue†	
29.....	15	+1	9304	7681.293	+	5.5	"	Green-blue†	
30.....	29	0	9389	7695.406	-	4.2	"	Orange*	
31.....	May 1	0	9401	7697.474	+	4.8	"	Ultra-violet*	

	1907	June 8	8 ^h 9 ^m	-4 ^m	9628	7735.337	-σ.002	2.3	Ba			
32.....			8 14	-4	9634	7736.340	+	0.6	"	Fair		
33.....		9	8 24	-5	9694	7740.347	+	2	"	Good		
34.....		19	9 7	-6	9868	7775.376	+	0.6	"	Good		
35.....		July 18	9 7	-6	9892	7779.376	-	4	"	Good		
36.....		22	9 7	-6	9910	7782.378	-	4	"	Good		
37.....		25	9 10	-6	9982	7794.387	-	6	"	Fair		
38.....		Aug. 6	9 23	-6	10012	7799.397	-	1	"	Uncertain		M.N., 69, 86
39.....		11	9 38	-6	10875	7943.369	+	6	"	Good		
40.....	1908	Jan. 2	8 45	+6	10947	7955.376	+	2	"	Fair		
41.....		14	8 56	+6	11373	8020.438	-	1	"	Fair		
42.....		Mar. 25	10 28	+3	11499	8047.457	-	2	"	Fair		
43.....		Apr. 15	10 56	+2	11565	8058.471	+	2	"	Good		
44.....		26	11 18	0	11607	8065.477	+	2	"	Good		
45.....		May 3	11 28	-1	11673	8076.487	+	2	"	Good		
46.....		14	11 44	-2	15808	8766.282	-	2	H	Good		Riv. di Astr., 4,
47.....	1910	Apr. 4	6 44	+2	15820	8768.281	-	5	"	Good		310
48.....		6	6 43	+2	15826	8769.292	+	5	"	Fair		A.N., 4542
49.....		7	6 58	+2	18572	9227.372	+	2	L	Good		
50.....	1911	July 9	9 2	-6	22870	9944.379	+	15	P	Uncertain		
51.....	1913	June 25	9 11	-5	22906	9950.385	+	15	"	Fair		
52.....		July 1	9 21	-6	22918	9952.366	+	6	"	Fair		
53.....		3	8 53	-6	22924	9953.372	-	1	"	Good		Mem. Sp. It.,
54.....		4	9 2	-6	22930	9954.372	-	1	"	Fair		2, 193
55.....		5	9 2	-6	22936	9955.357	-	17	"	Uncertain		
56.....		6	8 40	-6	22942	9956.366	-	9	"	Uncertain		
57.....		7	8 54	-6	22948	9957.381	+	5	"	Uncertain		
58.....		8	9 15	-6	26361	0526.726	-	5	J			Letter
59.....	1915	Jan. 28	17 19	+6	26367	0527.728	-	4	"			
60.....		29	17 22	+6	29083	0980.809	-	5	Sh	Good		
61.....	1916	Apr. 26	19 25	0	29099	0983.478	-	6	Bi	Fair		
62.....		29	11 28	0	29171	0995.490	-	5	"	Good		
63.....		May 11	11 48	-2	29207	1001.494	-	6	"	Good		This paper
64.....		17	11 53	-2	29219	1003.494	-	8	"	Good		
65.....		19	11 54	-2	29231	1005.497	-	7	"	Good		
66.....		21	11 59	-3	29273	1012.508	-	0.002	"	Fair		
67.....		28	12 14	-3								

* Good.

† Uncertain; minimum phase very flat.

‡ Uncertain.

§ Uncertain; light-curve discordant.

used only to indicate the representation in 1915 by the adopted elements.¹

For the designation of the observers the following abbreviations are used, the method of observation being indicated in parentheses: MK=Müller and Kempf (photometric); F=Fauth (estimates); PJ=Parkhurst and Jordan (photographic); T=Tikhoff (photographic); Ba=Baldwin (photometric); H=Horn (photographic); L=Lazzarino (photometric); P=Padova (photometric); J=Jordan (photographic); Sh=Shapley (photographic); Bi=Van der Bilt (photometric).

The following points are to be noted:

1. The minima published by Müller and Kempf and those by Parkhurst and Jordan have been taken as they stand. There are two misprints in the latter's comparison with Müller and Kempf's data;² the calculated time of the minimum of December 7 should be 19^h55^m, and the residual minus instead of plus.

2. The detailed observations of Fauth, made by Argelander's method, are not accessible. Though the results are probably less accurate than those obtained by photometric methods, we have not thought ourselves justified in rejecting them on account of this a priori assumption.

3. All the observations made by Tikhoff have been plotted and several minima derived which had not been discussed previously. Though the various filters seem to give systematic differences in the time of minimum, the accompanying tabulation, based on data

Color Screen	Residuals (Unit is 0.0001)	Mean Residual
Orange.....	+23, -33, -1, +15, -29	-5
Green-blue.....	+6, +46, +38	+30
Ultra-violet.....	+34, +34, +46, +43, +33	+38

¹ For the sake of completeness, times of minima have also been computed for the new Harvard photographic observations from the data given by Russell, Fowler, and Burton in Table IX, *Astrophysical Journal*, 45, 321, 1917. The representation of these times by our adopted light-elements is as follows:

Mean Epoch.....	1,120	7,380	16,000	24,600
O.-C.....	+0.0015	+0.0008	+0.0016	-0.0004

² *Astrophysical Journal*, 23, 84, 1906.

in Table IV, shows that these differences hardly exceed the observational uncertainty.¹ The individual measures have a considerable accidental error, and as only one of the color-filters was used at each minimum systematic errors may be involved if the period is not strictly uniform.

4. All the observations by Baldwin have also been plotted, and with the aid of the mean light-curve given by him several new minima were derived from incomplete light-curves and are available for the discussion.

5. Horn, working with photographic trails, gives the times of three maxima. Since it is uncertain whether the minima occur exactly half-way between the maxima, and since the latter can usually not be determined with the same accuracy as the minima, we have made a new grouping of Horn's observations for an independent derivation of the minima. A striking feature of his mean light-curve, entirely differing from the results of other observers, is that the horizontal line through the points of median magnitude is divided by the light-curve into approximately equal parts. The curves obtained by other observers all indicate that the part of this line below the maximum is nearly three times as long as that above the minimum. The actual figures are:

Müller and Kempf.....	0 ^d .125	and	0 ^d .042
Baldwin.....	0.121	"	0.046
Van der Bilt.....	0.125	"	0.042
Horn.....	0.097	"	0.070

Further, one (and probably two) of Horn's observed curves shows a constant minimum of more than 25 minutes. The explanation of both phenomena may be that the photographic trails at minimum were so faint that they could not be measured accurately.

6. The mean light-curve published by Padova was derived from a very non-homogeneous grouping. A new mean curve has been constructed and the separate minima determined as accurately as the observations allow.

¹ For a note on the interpretation of Tikhoff's results see Kron, *Potsdam Publication*, 22, Pt. 3, p. 56, 1912.

The 64 minima used in the computation yield the following new light-elements:

Heliocentric Minimum = J.D. 2418075.8176 + 0^d.1668196*E*, Gr.M.T.,
the period of revolution being double the interval between minima.

The residuals to which the foregoing formula leads are given, both in fractions of a day and in minutes of time, in the sixth and seventh columns. The mean residual is 0^d.0050. The algebraic sum of the residuals is -0^d.0014. There are 30 positive and 34 negative residuals, 37 recurrences of sign, and only 26 changes, which points to some systematic change in the period. The evidence for this is strengthened by the fact that the first and the last six residuals are all negative, with practically the same mean value, viz.:

$$\begin{array}{ll} \text{Mean epoch} = & 148, \quad \text{Mean residual} = -0^{\text{d}}.0066 \\ \text{" " " " } = & 29,200, \quad \text{" " " " } = -0.0053. \end{array}$$

Were the time of normal minimum displaced by this amount, all but four of the intermediate residuals would become positive.

There is, however, no indication of a systematic difference between the odd and even minima. We derive:

$$\begin{array}{l} \text{Mean residual of 39 even minima} = +0^{\text{d}}.0005 \\ \text{Mean residual of 25 odd minima} = -0.0009 \end{array}$$

a difference wholly covered by the observational errors.

From the present study a perturbation of some character appears certain, but the data are as yet insufficient to determine the law of variation.

SUMMARY

1. Mount Wilson photographic observations of W Ursae Majoris, the star of highest known density, give a color-index in agreement with its spectral type (G), and show no conspicuous change of color between minimum and maximum light.

2. From a series of photometric measures made at the Utrecht Observatory (Table III) the times of six minima have been derived.

3. A study of 64 minima, distributed throughout an interval that includes 29,000 recurrences of minimum light, reveals a small but distinct perturbation of the period (Table IV).

MOUNT WILSON SOLAR OBSERVATORY
June 1917