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THE ECLIPSING VARIABLE *u* HERCULIS.*

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Variations in the light of 68*u* Herculis ($\alpha = 17^h\ 14^m$, $\delta = +33^\circ\ 12'$) seem to have been suspected as early as 1848 by Schmidt of Athens; but it was not until 1869 that he announced it definitely as a variable.† The character of its fluctuations greatly puzzled him and he continued to observe the star for many years, finally concluding that the period is irregular and in the neighborhood of forty days, and that during minimum the light waxed and waned rapidly. These conclusions, now known to be incorrect, were nevertheless confirmed by the numerous observers who studied the star from Schmidt's day up to 1908.

In 1903 Frost and Adams‡ at the Yerkes Observatory secured four spectrograms of this object and proved it to be a spectroscopic binary. In 1908 it was put upon the observing programme of the Mellon Spectrograph of the Allegheny Observatory for a determination of its orbit. At the same time Director Pickering kindly agreed to have photometric observations made at Harvard College Observatory, as nearly as possible at the same hours when the star was being spectrographed here. These observations were carried out by Professor Wendell with a polarizing photometer attached to the 15-inch equatorial. After a number of spectrograms had been secured and measured, it appeared at once that the period is much shorter than the earlier photometric observations had indicated, and that it is in fact only 2.05 days. This period was accordingly communicated to Harvard, with the suggestion that the star might be an Algol variable. When Professor Wendell collected his observed magnitudes upon this period he discovered the true character of the light changes. As we may see from Figure 2, page 55, there is a principal minimum of about 0.7 magnitude, lasting for about 15 hours; and a secondary minimum of the same duration but much less deep. In the two intermediate intervals of about 10 hours each, the light remains nearly constant. The star is then a variable of the β Lyræ type, or an Algol variable with a secondary minimum.

It is instructive to inquire how the earlier observers came to be so greatly

* Read at the Cambridge Meeting of the Astronomical and Astrophysical Society of America, August, 1910.
† Astronomische Nachrichten, 74, 230.
‡ Astrophysical Journal, 17, 381.

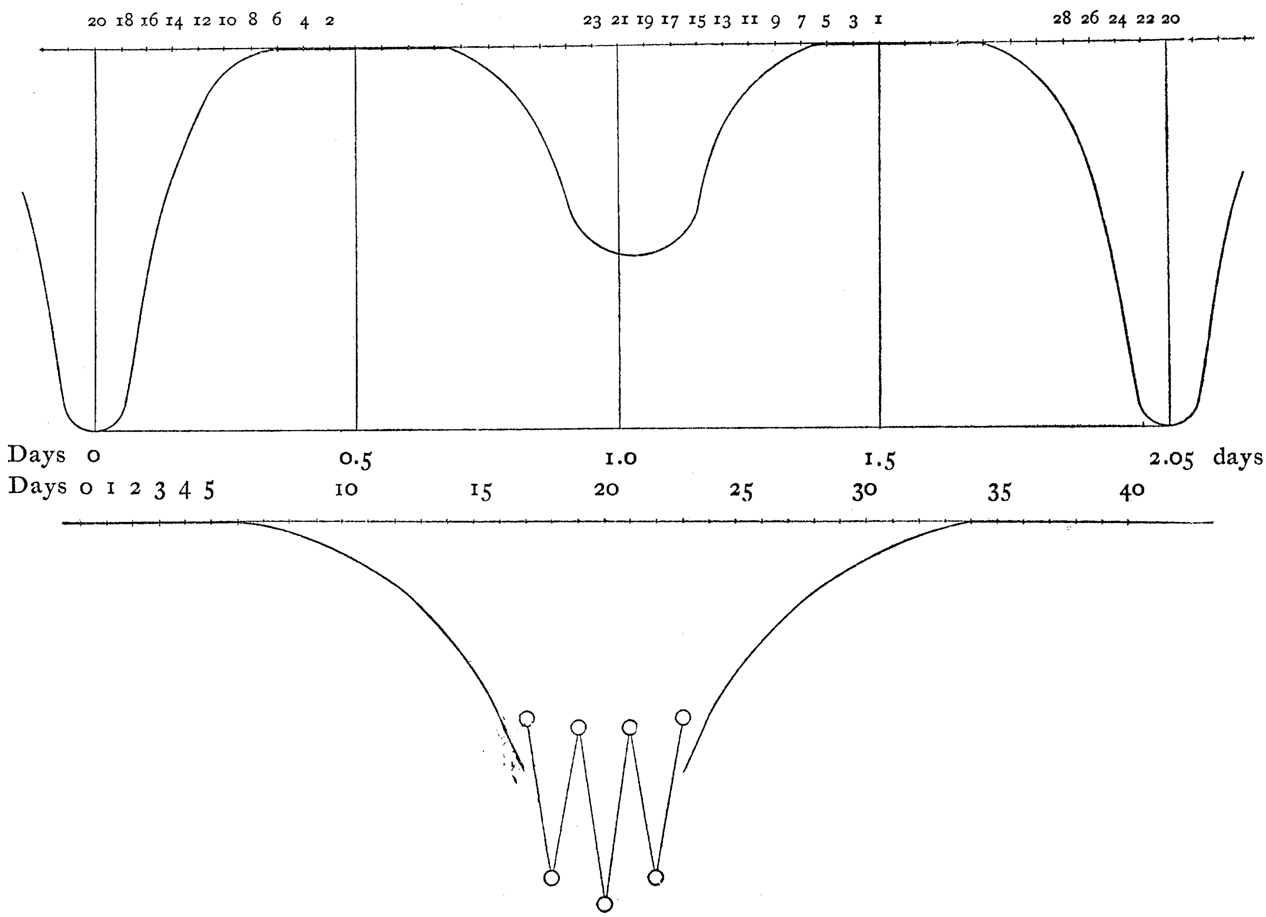

Figure 1. Explanation of Schmidt's Light-Curve.

mistaken as to the character of the light changes. The upper curve in Figure 1 shows the essential features of the true light-curve; we have, however, slightly modified some of the details in order to bring out the point in question more clearly. Let us suppose that observations have been made at the same hour on a long succession of nights, and that the first observation falls at the phase indicated by (1) at the top of the figure; the second observation will then come at (2), nearly opposite in phase; (3) marks the next observation, 0.05 days to the left of (1), since the period is 2.05 days; (4) lags similarly behind (2) and so on until 41 days have elapsed, when the point (1) will again be reached. In the lower part of the figure we have plotted these observations, scaled from the upper curve, in chronological order; and we find that we get a smooth curve (except near the center) that indicates a period of about 41 days. This accords well with what Schmidt derived and the agreement would have been even closer if we had used the more exact period, 2.05102 days. The magnitudes from about the 17th to the 23rd day do not fall on the curve indicated by the others, but come alternately above and below. Thus for example the star is at the principal minimum on the 20th, but the day before and the day after it is at the secondary minimum.

We accordingly have the "rapid fluctuations at minimum" that Schmidt remarks, and the explanation for his conclusions is complete. The assumption that his observations were made at the same hour on successive nights cannot of course be strictly true; but on the other hand it cannot be far wrong, since the star is close to 18 hours right ascension and it therefore culminates at or near midnight when the nights are shortest. Had *u* Herculis been in the opposite half of the heavens, where it could have been observed during a longer interval each night, it is probable that its true character would have been discovered much earlier.

It may be that there are other variables whose periods have been similarly misinterpreted, and it would be well to carry this case in mind. However, it should be remarked that a number of curious circumstances have conspired in this star to make the mistake possible, and that it is not very likely that they are repeated in any other of the variables now known. A somewhat more plausible case for this possibility would be presented by a variable with only one minimum, and with a period a little greater than or a little less than one day.

The presence of the secondary minimum in the light-curve and the fact that the fainter spectrum is visible upon our plates, afford an opportunity that is at present unique for ascertaining the relative densities of the two stars in the system, and other data that have a very direct bearing upon the question of double-star evolution. In an ordinary spectroscopic binary it is impossible to state anything as to the mass of either star; first, because the only expression that we can derive for the mass involves the undeterminable inclination of the orbit; and secondly, even if this inclination were known, the expression for the masses is such that we cannot state what either is, or even what their sum is, unless we also know their ratio. The first of these obstacles is removed in case the spectroscopic binary is an eclipsing variable as well, for then the inclination is not far from 90° and we can determine it with considerable accuracy from a study of the light-curve. The second obstacle is removed if plates taken with a slit spectrograph show the spectra of both components. Among the stars that are bright enough to be readily observed with instruments of the present time, there are only three that fulfill these conditions, V Puppis, β Lyræ and *u* Herculis; but for only the last are the necessary data forthcoming at this writing. V Puppis is an eclipsing variable with two minima, and Pickering has shown, from objective-prism plates, that it is a binary of the class in which both spectra are bright. But it is 49° south of the equator and it has not yet been observed with a slit spectrograph; so that the ratio of the masses, and therefore the masses themselves, remain unknown. In the case of β Lyræ the interpretation of the spectrograms is greatly complicated by the presence of a third (bright line) spectrum; a series of plates of this star was obtained at this observatory in 1907 and they are now being studied by Dr. R. H. Curtiss.

The spectrographic data used in the present paper are taken from Baker's orbit on page 82, Volume 1 of these publications. In order to facilitate reference the elements that are of interest in the present connection are repeated here:

$P = 2.05102$ days, $\pm .00003$ " $e = 0.053$, $\pm .010$ $\varpi = 66^\circ.15$, $\pm ^\circ.54$ $T =$ 1908 July 2.80 = Julian Day 2,418,125.80

$a \cdot \sin i = 2{,}800{,}000$ km. $\pm$ 28,000 " $a_s \cdot \sin i = 7{,}120{,}000$ km. $\pm 340{,}000$ "

$m \cdot \sin^3 i = 6.8$ $m_s \cdot \sin^3 i = 2.6$

The probable error given for ϖ results from assuming that T is known before entering the least-squares solution. The subscript s refers to the fainter star.

TABLE I.—THE LIGHT-CURVE OF *u* HERCULIS.

	Limits of Phase.			Mean Phase.	Mean Mag. Observed.	r	$u = v + \varpi$.	Computed Magnitude.	Observed *minus* Computed.
(1)	(2)		(3)	(4)	(5)	(6)	(7)	(8)	(9)
	days	days		days					
1	0.058 to	0.074	7	0.068	5.13	0.948	79°.4	5.18	−0.05
2	0.085	0.105	5	0.094	5.20	0.950	84 .5	5.26	− .06
3	0.109	0.113	3	0.111	5.26	0.951	87 .8	5.28	− .02
4	0.117	0.122	4	0.120	5.30	0.951	89 .5	5.29	+ .01
5	0.126	0.128	3	0.127	5.34	0.952	90 .9	5.29	+ .05
6	0.135	0.153	4	0.143	5.24	0.952	94 .0	5.27	− .03
7	0.160	0.174	5	0.168	5.31	0.954	98 .8	5.21	+ .10
8	0.177	0.200	3	0.188	5.10	0.956	102 .6	5.14	− .04
9	0.218	0.244	4	0.233	4.94	0.961	111 .2	4.98	− .04
10	0.276	0.288	4	0.281	4.89	0.967	120 .3	4.83	+ .06
11	0.367	0.379	4	0.373	4.81	0.980	137 .3	4.66	+ .15
12	0.411	0.446	4	0.434	4.64	0.990	148 .3	4.64	.00
13	0.463	0.516	7	0.485	4.67	0.998	157 .4	4.64	+ .03
14	0.546	0.665	8	0.604	4.60	1.018	177 .9	4.64	− .04
15	0.722	0.813	6	0.775	4.61	1.040	206 .2	4.64	− .03
16	0.855	0.871	3	0.863	4.67	1.047	220 .4	4.64	+ .03
17	0.895	0.976	6	0.946	4.66	1.052	233 .6	4.67	− 0.1
18	1.015	1.025	4	1.020	4.74	1.053	245 .3	4.72	+ .02
19	1.069	1.072	2	1.070	4.75	1.053	253 .2	4.77	− .02
20	1.112	1.152	6	1.129	4.85	1.051	262 .5	4.82	+ .03
21	1.161	1.185	7	1.173	4.85	1.049	269 .6	4.83	+ .02
22	1.198	1.243	6	1.228	4.78	1.044	278 .4	4.81	− .03
23	1.288	1.288	1	1.288	4.79	1.039	288 .0	4.76	+ .03
24	1.336	1.397	4	1.371	4.71	1.028	301 .7	4.69	+ .02
25	1.439	1.508	5	1.480	4.64	1.012	320 .0	4.64	.00
26	1.601	1.702	10	1.653	4.60	0.984	350 .5	4.64	− .04
27	1.817	1.817	1	1.817	4.65	0.962	20 .9	4.64	+ .01
28	1.927	1.972	6	1.953	4.69	0.950	47 .0	4.70	− .01
29	2.018	2.030	4	2.024	4.76	0.947	60 .9	4.86	− .10
30	0.006	0.013	2	0.010	4.87	0.947	68 .1	4.98	− .11

The photometric observations made by Professor Wendell have very kindly been placed at our disposal for the purpose of this paper by Director Pickering,

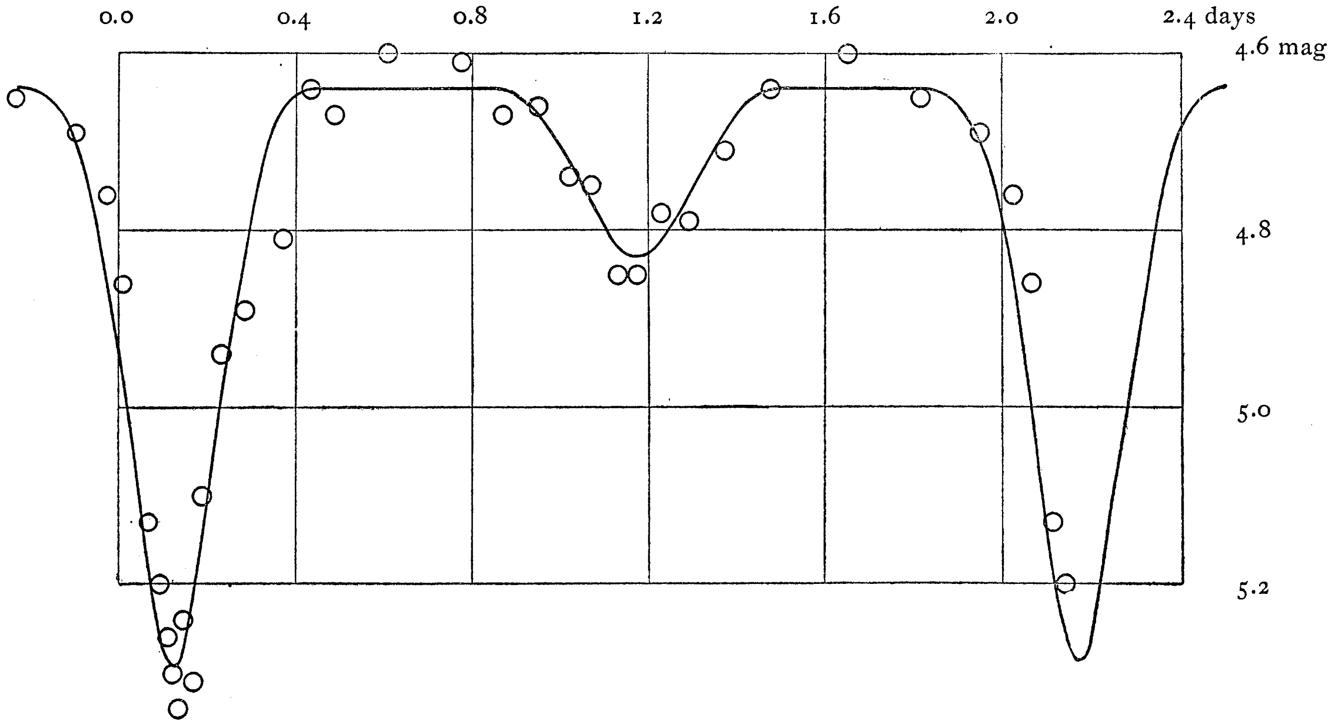

Figure 2. The Light-Curve of *u* Herculis.

in advance of their definitive publication. The 138* observed magnitudes are combined into 30 normal places as shown in the accompanying table. Those in nearly equal phases, as indicated in column (2), are gathered into the means in columns (4) and (5). The number of observations in each mean appears in column (3). The phases are computed *from the spectrographic data* just quoted, and indicate the days that have elapsed since the preceding periastron passage. From the same data we next compute columns (6) and (7); the former being the ratio of the radius vector to the major axis in the orbit of either star, and the latter the so-called "argument of the latitude," the angle between the line of nodes and the radius vector at the times in column (4).

In what follows we assume that the two stars are spherical in form and that each presents to us a disk that is uniformly illuminated from center to circumference. With these restrictions we proceed to compute the inclination of the orbits, the radii of the two stars and their relative surface luminosities, that is, the amount of light emitted from each unit of surface. In addition we assume for the present that the two minima in the light-curve take place when $u = 90°$ and $u = 270°$ respectively, as computed from the spectrographic data.

From an inspection of column (5), or of the plot of these observations in Figure 2, we see that at its brightest the system has a magnitude of about 4.60, a prin-

* Professor Wendell states that the comparison star used in these observations, D. M. + 33°2871, may be itself variable by 0.1 to 0.2 magnitudes. For this reason observations of 4 nights, all indicating *u* Herculis to be fainter than normal by 0.2 to 0.3 magnitudes, have not been included in the present discussion.

cipal minimum of 5.33 and a secondary minimum of 4.84. From these we may at once deduce a fair approximation to the relative surface luminosities of the stars (λ_s/λ), without making any assumption as to their relative size or the fraction of their disks that is covered at the two minima. For, translating these magnitudes into quantities of light we find that at the critical phases these are in the ratios 100 : 51 : 80. That is, the system loses 0.49 of its total light at principal minimum and 0.20 at secondary minimum. These two losses are in the ratio 0.40, and since the eccentricity of the orbits is small, this must be a close approximation to λ_s/λ. It would of course be just this if the orbits were circles, for then the area of the brighter disk that is covered at principal minimum would be exactly equal to the area of the fainter that is covered at secondary minimum.

This immediately enables us to approximate the value of the ratio of the radii of the two stars, ρ_s/ρ. The difference in magnitude ($M_s - M$) between the two components is obviously given by

$$M_s - M = 2.5 \log \frac{\rho^2}{0.40\,\rho_s^2} = 1.00 - 5 \log \frac{\rho_s}{\rho} \qquad (1)$$

Now the spectra of both stars are visible upon our plates and it is possible to form an estimate of the difference of magnitude that would produce a double spectrum of corresponding appearance. For this purpose we secured a number of experimental plates of stars of the same spectral type as *u* Herculis, but in which only one spectrum is visible; after obtaining one exposure the plate was shifted a very little to the right or the left and a second exposure of shorter duration was made, overlapping the first. The experiment was varied by stopping down the aperture of the telescope instead of making the two exposures unequal. From an examination of these plates we conclude that the difference of magnitude in the case of *u* Herculis is about 1.00,* with a probable error not greater than 0.10, and is certainly included within the limits 0.80 to 1.20. The ratios of the radii of the two stars corresponding to various magnitudes within these limits are as follows:

Table II.

$M_s - M$	ρ_s/ρ
0.80	1.096
0.90	1.047
1.00	1.000
1.10	0.955
1.20	0.912

* It is of general interest to remark that the difference of magnitude in this case is probably greater than for any other of the double spectra that we have studied. The result just given is therefore in good agreement with the general conclusion (not however founded upon experiment so far as we know), expressed by other astronomers, that when the difference of brightness between the two components of a binary exceeds a magnitude, only the brighter spectrum will be visible upon plates taken with a slit spectrograph.

It therefore appears that the two stars are of nearly equal radii, certainly within ten per cent. and probably within five per cent. We assume provisionally that they are just equal in the computations that follow.

To determine the remaining elements of the system we may now employ the method described by Schlesinger on page 131, Volume 1 of these publications. The fraction (σ_0) of the brighter disk that is eclipsed at the instant of principal minimum is given, in this case, by

$$2.5 \log \frac{1 + 0.40}{1 - \sigma_0 + 0.40} = \text{magnitude at minimum minus magnitude at maximum} = 0.73\,; \qquad (2)$$

whence $\sigma_0 = 0.69$. As the two stars have equal radii, it follows from this value of σ_0 that the *projected* distance between centers is 0.49 of the radius of either star. We see from column (6) of the table above that the true distance between centers at the same instant is 0.95 $(a + a_s)$ and accordingly

$$\cos i = \frac{0.49\rho}{0.95\,(a + a_s)} \qquad (3)$$

Thus far we have made no use of the photometric observations except to determine the three magnitudes at maximum and minima. We are now ready to see how well the whole light-curve is represented under various assumptions as to the ratio between the radius of either star and the sum of the major axes of the orbits. We begin by assuming that the two stars are nearly in contact, or that $2\rho = 0.90$ $(a + a_s)$. Substituting in equation (3) we obtain $i = 76°.6$. The thirty magnitudes corresponding to the phases in Table I are then computed with the formulæ:

$$p = (r + r_s)\sqrt{1 - \sin^2 i \cdot \sin^2 u} \qquad (4)$$

$$\rho = 0.45\,(a + a_s) \qquad (5)$$

$$\cos\frac{\theta}{2} = \frac{p}{2\rho} \qquad (6)$$

$$\pi \cdot \sigma = \theta - \sin\theta \qquad (7)$$

$$\left.\begin{aligned}\text{Computed magnitude} &= 4.60 + 2.5 \log \frac{1.00 + 0.40}{1.40 - \sigma} \\ &= 6.21 - 2.5 \log (4.40 - \pi \cdot \sigma)\end{aligned}\right\}\ \text{Principal minimum} \qquad (8)$$

$$\left.\begin{aligned}\text{Computed magnitude} &= 4.60 + 2.5 \log \frac{1.00 + 0.40}{1.40 - 0.40\sigma} \\ &= 7.21 - 2.5 \log (11.00 - \pi \cdot \sigma)\end{aligned}\right\}\ \text{Secondary minimum} \qquad (9)$$

The resulting magnitudes are:

(1)	5.23	(7)	5.25	(13)	4.60	(19)	4.76	(25)	4.62
(2)	5.31	(8)	5.19	(14)	4.60	(20)	4.81	(26)	4.60
(3)	5.33	(9)	5.02	(15)	4.60	(21)	4.83	(27)	4.60
(4)	5.34	(10)	4.87	(16)	4.62	(22)	4.81	(28)	4.73
(5)	5.34	(11)	4.67	(17)	4.67	(23)	4.75	(29)	4.90
(6)	5.32	(12)	4.61	(18)	4.71	(24)	4.68	(30)	5.01

A comparison with the observed magnitudes in column (5) of Table I shows that the light-curve just computed, gives a principal minimum that is too prolonged and is too faint. It is therefore in order to make a second trial in which the ratio of the radii of the stars to the sum of the major axes is smaller, and in which a smaller fraction of the brighter disk is assumed to be covered at the instant of principal minimum. After several trials we arrive at magnitudes computed from the following formulæ in addition to (4), (6) and (7) above:

$$\rho = 0.40\,(a + a_{\prime}) \qquad (10)$$
$$i = 75^{\circ}.4 \qquad (11)$$

$$\left.\begin{aligned}\text{Computed Magnitude} &= 4.64 + 2.5 \log \frac{1.385}{1.385 - \sigma} \\ &= 6.24 - 2.5 \log (4.35 - \pi \cdot \sigma)\end{aligned}\right\} \text{Principal minimum} \qquad (12)$$

$$\left.\begin{aligned}\text{Computed Magnitude} &= 4.64 + 2.5 \log \frac{1.385}{1.385 - 0.385\,\sigma} \\ &= 7.27 - 2.5 \log (11.30 - \pi \cdot \sigma)\end{aligned}\right\} \text{Secondary minimum} \qquad (13)$$

The resulting magnitudes appear in column (8) of Table I, and the corresponding light-curve is drawn in Figure 2. Other trials were made in which the two stars were assumed slightly unequal in size and curves were derived that do not differ materially from the above; the photometric observations do not enable us to determine the ratio of the two radii any more closely than the considerations that led to Table II. Furthermore it appears probable, from an inspection of Figure 2, that the light of the system does not remain quite constant between the two eclipses. This could be accounted for, without changing the assumptions in equations (10) to (13), by supposing that the two stars are prolate toward each other, and therefore present to us more of their uniformly brilliant surfaces at the times that are midway between the two eclipses. But as the quantities involved amount to only a few hundredths of a magnitude it would be futile to attempt to determine the amount of this prolateness and to correct the computed magnitudes accordingly. So far then as the present observational material is concerned, we regard as definitive formulæ (10) to (13) and the resulting magnitudes in column (8). On this basis the system is constituted as follows:

The diameter of either star is 8,200,000 km., or nearly six times that of our sun. The brighter star (visual magnitude 5.0) is 7.5 times as massive, but only 1/27 as dense, as the sun. The fainter star (visual magnitude 6.0) is 2.9 times as massive as the sun, and 1/70 as dense. The center of gravity of the system is situated well within the surface of the brighter star, at a mean distance of 2,900,-000 km. from its center, and 7,300,000 km. from the center of the faint star. The latter has a surface brightness equal to two-fifths of that for the massive star. The orbits are inclined 75° to the plane of sight. At the instant of prin-

cipal minimum 0.62 of the area of the brighter disk is eclipsed by the fainter, and at the instant of secondary minimum 0.59 of the fainter disk is eclipsed by the brighter.

We have thus far assumed that the instants of maximum eclipse occur when $u = 90°$ and $u = 270°$ as computed from the spectrographic data. We now inquire whether the photometric observations conform with this, or whether there is present a discrepancy of the kind that Schlesinger found to be the case with Algol and δ Libræ. An inspection of Figure 2 indicates at once that if the computed light-curve were moved to the right the agreement with the observed magnitudes would on the whole be improved. In order to test this matter more definitely, we shall make a least-squares solution in the following way: Let s represent the slope of the curve (that is the tangent of the inclination to the horizontal axis), and ΔT, the amount by which it is necessary to shift the curve to the right in order to secure the best agreement with the observations. Then each of the numbers in column (9) of Table I furnishes an equation of the form

$$s_1 \cdot \Delta T - (o - c)_1 = v_1$$

With ample accuracy for the purpose s may be scaled from Figure 2, in units of magnitudes *per diem*. Weights being assigned to these equations in accordance with the number of observations in column (3), their solution yields

$$\Delta T = +0.0160 \text{ days} = +23 \text{ minutes}$$
$$\pm .0038 \text{ “} \quad \pm 5.5 \text{ “}$$

A further inspection of Figure 2 indicates that the value of ΔT might be reduced by raising the entire curve, especially at principal minimum. Accordingly we undertake a second least-squares solution, in which the equations are of the form:

$$C + s_1 \, \Delta T - (o - c)_1 = v_1$$

The solution, in this way, of the twelve equations that concern only the principal minimum gives

$$C = -0.008 \text{ magnitudes}$$
$$\Delta T = +0.0169 \text{ days} = +24 \text{ minutes}$$
$$\pm .0047 \text{ “} \quad \pm 6.8 \text{ “}$$

The close agreement of the values of ΔT, from these two solutions, indicates that it is not greatly dependent upon the particular form of light-curve we may adopt. In addition to the probable errors given above, there is that arising from the spectrographic data, in the determination of the exact instant when $u = 90°$. For an orbit so nearly circular as the present, this probable error is equal to that for ϖ ($\pm$ 0°.54) given on page 54 above, since the value of T itself was assumed by

Baker, before entering the least-squares solution. Translating this probable error into days we have

$$\frac{\pm 0.^\circ 54}{360^\circ} \times 2.05 \text{ days} = \pm 0.0030 \text{ days} = \pm 4.3 \text{ minutes}$$

and combining this with our first solution for ΔT we adopt as definitive:

$$\Delta T = + 23 \text{ minutes} \quad \pm 7.0 \text{ minutes}$$

We conclude therefore *that the discrepancy between the light and the velocity phases already noticed in Algol and δ Libræ is very probably present in u Herculis as well, and is in the same sense.*

It should be remarked that the observations here discussed offer an unusually good opportunity to determine the amount of this discrepancy; for first, the two series, photometric and spectrographic, were made for the most part at the same time, and the mean epochs of the two do not differ by as much as 100 days. Consequently uncertainties as to periods or variations of period, such as had to be considered in the case of other eclipsing variables, have no bearing on the present discussion. Again, the photometric data are of a high degree of precision, the probable error of one observation being between 0.06 and 0.07 magnitudes. Finally, the range in velocity being large (200 km. for the massive star), and a large number of spectrograms (83) having been measured, the instant at which $u = 90^\circ$ is determined with much precision. For these reasons, although the amount of the discrepancy comes out somewhat less for *u* Herculis than for either Algol or δ Libræ, the probability for its reality is perhaps not less.

For the density of the brighter star in this system we have obtained a value 2.6 times as great as for the fainter. It may be well to estimate the uncertainty to which this quantity is liable. In the first place the ratio of the radii is determined with a probable error of perhaps 4 per cent.; the ratio of the volumes will therefore have a corresponding uncertainty of about 12 per cent. In addition we have the probable error of the ratio of the masses, which comes out equal to 5 per cent. Combining these two we obtain:

$$\text{The Ratio of the Densities} = 2.6 \pm 0.34$$

Thus we see that the brighter star is probably at least twice, and may be as much as three times as dense as the fainter star. This result has an important bearing upon questions of double-star evolution and it is desirable that it should be tested in the case of the few other stars having the necessary character, that are within the reach of present day instruments, or of the more powerful ones that will doubtless be available for this work in the near future.

In discussing other eclipsing variables astronomers have sometimes made the assumption, in default of positive knowledge, that the densities of the two bodies concerned are equal. It would appear from the present results that such an assumption may lead to very erroneous ideas as to the constitution of these systems, and should hereafter be used with caution. Suppose for example we had not been able to measure the fainter spectrum in the case of *u* Herculis and had therefore had no information as to the relative mass and density of the fainter star. If then we had assumed the two stars to be of equal density, we should have deduced masses that are respectively only one-eighth and one-third of their true values. The dimensions of the systems and of the two bodies in it would have come out only a little more than one-half those derived above. The mean density of the entire system would of course not have been changed, but for one body it would have been estimated much too low, and for the other much too high.

Another result that is of general interest is the fact that the surface brightness of the massive star is 2.6 times that of the other, with a probable error of perhaps ± 0.2, the presence of the secondary minimum in the light-curve enabling us to determine this ratio with unusual precision.

So close an equality in surface brightness could hardly have been expected, for in considering stars having a common origin and therefore the same age, it has been customary to associate a difference in surface brightness with a difference in spectrum. But the two components of *u* Herculis seem to have precisely the same spectrum,* both being of the strong helium type; in fact each of the nine lines used to determine the orbit of the brighter star, including the carbon line at $\lambda 4267$, has also been measured in the fainter spectrum. This result (so far as it goes) indicates that relative density, rather than age, is the controlling factor in determining the surface brightness of stars.

The question whether a solar star or a helium type star has the greater surface brightness was raised many years ago by Huggins and has recently been the subject of considerable discussion.* The present star offers the possibility for deciding this question by means of direct measurement. The visual magnitude of any star, as seen from the solar system, isgiven by the formula,

$$M = -0.60 - 5 \log \rho \cdot \pi \cdot \sqrt{\lambda}$$

where ρ and λ are the radius and the surface brightness of the star, those of our sun being the units; and π is the parallax expressed in seconds of arc. The constant in the right-hand member follows from Schwarzschild's determination of the visual magnitude of the sun, -27.17. Had we used Ceraski's value, -26.6,

* Publications of the Allegheny Observatory, 1, 75, 1909.

* See for example our paper on "A Comparative Study of Spectroscopic Binaries," Publications of the Allegheny Observatory, 1, 135, 1910.

the constant would, curiously enough, have reduced to zero. Now in this equation we know both ρ and M for each star in u Herculis. We have consequently:

$$\pi\sqrt{\lambda} = +0.''013 \text{ for the brighter star}$$
$$\pi\sqrt{\lambda_s} = +0.''008 \text{ `` `` fainter ``}$$

With Ceraski's constant these would become $+0''.016$ and $+0''.010$ respectively. If therefore the parallax of the system were accurately determined, we should be able to say whether λ and λ_s are greater than unity; or in other words, whether these helium stars have a greater surface luminosity than our sun. It would be extremely difficult to say with confidence, even with the accuracy that such determinations have recently attained, that the absolute parallax of any star is less than the small amounts here indicated. But if the parallax is really about $0''.02$ or over, it ought to be possible to establish the fact; and in this case it would be very probable, for at least the fainter star in the system, that the surface brightness is less than for our sun. The bearing of this consideration upon the question of stellar evolution is so important, that it would seem worth while for those engaged in parallax work to put this star upon their programmes, even if the chance of securing only negative evidence be large.