ON THE NATURE AND CAUSE OF CEPHEID VARIATION[1]

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The purpose of the present discussion is an attempt to investigate the question of whether or not we should abandon the usually accepted double-star interpretation of Cepheid variation. In addition to the brief statement of some general considerations and correlations of the many well known characteristics of Cepheid and cluster variables, certain recently discovered properties of these stars are discussed in greater detail, because chiefly upon them are based the conclusions reached in this study.

It seems a misfortune, perhaps, for the progress of research on the causes of light-variation of the Cepheid type, that the oscillations of the spectral lines in nearly every case can be so readily attributed, by means of the Doppler principle, to elliptical motion in a binary system. The natural conclusion that all Cepheid variables are spectroscopic binaries has been the controlling and fundamental assumption in all the recently attempted interpretations of their light-variability, and the possibility of intrinsic light-fluctuations of a single star has received little attention.

From the very first there have been serious troubles with each new theory. Considered from the spectroscopic side alone, the Cepheids stand out as unexplainable anomalies. There are persistent peculiarities in the spectroscopic elements, such as the low value of the mass function, the universal absence of a secondary spectrum, and the minute apparent orbits. Practically the only thing they have in common with ordinary spectroscopic binaries is the definitely periodic oscillation of the spectral lines, which permits, with some well known conspicuous exceptions,[2] of

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 92. Read at the seventeenth meeting of the American Astronomical Society, August 1914.

[2] The irregularities in the velocity-curve of ζ Geminorum have been discussed by Campbell (*Astrophysical Journal*, **13**, 94, 1901), Russell (*Astrophysical Journal*, **15**, 260, 1902), and Plummer (*Monthly Notices*, **73**, 661, 1913). The deviations from purely elliptical motion in the case of W Sagittarii have been considered by Curtiss (*Lick Observatory Bulletins*, **3**, 36, 1904; *Astrophysical Journal*, **20**, 149, 1904); in the

interpretation as periodic orbital motion. Adding, then, to the spectroscopic abnormalities the curious relations between light-variation and radial motion, the difficulties in the way of all the proposed simple solutions seem insurmountable. Geometrical explanations of the light-variation fail completely, and little better can be said of the hypotheses that involve partly meteorological and partly orbital assumptions.

The writer can offer no complete explanation of Cepheid variability as a substitute for the existing theories that are shown to be more and more inadequate. At most, only the direction in which the real interpretation seems to lie can be pointed out, and an indication given of the strength of the observational data that would support the theory developed along the lines suggested. The principal results of a rather extensive investigation, further details of which it is hoped can be published in subsequent papers in the near future, are outlined in the following paragraphs. The main conclusion is that the Cepheid and cluster variables are not binary systems, and that the explanation of their light-changes can much more likely be found in a consideration of internal or surface pulsations of isolated stellar bodies.

THE ESSENTIAL IDENTITY OF CEPHEID AND CLUSTER VARIABLES

The subdivision of the short-period variables into the cluster type and the Cepheid type is an artificial one. This proposition scarcely needs proof, although the assumption of the essential similarity of the two groups is important in the following discussion. Practically all writers on the subject are more or less inclined to accept this view.[1] The definition of the cluster-type variable is,

case of Y Ophiuchi by Albrecht (*Lick Observatory Bulletins*, **4**, 134, 1907) and later by Zurhellen (*Astronomische Nachrichten*, **177**, 329, 1908) and Miss Udick (*Publications of the Allegheny Observatory*, **2**, 151, 1912); in the case of RT Aurigae by Duncan (*Lick Observatory Bulletins*, **5**, 120, 1909). For many Cepheids the total range of velocity variation is so small that secondary oscillations and other irregularities of considerable relative importance may easily be lost in the accidental errors (Curtiss, *op. cit.*, p. 39).

[1] See, for instance, Nijland, *Hemel en Dampkring*, April 1913; Williams, *Journal of the British Astronomical Association*, **23**, 134, 1912; Kiess, *Publications of the Astronomical Society of the Pacific*, **24**, 191, 1912, and **25**, 121, 1913; Townley, *Publications of the Astronomical Society of the Pacific*, **25**, 239, 1913.

in fact, by some merely "short-period Cepheid." Others, including Hartwig[1] and Kron,[2] have considered only those with rapidly decreasing brightness and constant light at minimum as "antalgol" or cluster-type variables. Kron calls the shortest-period variable known a Cepheid,[3] and Hertzsprung[4] designates as Cepheids only those variables whose periods are greater than a day. The writer proposes to adopt, merely as a convenience, the latter practice, arbitrarily calling the Cepheids of periods less than a day cluster-type variables; for there is at present no evidence of real difference between the two classes in the nature or probable causes of the light and velocity variations.[5] Hertzsprung[6] calls attention to the maxima in the frequency-curve of the periods at twelve hours and at seven days, and notes also that the longer-period Cepheids are

[1] *Vierteljahrsschrift der Astronomischen Gessellschaft*, **37**, 284, 1902.

[2] *Publikationen des Astrophysikalischen Observatoriums zu Potsdam*, **22**, Pt. III, 53, 1912. See also Newcomb-Engelmann, *Populäre Astronomie*, 5th ed., p. 623, Leipzig, 1914.

[3] XX Cygni, period $3^h 14^m$.

[4] *Astronomische Nachrichten*, **192**, 262, 1912.

[5] It is hardly necessary to remark that Cepheids and Geminids are physically identical. The latter term merely signifies that the rise and decline of brightness require approximately equal intervals of time. There are numerous types of variation intermediate between the chosen typical curves of ζ Geminorum and δ Cephei. A significant feature that has not been pointed out explicitly heretofore is that the smaller the ratio of interval of increasing light to interval of decreasing light the greater the eccentricity. Orbits of Geminids therefore have smaller eccentricities. This really amounts to observing that the light-curves and velocity-curves of all classes of Cepheids are generally identical in form. See the study by Luizet, "Les Céphéides considérées comme étoiles doubles," *Annales de l'Université de Lyon*, Nouvelle Série, I, Fascicule 33, 67–148, 1912.

Some Geminid curves, however, permit of hypothetical interpretations that cannot be applied to the more typical Cepheid. Russell has found (*Popular Astronomy*, **22**, 142, 1914) that, if the time of rise to maximum is greater than one-fourth the period, the light-variations may be interpreted as the rotation of a spotted body, but such an explanation is otherwise untenable. The writer has shown (*Laws Observatory Bulletin*, **2**, 71, 1913; *Astronomische Nachrichten*, **194**, 353, 1913) that for certain symmetrical curves of the Geminid type the light-variations may be due entirely to the rotation of a single ellipsoidal star. This explanation is a possible and plausible one, but for SZ Tauri, one of the stars suitable for such an interpretation, Haynes has found a typical Cepheid velocity variation (*Lick Observatory Bulletins*, **8**, 85, 1914).

[6] *Astronomische Nachrichten*, **179**, 376, 1909; **192**, 262, 1912; **196**, 205, 1913. Chandler reached some of the same conclusions twenty-five years ago (*Astronomical Journal*, **9**, 1, 1889).

in the galaxy, while the shorter-period Cepheids or cluster variables are apparently distributed more at random over the sky. Making the reasonable assumption that the data, though rather meager, are sufficient, nevertheless, to establish the reality of both phenomena, these conditions do not impeach the hypothesis that the light and velocity variations of the long- and short-period Cepheids are attributable to the same causes, and that the only modifications necessary in an explanation of one, to make it applicable to the other, are those depending on the length of the periods and other gradative characteristics, such as differences of spectral type and relative speed of light-change at corresponding phases. Among the several arguments that tend to prove the inherent similarity of the two groups of Cepheids, the following are the most important.

a) For RR Lyrae, period 13.6 hours, which is commonly classified as a cluster-type variable, the spectroscopic orbit by Kiess[1] resembles in all details the peculiar orbits characteristic of the longer-period Cepheids. The light-curve is typical of cluster variables in all its properties.[2]

b) From the photometric standpoint, Graff and Bottlinger[3] have found no essential differences between light-curves of cluster and Cepheid types, and insist on the artificiality of the division into two classes. Very few, if any, of the cluster-type variables have rigorously constant light at minimum phase, as Plummer,[4] among others, has shown. In fact, it was partly for this reason that Hartwig abandoned, in the *Vierteljahrsschrift* catalogue, the former term "antalgol" and the former distinction between cluster and Cepheid variables.[5]

c) Russell's harmonic analyses of the mean light-curves of typical cluster variables and typical Cepheids indicate the necessity of analogous interpretations of the two.[6]

d) An unpublished investigation by the writer of the relation between the periods and spectral types of all variables shows the

[1] *Lick Observatory Bulletins*, **7**, 140, 1913.

[2] Kiess, *Publications of the Astronomical Society of the Pacific*, **24**, 189, 1912.

[3] *Astronomische Nachrichten*, **196**, 113, 1913. [4] *Monthly Notices*, **73**, 657, 1913.

[5] *Vierteljahrsschrift der Astronomischen Gessellschaft*, **48**, 287, 1913.

[6] An abstract is printed in *Popular Astronomy*, **22**, 142, 1914.

existence of a continuous property from the longest-period Cepheids to the shortest-period cluster variables.

e) The shift of the maximum intensity in the spectra toward the violet with increasing light is a property common to both classes.[1]

IRREGULARITIES IN THE LIGHT-ELEMENTS

Starting, then, with the apparently well-grounded assumption that the reasoning relative to the nature of the cluster-type variation applies equally well to Cepheid variation, the first argument presented against the binary character of Cepheids deals with the irregular oscillations in the photometric period. In a paper presented to the American Astronomical Society at its last meeting, the writer reported on the oscillations in the periods of several cluster-type stars.[2] The study of such irregularities must necessarily, for the present at least, be confined to photometric observations, for the faintness of the stars, and the consequent length of spectroscopic exposure would conceal irregular oscillations in the velocity measurements.[3] The investigation, as is also obvious, must succeed first with the stars of shortest period, for with them the light-change is of sufficient rapidity to permit the determination of points on the steep ascending branch of the light-curve with high precision.

Further observations of SW Andromedae, made since the last report, have confirmed the previous results, showing that the time of the rise to maximum light varies from the mean predicted time by ten or fifteen minutes within the short interval of two or three days, but evidently without exhibiting regular periodicity. The uncertainty of the determination does not exceed three or four minutes. The similar oscillations in the light-curve of RR Lyrae

[1] For the Cepheids see the work of Albrecht (*Lick Observatory Bulletins*, 4, 131, 1907) and of other Lick observers. For the cluster-type variables see the work of Kiess, referred to above, and the indirect determinations of the maximum intensity shifts presented in a later section of the present paper.

[2] An abstract is printed in *Popular Astronomy*, **22**, 144, 1914.

[3] The average length of exposure on RR Lyrae with a one-prism spectrograph attached to the 36-inch refractor of the Lick Observatory was more than two hours—nearly one-sixth of the entire period. RR Lyrae is the brightest cluster variable known (excepting β Cephei).

were apparently periodic throughout the interval of two or three years covered by the earlier series of Harvard observations,[1] but the later work at Harvard[2] and at the Lick Observatory[3] shows a different amplitude and perhaps no periodicity at all. The Lick and Harvard observations were made with visual photometers. In a recent letter Professor Hertzsprung writes that he also finds irregularities from night to night in photographic observations of the star.

The oscillation in the time of brightening to maximum seems to be a pretty general characteristic, though possibly not universal. It is shown definitely for several other stars besides those mentioned above,[4] and perhaps most noticeably in the case of XX Cygni, which is discussed in the next section. The most remarkable feature of the oscillation, which from night to night is conspicuously large in some cases, is that a change in the mean period of the light-variation has been recorded for only two stars, and these changes are relatively minute.[5] If the observed oscillations were definitely periodic, it would perhaps be possible to attribute them in some kind of a binary system to orbital changes, such as the rotation of the line of apsides. But the sudden and unpredictable changes in the light-variation, very likely accompanied by analogous oscilla-

[1] *Harvard Annals*, **69**, Pt. I, 45, 1909.

[2] The manuscript of these observations was kindly sent to the writer by Professor Pickering; more recently the work has been published in *Harvard Annals*, **69**, Pt. II, 124, 1913.

[3] *Lick Observatory Bulletins*, **7**, 141, 142, 1913.

[4] *Popular Astronomy*, **22**, 144, 1914.

[5] According to Kron the mean period of XX Cygni is decreasing by about a tenth of a second a year (*op. cit.*, p. 47). Roberts finds that the mean period of S Arae is decreasing by four-hundredths of a second a year (*Astrophysical Journal*, **33**, 200, 1911). The long-accepted secular change in the period of δ Cephei, first established with some uncertainty by Chandler (*Astronomical Journal*, **13**, 101, 1893) and later maintained by Nijland (*Astronomische Nachrichten*, **161**, 229, 1903), has recently been completely rejected by Luizet in his monograph on the light-variations (*Annales de l'Université de Lyon*, Nouvelle Série, Fascicule 33). Belopolsky, however, finds an oscillation in the spectroscopic period (*Mitt. Pulk.*, **3**, 63, 1909). W. J. S. Lockyer has found (Dissertation, Göttingen, 1896) that, while the mean period of η Aquilae is constant, there is an oscillation in the epoch of maximum through an amplitude of ten hours. Hellerich (Dissertation, Berlin, 1913) has studied the periods of ten Cepheids and finds no necessity of second-order terms.

tions in the velocity-curve, introduce another difficulty into the binary system theory.

IRREGULAR CHANGES IN THE LIGHT-CURVES

The second argument against the double-star explanation of Cepheid variation lies in the continually changing form of the light-curves from one maximum to the next. Again we will consider mainly the cluster-type stars. For the Cepheids the length of the period prohibits, in general, continuous observations throughout successive epochs, and in the long run the irregularities smooth out in a mean light-curve. Curtiss[1] has noticed, however, that the light-curve of W Sagittarii, period 7.6 days, changes shape with the time, and similar results are suggested for other Cepheids by various observers.[2] For the stars with periods less than a day, however, many long series of observations have been made, and although the work was rarely if ever undertaken for the purpose of seeking short-period changes in the form of the light-curve, nevertheless irregularities have often been found. Many observers have noticed that the errors are larger in observing short-period variables than in any other class. Roberts[3] and Innes[4] were suspicious of the large deviations in their measures of S Arae. Sperra[5] concluded from an extensive treatment of his visual observations on SW Draconis and SU Draconis that the shapes and durations of both maxima and minima varied from night to night. This was, I believe, the first and only serious attempt that has been made to question the supposed clocklike precision of short-period variation. Plummer and Martin[6] are disinclined to accept Sperra's results without further proof, for the photographic observations at Dunsink (exposure times from thirty minutes to an hour) do not confirm such irregularities (though they do show remarkable irregularities,

[1] *Lick Observatory Bulletins*, **3**, 168, 1905.

[2] For instance, the light-curve of δ Cephei, as pointed out by Luizet (*op. cit.*, pp. 58–60).

[3] *Astrophysical Journal*, **33**, 201, 1911.

[4] *Annals of the Cape Observatory*, **9**, 126B, 1903.

[5] *Astronomische Nachrichten*, **184**, 241–252, 1910.

[6] *Monthly Notices*, **73**, 440, 1913.

supposedly permanent, in the mean curves). The proof is now at hand, however, and with amazing clearness in the observations of XX Cygni, published at Potsdam.[1] The three-hour period of this star is obviously a great advantage in the study of the changing form of the light-curve. If the variations of SW Andromedae and RR Lyrae referred to above could have been followed regularly throughout their entire periods, there is little doubt that the oscillations in the time of the rise to maximum would have been found to constitute only a part of the irregularities.

In his discussion of nearly three thousand observations of XX Cygni, Kron finds a small secular change in the mean period, but does not consider real the deviations from the mean curves. The observations of ten observers are included. Both Schwab and Guthnick considered the irregularity of form a real phenomenon, and the former notes that, while the maximum and minimum magnitudes remain sensibly constant, the times between the ascent and descent past magnitude 11.2 vary from 35 minutes to an hour.[2] A study of the observations shows the same phenomenon in the work of all the observers.[3] The various forms of light-curve cannot be attributed to night errors. The differences between the curves at different epochs is distinctly larger than the errors of the observations. This is particularly true for Kron's photometric work. The average deviation of his measures from a normal curve, based on his own observations, is much larger than that of measures on the comparison stars, and probably more than twice as large as the average deviation of the observations from separate nightly curves. There appears, however, to be no definite periodicity in the changing shape of the curve; as a rule sharp maxima follow each other

[1] *Publikationen des Astrophysikalischen Observatoriums zu Potsdam*, **22**, Pt. III, 1912.

[2] *Astronomische Nachrichten*, **170**, 369, 1906.

[3] This and other points relative to the anomalies of the light-variation will be discussed more fully in a later communication. Contrary to all other experience with Cepheid variables the photographic range measured by Parkhurst and Jordan (*Astrophysical Journal*, **23**, 84, 1906) is less than the visual range. To examine this question more closely a series of simultaneous photographic and photovisual observations has recently been made with the 60-inch reflector. This will furnish a definitive color-curve, as well as serve as a control on the secular change in the period. (Parkhurst and Jordan also suspected oscillations in the period; *op. cit.*, p. 86.)

for a few days, to be succeeded by an intermediate type and then by a series of relatively wide, flat-topped maxima. Sometimes the extreme change in form occurs on successive nights. In the variations of the light-curve, as in the oscillations of the time of the rise to maximum light, the regularity and continuity of the phenomena that would be demanded by an orbital explanation is apparently lacking.

CHANGES IN COLOR AND SPECTRAL TYPE

A third argument against the binary interpretation of Cepheids is the difficulty such theories would have in explaining the periodic change of the spectral type, though it must be admitted that to a certain extent Duncan's hypothesis,[1] if otherwise acceptable, could account for spectral changes through the medium of atmospheric absorption. The evidences of the change of spectral type with changing light, though not well known nor generally recognized, are decisive and important. Schwarzschild,[2] Wirtz,[3] and more particularly Wilkens[4] have demonstrated for Cepheids of longer period that the range of light-variation is greater in the photographic than in the visual part of the spectrum. The photographic work of Martin and Plummer[5] suggests similar results for cluster-type variables, while the recent simultaneous photographic and photovisual observations by Mr. Seares and the writer at

[1] *Lick Observatory Bulletins*, **5**, 91, 1909; *Publications of the Astronomical Society of the Pacific*, **21**, 123, 1909.

[2] *Publikationen der v. Kuffnerschen Sternwarte*, **5**, C100, 1900. The photographic range of η Aquilae is found to be double the visual range. More recently Kohlschütter has repeated the photographic work and finds that the color-curve has an amplitude of four-tenths of a magnitude (*Astronomische Nachrichten*, **183**, 265, 1910).

[3] *Astronomische Nachrichten*, **154**, 327, 1901. Wirtz measures the photographic ranges of δ Cephei and ζ Geminorum.

[4] *Astronomische Nachrichten*, **172**, 316, 1906. An average value of 1.6 is found by Wilkens for the ratio of photographic to visual range for the Cepheid variables SU Cygni, X Cygni, T Vulpeculae, S Sagittae, and U Vulpeculae. The visual ranges, it should be remarked, are collected by him from various sources and can hardly be considered homogeneous or reliable. The results, however, are qualitatively dependable, but more work along this line is desired. The Cepheids mentioned in the three last notes are of spectral types F to K5, with an average very close to the solar type.

[5] SU Draconis, *Monthly Notices*, **73**, 166, 1912; SW Draconis, *ibid.*, **73**, 440, 1913; Cygni, *ibid.*, **74**, 225, 1914.

Mount Wilson establish the fact definitely.[1] The shift of the maximum intensity in the spectra of Cepheids, discovered by Albrecht,[2] has been confirmed by Kiess[3] and other Lick observers. These two factors—the greater photographic range and the shift of the maximum intensity—would suggest as an underlying and common cause a change in the spectral type. Albrecht and Duncan[4] have observed that Wright's spectrograms of η Aquilae suggest a later type of spectrum at minimum than at maximum. The Harvard classification of TT Aquilae[5] at maximum is G, at minimum, K.

For the cluster-type variables there is more direct evidence of distinct and continuous change. At the writer's request Miss Cannon has examined some of the Harvard spectrograms of certain cluster variables. For RR Lyrae no definite change was recorded on the plates examined, and similarly for XZ Cygni, but the spectrum, when faint, was extremely uncertain. For SW Andromedae the spectrum was of type A at maximum and clearly of a redder type at minimum. The most conclusive results, however, are obtained from the series of spectrograms taken by Mr. Pease[6] in July of this year with the 60-inch reflector of the Mount Wilson Observatory. The variable RS Boötis, period $9^{h}.1$, shows a continuous change of spectral type from F0 at minimum to B8 at maximum. One consequence of this result is that hereafter the classification of all Cepheid and cluster-type spectra must be made with due specification of the corresponding phase of light-variation.[7]

[1] A report on this work was presented at the meeting of the American Astronomical Society at Evanston, Illinois, August 25–28, 1914.

[2] *Lick Observatory Bulletins*, **4**, 131, 1907. [3] *Ibid.*, **7**, 140, 1913.

[4] *Ibid.*, **5**, 93, 1909. [5] *Harvard Annals*, **55**, 285, 1909.

[6] A report on this work was presented at the meeting of the American Astronomical Society at Evanston, Illinois, August 25–28, 1914.

[7] Miss Clerke writes: "The spectrum [of δ Cephei] is of the solar type, and does not change with the brightness" (*Problems in Astrophysics*, p. 320, London, 1903). This, however, should not discourage new attempts to classify the spectrum of the type-star and of other longer-period Cepheids at various phases of their light-changes. It is possible, of course, that the changes in color index and shifts of maximum intensity are not generally accompanied in the longer-period Cepheids by those changes in the absorption lines that are necessary to give a different spectral classification under the present system, in which the absorption lines receive much attention and the background intensities but little. But in the early study of the spectrum of δ Cephei, published by Belopolsky in *Bulletin*, No. 3 of the Imperial Academy of Sciences of

Another difficulty in the spectral changes, that must not be overlooked in attempting a complete explanation of the Cepheid phenomena, is a peculiarity observed by Albrecht[1] on his plates of Y Ophiuchi and T Vulpeculae. Various lines showed large irregular shifts, which are not progressive with the phase of the star in its light-period.

CONCERNING EXISTING HYPOTHESES

The fourth principal argument against the binary interpretation of Cepheids is the inadequacy of all the existing double-star hypotheses. To many this is not only the best argument but is sufficient in itself.[2] A detailed criticism of these attempted explanations is unnecessary, for this has been generously provided by the proposers of the theory themselves, as well as by others, including Campbell,[3] Plummer,[4] Brunt,[5] Kiess,[6] and Ludendorff.[7] There is

St. Petersburg, 1894, the variations in the relative intensities of several lines are suggested and certain deviations from the solar spectrum are explicitly pointed out on many spectrograms. The question is one to be answered definitely by future researches. (A very recent study of the changes in the spectra of δ Cephei and ζ Geminorum has been made at St. Petersburg by Lohmann, but the paper is not yet available to the writer.)

[1] *Lick Observatory Bulletins*, **4**, 131–132, 1907.

[2] There has been a growing and but half-concealed discontent with the double-star explanations of Cepheids. Ludendorff writes (*Astronomische Nachrichten*, **193**, 304, 1912): "Freilich kann man sich aus verschiedenen Gründen des Eindrucks kaum erwehren, dass die in den Spektren der δ Cephei-Sterne beobachteten periodischen Linienverschiebungen nicht durch Radialbewegungen der Sterne, sondern durch irgendwelche andere Ursachen hervorgerufen werden." On the other hand, Paddock interprets Ludendorff's data in a manner favorable to Duncan's double-star hypothesis (*Publications of the Astronomical Society of the Pacific*, **25**, 180, 1913).

Plummer considers, in a recent paper on the nature of the Doppler principle when based on the Ritz theory of light, the possibility of getting around the difficulties presented by the velocity variations of certain variables (obviously Cepheids are meant) by abandoning the binary interpretation altogether, but he is led rather to abandon the Ritz theory (*Monthly Notices*, **74**, 660, 1914). Until the velocity variation of RR Lyrae was discovered he was inclined to suggest that cluster-type variation might be "a prominence effect on a large scale" (*ibid.*, **73**, 658, 1913).

[3] *Stellar Motions*, pp. 305 ff., New Haven, 1913; *Lick Observatory Bulletins*, **6**, 51, 1910.

[4] See various papers cited above.

[5] *Observatory*, **36**, 59, 1913.

[6] *Publications of the Astronomical Society of the Pacific*, **24**, 186, 1912; see also other papers cited above.

[7] *Astronomische Nachrichten*, **184**, 384, 1910.

one point, however, that has not been considered, which is of prime importance in the discussion of Cepheid phenomena. Russell[1] and Hertzsprung[2] have independently shown that the Cepheids are stars of small peculiar motions and small parallaxes, and hence of great absolute brightness. The former finds a mean absolute magnitude of -2.4 and the latter of -2.3, that is, the average Cepheid (the spectrum is of solar type) is nearly 700 times as bright as the sun. It is reasonable to assume that the Cepheids and the sun have a comparable surface brightness. The average Cepheid, then, has a volume between fifteen and twenty thousand times as great as that of the sun.

Interpreted as spectroscopic binaries these giant stars move in orbits whose apparent radii average less than one-tenth the radii of the stars themselves.[3] In order that the radii of the real orbits may greatly exceed those of the apparent orbits, the inclinations must be very small, a condition which cannot be supposed to exist generally for Cepheid orbits. The difficulty in applying the hypotheses of Eddie,[4] Loud,[5] Duncan,[6] and Roberts[7] is therefore immediately apparent. Moreover, if the mass of the average Cepheid is admitted to be as much as five times the solar mass, the density is still astonishingly low—hardly three ten-thousandths that of the sun. Considering the low average value of the mass function[8] derived from the orbits of the Cepheids, and taking a random distribution of the orbital inclinations, the non-luminous second body, to which Duncan's theory assigns the extensive atmosphere that must envelope the giant primary, has about one-tenth of the mass and therefore must move with an average apparent

[1] *Science*, N.S., **37**, 652, 1913.

[2] *Zeitschrift für wissenschaftliche Photographie*, **5**, 107, 1907; *Astronomische Nachrichten*, **196**, 201, 1913.

[3] The average value of $a \sin i$ for 15 Cepheids is 1,116,000 km. The greatest value is 2,000,000 km, and the least is 45,000 km.

[4] *Astrophysical Journal*, **3**, 227, 1896. [5] *Ibid.*, **26**, 369, 1907.

[6] *Lick Observatory Bulletins*, **5**, 91, 1909.

[7] *Astrophysical Journal*, **33**, 197, 1911; *Monthly Notices*, **66**, 329, 1906.

[8] The average value of $\frac{m_1 \sin^3 i}{(m_1+m)^2}$ for 15 Cepheids is 0.0025. The greatest value is 0.0058, and the least is 0.00001 for Polaris.

orbital velocity of about 200 km a second. Remembering the size of the primary star compared with its orbit, we know that the mass of the secondary must be still smaller and the velocity higher to separate the stars.

A SUGGESTED EXPLANATION OF CEPHEID VARIATION

In the face of all these difficulties, it seems appropriate to abandon completely the attempts to interpret Cepheids on the basis of a binary-star assumption. It has been shown by Russell[1] that the light-variations cannot be explained satisfactorily by the uniform rotation of a single spotted star; the light-change must be intrinsic, and not just apparent. The explanation that appears to promise the simplest solution of most, if not all, of the Cepheid phenomena is founded on the rather vague conception of periodic pulsations in the masses of isolated stars. The vagueness of the hypothesis lies chiefly in our lack of knowledge of the internal structure of stellar bodies, and not in the difficulty of explaining the observed facts if once we assume the stars to be ideally gaseous figures of equilibrium. Moulton[2] has considered the matter of explaining certain types of stellar variation from this point of view, but his conclusions are scarcely applicable to Cepheid variables in the light of our present knowledge of their peculiar properties.[3] According to him, the light-change should be due to the heat generated by the oscillation of a spherical star from an oblate to a prolate form, there being a maximum of light-emission every time the star passes through its mean spherical figure. The period of velocity variation, then, should be double that of the light-change,[4] and

[1] *Popular Astronomy*, **22**, 142, 1914. See the footnote on a preceding page relative to this work. Russell's investigation, then, opposes the explanation suggested by Hellerich (Dissertation, Berlin, 1913).

[2] *Astrophysical Journal*, **29**, 257, 1909.

[3] The hypothesis also is not obviously applicable as a complete or even partial explanation of the variation of elliptical eclipsing binaries, considering the present state of the orbital theory. The presence of the secondary spectrum, the very large periodic shift of the spectrum lines, and many other factors are almost unimpeachable proofs of the binary character of eclipsing variables. Very little if any effect on the light-variation can be attributed in any of the systems studied to pulsations, or even to tidal disturbances.

[4] That is, if the shift of the spectrum lines is to be attributed to a radial motion of the source.

this, of course, does not conform with known conditions.[1] It is to this phenomenon of pulsating stellar masses, however, that the writer would ascribe the light and velocity variation of Cepheid and cluster variables, and the theoretical work of Moulton,[2] Jeans,[3] Emden,[4] and others[5] on the properties of gaseous spheres already justifies the conclusion that such oscillations are both possible and probable. They might arise, as Moulton suggests, from the collision with masses of only planetary dimensions, from the near approach of two stars, or in other ways.[6]

Without any pretense of explaining clearly or fully on this hypothesis all the properties of Cepheid variation that have given the double-star theories such hopeless difficulty, a few points favoring the pulsation suggestion will be summarily stated. There will exist originally, as the result of the initial disturbance, a great number of oscillations with different periods. The character of these various vibrations will depend on the nature of the stellar structure. For the ideal homogeneous fluid mass investigated by Kelvin,[7] and for the polytropic gaseous sphere defined and studied by Emden,[8] the period of vibration of each type is independent of the volume and mass and depends only on the mean density and the order of the harmonic term defining the oscillation.[9] For any given

[1] It is well to keep in mind, however, the secondary maxima in the light-curves of η Aquilae and similar variables. Therein perhaps is a visible trace of the secondary heating of each oscillation period.

[2] *Op. cit.*

[3] *Philosophical Transactions of the Royal Society of London*, **199 A**, 1, 1902; *ibid.*, **201 A**, 157, 1903; *ibid.*, **213 A**, 457, 1914.

[4] *Gaskugeln*, Leipzig, 1907.

[5] The numerous papers by Ritter, *Wiedemanns Annalen*, **5–20**, 1878–1883, are of fundamental importance in this problem. His consideration of vibrational variable stars has been briefly discussed by Moulton (*op. cit.*).

[6] Perhaps then we should expect to find great numbers of these variables in condensed regions, such as the globular clusters and the Magellanic clouds.

[7] *Mathematical and Physical Papers*, **3**, 384, 1890.

[8] *Op. cit.*, pp. 13, 37, 448 ff.

[9] The vibration period in seconds is for the former

$$\tau = 2\pi\left(\frac{2m+1}{2m(m-1)}\right)^{\frac{1}{2}}\left(\frac{R}{G}\right)^{\frac{1}{2}}$$

and for the latter

$$\tau = 2\pi\left(\frac{R}{mg}\right)^{\frac{1}{2}}$$

where m is the order of the spherical harmonic defining a given oscillation, R is the radius, and g, the surface gravity, is proportional to radius times mean density.

mean density the most important oscillation is that corresponding to the second-order harmonic. Its period is the longest, its amplitude the greatest, and it may persist with inappreciable change in period almost indefinitely, while the oscillations of higher order are more rapidly destroyed by friction.

If, then, we attribute the principal light-change in a Cepheid variable to this principal oscillation, and if we are willing to adopt Emden's polytropic gaseous sphere as a stellar model, we can compute at once the density of each individual variable. Obtained in this way the densities are probably of the right order of magnitude, whatever function of the radius, within reasonable limits, the density is assumed to be; and for an incompressible homogeneous fluid they would be only 2.5 times as large. The densities in terms of the sun for all the Cepheids of known periods and spectra[1] have been derived in this manner, with the results given in Table I.[2]

[1] It is impossible to say, of course, whether the spectra listed were obtained near maximum or near minimum. Errors in the grouping, for the brighter Cepheids at least, will probably balance in the means.

[2] The data for this computation have been derived mainly from *Harvard Annals*, **56**, 191–195, 1912. In the table of "short-period variables" given in that publication are included many stars that are now known to be eclipsing binaries. It is very likely that further study will show that others are not Cepheids, but in the means in Table I are included all variables not known to be eclipsing stars.

The densities in the table are computed for the mean periods of each class. If the means of the individual densities were taken one badly discordant period would greatly distort the result.

A striking fact shown by Table I is the progressive relation between spectrum and period, and hence, between spectrum and density. "The redder the tint, the longer the period" was observed by Chandler in 1888, but referred mainly to long-period variables (*Astronomical Journal*, **8**, 137, 1888). He called attention to the importance of this correlation to variable star hypotheses. Campbell noticed that "the length of periods seems to increase with the spectral types, but the relationship is not strongly marked" (*Lick Observatory Bulletins*, **6**, 51, 1910).

Russell's theory of the order of stellar evolution is supported by the results of Table I when we remember that the Cepheids are giant stars. For it is obviously fair to conclude from the progression of the densities that, if the present Cepheid hypothesis approximates the truth, the order of evolution is in the direction from type M to type B. The Cepheids then are young stars. Perhaps the long-period redder variables of the Mira Ceti type, with spectra of types M*a*,....M*d*1, M*d*2,....M*d*10, N, etc., are still earlier in their evolution. Are they merely Cepheids of very long period? In many respects they appear to be—the light-curves are similar, the shape and time of successive maxima often oscillate around probably rigorously constant mean values, the spectra change progressively throughout the light-period, and finally

The extremely low densities for the Cepheids of the redder spectral types until recently might have thrown serious doubt on the hypothesis that demands such abnormally low values. But now for two reasons we are ready to accept as possible these supposedly impossible densities. In the first place, as Hertzsprung has pointed

TABLE I

Type	No. Stars	Mean Period	Mean Density
M..........................	3	33	0.000006
K..........................	9	18	0.000020
G..........................	31	11	0.000056
F..........................	31	6	0.000200
A..........................	9	0.4	0.04
B*..........................	1	0.19	0.2

* β Cephei is properly to be assigned to the Cepheid type. Notwithstanding the very short period ($4^{h}5$), the preliminary orbit by Frost (*Astrophysical Journal*, **24**, 259, 1906) is typical of the Cepheids. Guthnick's discovery (*Astronomische Nachrichten*, **196**, 357, 1913) of the light-variation and its nature (a typical cluster-variable curve of small amplitude) further supports this classification, and the harmony of the hypothetical density with that for the other groups of Cepheids is probably a third favorable argument. The presence of a second spectrum, however, is suspected on certain plates, according to the preliminary announcement (*Astrophysical Journal*, **24**, 261, 1906).

out[1] in his proof that the Cepheids of solar spectral type are giant stars, the average mean density must be of the order of 6×10^{-5}, if the masses are comparable with that of the sun. They may be larger, but from our knowledge of stellar masses in general we are inclined to believe that they are not more than ten times that of the sun,[2] which is sufficient to prove the point. In the second place, the densities of several long-period eclipsing binaries of types G and K are now available for comparison.[3] For instance: RX Cassiopeiae, type Ko, mean density 5×10^{-4}; W Crucis, type Gp, mean density 3×10^{-6}; SX Cassiopeiae, type G3, mean density 5×10^{-4}; RZ Ophiuchi,[4] type F8, density of one component 10^{-3}.

they are apparently stars of great absolute luminosity. Long ago, however, Chandler gave several good reasons for distinctly separating "long period" and "short period" variables (*Astronomical Journal*, **9**, 1, 1889), and to a certain extent these distinctions are still to be maintained.

[1] *Astronomische Nachrichten*, **196**, 203, 1913 (footnote).

[2] Russell, *Nature*, **93**, 283, 1914; *Popular Astronomy*, **22**, 294, 1914. Ludendorff, *Astronomische Nachrichten*, **189**, 151, 1911.

[3] *Contributions from the Princeton University Observatory*, No. 3, 1914.

[4] The spectrum has been reclassified recently by Miss Cannon at the writer's request. The components are of nearly equal brightness (visually), but it is very likely that the photographic spectrum classified is that of the smaller component which has high relative surface brightness, and density as given above. The density of the fainter component is 2×10^{-5} (*Astronomische Nachrichten*, **194**, 225, 1913).

For the A- and B-type spectra, the Cepheid densities above are, of course, entirely normal compared with eclipsing star densities.[1]

As previously stated, the Cepheids without doubt are enormously large. Their small observed velocity variations, even if attributed altogether to motion in the line of sight and not at all to pressure-shifts, are not larger than might arise from a radial oscillation through but a small fraction of their mean diameters.[2] In the central mass of the star the period of the supposed pulsation should, of course, be perfectly regular, but its effect need by no means be regular on the radiating surface.

We may suppose that, because of the internal vibration, the photosphere of the star is periodically scattered or broken through by the rush of hotter gases from the interior. Maximum light and maximum velocity of approach would obviously be approximately synchronous, and their coincidence would naturally be independent of the direction of the observer in space. Ludendorff's correlation of range of light and range of velocity is highly significant in this connection.[3] The essentially harmonic nature of the oscillation at the surface of the star would easily lend itself to interpretation as elliptic motion, though non-elliptic motion need not be unexpected, nor the anomalous behavior of certain spectral lines. In stars in which the initial disturbance is of recent origin, the presence of secondary oscillations could be expected, which would affect the light as well as the velocity.[4]

It should be noted as an important factor in the explanation of Cepheid variation, that a change in the spectrum of a given radiating surface from one type to the next will change the visual brightness of that surface by approximately one stellar magnitude,

[1] *Astrophysical Journal*, **38**, 173, 1913.

[2] The radial motions observed in sun-spots are suggestive in this connection.

[3] *Astronomische Nachrichten*, **193**, 301, 1913. He finds that $2K=47.3A$ with close approximation, where A is the amplitude of magnitude variation and $2K$ is the total range of velocity variation in kilometers.

[4] By means of a detailed periodogram analysis of the light-curve of the famous irregular variable SS Cygni, Gibb has recently found a prominent underlying periodicity of 40.86 days (*Monthly Notices*, **74**, 678, 1914). The spectrum of SS Cygni is extremely peculiar in that it varies through a number of different types, occasionally showing bright lines and at other times an apparently continuous spectrum (*Harvard Annals*, **56**, 211, 1912).

and the color range by four-tenths of a magnitude. These quantities correspond very closely to what is observed in cluster variables, and suggest that, if desired, it is unnecessary to go farther for the explanation of the light-variation than to suppose that a surface of approximately constant area progressively changes its spectral type as the result of a periodic flow and ebb of heat.[1] That the light-change should be of a more explosive character for the cluster variables than for the longer-period Cepheids would be expected because of their higher mean densities.

Various other details suggesting the possibility of the above interpretation could be cited, but this sketch of the pulsation argument will suffice for the present, since the purpose of the paper is not so much to advance an alternative theory as to question the validity of the spectroscopic binary hypothesis.

MOUNT WILSON SOLAR OBSERVATORY
August 13, 1914

[1] Cf. the hypothesis proposed by Schwarzschild as an alternative explanation of the change by six-tenths of a magnitude in the color index of η Aquilae (*Publikationen der v. Kuffnerschen Sternwarte*, **5**, C125, 1900).