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A STATISTICAL INVESTIGATION

OF

COMETARY ORBITS

BY

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CONTENTS OF PART III.

PAGE

CHAPTER XI. CLASSIFICATION OF ORBITS 167

Distinction between Classes and Types, 170. Chronological Catalogue of Comets, 174. Elements of orbits published since Galle's Catalogue, 179. Data relating to the more accurately known Non-periodic Comets, 183. Summary of Chronological Catalogue, 186. Classification according to Brilliancy, 187. Summary of facts relating to recent comets, 188.

CHAPTER XII. APHELION DISTANCES. HYPERBOLIC ORBITS 189

Aphelion Distances, 189. Catalogue of Hyperbolic orbits, 196. Possible return of Hyperbolic Comets, 197. Computation of the Mass of the assumed planet *Q*, 198.

CHAPTER XIII. CLASSES *A* AND *B*. THE PERIODIC COMETS. PERTURBING FORCES 202

Catalogue of the aphelia of class *A*, 202. Catalogue of the Aphelia of class *B*, 204. Appearance and Disappearance of the comets of class *A*, 205. Invisible comets, 209. Relation of cometary discovery to solar activity, 210. Perihelion passages of Halley's Comet, 212. Catalogue of all the Aphelia of all the Periodic comets, 214. Changes in the Aphelia, 216. Perturbing Forces between four independent bodies, 217.

CHAPTER XIV. CLASS *C*. COMETS OF MODERATE ECCENTRICITY . . . 221

Catalogue of the aphelia of class *C*, 222. Explanation of the table, 227. Elements of the Reference Circle of type *P*, 229. Probability that the distribution of the Aphelia is due to chance, 232. Importance of this type, 233. History of the search by this method for an Ultra-Neptunian Planet, 234.

CHAPTER XV. CLASSES *D* TO *I*. COMETS OF HIGH AND OF UNKNOWN ECCENTRICITY. SUMMARY 236

Catalogue of the aphelia of classes *D* and *G*, 237. Why the comets of classes *E* and *F* are divided between types *Q* and *R*, 242. Elements of the Reference Circle of type *Q*, 245. Catalogue of the aphelia of classes *E* and *F*, 247. Elements of the Reference Circle of type *R*, 252. Catalogue of the aphelia of the Comets belonging to the Sub-types, 253. Catalogue of the aphelia of classes *H* and *I*, 256. Summary of the Classes and Types, 260. Classification according to Brilliancy, 262. Displacements and Densities of the aphelia, 263. Distribution within the Cometary belts, 264.

PAGE

CHAPTER XVI. COMETARY GROUPS. IDENTIFICATION OF COMETS . . . 267

A Catalogue of the Elements of certain Cometary groups, 274. Possible identity of the comets of −372 and 1843 III, 282. Catalogue of the Elements of the Periodic Comets, 286. Catalogue of the Elements of those comets of long period not previously grouped, 287. Possible early appearance of some of the Periodic Comets, 293. Identification at two returns of some of the Non-periodic Comets, 295.

CHAPTER XVII. ELEMENTS. ASTEROIDS. RESISTING MEDIUM. MISCELLANEOUS CHARACTERISTICS 300

Catalogue of small and of large Perihelion Distances, 302. Eccentricity of certain Cometary orbits, 303. Possible origin of the Asteroids, and of the Zodiacal Light, 303. Inclination of Cometary orbits, 305. Explanation of the retardation of Encke's comet, and that of 1882 II, 306. Various statistics of certain exceptional comets, 308.

CHAPTER XVIII. APHELION DISTANCES. PLANET *P*. PERIHELION. ECCENTRICITY 315

Catalogue of the Aphelion Distances of types *P*, *Q*, and *R* 315. Relation of the aphelion distances of the comets of type *P* to the arguments of their aphelia upon the orbit of the planet, 318. Orbit of Jupiter determined from its associated comets, 320. Comparison of the results with their true values, 324. Orbit of *P* determined by means of its associated comets, 325.

CHAPTER XIX. DISTRIBUTION OF THE APHELIA. PLANETS *Q* AND *R* . 330

Distribution of the Aphelia along the Reference Circles, 330. Eccentricity and Longitude of Perihelion of the orbit of *Q*, 338. Eccentricity and Longitude of Perihelion of the orbit of *R*, 339. Convenient method of drawing ellipses, 339. List of accurately observed comets whose eccentricities should be more accurately determined, 341. Discussion of the question as to whether any comets reach us from outside of the Solar System, 342. Direction of revolution of planet *Q*, 344. Present location of planet *Q*, 346. Its present distance and its mass, 353. Possible effect of *Q* on the orbits of the known planets, 353.

CHAPTER XX. PLANET *R*. LONGITUDE OF *P*. INCLINATION OF THE AXIS OF THE SUN. FACTS INDICATING THE EXISTENCE OF *Q*. SUMMARY 356

Aphelia of certain Asteroids of high eccentricity, 356. Advance in longitude of the asteroidal and cometary aphelia, 357. Explanation of the concentration of the aphelia of types *N* and *O*, 359. Mass of *R*, 360. Present location of planet *P*, 360. Total number of visible comets in the Solar System, 362. Cause of the Inclination of the Axis of the Sun, 363. Facts indicating the the existence of planet *Q*, 364. Summary of the results contained in Part III, 365. Elements of the Orbits of the three assumed planets *P*, *Q*, and *R*, 368.

APPENDIX. A PHOTOGRAPHIC SEARCH FOR PLANET *Q* 369

A STATISTICAL INVESTIGATION OF COMETARY ORBITS.

CHAPTER XI.

CLASSIFICATION OF ORBITS.

NEARLY 500 comets and reappearances of comets have been recorded in the last 2000 years with sufficient accuracy to enable us to determine the elements of their orbits. It has been thought that a statistical study of this mass of material might lead to the discovery of some new facts, and also that certain unexplained facts already known might be presented in such a manner as to admit of a plausible explanation.

A general classification of these orbits was first undertaken. In Table XXXIV the first column gives the catalogue number of the comet. Up to 411 this number is taken from the list published by Galle in his Verzeichniss der Elemente Cometenbahnen. The numbers beyond that have been applied to successive comets taken from the lists which have appeared in the Vierteljahrschrift der Astronomischen Gesellschaft. In the second column is given the year and numerical designation of the comet. In 1899 the small letters now universally employed as a temporary designation came into more general use than before, and from this date the letter is also given. It did not seem necessary to make this designation complete by adding the year of discovery, except in a few cases where it could not be readily determined by inspection.

If the comet is periodic and has appeared twice, its name is given in the third column, which also contains the date of discovery when given. When a comet has a compound name, such as Pons-Brooks or De Vico-E. Swift, the first part of the name only is retained. The comet of 1366 is clearly an early appearance of 1866 I discovered by Tempel, as was first pointed out by Hind, M.N. 1873, 33, 48. It is therefore designated in the table under the name $Tempel_4$.

The fourth and fifth columns give the longitude and latitude of the comet's aphelion to tenths of a degree. The same quantities will be found in the following chapters expressed to minutes of arc. The first catalogue of the location upon the celestial sphere of the cometary aphelia was published by W. H. S. Monck in the Journ. Liverpool Astron. Soc. 1889, 7, 252. It was based on the manuscript of Chambers' catalogue of comets which was at that time unpublished, and was reduced to the equinox of 1890. Quite independently, and without any knowledge of this work, a similar catalogue of perihelia was prepared a few years later at this observatory by the Director, based on Galle's publication. The Harvard catalogue was never published. Neither Monck's nor the Harvard catalogue were complete. The former extended through comet number 384, the last given by Chambers, but generally gave the location for only one appearance of each of the periodic comets. The Harvard catalogue began with comet number 136, the first to appear in the last century, and gave the aphelia of every appearance of every comet through number 411, the last one in Galle's catalogue.

The present catalogue combines the results given in the other two, reducing all longitudes to the equinox of 1900, corrects all errors that have been found, and completes the computation through the year 1909. The elements of orbits 1, 8, and 25 are so indefinite that the longitudes and latitudes of their aphelia have been omitted. In a number of the earlier orbits Galle gives later elements differing by several degrees from those of Chambers, catalogue. In these cases the aphelia have been recomputed. In all cases between orbits 136 and 384 inclusive, which are the only ones common to the two original catalogues, if the two results differed by as much as one degree they were recomputed. Deviations of less than this amount are for our present purposes of little consequence, and the Harvard results have been used directly. These results were preferred because when they differed from those of Monck's catalogue by minutes only, it was usually found to be due to the fact that elements by different computers had been employed, and the later results were given the preference.

Orbits previous to and following numbers 136 to 384 could not be as readily checked as those lying between these numbers, but fortunately a new catalogue of aphelia by Dr. F. Nölke has recently appeared, entitled "Neue Erklärung des Ursprungs der Kometen" published in the Sonder-Abdruck Abh. Nat. Ver. Bremen 20, 57. This catalogue gives the position for only one appearance of each of the periodic comets, and does not give the position of comet number 444, nor of any comet following 462. The positions are given to degrees only, and no correction is made for precession. The catalogue is sufficiently accurate for our purpose, however, and the correction for precession is easily applied. This catalogue enabled us to check nearly all the comets whose aphelia had previously depended on the work of a single computer. In several cases errors in the published elements of the orbits were detected. In some cases errors had been made in the computations. The periodic comets rarely needed checking, since the aphelia for successive appearances usually nearly coincided. Since many of the aphelia were computed, new elements have been published, but it is thought that in very few cases would the deviation introduced by a fresh computation amount to more than a few minutes of arc.

In the sixth column of the table the comets have been divided into five groups according to the assumed certainty with which their orbits are known. Group 1 includes all those whose aphelion distances do not exceed sixty units, or twice that of the planet Neptune. These distances are consequently as a general thing known with considerable accuracy. When the comet has appeared twice the second place of decimals may generally be trusted, otherwise an error of a unit is quite possible even among the comets of short period. Comets which have appeared but once, or whose elements are a little doubtful, are entered in the subgroup 1*a*. The fact that the comet, since it was last observed, may have faded out, or even changed its orbit is not considered material, since this is an investigation primarily not of comets, but of orbits.

Groups 2 and 3 include only those orbits that are based on observations made during an interval exceeding thirty days. No orbits are admitted into group 2 unless corrected for the perturbations that they may have suffered from any planets likely to affect them, or unless their reduction is sufficiently complete to permit of their being classed as "definitive." New star places have been determined in many of these cases, or the older ones have been especially corrected and brought up to modern standards of accuracy. In this group, although not strictly definitive, are also included Bessel's reduction of the comet of 1807, Frischauf's reduction of 1863 II, and Berberich's reduction of 1889 I. It will be noted that because an orbit is recorded as definitive, that that is not sufficient to permit it to be entered in group 2. To enter that group the computations upon which it is based must be founded on observations extending over a period exceeding thirty days, and moreover these observations must be modern, that is to say made since 1800.

Group 3 includes within the time limit above mentioned those orbits that have been less completely reduced. All orbits prior to the year 1650 have been omitted, and before 1750 only those comets are included which were observed for an interval exceeding three months. An exception to this rule was made in the case of the comet of 1742, of which numerous observations were taken extending over a period of two months. A careful reduction of them has recently been made by Cohn. The earliest orbit included in group 3 is that of the great comet of 1664 reduced by Lindelöf. The earliest orbit in group 2 is that of the comet of 1771 by Kreutz.

Groups 4 and 5 include all orbits not previously classified. Group 4 includes those which are based on observations covering an interval of rather less han thirty days. It is not thought that the aphelion distances of any of these orbits are sufficiently well known to permit us to draw valuable conclusions from them. Neither is it known in most cases whether the orbits themselves are elliptical, parabolic, or hyperbolic. On the other hand their elements are generally sufficiently well known to enable us to locate both the orbits and aphelia with considerable accuracy upon the celestial sphere. When the comet has only been observed through a range of a few days under unfavorable circumstances, or when it appeared before the days of accurate observation, and its elements are accordingly somewhat uncertain, it is entered in group 5, and errors of even 30° or more in the location of its aphelion may occur. Most of the comets appearing before 1550 will be found in this group. Besides the comets included in Table XXXIV numerous others are known to have appeared, and others still are only suspected, from the vague descriptions of them. Of these orbits we know little or nothing, save the date of appearance of the comet.

In the seventh column of the table the orbits are divided into nine classes and two subclasses designated by letters. This classification is based, when possible, simply on their aphelion distances. These distances are always computed from the last elements given in Galle's catalogue or in the Vierteljahrschrift. These elements were selected as being in general those based on the most complete information, and therefore considered to be the most reliable. The reason for deciding on the exact limits of certain of these classes will be described later, but it may be stated here that class *A* includes the orbits of the periodic comets of short period, whose aphelia range from 4 to 8 units, class *B* those of long period, whose aphelia range from 8 to 60 units, class *C* includes those orbits with aphelia extending from 60 to 220 units, class *D* includes those orbits whose aphelia range from 220 to 4,000 units, and class *E* those whose computed aphelia exceed 4,000 units. When the orbit is given as parabolic, and is sufficiently well known to be ranked in groups 2 or 3 of the previous column, it is entered in class *F*. Class *G* contains the hyperbolic orbits, and class *H* those whose orbits are not well known, but which have appeared since 1700. These comets are therefore generally faint objects. Class *I* contains those comets whose orbits are not well known and which appeared prior to 1700. They are generally therefore brilliant objects. It also contains four bright comets which have appeared since that date. Twenty comets are recorded in our list as having appeared between 1600 and 1700. In the nineteenth century 81 naked eye comets appeared, 14 of them being brilliant, so it is safe to assume that the 20 seen in the seventeenth century were all fairly bright objects. With the advent of the large telescope, and the multiplication of observatories during the eighteenth century, few bright comets could have escaped careful observation. The sub-classes *a* and *b* contain the earlier appearances of the comets belonging to the first two classes.

In the eighth column the orbits are classified in five types and three subtypes. In orbits of the same type the aphelia will all be found lying near some particular circle on the celestial sphere. While the classes depend therefore on the aphelion distances, the types depend on the longitudes and latitudes of the aphelia. In many cases the aphelion distances of comets of the same type are found to be of the same order of size. Thus all the comets of classes *A* and *a* belong to type *N*, and all those of classes *B* and *b* to type *O*. It is of interest to note, that in the majority of cases a similar identity holds for the other classes and types. In a certain number of cases, as we shall see later, a comet may be classed under either type *Q* or type *R*. The subtypes contain those orbits which cannot be properly classified in the five main types.

A special group of comets, insignificant in numbers, must yet theoretically exist. It consists of those that are driven into hyperbolic orbits by some planet, and which then pass out of its influence. If no other planet interferes, the comet will leave the Sun never to return to it. To this theoretical group of interstellar objects must be added those comets that come to us from the stars. The hyperbolic excess of their perihelion velocities would in general be enormous, so that they could be readily distinguished, if seen, from the true solar comets. The chance that one should pass at perihelion within two or three units of the Sun, however, is extremely small, and in point of fact no high hyperbolic velocities have hitherto been detected. Nevertheless such bodies must theoretically exist, and it has been suggested that certain meteors which have passed through our atmosphere at high speed possessed hyperbolic velocities with regard to the Sun. A fireball which fell August 19, 1847 is mentioned by Monck, Journ. Liverpool Astron. Soc. 1889, 7, 221, the eccentricity of whose orbit fell little short of 4. Three meteors are mentioned by Chambers, Handbook of Astronomy 1889, I, 604, whose velocities with regard to the Earth equalled or exceeded 44.7 miles per second, the maximum parabolic velocity. Two meteors are mentioned by Denning, M.N. 1885, 45, 408 whose apparent velocities were so high that they may have had hyperbolic orbits.

The last column of the table refers directly to the comet itself, rather than to its orbit. The comets are here divided into five grades of brillancy. A *I* indicates that the comet is known to have been an unusually fine one, and a *II* that it was notable, but inferior to those in the first rank. A *III* indicates that the comet was visible to the naked eye, and possessed a tail which could be traced visually in the telescope or without it for a distance of over one degree. A *IV* indicates that the comet was visible to the naked eye, but that its tail, if it had one, did not visually exceed one degree in length. A blank after the year 1700 indicates that the comet was telescopic. The numbers *II*, *III*, and *IV* are applied only to comets appearing since that date. It is almost certain that all of those comets following number 411, or the year 1893, which are not marked with a Roman numeral, were invisible to the naked eye. This is generally true also of the others seen in the last century, but in the earlier portion of this period the brilliancy of the fainter comets was not so carefully recorded, and an occasional comet unmarked with a Roman numeral may yet have been fairly conspicuous.

In the first half of the nineteenth century 93 comets were recorded, of which we find that 31, or 33 per cent, were visible to the naked eye. In the latter half of the century 219 comets were recorded, of which 50, or 23 per cent, were naked eye comets. There were thus 19 more naked eye comets visible in the last half of the century than in the first half. This difference occurred chiefly in comets of brilliancy *III*, those furnished with tails over 1° in length. Twelve more comets of this brilliancy were found in the last half century than in the first half. These could hardly have been overlooked had they existed, nor would the fact of their having tails have been likely to have been omitted in the description of them in so many cases. It is probable that the difference is due, at least in part, to the more careful study that is now made of the southern skies.

The classification in this column is necessarily not only somewhat uncertain, but also obviously depends largely on the proximity of the comet, on the phase of the Moon, and whether the comet was well situated in its orbit for observation. Bearing well in mind its limitations, however, we shall yet find that interesting conclusions may be derived from the results recorded in this column.

As far as the very brightest comets are concerned, if impartially recorded, doubtless nearly all of the earlier comets on the list would be marked with either a *I* or a *II*. But the object of this marking for the earlier centuries is rather to call attention to the more celebrated comets, details of whose appearance have descended to the present time. For information regarding the conspicuousness of these bodies Herschel's Outlines, Chambers' table of comets, and the Markree Catalogue by Cooper have been consulted.

Although ranked in Subclass *a* the identification of De Vico's comet number 64, and Winnecke's number 100 are not absolutely assured, but of the two, De Vico's is probably the more certain. The last appearance of Faye's comet for which elements have been computed was number 382. It has accordingly been marked *A*, although the comet appeared again later,— number 421. This latter is marked *a*. The comets of −12, +66, 141, 837, 989, 1066, and 1301, together with several others not recorded by Galle because of insufficient elements, and therefore not included in this list, have been recently identified by Messrs. Cowell and Crommelin as undoubted appearances of Halley's comet. They have therefore been marked in the table with certainty 1*a*, because their aphelion distances are well known, although the elements given by Galle undoubtedly exhibit large errors. The later elements for number 17 will be found in the M.N. 1908, **68**, 378. Orbit number 107 is hyperbolic, and not parabolic as erroneously given by Galle. Burckhardt, its computer, gives its eccentricity as 1.0282955, the largest value of any orbit in this list.

TABLE XXXIV.

CLASSIFICATION OF ORBITS.

No.	Year.	Name.	Long. 1900.	Lat.	Cert.	Cl.	Ty.	Br.	No.	Year.	Name.	Long. 1900.	Lat.	Cert.	Cl.	Ty.	Br.
			°	°								°	°				
1	−372		—	—	5	*I*	—	*I*	47	1577		344.8	+69.4	4	*I*	—	
2	−137		50.1	+ 3.4	5	*I*	—		48	1580		292.3	−64.5	4	*I*	—	
3	− 69		154.5	−28.0	5	*I*	—		49	1582		67.7	+23.3	5	*I*	—	
4	− 12	Halley	126.4	− 9.5	1*a*	*b*	*O*	*I*	50	1585		193.7	+ 2.9	4	*I*	—	
5	+ 66	Halley	176.6	−36.9	1*a*	*b*	*O*		51	1590		38.4	+22.9	4	*I*	—	
6	141	Halley	95.3	−14.5	1*a*	*b*	*O*		52	1593		349.0	−12.0	4	*I*	—	
7	240		111.1	−43.5	5	*I*	—		53	1596		108.4	−42.7	4	*I*	—	
8	539		—	—	5	*I*	—		54	1607	Halley	124.1	−16.3	1	*b*	*O*	
9	565		287.9	−57.4	5	*I*	—		55	1618 I		140.7	− 8.9	5	*I*	—	
10	568		157.1	− 1.7	4	*I*	—		56	" II		191.0	+35.2	4	*I*	—	*I*
11	574		337.5	−11.1	5	*I*	—		57	1652		254.1	+58.2	5	*I*	—	
12	770		188.6	−61.6	5	*I*	—		58	1661		294.2	−17.4	4	*I*	—	*I*
13	837	Halley	123.7	+10.9	1*a*	*b*	*O*		59	1664		311.8	+16.0	3	*F*	*R*	*I*
14	961		129.5	−77.2	5	*I*	—		60	1665		237.4	−23.1	4	*I*	—	
15	989	Halley	96.7	0.0	1*a*	*b*	*O*		61	1668		102.5	−35.4	4	*I*	—	
16	1006		136.3	−17.4	5	*I*	—		62	1672		282.6	−69.4	4	*I*	—	
17	1066	Halley	95.4	−14.5	1*a*	*b*	*O*	*I*	63	1677		289.5	−75.7	4	*I*	—	
18	1092		344.4	−14.3	4	*I*	—		64	1678	De Vico	145.9	− 1.0	1*a*	*a*	*N*	
19	1097		196.6	−51.8	5	*I*	—		65	1680		90.6	+ 8.1	3	*D*	*Q*	*I*
20	1231		324.2	− 5.2	5	*I*	—		66	1682	Halley	124.1	−16.7	1	*b*	*O*	
21	1264		130.1	− 5.7	4	*I*	—	*I*	67	1683		284.3	−82.9	4	*I*	—	
22	1299		171.2	−65.0	5	*I*	—		68	1684		78.2	+26.6	4	*I*	—	
23	1301	Halley	140.5	+ 1.4	1*a*	*b*	*O*		69	1686		259.4	−31.3	4	*I*	—	
24	1337		190.0	−40.5	5	*I*	—	*I*	70	1689		167.4	−60.9	5	*I*	—	
25	1351		—	—	5	*I*	—		71	1695		241.3	+ 8.8	5	*I*	—	
26	1362		56.0	− 5.3	5	*I*	—		72	1698		97.1	− 5.2	5	*I*	—	
27	1366	$Tempel_4$	234.3	− 4.9	1*a*	*b*	*O*		73	1699		9.9	−62.1	4	*I*	—	
28	1378	Halley	126.0	−17.0	1	*b*	*O*		74	1701		312.8	− 9.9	4	*H*	—	
29	1385		283.9	−10.4	5	*I*	—		75	1702		321.6	+ 3.4	5	*H*	—	*IV*
30	1402		35.7	−55.0	5	*I*	—	*I*	76	1706		239.7	−44.9	4	*H*	—	
31	1433		100.6	+ 9.0	4	*I*	—		77	1707		236.2	−27.1	4	*I*	—	*II*
32	1449		90.4	+ 1.3	5	*I*	—		78	1718		306.0	− 3.2	4	*I*	—	*II*
33	1456	Halley	124.4	−17.0	1	*b*	*O*	*I*	79	1723		216.0	+21.5	4	*H*	—	*III*
34	1457 I		270.4	+ 3.4	5	*I*	—		80	1729		135.2	− 9.6	3	*F*	*R*	*IV*
35	" II		195.6	+ 0.9	5	*I*	—		81	1737 I		148.7	−18.1	4	*H*	—	*III*
36	1468		192.2	−35.3	5	*I*	—		82	" II		91.9	−24.6	5	*H*	—	
37	1472		226.0	+ 8.4	4	*I*	—	*I*	83	1739		274.6	−53.0	4	*H*	—	
38	1490		257.9	−37.0	5	*I*	—		84	1742		21.5	+29.2	3	*F*	*R*	*II*
39	1499		183.8	−11.4	5	*I*	—		85	1743 I		275.5	− 0.2	1*a*	*A*	*N*	
40	1500		130.2	−19.3	5	*I*	—		86	" II		59.8	+38.7	5	*H*	—	*IV*
41	1506		85.0	+38.7	5	*I*	—		87	1744		27.6	−20.5	3	*F*	*r*	*I*
42	1531	Halley	125.7	−16.4	1	*b*	*O*		88	1747		136.6	+49.0	3	*F*	*P*	
43	1532		293.4	−12.9	5	*I*	—	*I*	89	1748 I		53.6	−17.4	4	*H*	—	*III*
44	1533		43.9	+27.9	5	*I*	—		90	" II		76.0	+57.0	4	*H*	—	*IV*
45	1556		100.4	−29.8	4	*I*	—	*I*	91	1757		304.9	+12.8	4	*H*	—	*IV*
46	1558		11.9	−54.3	5	*I*	—		92	1758		68.3	−33.8	3	*F*	*Q*	

No.	Year.	Name.	Long. 1900.	Lat.	Cert.	Cl.	Ty.	Br.
			°	°				
93	1759 I	Halley	124.2	−16.4	1	b	O	II
94	" II		250.6	+78.4	3	F	Q	III
95	" III		320.3	+ 4.1	4	H	—	III
96	1762		341.4	−64.2	4	H	—	IV
97	1763		263.5	−72.4	3	D	Q	
98	1764		188.2	−50.4	4	H	—	III
99	1766 I		321.7	−40.0	4	H	—	
100	" II	Winnecke	73.1	+ 0.4	1a	a	N	III
101	1769		332.5	+19.6	3	D	R	I
102	1770 I		178.1	+ 1.1	1a	A	N	II
103	" II		31.8	+30.9	4	H	—	
104	1771		285.6	−10.9	2	F	Q	II
105	1772	Biela	290.6	+10.7	1	a	N	
106	1773		276.4	+39.0	3	F	QR	IV
107	1774		176.2	−42.9	3	G	R	
108	1779		264.7	−28.4	3	F	Q	
109	1780 I		83.4	+43.1	3	D	Q	
110	" II		95.3	+66.2	4	H	—	
111	1781 I		81.0	−23.6	4	H	—	III
112	" II		200.6	−23.6	4	H	—	III
113	1783		233.0	+ 4.2	1a	A	N	
114	1784		254.0	+18.4	3	F	QR	III
115	1785 I		275.0	+24.0	4	H	—	
116	" II		69.4	−52.8	3	D	Q	II
117	1786 I	Encke	338.2	+ 0.6	1	a	N	
118	" II		352.2	+26.4	3	F	R	
119	1787		184.9	−47.4	3	F	r	
120	1788 I		281.3	−10.5	4	H	—	III
121	" II		188.1	−27.2	4	H	—	
122	1790 I		237.8	−28.4	4	H	—	
123	" II	Tuttle	282.6	+20.4	1	b	O	
124	" III		72.6	−51.4	3	F	Q	III
125	1792 I		212.6	−16.1	4	H	—	
126	" II		307.6	−24.1	4	H	—	
127	1793 I		69.6	+48.7	4	H	—	
128	" II		243.0	−47.3	3	C	P	
129	1795	Encke	338.1	+ 0.5	1	a	N	IV
130	1796		16.6	+ 3.9	4-	H	—	
131	1797		225.4	+49.7	4	H	—	IV
132	1798 I		291.0	+11.8	4	H	—	
133	" II		223.6	+22.8	4	H	—	
134	1799 I		181.6	−50.6	3	F	r	III
135	" II		340.3	−42.1	4	H	—	III
136	1801		6.0	+13.1	4	H	—	
137	1802		144.0	−18.2	4	H	—	
138	1804		341.7	+23.1	4	H	—	
139	1805	Encke	338.0	+ 0.6	1	a	N	III
140	1806 I	Biela	290.0	+ 8.4	1	a	N	IV
141	" II		284.1	+24.1	3	G	Q	
142	1807		89.9	− 3.7	2	D	Q	II

No.	Year.	Name.	Long. 1900.	Lat.	Cert.	Cl.	Ty.	Br.
			°	°				
143	1808 I		256.9	+43.4	4	H	—	
144	" II		66.6	−28.3	5	H	—	
145	1810		265.7	−53.8	4	H	—	
146	1811 I		289.2	−60.4	2	D	Q	I
147	" II		233.2	+21.8	3	C	R	IV
148	1812	Pons	259.3	+18.5	1	b	O	III
149	1813 I		250.6	− 3.4	3	F	R	
150	" II		39.8	+24.7	4	H	—	IV
151	1815	Olbers	322.2	−39.6	1	b	O	
152	1816		97.5	+34.3	5	H	—	
153	1818 I		257.4	+ 0.2	5	H	—	
154	" II		70.9	−67.7	3	F	Q	
155	" III		276.6	+10.5	3	G	Q	
156	1819 I	Encke	338.0	+ 0.6	1	a	N	
157	" II		97.0	−13.2	2	F	QR	II
158	" III	Winnecke	96.1	− 3.4	1	a	N	
159	" IV		248.8	+ 1.6	1a	A	N	
160	1821		52.9	−10.4	3	F	R	III
161	1822 I		7.7	+12.2	4	H	—	IV
162	" II	Encke	338.2	+ 0.6	1	a	N	
163	" III		46.9	+30.0	4	H	—	
164	" IV		93.2	+ 0.8	2	D	Q	III
165	1823		116.8	−27.6	2	F	R	III
166	1824 I		71.2	+20.9	4	H	—	
167	" II		182.2	−54.3	3	F	r	
168	1825 I		83.3	−53.4	2	D	Q	
169	" II		194.0	− 2.7	4	H	—	
170	" III	Encke	338.2	+ 0.6	1	a	N	
171	" IV		142.3	+32.6	2	D	P	II
172	1826 I	Biela	290.0	+ 8.4	1	a	N	
173	" II		300.8	+39.4	3	F	R	
174	" III		216.8	− 0.4	5	H	—	
175	" IV		237.5	− 6.0	3	D	R	
176	" V		59.9	+80.4	2	F	Q	IV
177	1827 I		192.3	−28.3	2	F	R	
178	" II		123.9	−14.1	4	H	—	
179	" III		79.5	+52.6	3	D	Q	
180	1829	Encke	338.2	+ 0.6	1	a	N	IV
181	1830 I		32.8	− 2.1	2	F	R	III
182	" II		139.0	−18.6	3	F	R	III
183	1832 I	Encke	338.2	+ 0.6	1	a	N	
184	" II		55.0	+16.6	3	F	R	
185	" III	Biela	290.2	+ 8.8	1	a	N	
186	1833		45.2	+ 7.2	4	H	—	
187	1834		97.4	− 4.6	3	F	QR	
188	1835 I		28.9	+ 4.6	2	F	R	
189	" II	Encke	338.2	+ 0.6	1	a	N	
190	" III	Halley	124.5	−16.6	1	B	O	II
191	1838	Encke	338.2	+ 0.6	1	a	N	IV
192	1840 I		2.7	−49.6	3	E	q	IV

No.	Year.	Name.	Long. 1900.	Lat.	Cert.	Cl.	Ty.	Br.	No.	Year.	Name.	Long. 1900.	Lat.	Cert.	Cl.	Ty.	Br.
			°	°								°	°				
193	1840 II		250.2	−20.0	2	D	P		243	1854 III		119.1	−66.0	3	F	Q	III
194	" III		177.9	−41.2	4	H	—	III	244	" IV		283.1	−30.2	2	C	P	
195	" IV		220.6	−37.9	2	C	P		245	" V		346.3	+13.5	3	C	P	
196	1842 I	Encke	338.2	+ 0.6	1	a	N		246	1855 I		35.5	+28.0	3	C	P	
197	" II		182.0	+56.6	2	F	q		247	" II		60.0	− 8.7	4	H	—	
198	1843 I		101.2	−35.3	2	C	Q	I	248	" III	Encke	338.4	+ 0.4	1	a	N	
199	" II		116.4	−41.2	3	G	Q		249	IV		266.2	+ 5.8	3	F	QR	IV
200	" III	Faye	230.0	+ 3.9	1	a	N		250	1857 I		310.4	−58.4	2	F	Q	
201	1844 I	De Vico	163.3	+ 2.9	1	a	N	IV	251	" II	Brorsen	294.6	− 6.9	1	a	N	
202	" II		10.6	+22.9	2	E	R	IV	252	" III		52.4	−38.0	4	H	—	
203	" III		117.5	− 1.6	2	D	r	III	253	" IV		202.2	+ 0.5	2	C	P	
204	1845 I		281.3	−41.6	2	G	Q		254	" V		54.3	−42.9	3	D	q	III
205	" II		2.6	+21.0	2	E	R		255	" VI		223.5	−37.6	3	D	P	
206	" III		89.6	−46.7	4	H	—	III	256	" VII	d'Arrest	143.8	− 1.3	1	a	N	
207	" IV	Encke	338.4	+ 0.8	1	a	N		257	1858 I	Tuttle	286.0	+21.5	1	b	O	
208	1846 I		276.6	+16.0	3	F	QR		258	" II	Winnecke	96.6	− 3.3	1	a	N	
209	" II	Biela	289.1	+ 8.6	1	a	N		259	" III		20.0	− 8.3	1a	A	N	
210	" III	Brorsen	295.3	− 7.0	1	a	N		260	" IV		13.5	−76.7	4	H	—	
211	" IV		259.4	−12.8	1a	B	O		261	" V	Faye	230.0	+ 3.9	1	a	N	
212	" V		272.4	−55.9	3	F	Q		262	" VI		195.0	−43.8	2	D	p	I
213	" VI		63.4	+10.2	1a	B	O		263	" VII		183.3	− 8.6	3	D	R	
214	" VII		341.4	−28.8	3	C	P	IV	264	" VIII	Encke	338.4	+ 0.4	1	a	N	IV
215	" VIII		281.6	−49.5	3	F	Q		265	1859		202.0	+76.8	4	H	—	
216	1847 I		89.4	+46.3	3	D	Q	II	266	1860 I		330.5	+29.2	4	H	—	
217	" II		348.0	−31.8	2	F	p		267	" II		219.7	−29.4	3	F	P	
218	" III		55.8	−83.2	2	D	Q		268	" III		303.7	−73.1	2	F	Q	III
219	" IV		206.8	−26.4	2	F	R		269	" IV		182.2	+23.4	5	H	—	
220	" V		261.5	−14.7	1a	B	O		270	1861 I		37.2	+32.8	2	C	P	III
221	" VI		81.2	+70.7	2	G	Q	IV	271	" II		96.9	+29.8	2	C	P	I
222	1848 I		180.6	+79.4	4	H	—		272	" III		347.6	+18.6	4	H	—	
223	" II	Encke	338.5	+ 0.5	1	a	N	III	273	1862 I	Encke	338.5	+ 0.5	1	a	N	
224	1849 I		218.6	+27.9	3	F	r		274	" II		120.1	− 3.6	4	H	—	IV
225	" II		37.5	−30.3	2	G	r		275	" III		149.6	−24.8	1a	B	O	II
226	" III		62.0	+50.2	3	D	P		276	" IV		314.4	+31.4	4	H	—	
227	1850 I		93.8	+ 0.5	2	D	Q	III	277	1863 I		313.6	−73.8	2	G	Q	
228	" II		263.3	+35.1	2	F	QR		278	" II		70.2	− 3.7	2	F	R	III
229	1851 I	Faye	230.0	+ 3.9	1	a	N		279	" III		77.2	−55.4	2	D	Q	III
230	" II	d'Arrest	143.7	− 1.3	1	a	N		280	" IV		277.4	+ 2.7	2	D	Q	III
231	" III		130.9	−38.1	2	E	R		281	" V		263.4	−54.4	3	F	Q	IV
232	" IV		193.8	+61.1	4	H	—		282	" VI		314.4	−76.4	2	G	Q	
233	1852 I	Encke	338.4	+ 0.4	1	a	N		283	1864 I		5.4	+ 9.8	4	H	—	
234	" II		111.4	−27.1	4	H	—		284	" II		124.7	− 0.9	2	D	r	III
235	" III	Biela	289.1	+ 8.6	1	A	N		285	" III		8.6	+48.3	2	D	r	
236	" IV		216.4	−33.4	1a	B	O	IV	286	" IV		153.2	−41.5	3	F	R	
237	1853 I		334.0	+20.1	2	F	R		287	" V		342.8	− 0.4	3	F	P	
238	" II		31.1	+16.2	2	C	P	III	288	1865 I		259.7	−68.1	3	F	Q	III
239	" III		136.6	− 8.4	3	G	R	III	289	" II	Encke	338.5	+ 0.5	1	a	N	
240	" IV		114.9	+60.0	3	G	Q	IV	290	1866 I	Tempel$_4$	240.5	− 2.7	1a	B	O	
241	1854 I		231.4	− 8.3	3	F	R		291	" II	Faye	230.0	+ 3.9	1	a	N	
242	" II		348.4	−76.2	4	H	—	III	292	1867 I		256.6	+ 0.8	1a	B	O	

No.	Year.	Name.	Long. 1900.	Lat.	Cert.	Cl.	Ty.	Br.
			°	°				
293	1867 II	Tempel$_1$	56.8	− 4.5	1	*a*	*N*	
294	“ III		69.4	−31.2	3	*F*	*QR*	
295	1868 I	Brorsen	294.6	− 7.2	1	*a*	*N*	
296	“ II		94.4	−36.9	2	*F*	*QR*	
297	“ III	Encke	338.6	+ 0.6	1	*a*	*N*	
298	1869 I	Winnecke	96.6	− 3.2	1	*a*	*N*	
299	“ II		308.9	+ 7.6	3	*F*	*R*	
300	“ III	Tempel$_3$	223.5	− 5.2	1	*a*	*N*	
301	1870 I		132.3	+15.4	2	*F*	*P*	
302	“ II		194.2	+ 5.0	2	*F*	*R*	
303	“ III	d'Arrest	139.4	− 2.1	1	*a*	*N*	
304	“ IV		184.4	−32.7	4	*H*	—	
305	1871 I		281.9	+42.5	3	*D*	*Q*	
306	“ II		274.2	−76.5	2	*F*	*Q*	
307	“ III	Tuttle	286.1	+21.5	1	*b*	*O*	
308	“ IV		131.7	+61.7	2	*D*	*Q*	
309	“ V	Encke	338.5	+ 0.5	1	*a*	*N*	*IV*
310	1873 I	Tempel$_1$	58.7	− 3.4	1	*a*	*N*	
311	“ II	Tempel$_2$	126.4	+ 1.1	1	*a*	*N*	
312	“ III	Faye	230.6	+ 3.9	1	*a*	*N*	
313	“ IV		229.5	+13.7	2	*D*	*R*	
314	“ V		141.6	+43.4	3	*D*	*P*	*III*
315	“ VI	Brorsen	294.6	− 7.2	1	*a*	*N*	
316	“ VII		264.5	+ 7.7	4	*H*	—	
317	1874 I		119.7	+58.9	4	*H*	—	
318	“ II		119.1	+14.4	3	*F*	*P*	
319	“ III		107.2	−25.1	2	*D*	*Q*	*II*
320	“ IV		190.3	−16.5	2	*C*	*P*	
321	“ V		165.4	−41.8	3	*D*	*R*	
322	“ VI		99.6	−16.1	3	*F*	*QR*	
323	1875 I	Winnecke	97.3	− 2.9	1	*a*	*N*	
324	“ II	Encke	338.5	+ 0.5	1	*a*	*N*	
325	1877 I		19.0	+ 5.8	2	*F*	*R*	*IV*
326	“ II		91.3	−49.8	2	*D*	*Q*	*IV*
327	“ III		322.7	−60.5	2	*D*	*Q*	
328	“ IV	d'Arrest	139.7	− 1.9	1	*a*	*N*	
329	“ V		246.2	−61.2	4	*H*	—	
330	“ VI		260.3	−35.8	3	*F*	*Q*	
331	1878 I		102.1	− 2.4	4	*H*	—	
332	“ II	Encke	338.5	+ 0.5	1	*a*	*N*	
333	“ III	Tempel$_2$	126.3	+ 1.1	1	*a*	*N*	
334	1879 I	Brorsen	294.7	− 7.2	1	*A*	*N*	
335	“ II		225.0	− 3.6	2	*F*	*R*	
336	“ III	Tempel$_1$	58.8	− 3.4	1	*A*	*N*	
337	“ IV		141.0	−71.4	4	*H*	—	
338	“ V		62.4	−61.7	2	*F*	*Q*	
339	1880 I		101.0	−35.2	4	*I*	—	*II*
340	“ II		278.3	−28.6	3	*F*	*Q*	
341	“ III		256.2	+21.7	2	*F*	*QR*	*III*
342	“ IV	Tempel$_3$	223.4	− 5.2	1	*a*	*N*	

No.	Year.	Name.	Long. 1900.	Lat.	Cert.	Cl.	Ty.	Br.
			°	°				
343	1880 V		75.4	−10.2	3	*F*	*QR*	*IV*
344	1881 I	Faye	230.7	+ 4.1	1	*a*	*N*	
345	“ II		125.4	− 6.0	4	*H*	—	
346	“ III		88.6	+ 5.1	2	*D*	*Q*	*III*
347	“ IV		148.1	−32.8	2	*F*	*R*	*III*
348	“ V		198.9	+ 5.0	1a	*A*	*N*	
349	“ VI		92.0	− 5.8	3	*F*	*QR*	
350	“ VII	Encke	338.6	+ 0.6	1	*a*	*N*	*IV*
351	“ VIII		238.6	−30.6	3	*C*	*P*	
352	1882 I		214.0	+27.8	2	*E*	*r*	*III*
353	“ II		101.6	−35.2	2	*D*	*Q*	*I*
354	“ III		228.5	+73.2	2	*G*	*Q*	
355	1883 I		249.9	−66.1	2	*G*	*Q*	
356	“ II		285.0	−36.8	4	*H*	—	*IV*
357	1884 I	Pons	259.8	+18.4	1	*B*	*O*	*III*
358	“ II		126.5	+ 4.7	1a	*A*	*N*	
359	“ III	Wolf	199.9	− 3.1	1	*a*	*N*	
360	1885 I	Encke	338.7	+ 0.7	1	*a*	*N*	
361	“ II		92.2	− 1.5	2	*G*	*Q*	
362	“ III		50.5	−36.2	2	*C*	*q*	
363	“ IV	Tuttle	286.3	+21.5	1	*b*	*O*	
364	“ V		110.3	−23.2	2	*F*	*QR*	
365	1886 I		26.8	−52.8	3	*F*	*q*	*III*
366	“ II		58.8	−59.9	2	*G*	*Q*	*III*
367	“ III		99.9	−37.8	3	*G*	*Q*	
368	“ IV		50.6	− 0.7	1a	*A*	*N*	
369	“ V		193.8	+21.2	3	*C*	*p*	
370	“ VI	Winnecke	96.6	− 2.0	1	*a*	*N*	
371	“ VII	Finlay	187.8	+ 2.1	1	*a*	*N*	
372	“ VIII		81.2	−31.8	2	*F*	*QR*	
373	“ IX		245.2	−77.8	2	*G*	*Q*	*III*
374	1887 I		101.6	−37.8	4	*I*	—	*II*
375	“ II		285.4	−19.9	2	*C*	*P*	
376	“ III		286.1	−22.6	2	*F*	*Q*	
377	“ IV		79.9	− 4.5	2	*D*	*Q*	
378	“ V	Olbers	321.9	−39.6	1	*B*	*O*	
379	1888 I		65.5	0.0	3	*D*	*R*	*III*
380	“ II	Encke	338.7	+ 0.7	1	*a*	*N*	
381	“ III		306.2	−55.7	3	*E*	*Q*	
382	“ IV	Faye	230.6	+ 4.1	1	*A*	*N*	
383	“ V		262.1	+51.1	2	*D*	*Q*	
384	1889 I		196.6	+ 4.5	2	*G*	*R*	*IV*
385	“ II		255.9	+13.4	3	*E*	*R*	
386	“ III		147.2	−26.7	1a	*B*	*O*	
387	“ IV		100.5	+12.9	2	*D*	*Q*	*IV*
388	“ V	Brooks	181.8	+ 1.7	1	*a*	*N*	
389	“ VI		220.1	− 9.6	1a	*A*	*N*	
390	1890 I		19.8	+16.6	2	*F*	*R*	
391	“ II		87.6	−53.5	2	*G*	*Q*	
392	“ III		274.8	−63.0	4	*H*	—	

No.	Year.	Name.	Long. 1900. °	Lat. °	Cert.	Cl.	Ty.	Br.
393	1890 IV		291.7	+12.0	3	D	Q	
394	" V	d'Arrest	139.6	− 1.9	1	a	N	
395	" VI		103.0	−16.8	2	D	Q	
396	" VII		238.0	− 2.9	1a	A	N	
397	1891 I		194.6	− 0.9	2	F	R	
398	" II	Wolf	200.0	− 3.1	1	a	N	
399	" III	Encke	338.7	+ 0.7	1	a	N	
400	" IV		301.0	+77.6	2	D	Q	
401	" V	Tempel$_3$	223.3	− 5.2	1	a	N	
402	1892 I		80.6	−15.0	3	D	Q	III
403	" II		253.2	−50.7	2	G	Q	
404	" III	Holmes	165.1	− 5.0	1	a	N	IV
405	" IV	Winnecke	96.6	− 2.0	1	a	N	
406	" V		198.5	− 5.0	1a	A	N	
407	" VI		335.7	+23.6	3	F	R	
408	1893 I		281.6	−36.0	3	F	Q	
409	" II		112.1	−14.6	2	D	Q	III
410	" III	Finlay	188.1	+ 2.1	1	a	N	
411	" IV		3.0	+ 9.4	2	D	R	
412	1894 I		310.6	− 4.0	1a	A	N	
413	" II		24.3	+35.7	3	C	P	IV
414	" III	Tempel$_2$	126.2	+ 1.1	1	a	N	
415	" IV	De Vico	165.4	+ 2.6	1	A	N	
416	1895 I	Encke	338.7	+ 0.7	1	a	N	
417	" II		158.0	− 0.6	1a	A	N	
418	" III		239.7	+58.4	4	H	—	
419	" IV		227.2	+38.3	3	F	r	
420	1896 I		30.4	+ 0.7	4	H	—	
421	" II	Faye	—	—	1	a	N	
422	" III		359.3	− 1.4	2	G	P	
423	" IV		332.4	−41.0	4	H	—	
424	" V		154.2	− 7.2	1a	A	N	
425	" VI	Brooks	182.0	+ 1.7	1	a	N	
426	" VII	Perrine	231.0	− 3.8	1	a	N	
427	1897 I		93.0	− 4.3	2	G	Q	
428	" II	d'Arrest	139.7	− 1.9	1	A	N	
429	" III		250.4	−59.1	2	F	Q	
430	1898 I		100.5	−44.5	2	C	Q	
431	" II	Winnecke	94.6	− 1.9	1	a	N	
432	" III	Encke	338.7	+ 0.7	1	a	N	
433	" IV	Wolf	200.1	− 3.0	1	A	N	
434	" V		76.4	− 5.0	2	F	QR	
435	" VI		268.4	+24.0	2	F	QR	
436	" VII		98.7	+48.8	2	G	Q	
437	" VIII		280.2	− 1.8	3	F	Q	
438	" IX		19.4	− 8.4	4	H	—	IV
439	" X		145.6	−32.1	2	E	R	

No.	Year.	Name.	Long. 1900. °	Lat. °	Cert.	Cl.	Ty.	Br.
440	1899 Ia		197.8	− 4.8	2	G	R	IV
441	" IId	Holmes	164.9	− 5.0	1	a	N	
442	" IIIb	Tuttle	286.1	+21.4	1	B	O	
443	" IVc	Tempel$_2$	126.4	+ 1.2	1	a	N	
444	" Ve		94.6	−10.2	4	H	—	
445	1900 Ia		199.7	−13.2	4	H	—	
446	" IIb		153.8	−11.0	2	G	R	IV
447	" IIIc		189.0	− 4.3	1a	A	N	
448	1901 Ia		94.0	+17.2	2	D	Q	II
449	" IIb	Encke	338.7	+ 0.7	1	a	N	
450	1902 Ia		76.4	+43.3	4	H	—	
451	" IIc		340.4	+14.2	4	H	—	
452	" IIIb		74.4	−10.5	3	F	QR	IV
453	1903 Ia		320.3	−21.8	2	D	P	IV
454	" IId	(1902)	301.6	− 4.0	3	F	p	
455	" IIIb		215.8	+ 6.1	2	F	R	
456	" IVc		287.0	−52.4	4	H	—	III
457	" Vd	Brooks	181.7	+ 1.7	1	A	N	
458	1904 Ia		57.8	−41.1	3	F	q	
459	" IId		30.2	−40.0	4	H	—	
460	" IIIc	Tempel$_2$	126.6	+ 1.2	1	A	N	
461	1905 Ib	Encke	338.9	+ 1.0	1	a	N	
462	" IIe	(1904)	249.9	+ 3.9	1a	A	N	
463	" IIIa	(1905)	336.0	+ 1.2	3	C	P	
464	" IVb	(1906)	320.9	− 1.6	3	F	P	
465	" Vb	(1905)	262.8	−27.8	3	G	Q	IV
466	" VIa	(1906)	16.7	−53.5	4	H	—	
467	1906 Ic	(1905)	106.2	+13.2	3	F	Q	IV
468	" IIc	(1906)	206.8	+80.8	4	H	—	
469	" IIIf	Holmes	165.1	− 5.0	1	A	N	
470	" IVe		103.1	− 2.9	1a	A	N	
471	" Vd	Finlay	188.1	+ 2.1	1	A	N	
472	" VIh		211.2	+ 4.2	1a	A	N	
473	" VIIg		269.7	− 7.3	3	F	Q	
474	1907 Ia		313.2	+25.0	3	F	R	
475	" IIb		20.5	+29.2	3	F	R	
476	" IIIc		19.4	− 9.4	4	H	—	
477	" IVd		257.6	+ 8.2	3	F	QR	III
478	" Ve		282.0	+52.4	3	F	Q	
479	" VIa	(1908)	176.0	− 6.6	4	H	—	
480	1908 Ib	Encke	338.8	+ 1.0	1	A	N	
481	" IId	Tempel$_3$	223.8	− 5.0	1	A	N	
482	" IIIc		109.5	− 5.4	3	F	R	III
483	1909 Ia		128.6	− 4.0	4	H	—	
484	" IId	Winnecke	91.8	− 2.4	1	A	N	
485	" IIIb	Perrine	229.5	− 3.5	1	A	N	
486	" IVe		254.2	− 1.2	1a	A	N	

In Table XXXV is given a reference list of the later elements of the various orbits that have been published in the Vierteljahrschrift. The eccentricity of number 375, 1887 II, is incorrectly stated there. It should be 0.9836922. Aditional elements for Holmes' comet 1892 III by Corrigan are given in a note by Galle on p. 311 of his work. The orbits published in the Vierteljahrschrift 1910, 45, 334, appeared too late to be discussed in this volume.

TABLE XXXV.

LIST OF ELEMENTS NOT INCLUDED IN GALLE'S TABLE.

No.	Year.	Name.	V. J. S.			Computer.
14	961		1898	33	326	Ravené
84	1742 I		1906	41	298	Cohn
130	1796		1908	43	394	Peck
147	1811 II		1910	45	334	Nekrassow
149	1813 I		1908	43	394	Peck
157	1819 II		1906	41	298	"
"	" "		1908	43	394	"
159	1819 IV		"	"	"	Lagarde
163	1822 III		"	"	"	Peck
164	" IV		1898	33	326	Stichtenoth
165	1823		1908	43	394	Hnatek
166	1824 I		1896	31	310	Doberck
168	1825 I		"	"	"	Martin
"	" "		1908	43	394	Boegehold
173	1826 II		"	"	"	Cowley, Whiteside
175	" IV		"	"	"	Klug
176	" V		1906	41	298	Hnatek
"	" "		1908	43	394	"
177	1827 I		1904	39	229	Strömgren
188	1835 I		1896	31	310	Rechenberg
197	1842 II		"	"	"	Schwarzschild
198	1843 I		"	"	"	Kreutz
"	" "		1902	37	275	"
202	1844 II		1906	41	298	Ross
203	" III		"	"	"	Fayet
205	1845 II		1900	35	357	Scheller
"	" "		1902	37	275	"
206	" III		1904	39	229	Peck
214	1846 VII		1910	45	334	Krause
220	1847 V		1896	31	310	Hind
"	" "		1898	33	326	Schobloch
226	1849 III		1910	45	334	Respondek
228	1850 II		1896	31	310	Rechenberg

No.	Year.	Name.	V. J. S.			Computer.
231	1851 III		1896	**31**	310	Spitaler
236	1852 IV		1910	**45**	334	Hnatek
237	1853 I		1900	**35**	357	Cohn
243	1854 III		1904	**39**	229	v. Hillmayer
244	" IV		1900	**35**	357	Buschbaum, Steiner
277	1863 I		1904	**39**	229	v. Flotow
285	1864 III		1906	**41**	298	Schroeter
287	1864 V		1910	**45**	334	Wesely
302	1870 II		1896	**31**	310	Schobloch
308	1871 IV		1898	**33**	326	Lagarde
314	1873 V		1896	**31**	310	Kreutz
318	1874 II		1906	**41**	298	Burggraf
338	1879 V		1896	**31**	310	Laves
339	1880 I		1902	**37**	275	Kreutz
345	1881 II		1900	**35**	357	Parizek, Sulc
"	" "		1902	**37**	275	Kreutz
346	" III		1896	**31**	310	Riem
354	1882 III		"	"	"	de Ball
355	1883 I		1906	**41**	298	Hellebrand
356	1883 II		1910	**45**	334	Moravi
362	1885 III		1896	**31**	310	Klumpke
363	" IV	Tuttle	"	"	"	Rahts
364	" V		"	"	"	Cohn
365	1886 I		1910	**45**	334	Redlich
367	1886 III		1908	**43**	394	Furness, Waterman
"	" "		"	"	"	Kobold
"	" "		1910	**45**	334	Kobold
369	" V		1898	**33**	326	Klumpke
"	" "		1908	**43**	394	Bucht
372	" VIII		1906	**41**	298	Fagerholm
374	1887 I		1902	**37**	275	Kreutz
375	" II		1904	**39**	229	Stechert
383	1888 V		"	"	"	Dinter
387	1889 IV		"	"	"	Horn
389	" VI		1896	**31**	310	Coniel
390	1890 I		"	"	"	Seydler
"	" "		"	"	"	Radelfinger
391	" II		"	"	"	Strömgren
392	" III		1904	**39**	229	Rheden
393	" IV		1896	**31**	310	Venturi
396	" VII		1898	**33**	326	Spitaler
397	1891 I		1896	**31**	310	Lamp
400	" IV		1904	**39**	229	Peck
403	1892 II		1898	**33**	326	Steiner
404	" III	Holmes	1896	**31**	310	Kohlschütter
"	" "	"	"	"	"	Zwiers

No.	Year.		Name.	V. J. S.			Computer.
406	1892	V		1896	**31**	310	Coniel
409	1893	II		"	"	"	Kromm
411	"	IV		"	"	"	Peyra
412	1894	I		1904	**39**	229	Gast
413	"	II		1902	**37**	275	Peck
414	"	III	Tempel$_2$	1895	**30**	127	Schulhof
415	"	IV	De Vico	1896	**31**	310	Chandler
"	"	"	"	1900	**35**	357	Seares
416	1895	I	Encke	1895	**30**	129	Backlund
417	"	II		1900	**35**	357	Morgan
418	"	III		1898	**33**	326	Wassilief
419	"	IV		1897	**32**	58	Aitken
420	1896	I		"	"	59	Buchholz
421	"	II	Faye	"	"	60	—
422	"	III		1900	**35**	357	Aitken
423	"	IV		1902	**37**	275	Peck
424	"	V		1897	**32**	62	Giacobini
425	"	VI	Brooks	"	"	63	Bauschinger
426	"	VII		1899	**34**	82	Osten
427	1897	I		1902	**37**	275	Möller
428	"	II	d'Arrest	1898	**33**	90	Leveau
429	"	III		1900	**35**	357	Wessell
430	1898	I		1902	**37**	275	Curtis
431	"	II	Winnecke	1899	**34**	74	Haerdtl
432	"	III	Encke	"	"	75	Iwanow
433	"	IV	Wolf	"	"	76	Thraen
434	"	V		1902	**37**	275	Hnatek
435	"	VI		1899	**34**	77	Perrine
"	"	"		1910	**45**	334	Curtis
436	"	VII		1902	**37**	275	Merfield
437	"	VIII		1900	**35**	71	Sprague
438	"	IX		1904	**39**	229	Peck
439	"	X		"	"	"	Scharbe
440	1899	I		1902	**37**	275	Merfield
"	"	"		1904	**39**	229	Wedemeyer
441	"	II	Holmes	1900	**35**	74	Zwiers
442	"	III	Tuttle	"	"	"	Rahts
443	"	IV	Tempel$_2$	"	"	76	Schulhof
444	"	V		"	"	"	Giacobini
445	1900	Ia		1901	**36**	62	Berberich
446	"	IIb		1904	**39**	229	de Mello
"	"	"		"	"	"	Poor
447	"	IIIc		1902	**37**	61	Giacobini
"	"	"		1908	**43**	394	Abold, Scharbe
448	1901	Ia		1904	**39**	229	Merfield
449	"	IIb	Encke	1902	**37**	64	Thonberg

No.	Year.		Name.	V. J. S.			Computer.
450	1902	Ia		1903	**38**	65	Kreutz, Strömgren
451	"	IIc		"	"	"	Grigg
452	"	IIIb		1904	**39**	43	Aitken
453	1903	Ia		"	"	44	Ebell
"	"	"		1908	**43**	394	Bruck
454	"	IId	(1902)	1904	**39**	45	Aitken
455	"	IIIb		1906	**41**	298	Peck
456	"	IVc		1904	**39**	48	Aitken
457	"	Vd	Brooks	"	"	49	Bauschinger
458	1904	Ia		1905	**40**	83	Nijland
459	"	IId		"	"	84	Aitken
"	"	"		1910	**45**	334	Sedlacek
460	"	IIIc	$Tempel_2$	1905	**40**	84	Coniel
461	1905	Ib	Encke	"	"	86	Kamensky, Okulitsch
462	"	IIe	(1904)	"	"	"	Fayet
463	"	IIIa	(1905)	1906	**41**	75	Banachiewicz
"	"	"	"	1907	**42**	103	Giacobini
"	"	"	"	1908	**43**	394	"
464	"	IVb	(1906)	1907	**42**	97	Weiss
"	"	"	"	1909	**44**	157	"
465	"	Vb	(1905)	1906	**41**	77	Wedemeyer
"	"	"	"	1908	**43**	394	Zappa
466	"	VIa	(1906)	1907	**42**	96	Ebell
467	1906	Ic	(1905)	1906	**41**	77	Wedemeyer
"	"	"	"	1907	**42**	95	Schönberg, Büss
"	"	"	"	1908	**43**	394	Terkán, Czuczy
468	"	IIc	(1906)	1907	**42**	98	Lamson
469	"	IIIf	Holmes	"	"	100	Zwiers
470	"	IVe		"	"	"	Ebell
471	"	Vd	Finlay	"	"	98	Schulhof
472	1906	VIh		"	"	102	Crawford
473	"	VIIg		"	"	101	Dybeck
474	1907	Ia		1909	**44**	159	Tringali
475	"	IIb		"	"	160	Weiss
476	"	IIIc		"	"	161	Strömgren
477	"	IVd		"	"	164	Kritzinger
478	"	Ve		"	"	165	Kobold
479	"	VIa	(1908)	"	"	167	Ebell
"	"	"	"	1910	**45**	334	Matkiewitch
480	1908	Ib	Encke	1909	**44**	167	Kamensky
481	"	IId	$Tempel_3$	"	"	171	Maubant
482	"	IIIc		"	"	"	Kobold
483	1909	Ia		1910	**45**	108	Kobold
484	"	IId	Winnecke	"	"	111	Hillebrand
485	"	IIIb	Perrine	"	"	109	Kobold
486	"	IVe		"	"	112	Ebell

Table XXXVI gives a summary of the more important facts, according to Galle and the Vierteljahrschrift, relating to the computation of the various orbits in class 2, that is of what appear to be all the best known orbits outside of the periodic comets. The first two columns give the catalogue number and the year and designation of the comet, the third gives the computer's name, the fourth the duration in months of the observations used in computing the orbit, the fifth the number of observations that were employed when this is stated, and the sixth the number of normal places selected. In the seventh column a + indicates that the perturbations produced by the more important planets have been determined, in the eighth it indicates that the orbit has been described as "definitive," and in the ninth that the places of the comparison stars have been recomputed and generally re-observed. The tenth column indicates the class to which the comet belongs. When the orbit is elliptical the period $U = \left(\frac{q}{1-e}\right)^{\frac{3}{2}}$ is given in the last column to three significant figures. Even for the best determined orbits a period in excess of 500,000 years must be considered simply as a result of computation, and as indicating merely that the period is surely very long.

TABLE XXXVI.

DATA PERTAINING TO ORBITS OF CERTAINTY 2.

No.	Year.	Computer.	Dur.	Obs.	Nor.	Per.	Def.	Star.	Cl.	Period.
104	1771	Kreutz	3.5	–	–	+	–	–	*F*	–
142	1807	Bessel	6	–	–	–	–	–	*D*	1720
146	1811 I	Herz	17	1000±	–	+	–	+	*D*	3090
157	1819 II	Peck	3.5	–	10	–	+	–	*F*	–
164	1822 IV	Stichtenoth	4.5	–	6	–	+	–	*D*	5450
165	1823	Hnatek	3(?)	800+	8	+	–	–	*F*	–
168	1825 I	Boegehold	2	97	6	+	+	–	*D*	3860
171	" IV	Hubbard	12	–	20	+	–	–	*D*	4470
176	1826 V	Hnatek	2(?)	32	7	–	+	–	*F*	–
177	1827 I	Strömgren	1.0	13	5	–	+	–	*F*	–
181	1830 I	Schulze	4.5	–	8	+	–	–	*F*	–
188	1835 I	Rechenberg	1.2	–	5	–	+	–	*F*	–
193	1840 II	Kowalczyk	2	–	10	+	–	–	*D*	3790
195	" IV	Schultz, Steinheil	3.5	–	5	+	–	+	*C*	367±4
197	1842 II	Schwarzschild	1.0	–	4	–	+	–	*F*	–
198	1843 I	Kreutz	1.5	–	9	+	+	–	*C*	512±71

No.	Year.	Computer.	Dur.	Obs.	Nor.	Per.	Def.	Star.	Cl.	Period.
202	1844 II	Ross	8	–	9	+	+	–	*E*	98200 ± 3000
203	" III	Fayet	2.5	–	–	–	+	–	*E*	28300
204	1845 I	Doberck	3	244	5	+	–	–	*G*	–
205	" II	Scheller	2	–	8	–	+	–	*E*	115000
217	1847 II	Engström	8	–	9	+	–	–	*F*	–
218	" III	Gautier	9	–	8	+	–	–	*E*	44300
219	" IV	Schur	3	–	8	–	+	+	*F*	–
221	" VI	Palmer	2.5	–	6	+	–	+	*G*	–
225	1849 II	Weyer	5.5	–	10	+	–	–	*G*	–
227	1850 I	Carrington	2.5	–	9	–	+	+	*E*	28800
228	" II	Rechenberg	2.5	–	9	–	+	+	*F*	–
231	1851 III	Spitaler	2	–	4	–	+	–	*E*	1250000
237	1853 I	Cohn	1.2	–	6	–	+	–	*F*	–
238	" II	Rümker	1.2	–	9	+	–	–	*C*	782
244	1854 IV	Buschbaum, Steiner	2	–	10	–	+	–	*C*	1090 ± 65
250	1857 I	Loewy	2.5	222	12	–	+	+	*F*	–
253	" IV	Möller	3	–	–	+	–	–	*C*	235
262	1858 VI	Hill	9	–	16	+	–	–	*D*	1950 ± 6
268	1860 III	Auwers	4	–	–	+	–	+	*F*	–
270	1861 I	Oppolzer	5	–	7	–	+	+	*C*	417
271	" II	Kreutz	11	1156	31	+	–	–	*C*	409 ± 04
277	1863 I	v. Flotow	3.5	–	5	–	+	–	*G*	–
278	" II	Frischauf	7(?)	–	8	–	–	–	*F*	–
279	" III	Ericsson	2	–	6	+	+	–	*E*	17800
280	" IV	Svedstrup	3(?)	–	12	–	+	+	*E*	18200
282	" VI	Rosén	6	–	–	+	–	–	*G*	–
284	1864 II	Kowalczyk	3	–	7	–	+	–	*D*	3930
285	" III	Schroeter	7	57	7	+	+	–	*E*	55200
296	1868 II	Karlinski	1.1	201	7	–	+	–	*F*	–
301	1870 I	Seydler	1.4	74	7	–	+	–	*F*	–
302	" II	Schobloch	4	–	10	–	+	–	*F*	–
306	1871 II	Cramer	3	185	10	+	+	–	*F*	–
308	" IV	Lagarde	3.5	–	6	–	+	–	*D*	2060 ± 150
313	1873 IV	Gautier	1.0	–	5	+	–	+	*D*	3380
319	1874 III	v. Hepperger	6	638	17	+	+	+	*E*	13700
320	" IV	Holetschek	3	–	7	+	+	–	*C*	306 ± 14
325	1877 I	Thraen	1.1(?)	94	5	–	+	–	*F*	–
326	" II	Plath	3	274	7	+	+	–	*E*	19800
327	" III	Poenisch	1.5	173	6	–	+	–	*D*	10700
335	1879 II	Kremser	2	86	6	–	+	–	*F*	–
338	" V	Laves	2	–	6	–	+	–	*F*	–
341	1880 III	Molien	2	–	7	–	+	–	*F*	–
346	1881 III	Riem	9	–	18	–	+	–	*D*	2430
347	" IV	Stechert	3	–	12	+	+	–	*F*	–
352	1882 I	v. Rebeur, Paschwitz	5	1070	23	+	–	+	*E*	1170000

No.	Year.	Computer.	Dur.	Obs.	Nor.	Per.	Def.	Star.	Cl.	Period.
353	1882 II	Kreutz	9	—	19	+	+	+	*D*	1240
354	" III	de Ball	3	—	9	—	+	—	*G*	—
355	1883 I	Hellebrand	1.7	—	6	—	+	—	*G*	—
361	1885 II	Berberich	1.5(?)	—	6	—	+	+	*G*	—
362	" III	Klumpke	1.0	77	6	+	+	—	*C*	274
364	" V	Cohn	2	—	5	—	+	—	*F*	—
366	1886 II	Thraen	7.5	760	14	+	+	+	*G*	—
372	" VIII	Fagerholm	4	86	6	—	+	—	*F*	—
373	" IX	Buschbaum	8	323	10	+	+	+	*G*	—
375	1887 II	Stechert	3	—	11	—	+	—	*C*	999
376	" III	Heinricius	2	—	9	+	+	+	*F*	—
377	" IV	Muller	3(?)	—	7	+	+	+	*D*	6730
383	1888 V	Dinter	7	250	16	—	+	—	*D*	2367
384	1889 I	Berberich	13	—	12	—	—	—	*G*	—
387	" IV	Horn	4	—	7	—	+	—	*D*	9740
390	1890 I	Radelfinger	1.1	—	6	—	+	—	*F*	—
391	" II	Strömgren	10.5	—	16	+	+	—	*G*	—
395	" VI	Bobrinskoy	2	—	—	+	—	—	*E*	57500
397	1891 I	Lamp	3.5	—	6	+	+	—	*G*	—
400	" IV	Peck	2	—	4	—	+	—	*E*	54400
403	1892 II	Steiner	9.5	—	12	—	+	—	*G*	—
409	1893 II	Kromm	5.5	—	9	—	+	—	*E*	44400
411	" IV	Peyra	3	—	4	—	+	—	*D*	3520
422	1896 III	Aitken	2	—	7	—	+	—	*G*	—
427	1897 I	Möller	6	—	13	—	+	—	*G*	—
429	" III	Wessell	1.2	—	5	—	+	—	*F*	—
430	1898 I	Curtis	8	—	9	—	+	—	*C*	417
434	" V	Hnatek	2	—	6	—	+	—	*F*	—
435	" VI	Curtis, Richardson	2	122	7	+	+	—	*F*	—
436	" VII	Merfield	18	—	16	—	+	—	*G*	—
439	" X	Scharbe	1.2	266	6	—	+	—	*E*	159000
440	1899 I	Wedemeyer	5.5	680	24	—	+	—	*G*	—
446	1900 II	Poor	3	—	8	—	+	—	*G*	—
448	1901 I	Merfield	1.5	—	6	—	+	—	*E*	39000
453	1903 I	Bruck	1.5	507	8	—	+	—	*E*	43100
455	" III	Peck	1.0	—	4	—	—	—	*F*	—

In Table XXXVII is given an enumeration of the orbits contained in Table XXXIV. In the first two columns they are tabulated according to the certainty with which we know them, the second column giving the number of orbits of the certainty described in the first. In the third, fourth, fifth, and sixth columns the orbits are tabulated by classes, and the number of orbits in each class is given. The seventh, eighth, and ninth columns refer similarly to the accurately known orbits of certainty 2, and the last three columns to the less accurately known orbits of certainty 3.

TABLE XXXVII.

SUMMARY OF TABLE XXXIV.

Cert.	No.	Class.	No.	Class.	No.	2	No.	Per cent.	3	No.	Per cent.
1	106	*A*	34	*G*	26	*C*	11	11	*C*	9	9
1*a*	40	*B*	12	*H*	87	*D*	29	30	*D*	19	20
2	96	*C*	20	*I*	61	*E*	5	5	*E*	3	3
3	96	*D*	48	*a*	79	*F*	33	35	*F*	58	60
4	105	*E*	8	*b*	20	*G*	18	19	*G*	8	8
5	43	*F*	91								
					486		96	100		97	100

A large proportion of the comets of this certainty, a little short of two-thirds in fact, belong to class *F*, and therefore have parabolic orbits. Of the more accurately known orbits of certainty 2, only one-third were indistinguishable from parabolas. A longer observation of their paths would diminish this fraction still further. It is evident therefore that many comets classed as parabolic under certainty 3 would really be placed in the other classes if their orbits were better known. If accurately known the path of every comet must obviously be either an ellipse or a hyperbola. There is strictly speaking no such thing as a parabolic orbit, except as a mathematical conception.

It is probable, judging from the ninth column, if all the orbits now assigned to class *F* were accurately known that we should find that two-thirds of them were elliptical, and the rest hyperbolic. Excluding the orbits of the known periodic comets in classes *A*, *B*, *a*, and *b*, and the comparatively unknown orbits in classes *H* and *I*, there remain in classes *C*, *D*, *E*, *F*, and *G* 193 orbits. Of these we conclude that about 10 per cent belong in class *C*, 40 in *D*, 20 in *E*, and 30 in *G*.

Table XXXVIII refers solely to the comets of the nineteenth century. The first two columns give the different classes, and the number of comets of each class that appeared in that century. The next three columns refer to the five great comets of brilliancy *I*. Three of these, or 60 per cent were in class *D*, as is shown by columns three and four. Since by the second column we find that there were 41 comets in this class, it appears that 7 per cent of them were comets of the first quality, as is shown by the fifth column. The next nine columns are similarly arranged for comets of brilliancy *II*, *III*, and *IV*. The last three columns give the same results for all of the naked eye comets taken together. Of the 14 comets of the two highest grades 7 belong to class *D*, and 51 per cent of all the comets of this class were visible to the naked eye. On the other hand class *F* of the parabolic orbits stands, as shown by the last column, rather low on the scale, while class *H* of the poorly known orbits, stands lower still. This is clearly due to the fact that faint comets could not be followed for long, and would therefore naturally fall into this class. The comets of short period that were visible to the naked eye are represented as we see by their early appearances only. Not a single last appearance, which would be entered under class *A*, was bright enough to be included in the table. The last figure of the last line shows that one-quarter of all the comets observed were visible to the naked eye. We notice also that one comet out of every sixty is of the very first class.

TABLE XXXVIII.

CLASSIFICATION OF COMETS ACCORDING TO THEIR BRILLIANCY.

Cl.	No.	*I.*	Pc.	Pc.	*II.*	Pc.	Pc.	*III.*	Pc.	Pc.	*IV.*	Pc.	Pc.	Total.	Pc.	Pc.
A	19															
B	12				2	22	17	1	3	8	1	3	8	4	5	33
C	18	2	40	11				2	6	11	3	9	17	7	9	39
D	41	3	60	7	4	45	10	12	33	29	2	7	5	21	26	51
E	8							1	3	12	2	7	25	3	4	37
F	64				1	11	2	11	30	17	5	16	8	17	21	27
G	24							3	8	12	5	16	21	8	10	33
H	47							3	8	6	5	16	11	8	10	17
I	2				2	22	100							2	2	100
a	72							2	6	3	8	26	11	10	12	14
b	5							1	3	20				1	1	20
	312	5	100	2	9	100	3	36	100	11	31	100	10	81	100	26

Table XXXIX refers only to the comets and returns of comets observed in the last half of the nineteenth century. The first three columns indicate the certainty with which we know their orbits, the number of comets for each grade of certainty, and the corresponding percentage. Certainty 1 and

1*a* are combined in these columns. The next three columns give similar results for the classes of orbits, and the last three for the different brilliancies of the comets.

TABLE XXXIX.

ENUMERATION AND PERCENTAGES.

Cert.	No.	Per cent.	Classes.	No.	Per cent.	Brilliancy.	No.	Per cent.
1	82	37	*A B*	26	12	*I*	3	1
2	67	31	*C D*	42	19	*II*	4	2
3	40	18	*E F G*	65	30	*III*	24	11
4	29	13	*H I*	30	13	*IV*	19	9
5	1	1	*a b*	56	26	*V*	169	77
	219	100		219	100		219	100

The most interesting features exhibited by this table are first, that for only one-seventh of the orbits, as indicated by the third column, are we so uncertain regarding the aphelion distances that we are unable to classify them. Second, as shown by the sixth column, that over one-third of the comets were in classes *A*, *B*, *a*, and *b*, *i. e.* periodic comets, one-fifth in classes *C* and *D*, moved in determinable ellipses, and about one-third in classes *E*, *F*, and *G* moved in extremely long ellipses and hyperbolas. Third, as shown by the last column, about one-quarter of all the comets observed were visible to the naked eye.

During the earlier centuries since the Christian era numerous comets were recorded, but with no mention made of their position in the heavens. Many such instances are collected in Chambers' Handbook of Astronomy, and his later work "The Story of the Comets" from which latter publication, the following enumeration has been taken. The figures indicate the total number of comets recorded, whether accurately observed or not, in each century up to the year 1700:—22, 22, 39, 22, 19, 26, 33, 17, 41, 30, 38, 31, 30, 34, 45, 40, 35. In all 524 comets were mentioned in 17 centuries, or 31 comets per century. At this rate judging by the totals in Table XXXVIII we should find that all the comets of brilliancy *I* and *II*, and about half those of brilliancy *III* were sufficiently conspicuous to secure a record from the Chinese astronomers and our own predecessors. Every return of Halley's comet was recorded.

CHAPTER XII.

APHELION DISTANCES. HYPERBOLIC ORBITS.

IN Table XL the comets of classes *A* and *B* are given in the order of their aphelion distances. This order it will be noted differs materially from that of their mean distances, or periods, in which comets are more usually classified. The reason for this change is that the relation between the aphelion distances of these comets and the four major planets with which they are associated seems to be clearly marked, as was first pointed out by Laplace, while the relation between the planets and the mean distances, or periods of the comets, seems to be only secondary in its nature.

TABLE XL.

APHELION DISTANCES OF CLASSES *A* AND *B*.

No	No.	n.	Name.	Computer.	Cert.	Per.	Dist.	Log. dist.
				CLASS *A*.				
480	117	31	Encke	Kamensky	1	3.30	4.09*	0.6117
460	311	5	$Tempel_2$	Schulhof	1	5.28	4.67	0.6693
336	293	3	$Tempel_1$	Gautier	1	5.98	4.83	0.6839
358	358	1	Barnard	Berberich	1*a*	5.40	4.87	0.6875
368	368	1	Brooks	Oppenheim	1*a*	5.60	4.97	0.6964
159	159	1	Blanpain	Lagarde	1*a*	5.10	5.02	0.7007
113	113	1	Pigott	Peters	1*a*	5.88	5.06	0.7042
396	396	1	Spitaler	Tennant	1*a*	6.40	5.08	0.7059
469	404	3	Holmes	Zwiers	1	6.86	5.10	0.7076
415	64	3	De Vico	Chandler	1	5.86	5.11	0.7084
481	300	4	$Tempel_3$	Maubant	1	5.68	5.22	0.7177
85	85	1	Grischow	Clausen	1*a*	5.44	5.32	0.7259
470	470	1	Kopff	Ebell	1*a*	6.62	5.36	0.7292
457	388	3	Brooks	Neugebauer	1	7.10	5.43	0.7348

* Computed from the elements of M. Thonberg founded on the appearance number 449. Astr. Nach. 166, 27.

No.	No.	n.	Name.	Computer.	Cert.	Per.	Dist.	Log. dist.
			CLASS *A* (continued).					
486	486	1	Daniel	Ebell	1*a*	6.40	5.52	0.7419
406	406	1	Barnard	Coniel	1*a*	6.52	5.55	0.7443
484	100	9	Winnecke	Hillebrand	1	5.89	5.56	0.7451
334	210	5	Brorsen	Lamp	1	5.46	5.61	0.7490
433	359	3	Wolf	Thraen	1	6.84	5.61	0.7490
102	102	1	Lexell	Le Verrier	1*a*	5.60	5.63	0.7497
424	424	1	Giacobini	Giacobini	1*a*	6.86	5.76	0.7604
485	426	2	Perrine	Kobold	1	6.45	5.76	0.7604
259	259	1	Tuttle	Schulhof	1*a*	6.61	5.89	0.7701
428	230	6	d'Arrest	Leveau	1	6.68	5.90	0.7709
462	462	1	Borrelly	Fayet	1*a*	7.04	5.94	0.7738
382	200	8	Faye	Möller	1	7.56	5.96	0.7752
471	371	3	Finlay	Schulhof	1	6.54	6.02	0.7796
447	447	1	Giacobini	Abold, Scharbe	1*a*	6.52	6.05	0.7818
417	417	1	Swift	Morgan	1*a*	7.22	6.17	0.7903
235	105	6	Biela (B)	Hubbard	1	6.62	6.19	0.7924
412	412	1	Denning	Gast	1*a*	7.42	6.48	0.8116
472	472	1	Metcalf	Crawford	1*a*	8.24	6.54	0.8156
389	389	1	Swift	Coniel	1*a*	8.91	7.24	0.8597
348	348	1	Denning	Matthiessen	1*a*	8.69	7.73	0.8882
			CLASS *B*.					
213	213	1	Peters	Berberich	1*a*	13.4	9.7	0.9886
442	123	5	Tuttle	Rahts	1	13.7	10.4	1.0170
290	27	2	Tempel₁	Oppolzer	1*a*	33.2	19.6	1.2923
292	292	1	Stephan	Becker	1*a*	40.1	21.8	1.3385
236	236	1	Westphal	Westphal	1*a*	60.6	29.6	1.4713
378	151	2	Olbers	Ginzel	1	72.6	33.6	1.5263
357	148	2	Pons	Schulhof, Bossert	1	71.6	33.7	1.5276
211	211	1	De Vico	v. Hepperger	1*a*	75.7	35.1	1.5453
190	4	14	Halley	Westphalen	1	76.3	35.4	1.5490
220	220	1	Brorsen	Gould	1*a*	81.0	36.9	1.5670
275	275	1	Tuttle	Hayn	1*a*	119.9	47.6	1.6776
386	386	1	Barnard	Berberich	1*a*	128.4	49.8	1.6972

In the table the first column gives the number of the last appearance of the comet, the second that of its first appearance, and the third the total number of its appearances as recorded in Table XXXIV. The fourth column gives its generally accepted name, and the fifth that of the computer whose orbit has been adopted in this investigation. The sixth column gives the assumed certainty of our knowledge of the orbit, taken from the sixth column of Table XXXIV. The seventh gives the period, the eighth the aphelion distance $R = q \frac{(1+e)}{1-e}$, and the last the logarithm of this distance.

Table XLI is arranged like Table XL except that the second, third, and fourth columns are omitted.

TABLE XLI.

APHELION DISTANCES OF CLASSES *C*, *D* AND *E*.

No.	Computer.	Cert.	Per.	Dist.	Log. dist.
	Class *C*.				
253	Möller	2	235	75.5	1.8779
362	Klumpke	2	274	83.7	1.9227
463	Giacobini	3	297	88.1	1.9450
320	Holetschek	2	306	89.0	1.9494
195	Schultz, Steinheil	2	367	101.0	2.0043
128	Peters	3	390	105.3	2.0224
271	Kreutz	2	410	109.6	2.0398
430	Curtis	2	417	110.5	2.0434
270	v. Oppolzer	2	417	110.7	2.0441
246	Tiele	3	500	123.9	2.0931
214	Oudemans	3	497	124.9	2.0966
198	Kreutz	2	515	128.5	2.1089
351	Olsson	3	611	142.2	2.1529
147	Nekrassow	3	755	164.2	2.2154
369	Bucht	3	771	167.8	2.2248
238	Rümker	2	782	168.9	2.2276
375	Stechert	2	999	198.2	2.2971
245	Elkin	3	997	198.3	2.2973
244	Buschbaum, Steiner	2	1090	210.8	2.3239
413	Peck	3	1140	217.6	2.3377
	Class *D*.				
353	Kreutz	2	1240	231	2.3636
116	Krueger	3	1350	241	2.3820
142	Bessel	2	1720	286	2.4564
262	Hill	2	1960	311	2.4928
308	Lagarde	2	2060	323	2.5092
101	Bessel	3	2090	327	2.5145
379	Tennant	3	2150	336	2.5263
383	Dinter	2	2370	354	2.5490
	Class *D* (continued).				
346	Riem	2	2430	361	2.5575
254	Linsser	3	2450	364	2.5611
179	Clüver	3	2600	379	2.5786
146	Herz	2	3090	423	2.6263
313	Gautier	2	3380	449	2.6522
411	Peyra	2	3520	462	2.6646
193	Kowalczyk	2	3790	485	2.6857
168	Boegehold	2	3864	492	2.6917
284	Kowalczyk	2	3930	497	2.6964
171	Hubbard	2	4470	542	2.7340
305	Holetschek	3	5130	597	2.7760
164	Encke	3	5440	618	2.7910
263	Weiss	3	6040	659	2.8189
255	Auwers	3	6140	669	2.8254
175	Klug	3	6263	679	2.8317
377	Muller	2	6730	712	2.8525
97	Burckhardt	3	7510	754	2.8774
226	d'Arrest	3	8200	823	2.9154
65	Encke	3	8450	852	2.9304
387	Horn	2	9740	911	2.9595
216	Hornstein	3	10300	942	2.9741
327	Poenisch	2	10700	969	2.9863
393	Venturi	3	11000	990	2.9956
319	v. Hepperger	2	13700	1140	3.0569
279	Ericsson	2	17800	1360	3.1335
280	Svedstrup	2	18200	1380	3.1399
326	Plath	2	19800	1460	3.1644
402	Berberich	3	19900	1480	3.1703
321	Gruss	3	23400	1680	3.2253
203	Fayet	2	28300	1860	3.2695
227	Carrington	2	28800	1880	3.2742
448	Merfield	2	39000	2300	3.3617

No.	Computer.	Cert.	Per.	Dist.	Log. dist.	No.	Computer.	Cert.	Per.	Dist.	Log. dist.
Class *D* (continued).						Class *E*.					
453	Bruck	2	43100	2460	3.3905	202	Ross	2	98200	4260	3.6294
218	Gautier	2	44300	2500	3.3979	205	Scheller	2	116000	4750	3.6767
409	Kromm	2	44400	2510	3.3997	439	Scharbe	2	159000	5860	3.7679
314	Kreutz	3	53800	2850	3.4548	385	Millosewick	3	322000	9400	3.9731
400	Peck	2	54400	2870	3.4579	192	Rechenberg	3	486000	14200	4.1523
285	Schroeter	2	55200	2900	3.4624	381	Millosewick	3	971000	19600	4.2923
395	Bobrinskoy	2	57500	2980	3.4742	352	v. Rebeur, Paschwitz	2	1170000	22300	4.3483
109	Clüver	3	75300	3570	3.5527	231	Spitaler	2	1250000	23200	4.3655

An enumeration of the comets contained in Tables XL and XLI is made in Table XLII. The first column gives the logarithms of aphelion distances taken at regular intervals. A count was made of all the comets the logarithms of whose aphelia lay between 0.5 and 0.7 and the result was entered in the second column of the table against the number 0.6. The number of comets lying between 0.6 and 0.8 was similarly entered against 0.7 and so on.

TABLE XLII.

ENUMERATION OF ORBITS.

Log.	Orbit.	Log.	Orbit.	Log.	Orbit.	Log.	Orbit.	Log.	Orbit.	Log.	Orbit.	Log.	Orbit.	Log.	Orbit.
0.5	0	1.1	1	1.6	7	2.1	9	2.6	13	3.1	5	3.6	3	4.1	1
0.6	5	1.2	1	1.7	2	2.2	7	2.7	9	3.2	7	3.7	3	4.2	2
0.7	30	1.3	2	1.8	1	2.3	9	2.8	8	3.3	7	3.8	1	4.3	3
0.8	29	1.4	2	1.9	4	2.4	6	2.9	11	3.4	8	3.9	1	4.4	2
0.9	5	1.5	6	2.0	10	2.5	9	3.0	7	3.5	5	4.0	1	4.5	0
1.0	2														

These results are recorded in Figure 32 where the abscissas are taken from the first column of the table, and the ordinates from the second.

Beneath the heavy horizontal line are marked the intervals equal to the logarithms of the mean distances of the planets Earth, Jupiter, Saturn, Uranus, and Neptune, each being indicated by its initial. The distance of the hypothetical planet *O* is also indicated. The Jupiter family of comets is very clearly marked. The ordinates of the curve do not directly indicate the number in the different planet families, which for Jupiter is 34. Saturn has only 2 comets in its family, and these are absorbed in the curve of the Jupiter group. Uranus also has 2, while Neptune has 6. Why Neptune should be so well provided as compared with Saturn is not at first apparent, since its mass is less than one-fifth as great. The size of its family is probably due in part to its more isolated position and larger orbit, and in part

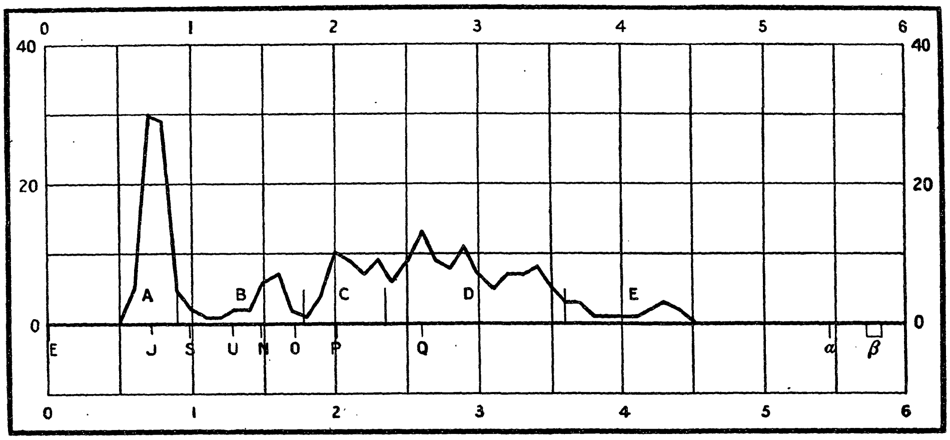

FIG. 32.

to the slow orbital motion that both it and the comets approaching it possess at that distance from the Sun. Neglecting any perturbations, a comet would remain nearly twice as long in its vicinity as it would in that of Saturn.

The four short vertical lines above the horizontal one indicate the divisions between the first five classes of cometary orbits described in Chapter XI. Beyond Neptune it will be seen that numerous cometary maxima occur. It was pointed out by Flammarion, L'Astronomie Populaire, 1879 that the existence of a planet beyond Neptune might be indicated by some of these maxima. The hypothetical planet *O*, described in Part II of this volume, agrees pretty closely in distance with the value which he suggested, 48 units.

At about the same time Professor G. Forbes, Proc. Roy. Soc. Edinburgh 1878–80, 10, 426, suggested that these maxima might indicate the existence of a planet at a distance of 100 units, with a period of 1,000 years, and of another at a distance of 300 units with a period of 5,000 years. These distances would correspond in the figure to abscissas of 2.00 and 2.48. In the light of the data now collected, and exhibited in Figure 32, the first of these positions appears at first sight quite plausible, but later we shall find reason to modify this value considerably, and to change the other one entirely. Too much stress must not be laid on the smaller maxima and minima shown in Figure 32, and it must be remembered that since the curve is logarithmic, the more remote aphelia are very much more concentrated than those lying nearer to the Sun. The abscissa provisionally adopted for the second planet *Q*, at 2.60, corresponds to a distance of 400 units, and a period of 8,000 years.

As the years go by, these maxima will all rise, but not all with equal speed. Thus from 1881.0 to 1901.0 there were discovered 15 new comets in class *A*, 1 in class *B*, 6 in *C*, 12 in *D*, and 4 in *E*. Except in class *B* the rise was roughly proportional to the areas bounded by the four short vertical lines. This will not remain true, however. After a few decades, all the comets in class *A* readily visible in our modern telescopes will have been found, and we shall merely watch their returns. Not so with the other classes, for new comets belonging to *C*, *D*, and *E* will be constantly appearing, and these classes will then furnish the maxima of the curve. This will be especially true of *D* and *E*, the eccentricity of whose orbits are so nearly equal to unity that many astronomers do not now consider the difference worth the trouble of computing.

Besides the 122 orbits represented by the curve, there are 91 parabolic orbits, and 26 hyperbolic ones, 117 in all. Among the 91 orbits classed as parabolic, doubtless there are many which should be entered somewhere upon the curve, if their aphelia were better known. As it now stands, it is clear that there is a well marked diminution in numbers shortly after passing abscissa 3.70. This diminution becomes more striking when we recollect that owing to the concentration of the aphelia already mentioned there is a tendency to increase the height of the ordinates as we advance towards the right.

The larger aphelion distances of class *E* have little individual numerical significance. That is to say the orbits are usually almost as well represented by a parabola as by the given ellipse, and the eccentricity is published chiefly for the purpose of indicating the close agreement of the observations with the parabola. It may be remarked in this place, however, that it is very desirable that the computers of cometary orbits should always give the eccentricity when the orbit has been well observed, indicating also its maximum and minimum probable values. See also a paper by Leuschner, Pub. Astr. Soc. Proc. 1907, 19, 67.

The distance of the nearest known star, α Centauri, 275,000 units, is indicated on the curve at α by the abscissa 5.44. The two lines marked β

indicate the limiting distances of the nine stars next nearest to the Sun, whose parallaxes range from 0″.4 to 0″.3.

While the parabolic orbits of class F cannot be advantageously classified in this chapter, the hyperbolic ones of class G may be compared with one another, and with the elliptic orbits, by taking advantage of a modification of Kepler's third law, namely: — The areal velocities of bodies revolving around the Sun are proportional to the square roots of the parameters of their orbits. The semi-parameter of a conic section expressed in terms of the perihelion distance q and the semi-major axis a is for an ellipse $p = \frac{2aq - q^2}{a}$, for a parabola $P = 2q$, and for a hyperbola $p = \frac{2aq + q^2}{a}$. The ratio of the areal velocities at perihelion of the ellipse or hyperbola to the parabola is therefore $r = \sqrt{\frac{p}{P}} = \sqrt{1 \pm \frac{q}{2a}}$, and where q is very small as compared to a, we have $r = 1 \pm \frac{q}{4a}$.

At the distance q we have in the case of the Sun the parabolic velocity $U = \frac{42.10}{\sqrt{q}}$ in kilometres per second. The difference between the elliptic or hyperbolic and the parabolic perihelion velocities in kilometres per second is therefore

$$k = \pm \frac{Uq}{4a} = \pm \frac{10.525\sqrt{q}}{a}.$$

For an ellipse $a = \frac{q}{1 - e}$ and for a hyperbola $a = \frac{q}{e - 1}$. Hence for an ellipse we have

$$k = -\frac{10.525\,(1 - e)}{\sqrt{q}} = \frac{10.525\,(e - 1)}{\sqrt{q}}$$

and for a hyperbola we have the same equation.

In Table XLIII the first column gives the catalogue number of the comet, the second its name if periodic, otherwise its date and designation, the third the name of the computer, the fourth the certainty of its elements, and the fifth the type under which it is classified. The sixth column gives the logarithm of the perihelion distance q, the seventh the eccentricity minus 1, the eighth the resulting deviation from the parabolic velocity at perihelion expressed in kilometres per second, and the ninth the same quantity expressed in kilometres per day. In the computation the value of $e - 1$ was always taken to three significant figures, but in the table it was thought that the differences between the various orbits would be better shown if all were carried to the same number of decimal places.

The last column of Remarks contains the aphelion distances in the case of the elliptic orbits. These were taken merely as representative examples, two being selected from class *A*, two from class *B*, one from *C*, four from *D*, and one from *E*. A comparison of the first two illustrates the effect on *K* of a diminished eccentricity, the negative value of *K* being considerably larger in the second case, although the aphelion distance is greater. The sixth and eighth orbits illustrate the effect of a difference in the perihelion distance, *K* being nearly identical in the two cases, although their aphelion distances are very unlike.

TABLE XLIII.

DEVIATIONS FROM PARABOLIC VELOCITY OF ELLIPTIC AND HYPERBOLIC COMETS.

No.	Name.	Computer.	Cert.	Type.	log q.	e — 1.	k.	K.	Rem.
					ELLIPTIC ORBITS.				
480	Encke	Kamensky	1	*N*	9.5291	−.152605	−2.76	−239000	4.09
382	Faye	Möller	1	*N*	0.2401	−.450983	−3.60	−311000	5.96
442	Tuttle	Rahts	1	*O*	0.0085	−.178290	−1.86	−161000	10.4
190	Halley	Westphalen	1	*O*	9.7683	−.032609	−0.448	−38700	35.4
271	1861 II	Kreutz	2	*P*	9.9151	−.014923	−0.173	−15000	109.6
353	1882 II	Kreutz	2	*Q*	7.8890	−.000067	−0.0080	−692	231.
262	1858 VI	Hill	2	*p*	9.7623	−.003707	−0.0513	−4430	311.
395	1890 VI	Bobrinskoy	2	*Q*	0.1004	−.000846	−0.00793	−685	2980.
109	1780 I	Clüver	3	*Q*	8.9836	−.000054	−0.00183	−158	3570.
352	1882 I	v. Rebeur, Paschwitz	2	*r*	8.7836	−.000005	−0.00023	−20	22300.
					HYPERBOLIC ORBITS.				
354	1882 III	de Ball	2	*Q*	9.9803	+.000074	+0.00080	+69	*i*
277	1863 I	v. Flotow	2	*Q*	9.9002	+.000071	+0.00084	+72	*i*
199	1843 II	Goetze	3	*Q*	0.2085	+.000180	+0.00149	+129	
465	1905 V	Zappa	3	*Q*	0.0221	+.000189	+0.00194	+168	
403	1892 II	Steiner	2	*Q*	0.2946	+.000345	+0.00259	+223	
204	1845 I	Doberck	2	*Q*	9.9567	+.000247	+0.00273	+236	*n*
391	1890 II	Strömgren	2	*Q*	0.2805	+.000410	+0.00312	+270	*p*
221	1847 VI	Palmer	2	*Q*	9.5172	+.000173	+0.00317	+274	
446	1900 II	Poor	2	*R*	0.0064	+.000329	+0.00344	+297	*n*

No.	Name.	Computer.	Cert.	Type.	log q.	e — 1.	k.	K.	Rem.
		Hyperbolic Orbits (continued).							
366	1886 II	Thraen	2	*Q*	9.6806	+.000229	+0.00348	+301	*s*
355	1883 I	Hellebrand	2	*Q*	9.8809	+.000344	+0.00415	+359	
373	1886 IX	Buschbaum	2	*Q*	9.8217	+.000382	+0.00494	+426	
239	1853 III	Krahl	3	*R*	9.4869	+.000261	+0.00496	+428	
282	1863 VI	Rosén	2	*Q*	0.1183	+.000650	+0.00597	+516	
440	1899 I	Wedemeyer	2	*R*	9.5140	+.000350	+0.00645	+557	*s*
422	1896 III	Aitken	2	*P*	9.7530	+.000476	+0.00666	+575	*s*
225	1849 II	Weyer	2	*r*	0.0642	+.000708	+0.00692	+598	
436	1898 VII	Merfield	2	*Q*	0.2309	+.001034	+0.00831	+718	*s*
384	1889 I	Berberich	2	*R*	0.2589	+.001086	+0.00852	+736	*p*
427	1897 I	Möller	2	*Q*	0.0264	+.000927	+0.00946	+818	*s*
361	1885 II	Berberich	2	*Q*	0.3993	+.002852	+0.0189	+1640	*s*
240	1853 IV	d'Arrest	3	*Q*	9.2372	+.001229	+0.0312	+2690	*s*
141	1806 II	Hensel	3	*Q*	0.0342	+.010182	+0.1032	+8920	*n*
155	1818 III	Rosenberger, Scherk	3	*Q*	9.9320	+.011617	+0.1320	+11400	
367	1886 III	Kobold	3	*Q*	9.9257	+.012893	+0.1478	+12770	
107	1774	Burckhardt	3	*R*	0.1562	+.028296	+0.2488	+21500	

Under the hyperbolic orbits an *i* in the last column indicates that the deviation from the parabola is so insignificant that practically both orbits fit the observations equally well. An *n* indicates that the deviation from the parabola is not proved. A *p* shows that it is due to perturbations of known planets, and an *s* that the evidence for it appears to be satisfactory. These conclusions are in most cases taken from the Vierteljahrschrift.

The fact that many so-called parabolic, and some hyperbolic orbits exist, has been used as an argument by some astronomers in favor of the belief that these comets come from interstellar space, and will visit the Sun but once. It is proposed now to show, not only that comets arriving from finite distances may, owing to the action of a remote and unknown planet, possess hyperbolic orbits, but also that owing to the continued action of the planet, as the comet recedes from the Sun, the orbit may be reconverted into an ellipse. The effect is therefore not the same as that of an ordinary perturbation due to a close approach to a planet, which simply changes the velocity of the comet in one direction. Although the two effects are practically quite dissimilar, there is really no distinction of kind, the difference being merely quantitative. It is convenient however to treat the present case from a different standpoint from that usually taken.

Assume a comet whose aphelion coincides with the orbit of an unknown planet, to be on its way to the Sun. Assume the planet in the diametrically opposite portion of its orbit. The velocity v of the comet at any point may be decomposed into three portions, its parabolic velocity U, the difference between its parabolic and elliptic velocities E, and the velocity P occasioned by the accelerating force of the planet. Hence we have $v = U - E + P$. When the comet recedes from the Sun the three signs are reversed, and if the planet be assumed stationary, its final effect on the aphelion distance will be zero. The presence of the planet nevertheless obviously affects the speed of the comet at perihelion, which is the only place where we can observe it, and our observations lead us to attribute a greater aphelion distance to it than is really the case, since no correction for this perturbation can be made.

An inspection of the equation shows that whether our observations lead us to believe the orbit to be elliptic or hyperbolic, depends merely on whether E or P is numerically the larger. If they are equal the orbit appears to us to be parabolic. The fact that a comet possesses a hyperbolic orbit is consequently not conclusive evidence that it may not in the future again return many times to the vicinity of the Sun.

If the planet is on the side of its orbit near the aphelion of the comet, it may under certain circumstances give the latter a greater hyperbolic velocity than if it is on the further side, but the effect in general will be to diminish the speed of approach to the Sun, and give the impression that the aphelion distance of the comet is less than is really the case. The known planets act on the parabolic comets for most of the time of their approach as if the former were themselves a part of the Sun. Unless very near to the comets the perturbations caused by them are therefore comparatively slight.

Let us now see what mass an unknown planet must have that would be capable of producing the hyperbolic excesses k that we find in the eighth column of Table XLIII. For reasons which will appear later in Chapter XIX it is thought that the higher excesses are not due to this cause. In the present computation therefore we shall adopt for the maximum value of k the very moderate value of 0.006. This velocity we shall assume to be due to the presence of an unknown planet situated nearly opposite to the aphelion of the comet, and we will further assume that its distance from the Sun is 400 units, that is to say the distance of the assumed planet Q. The calculation involves a solution of Kepler's problem, and therefore a general equation for it cannot be given, but an approximate value is readily obtained which will be quite accurate enough for our present purposes.

Let us assume a comet whose aphelion distance is 399 units, and its perihelion unity. Its eccentricity therefore $= \frac{199}{200} = 0.995$. Its velocity v in any portion of its orbit is given by the formula for elliptic motion $v^2 = U^2 \mu \left(\frac{1}{r} - \frac{1}{s}\right)$ where U is the parabolic velocity, and is therefore a constant for all ellipses having the same perihelion distance. When this distance is unity and the mass equal to that of the Sun, $U = 42.10$ km. per sec. In the above formula μ is the mass of the Sun, r the radius vector of the comet, and s its aphelion distance. A comet moving in an orbit whose semi-major axis is 200 units has a period of 2,828 years.

In the first column of Table XLIV the semi-period is divided into ten equal parts starting from the aphelion, and the year given at which it reaches each of the eleven dividing points of its orbit. The second and third columns give the corresponding mean and eccentric anomalies, the latter being taken directly from Åstrand's tables. The fourth column gives the natural cosines of the eccentric anomalies, and the fifth, obtained by multiplying by 200 and adding 200 to the product gives the distances of the dividing points from the perihelion. We shall assume that the comet moves

TABLE XLIV.

QUANTITIES REQUIRED IN ORDER TO DETERMINE THE MASS OF Q.

Years.	M.	E.	Cosine.	Dist.	x	Reciprocals.	$\frac{1}{r} - \frac{1}{s}$	v.	A.	$x + 399$.	a
	°	°									
0.0	180	180.0	+1.0000	400.0		0.0025063	0.0000000	0.000			
					399				0.167	798	0.0415
141.4	162	171.0	+0.9877	397.5		0.0025221	0.0000158	0.167			
					394				0.171	793	0.0420
282.8	144	161.8	+0.9500	390.0		0.0025707	0.0000644	0.338			
					384				0.181	783	0.0431
424.2	126	152.4	+0.8862	377.2		0.0026582	0.0001519	0.519			
					368				0.195	767	0.0449
565.6	108	142.6	+0.7944	358.9		0.0027941	0.0002878	0.714			
					347				0.222	746	0.0475
707.0	90	132.2	+0.6717	334.3		0.003000	0.000494	0.936			
					318				0.261	717	0.0514
848.4	72	120.9	+0.5135	302.7		0.003314	0.000808	1.197			
					283				0.331	682	0.0568
989.8	54	108.2	+0.3123	262.5		0.003824	0.001318	1.528			
					236				0.480	635	0.0655
1131.2	36	92.9	+0.0506	210.1		0.004782	0.002276	2.008			
					175				0.888	574	0.0802
1272.6	18	72.3	−0.3040	139.2		0.007236	0.004730	2.896			
					70				39.151	469	0.1201
1414.0	0	0.0	−1.0000	0.0		1.000000	0.997494	42.047			
									42.047		0.5930

with a uniform velocity in traversing each of these ten sections of its orbit. The distances of the middle points of these sections from the perihelion is given in the sixth column. The distances in the fifth column are each equal to $r+1$. Subtracting 1 and taking the reciprocals gives us the numbers in the seventh column. The first of these equals $\frac{1}{s}$, the others $\frac{1}{r}$. The eighth column is obtained by subtracting $\frac{1}{s}$ from the figures in the seventh. Multiplying these numbers by $U^2 \mu$ and extracting the square root gives us the values of v given in the ninth. These are the velocities attained by the comet at each of the dividing points. Their differences given in the tenth column are the accelerations acquired by the comet in km. per second per 141.4 years, and the sum of these ten accelerations is the total velocity acquired by the comet, by the time it reaches perihelion, owing exclusively to the action of the Sun.

Let us now imagine another large body, a planet, whose mass we will assume at first also equals unity, and we will locate it at a distance of 400 units from the Sun, in the opposite direction to the aphelion of the comet. Its mean distance from the comet during each of the ten periods is given in the eleventh column. Since the acceleration varies inversely as the square of the distance, the acceleration a due to the planet $= \frac{A\ (x-1)^2}{(x+399)^2}$ where A is the acceleration due to the Sun. $A\ (x-1)^2$ is a constant when these quantities are accurately determined. From the first eight rows of the table we derive log $A(x-1)^2 = 4.4218$. The next row shows a small systematic error, owing to these results being only approximate, and the last row a large one. These two values are therefore rejected, and the above value of the constant used in all cases. This equation gives us the accelerations due to the planet which are given in the last column of the table. The sum of these ten accelerations, 0.5930, is the velocity acquired by the comet at perihelion owing to the attraction of this body. For it let us now substitute the planet Q with a mass μ' capable of giving at perihelion a velocity v'. Then the elliptical velocity $42.047 + v' = U + k = 42.100 + 0.006$, whence $v' = 0.059$ km. per second. But the velocity varies as the acceleration, which varies as the mass, hence $\mu' = \frac{.059\,\mu}{.5930} = 0.0995\,\mu$ or practically just one-tenth that of the Sun.

If then we are to adopt the theory that the hyperbolic comets owe their excess of velocity to the attraction of an unknown planet, located at a distance of 400 units, we must admit that this body is of extraordinary size. In fact its mass would be more than one hundred times that of Jupiter. If the planet were located at a greater distance it would have a somewhat smaller mass. The reason for selecting Q rather than P for this computation will appear later. We notice that whether we deny or affirm the existence of hyperbolic orbits, it will make no great difference in the resulting mass of Q, our assumption as to the value of k entering only as a correction into the formula. Had we assumed $k = 0.031$ instead of 0.006, or about five time as great, the mass of Q would only have been increased to 0.142 μ. We shall find later that these hyperbolic orbits seem to be more closely related to the elliptical orbits of high eccentricity, than they do to those orbits that are parabolic.

CHAPTER XIII.

CLASSES *A* AND *B*. THE PERIODIC COMETS. PERTURBING FORCES.

IN the two following tables the comets are arranged in the order of their aphelion distances. Table XLV contains all the known comets of short period, that is all those that are associated with the planet Jupiter. The first five columns give the number of the orbit, its certainty, the grade of brilliancy of the comet, and the longitude and latitude of its aphelion, for the equinox of 1900. The sixth column gives the difference in longitude between the comet's aphelion and the nearer node of Jupiter's orbit. The seventh the latitude of the planet's orbit in minutes, in the longitude of the comet's aphelion. The eighth gives the deviation of the aphelion from the planet's orbit expressed in minutes of latitude, and the ninth the same result expressed in degrees and tenths. The last column gives the letter indicating the type. It will be noticed that only one comet of class *A*, Lexell's, number 102, was visible to the naked eye. Earlier appearances of these comets however, notably of Encke's, number 480, were brighter, and will be found recorded in Table L under subclass *a*.

TABLE XLV.

LOCATION OF THE APHELIA OF CLASS *A*.

No.	Cert.	Br.	Long.		Lat.		Δl.	Planet.	Dev.	Dev.	Type.
			°	′	°	′	°	′	′	°	
480	1		338	51	+1	00	60	−68	+128	+ 2.1	*N*
460	1		126	34	+1	15	28	+37	+ 38	+ 0.6	*N*
336	1		58	50	−3	24	40	−51	−153	− 2.6	*N*
358	1*a*		126	31	+4	40	28	+37	+243	+ 4.0	*N*
368	1*a*		50	34	−0	42	48	−59	+ 17	+ 0.3	*N*
159	1*a*		248	49	+1	33	30	+40	+ 53	+ 0.9	*N*
113	1*a*		233	00	+4	12	46	+57	+195	+ 3.2	*N*
396	1*a*		238	03	−2	54	41	+52	−226	− 3.8	*N*
469	1		165	04	−5	02	66	+72	−374	− 6.2	*N*
415	1		165	26	+2	39	66	+72	+ 87	+ 1.4	*N*

No.	Cert.	Br.	Long.		Lat.		Δl.	Planet.	Dev.	Dev.	Type.
			°	′	°	′	°	′	′	°	
481	1		223	51	−4	59	56	+65	−364	− 6.1	*N*
85	1*a*		275	30	−0	13	3	+ 4	− 17	− 0.3	*N*
470	1*a*		103	06	−2	54	4	+ 5	−179	− 3.0	*N*
457	1		181	44	+1	42	83	+78	+ 24	+ 0.4	*N*
486	1*a*		254	14	−1	12	25	+33	−105	− 1.8	*N*
406	1*a*		198	31	−5	00	80	+78	−378	− 6.3	*N*
484	1		91	51	−2	25	7	− 9	−136	− 2.3	*N*
334	1		294	41	−7	15	16	−22	−413	− 6.9	*N*
433	1		200	04	−3	01	79	+78	−259	− 4.3	*N*
102	1*a*	*II*	178	05	+1	06	79	+78	− 12	− 0.2	*N*
424	1*a*		154	15	−7	14	55	+65	−499	− 8.3	*N*
485	1		229	29	−3	31	50	+61	−286	− 4.8	*N*
259	1*a*		20	03	−8	19	79	−78	−421	− 7.0	*N*
428	1		139	44	−1	52	41	+52	−164	− 2.7	*N*
462	1*a*		249	56	+3	56	29	+38	+198	+ 3.3	*N*
382	1		230	35	+4	05	48	+59	+186	+ 3.1	*N*
471	1		188	08	+2	08	89	+79	+ 49	+ 0.8	*N*
447	1*a*		189	02	−4	19	90	+79	−338	− 5.6	*N*
417	1*a*		158	01	−0	38	59	+67	−105	− 1.8	*N*
235	1		289	07	+8	34	10	−14	+528	+ 8.8	*N*
412	1*a*		310	35	−4	00	32	−42	−198	− 3.3	*N*
472	1*a*		211	15	+4	11	68	+73	+178	+ 3.0	*N*
389	1*a*		220	06	−9	36	59	+67	−643	−10.7	*N*
348	1*a*		198	56	+5	02	80	+78	+224	+ 3.7	*N*

Figure 33 represents the portion of the celestial sphere near the ecliptic in rectangular coordinates. The scale of latitudes is four times that of the scale of longitudes. An examination of this figure shows that while the

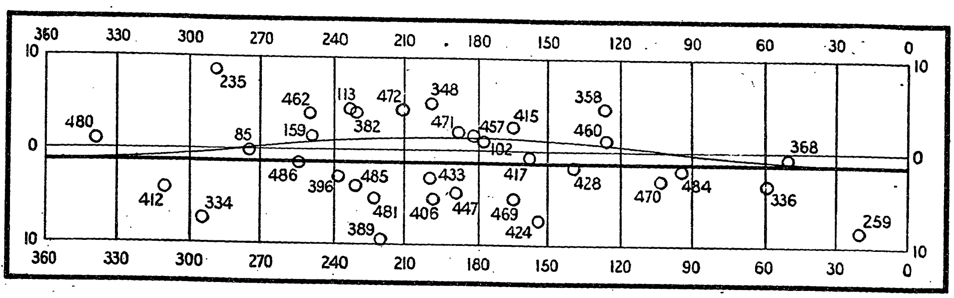

Fig. 33.

aphelia are more crowded together near longitude 200° than elsewhere, there is no marked tendency for them to lie north of the ecliptic in one longitude rather than in another. The curved line represents the orbit of Jupiter. Fourteen aphelia lie north of it and twenty south. Their mean deviation from it taken from the ninth column of Table XLV is 3°.64. Fourteen aphelia lie north of the ecliptic and twenty south. Their mean deviation from it is 3°.66, or practically the same as their deviation from the orbit of Jupiter. The mean excess of the southern over the northern deviations from the ecliptic is 0°.95, and their mean deviation from this latitude ±3°.49. While this quantity 0°.95 seems so small that it might readily be due to accidental errors, yet such is probably not the case. The sum of the fourteen positive deviations is 46°.0. The sum of the nineteen negative ones is 78°.3, giving an excess of −32°.3, and a ratio of 1 to 1.70. The highest individual deviation is but 9°.6, and the four highest negative deviations out of the nineteen would have to be eliminated in order to obtain a positive result. A possible explanation of this southern deflection of the aphelia will be given presently. The heavy line drawn parallel to the ecliptic and 0°.95 to the south of it best represents the positions of these various aphelia, and will be designated in what follows as their "reference circle."

Class *B* consists of only twelve orbits. Of these the first two are associated with Saturn, the next two with Uranus, the next six with Neptune, and the two remaining ones with the hypothetical planet *O*. In Table XLVI, the six columns give the number of the orbit, its certainty, the grade of brightness of the associated comet, the longitude and latitude of its aphelion, and the type to which it belongs. Four of these comets it will be seen are visible to the naked eye. The larger proportion of bright comets belonging to this type than to type *N* is doubtless due to their less frequent visits to the vicinity of the Sun. Halley's comet when favorably situated is much

TABLE XLVI.

LOCATION OF THE APHELIA OF CLASS *B*.

No.	Cert.	Br.	Long.		Lat.		Type.	No.	Cert.	Br.	Long.		Lat.		Type.
			°	′	°	′					°	′	°	′	
213	1*a*		63	26	+10	13	*O*	357	1	*III*	259	47	+18	26	*O*
442	1		286	06	+21	25	*O*	211	1*a*		259	24	−12	50	*O*
290	1*a*		240	32	− 2	41	*O*	190	1	*II*	124	30	−16	35	*O*
292	1*a*		256	35	+ 0	47	*O*	220	1*a*		261	28	−14	42	*O*
236	1*a*	*IV*	216	24	−33	21	*O*	275	1*a*	*II*	149	37	−24	49	*O*
378	1		321	55	−39	37	*O*	386	1*a*		147	14	−26	42	*O*

the finest member of this type, although for a considerable period during its appearance, in 1835 it was without a tail.

The positions of these twelve aphelia are plotted in Figure 34, the first four as crosses, the remainder as circles. This figure is on square projection. That is to say the ordinates and abscissas are on the same scale. To compare it with Figure 33 we must conceive the ordinates of the latter reduced four times. The difference between the two figures then at once becomes apparent. None of the thirty-four aphelia of Class *A* deviates as much as 10° from the ecliptic. In the case of the aphelia of class *B*, but two deviate as little as 10°, the maximum deviation being nearly 40°.

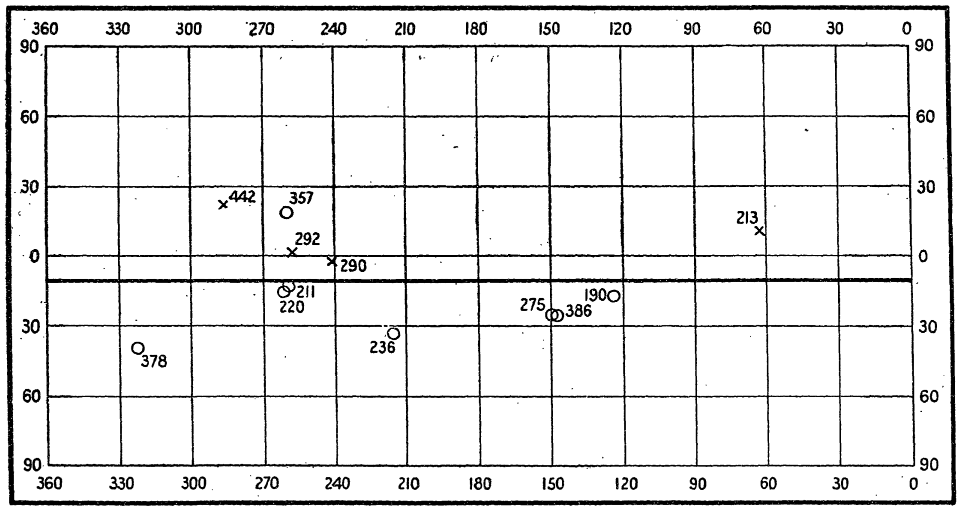

FIG. 34.

The aphelia of these comets lie still farther to the south of the ecliptic than do those of class *A*. The sum of the four positive latitudes is +50°.8. The sum of the eight negative latitudes is −171°.3, giving a ratio of 1 to 3.37. The mean latitude therefore is −10°.0, which on the assumption that the aphelia are distributed about a circle parallel to the ecliptic would be the latitude of their reference circle. Their mean deviation from it is ±16°.4.

In class *A* especially, new comets are constantly being discovered, while others frequently disappear. It is of interest in this connection to determine whether these comets are really changing their orbits, whether they are temporarily changing their brilliancy, or whether the observed facts depend merely upon more or less favorable conditions of observation. In some cases the new comet is so faint that at earlier perihelion passages it might readily have been overlooked. In other instances, such for example as Holmes' comet in 1892, it is impossible that so bright an object could have previously escaped observation. Here either a change of orbit or a change of brilliancy seems certain. At its last two appearances Holmes' comet has been extremely faint, so that a sudden accession of brilliancy in 1892 seems the most plausible explanation to account for its discovery in that year. Indeed, a second considerable accession of brilliancy occurred in 1893 just before the comet disappeared. It is well established that the discovery and subsequent loss of comet 102, otherwise known as Lexell's comet, was due to changes in its orbit by the planet Jupiter. All three of these causes are therefore influential in causing the appearance and disappearance of comets. The second cause, that of change of brilliancy, is probably responsible for more changes than either of the others at the present time, and the first, the change of orbit, the least of all.

In order to study this question in more detail with regard to type *N* we apparently need some sort of census of Jupiter's family. The longest period of any one of Jupiter's comets that has been known to return, that of Faye, is 7.56 years. Any comet that has not been recognized for an interval of twenty years will in what follows be considered as having permanently disappeared. There is one comet, however, where this rule has notably broken down, that of De Vico, which appeared in 1678, in 1844, and not again until 1894. Similarly Winnecke's comet probably first appeared in 1766, and was not found again till 1819. Such an interval would not be likely to elapse at the present day, unless the comet were very faint. Galle however does not consider the identity of the early appearances of these two comets as sufficiently assured to mark them as such in his table.

In Table XLVII each of the eight columns is headed by the first and last of the series of years to which it refers. So many comets were observed in the period 1881 to 1900 that it was found necessary to enter them in two columns. The comets in each period are divided into three groups, those under Obs. were observed for the first time, those under Ret. are returns of previously observed comets, and those under Dis. are the comets which have failed to appear for twenty or more years. The number of times that the comet returned during the period is indicated by the number following its name. The second and third appearances of De Vico's, and the second appearance of Winnecke's comet entered under Obs., also their three disappearances are entered in Italics. Including these appearances as returns, there were with the others 79 in all. If a comet has appeared but once, it is indicated merely by its number. The total number of comets appearing under each heading is indicated at the bottom of the columns.

TABLE XLVII.

JUPITER'S FAMILY OF COMETS.

1678 1800	1801 1820	1821 1840	1841 1860	1861 1880	1881 1900		1901 1909
OBS. De Vico 85 Winnecke 102 Biela 113 Encke	OBS. *Winnecke* 159	OBS.	OBS. Faye *De Vico* Brorsen d'Arrest 259	OBS. Tempel$_1$ Tempel$_3$ Tempel$_2$	OBS. 348 358 Wolf 368 Finlay Brooks 389 396 Holmes 406 412 *De Vico* 417 424 Perrine 447	RET. Encke 6 Faye 3 Winnecke 3 Wolf 2 d'Arrest 2 Tempel$_2$ 2 Finlay 1 Brooks 1 Tempel$_3$ 1 Holmes 1	OBS. 462 470 472 486
RET. Encke 1	RET. Encke 2 Biela 1	RET. Encke 6 Biela 2	RET. Encke 6 Faye 2 Biela 2 Brorsen 1 d'Arrest 1 Winnecke 1	RET. Encke 6 Winnecke 3 Faye 2 Tempel$_1$ 2 Brorsen 2 d'Arrest 2 Tempel$_3$ 1 Tempel$_2$ 1		DIS. Tempel$_1$ Brorsen	RET. Encke 3 Brooks 1 Tempel$_2$ 1 Holmes 1 Finlay 1 Tempel$_3$ 1 Winnecke 1 Perrine 1
DIS. *De Vico* 85 *Winnecke* 102	DIS. 113	DIS. 159	DIS.	DIS. *De Vico* Biela 259			DIS. 348 358 368 389
OBS. 7 RET. 1 DIS. 4	2 3 1	0 8 1	5 13 0	3 19 3	16 — —	— 22 2	4 10 4

Of the 34 different comets observed, 15 are named and 19 are numbered. Of the 12 that have disappeared, admitting the identity of Winnecke's and of De Vico's comets, 3 are named and 9 are numbered. Of the 22 comets still existing as far as known, 12 are named and 10 numbered. There has been at least one opportunity for 6 of these last to reappear, but so far they have not been detected. The fact that three times as many unnamed as named comets have disappeared, leads us to believe that they must appear in about the same proportion. This view is partially confirmed by the statistics for the period of 1881 to 1900. During the whole of this period the sky was carefully searched for new arrivals, yet only 5 new comets to which names have been assigned were discovered. It is possible that some of the remaining 10 unnamed ones may yet prove to belong to this class before the present period is over, although 4 of them have already been rejected. In general we may say that the chance that a new comet of short period will return is about one in three.

A statistical investigation has been made to learn if the comets which have returned had any orbital characteristics in common. The following quantities were investigated, aphelion distance, period, longitude of the aphelion, longitude of the node, deviation of the aphelion from the orbit of Jupiter, inclination of the orbit to the plane of the ecliptic, perihelion distance, and eccentricity. The only results obtained worthy of record were as follows: — No comet whose aphelion distance exceeds that of Biela, 6.19, has been known to return. Four such comets, or 12 per cent of the whole, are recognized. There are four comets whose inclination exceeds that of Brorsen, 29°.4. None of these has been known to return. The highest inclination, that of 113, is 45°.1.

But four comets of this type have been visible to the naked eye since 1800. Encke's comet was visible in 1805, 1828, 1838, 1848, 1858, 1871, and 1881. Astr. Nach. 1888, 119, 64. It was also visible in 1795. At its twenty-three other appearances no record is made that it was other than telescopic. Biela's comet was last seen with the naked eye in 1806. At its four subsequent returns it was telescopic. It has not been observed since 1852. De Vico's was visible to the naked eye in 1844. At only one return since then has it been detected, in 1894, and it was then extremely faint. Holmes' comet was a bright naked eye object in the year of its discovery, 1892. It was indeed the brightest naked eye comet of type N which had appeared since 1850, perhaps since 1800.

With a few possible exceptions comets are visible to us only by the incandescence, presumably electric, of the gases emitted by their nuclei. This incandescence occurs only in the immediate vicinity of the Sun, when the gases from the nucleus are escaping into outer space. It is clear from the facts above cited that this process of dispersion, in the case of a small comet such as those which seem to compose type N, is a rapid one, lasting comparatively but a few years. The comets of De Vico, Winnecke, and Encke are the only ones known of this type, whose existence has outlasted one hundred years, and since 1678 De Vico's has been detected but twice. In addition, Le Verrier has shown that Faye's comet must have adopted its present orbit at least as far back as 1747, although it was not discovered until 1843.

The various appearances of De Vico's comet seem to be best explained on the supposition that there were three outbursts of gas from the nucleus, which have at these times rendered it temporarily visible. Usually this incandescence seems to last through but one perihelion passage, which accounts for the large proportion of numbered to named comets. This accession of brilliancy is sometimes quite sudden, as in the case of Holmes' comet already mentioned, where after its perihelion passage in June 1892 it had nearly faded from sight in the most powerful telescopes by the following December. Yet on January 16 it suddenly appeared again with the brightness of a star of the seventh magnitude. Similarly Pons comet on September 22, 1883, while still distant from the Sun by over two units, increased its brightness eight-fold within twenty-four hours. Such an accession of brilliancy is always accompanied by a decrease of volume.

We need not suppose however that the comet itself really contracts, but what is perhaps more likely is that the source of light appears somewhere in its interior, somewhat as the *nova* did in the nebula of Andromeda. The bulk of a comet is therefore not necessarily measured by what we can see. The solid matter accompanying it may extend in all directions, but especially in the direction of its orbit, far beyond the illuminated gaseous region that is visible to our eyes. The duplication of Biela's comet may thus have been simply the formation of two nuclei in one cometary mass of meteors. The head of a comet may therefore be defined simply as that portion of a meteoric swarm which is at the time self luminous. In the case of the brighter comets, however, there is obviously a concentration of matter and energy at this point.

It thus appears that our heavens are perhaps full of invisible comets, which are only made visible to us occasionally, either when through some internal disturbance, they temporarily give out a greatly increased amount of light, or when we suddenly dash through one of them, as we have so frequently done in November through an invisible companion of the comet of 1866 I. This latter comet itself, it may be noted failed of detection at its last predicted return in 1899.

All the known comets excepting those connected with the planet Jupiter move in orbits of high eccentricity. In but three other cases is the minor axis as much as one half that of the major, and it is usually much less. Doubtless many of Jupiter's comets possess high eccentricity also, but this would bring them so near the Sun at perihelion that they would long ago have lost their gaseous envelope, and thus have become invisible. Very few even of the brightest comets, as is shown by the spectroscope, owe their visibility to reflected light. The highest known eccentricity of an orbit in class *A* is that of number 480, Encke, 0.8474, its minor axis being therefore but little over half its major.

On the other hand it is highly probable that a large number, perhaps the majority of the elliptical orbits belonging to the other classes are of comparatively small eccentricity. The reason that we do not see them being that their perihelion distances are too large. The greatest perihelion distance known, that of orbit number 80 is 4.05 units. It is on the whole probable that not more than a few per cent of all the comets that have passed perihelion within the last century have been detected.

In the Astr. Nach. 1888, 119, 64, Berberich suggests that the varying brilliancy of Encke's comet during the last century may be connected with the solar activity, as indicated by the sunspot period. As we have just seen it was visible to the naked eye on seven returns during that interval. Each of these returns occurred in the immediate vicinity of a sunspot maximum. At several similarly situated later returns, however, it was only telescopic.

It is sometimes convenient to divide an interval of several years equally between periods of minimum and maximum solar activity. This result may be accomplished approximately by means of Table XLVIII, where the dates of minimum and maximum activity are given in the first two columns, and the dates intermediate between the successive minima and maxima in the third. The two dates given on each line in this column may then be said to mark the beginning and end of the periods of maximum activity, and the intermediate intervals between the lines the minimum periods. While for an interval of a few years the sum of the periods of maximum and of minimum activity may not be quite equal, for longer intervals of time the agreement becomes more exact, and a correction to the result can always be readily applied if necessary. The date of the last maximum given in the table was computed by Dr. Wolfer, and is taken from Nature 1910, 82, 378. The other dates are from Clerke's History of Astronomy. Appendix.

If the unnamed comets of Jupiter's family owe their temporary brilliancy largely to unusual electrical activity on the part of the Sun, we should ex-

TABLE XLVIII.

PERIODS OF MAXIMUM SOLAR DISTURBANCE.

Min.	Max.	Maximum activity.		Min.	Max.	Maximum activity.	
1610.8	1615.5	1613.2	1617.2	1766.5	1769.7	1768.1	1772.6
1619.0	1626.0	1622.5	1630.0	1775.5	1778.4	1770.0	1781.6
1634.0	1639.5	1636.8	1642.2	1784.7	1788.1	1786.4	1793.2
1645.0	1649.0	1647.0	1652.0	1798.3	1804.2	1801.2	1807.4
1655.0	1660.0	1657.5	1663.0	1810.6	1816.4	1813.5	1819.8
1666.0	1675.0	1670.5	1677.2	1823.3	1829.9	1826.6	1831.9
1679.5	1685.0	1682.2	1687.2	1833.9	1837.2	1835.6	1840.4
1689.5	1693.0	1691.2	1696.0	1843.5	1848.1	1845.8	1852.0
1698.9	1705.5	1702.2	1708.8	1856.0	1860.1	1858.0	1863.6
1712.0	1718.2	1715.1	1720.8	1867.2	1870.6	1868.9	1874.8
1723.5	1727.5	1725.5	1730.8	1878.9	1884.0	1881.4	1887.1
1734.0	1738.7	1736.4	1741.8	1890.2	1894.0	1892.1	1898.0
1745.0	1750.3	1747.6	1752.8	1901.9	1906.4	1904.2	
1755.2	1761.5	1758.4	1764.0				

pect to find them coming in increasing numbers during the periods of increased solar activity. In the first column of Table XLIX are given the numbers of the nineteen unnamed comets of class *A*, and in the second the dates of their perihelion passages. In the third column a + indicates that this passage occurred during a period of maximum solar activity, and a − that it occurred during a minimum period. It will be seen that thirteen passages occurred during intervals of sunspot maxima and only six during intervals of minima. After 1910 we may expect to find a decided diminution in the number of comets discovered belonging to this class. The influence of solar activity on bright comets seems to be less important.

TABLE XLIX.

UNNAMED COMETS OF SHORT PERIOD.

No.	Year.	Per.	No.	Year.	Per.	No.	Year.	Per.	No.	Year.	Per.
85	1743.0	−	348	1881.7	+	406	1892.9	+	462	1905.0	+
102	1770.6	+	358	1884.6	+	412	1894.1	+	470	1906.3	+
113	1783.9	−	368	1886.4	+	417	1895.6	+	472	1906.8	+
159	1819.9	−	389	1889.9	−	424	1896.8	+	486	1909.9	+
259	1858.3	+	396	1890.8	−	447	1900.9	−			

With regard to the comets of class *B* little can be said in this connection, Number 213 discovered by Peters in 1846 has a period of 13 years. It has not been seen since the year of its discovery, and may be definitely classed with those that have disappeared. Number 442, Tuttle's, has appeared hitherto five times. It has a period of 14 years, and is next due in 1913. Number 290 $Tempel_4$, discovered in 1866, with a period of 33 years failed to be seen in 1899. This was however a year of minimum solar activity, so that it is possible that at some future return, when favorable circumstances conspire, it may yet be visible. In 1366 when first observed, it was obviously a brilliant naked eye object. Numbers 292 and 236 discovered by Stephan and Westphal have periods of 40 and 60 years respectively. Stephan's failed to appear when due in 1907. Westphal's should appear in 1914.5 Vierteljahrschrift 1910, 45, 332. Numbers 378 and 357 known as the comets of Olbers and Pons have each appeared twice. Their periods are 73 and 72 years, and they are not due again until 1960 and 1956. Numbers 211 and 220 discovered by de Vico and Brorsen have periods of 76 and 81 years, and are not due until 1922 and 1928. Halley's comet with a period of 76 years came to perihelion in 1910, April 19. It has appeared according to Cowell and Crommelin twenty-eight times in the past, and perhaps many more. Its various perihelion passages taken from Galle and from the Monthly Notices 1907–08, 68, are as follows:—

−240 May 15, (*−163 June*), *−87 Aug.–Sept.*, −12 Oct. 8, +66 Jan. 14, 141 Mar. 29, *218 Apr. 6*, *295 Apr.*, *373 Nov.*, *451 July 3*, *530 Nov. 15 ± 14*, *607 Mar. 26 ± 14*, *684 Nov. 6 ± 14*, *760 June 11*, 837 Mar. 1, *912 July 19*, 989 Sept. 12, 1066 Mar. 27, 1145 Apr. 19, 1222 Sept. 10, 1301 Oct. 22, 1378 Nov. 8, 1456 June 8, 1531 Aug. 25, 1607 Oct. 26, 1682 Sept. 14, 1759 Mar. 12, 1835 Nov. 15. The Italics indicate that the elements of these orbits are not known. Those for 1066, 1145, and 1222 are given by Cowell and Crommelin, the others including 1066 by Galle. The parenthesis indicates that this date is purely hypothetical, no comet having been recorded during that year. The mean period since −240 is 76.7 years, the longest individual period, that from 451 to 530, 79 yrs. 4 mos., and the shortest period, the last one, from 1835 to 1910, 74 yrs. 5 mos. Although there were one or more comets visible in 912, the identification is not satisfactory, and the date is given merely by computation. Some of the earlier dates may be in error by a couple of months. At some of the earlier appearances, notably those of 1066 and 1456 the comet was a very remarkable object.

The two comets of 1862 III, number 275, and 1889 III, number 386, as previously noted seem to be related to one another. Their periods are 142 and 128 years respectively, and they will be due next in the years 2004 and 2017. The former was moderately brilliant and is of interest as being connected with the well-known meteor shower of August 10.

On account of perturbations, due chiefly to the planet Jupiter, the nodes and apsides of all these orbits are constantly shifting, and consequently the aphelia are continually changing both in longitude and latitude. The comets of class *A* being associated with a very massive planet, the inclinations of their orbits are in consequence much less than those of the comets of class *B*. Hence a change of the apsides produces much less effect on the latitude of the aphelia of the former class than it does in the case of the latter, which accounts for the wider deviations of these latter aphelia from their reference circle.

There is no question but that when these comets were originally captured, and their orbits reduced to their present dimensions, the comets and their controlling planets must have passed very near one another. Since all the comets of class *B* have very eccentric orbits, this conjunction must have occurred near the comet's aphelion which must consequently at that time have been located near the plane of the ecliptic. It is therefore proper to locate their reference circle parallel to this plane, even if other reference circles can be found with which the aphelia more closely coincide. For instance, omitting number 378, the remaining eleven aphelia lie very near a reference circle whose inclination is 35° and its ascending node 260°. The maximum deviation in this case is about 15°.

Considering only the five comets associated with Neptune whose aphelia lie to the south of the ecliptic, we find from Table XLVI that their mean latitude is $-23°.4$. The sine of this angle at a distance from the Sun equal to that of Neptune, 30 units, gives their mean distance from the plane of the ecliptic as 11.9, or a distance somewhat greater than that of Saturn from the Sun. Crommelin has pointed out in the case of Halley's comet, number 190, that Neptune could never have retained it in the solar system had its orbit formerly occupied its present position. Journ. Brit. Astron., Assoc. 1907, 17, 215. This same remark clearly applies also to the other members of this group.

The shifting of the cometary aphelia in a north and south direction owing to planetary perturbations is a very slow process, amounting on the average for the comets of class *B* to only a few minutes per century. We therefore see that these comets must have been connected with our system for very long intervals of time, amounting to many thousands of years. The aphelia of the comets of class *A* are subject to a somewhat more rapid movement, amounting on the average to about half a degree per century.

Table L gives a list of all the returns of all the periodic comets of classes *A* and *B* which have appeared more than once, and have therefore been entered in the subtypes *a* and *b*. The first and second columns give the number of the comet, and the number of its return, the third its year and designation, the fourth its brilliancy, the fifth its name, and the sixth and seventh the longitude and latitude of its aphelion.

TABLE L.

APHELIA OF THE COMETS WHICH HAVE RETURNED AT LEAST ONCE.

No.	n.	Year.	Br.	Name.	Long.	Lat.	No.	n.	Year.	Br.	Name.	Long.	Lat.
					° ′	° ′						° ′	° ′
117	1	1786 I		Encke	338 09	+ 0 35	416	27	1895 I		Encke	338 41	+ 0 41
129	2	1795	*IV*	"	338 06	+ 0 29	432	28	1898 III		"	338 42	+ 0 42
139	3	1805	*III*	"	338 03	+ 0 34	449	29	1901 II		"	338 41	+ 0 41
156	4	1819 I		"	338 03	+ 0 34	461	30	1905 I		"	338 53	+ 0 53
162	5	1822 II		"	338 12	+ 0 38	480	31	1908 I		"	338 51	+ 1 00
170	6	1825 III		"	338 14	+ 0 39	311	1	1873 II		Tempel$_2$	126 21	+ 1 08
180	7	1829	*IV*	"	338 14	+ 0 39	333	2	1878 III		"	126 19	+ 1 08
183	8	1832 I		"	338 13	+ 0 39	414	3	1894 III		"	126 13	+ 1 07
189	9	1835 II		"	338 14	+ 0 39	443	4	1899 IV		"	126 27	+ 1 13
191	10	1838	*IV*	"	338 15	+ 0 39	460	5	1904 III		"	126 34	+ 1 15
196	11	1842 I		"	338 14	+ 0 39	293	1	1867 II		Tempel$_1$	56 49	− 4 32
207	12	1845 IV		"	338 26	+ 0 47	310	2	1873 I		"	58 42	− 3 26
223	13	1848 II	*III*	"	338 30	+ 0 30	336	3	1879 III		"	58 50	− 3 24
233	14	1852 I		"	338 25	+ 0 25	404	1	1892 III	*IV*	Holmes	165 07	− 5 00
248	15	1855 III		"	338 26	+ 0 26	441	2	1899 II		"	164 56	− 4 57
264	16	1858 VIII	*IV*	"	338 27	+ 0 27	469	3	1906 III		"	165 04	− 5 02
273	17	1862 I		"	338 28	+ 0 28	64	1	1678		De Vico	145 53	− 1 00
289	18	1865 II		"	338 28	+ 0 28	201	2	1844 I	*IV*	"	163 19	+ 2 53
297	19	1868 III		"	338 33	+ 0 33	415	3	1894 IV		"	165 26	+ 2 39
309	20	1871 V	*IV*	"	338 32	+ 0 32	300	1	1869 III		Tempel$_3$	223 29	− 5 11
324	21	1875 II		"	338 32	+ 0 32	342	2	1880 IV		"	223 25	− 5 11
332	22	1878 II		"	338 32	+ 0 32	401	3	1891 V		"	223 16	− 5 09
350	23	1881 VII	*IV*	"	338 39	+ 0 39	481	4	1908 II		"	223 51	− 4 59
360	24	1885 I		"	338 40	+ 0 40	388	1	1889 V		Brooks	181 49	+ 1 43
380	25	1888 II		"	338 40	+ 0 40	425	2	1896 VI		"	181 57	+ 1 41
399	26	1891 III		"	338 40	+ 0 40	457	3	1903 V		"	181 44	+ 1 42

No.	n.	Year.	Br.	Name.	Long.	Lat.	No.	n.	Year.	Br.	Name.	Long.	Lat.
					° ′	° ′						° ′	° ′
100	1	1766 II	*III*	Winnecke	73 07	+ 0 25	410	2	1893 III		Finlay	188 08	+ 2 07
158	2	1819 III		"	96 07	− 3 23	471	3	1906 V		"	188 08	+ 2 08
258	3	1858 II		"	96 33	− 3 18	105	1	1772		Biela	290 37	+10 41
298	4	1869 I		"	96 39	− 3 15	140	2	1806 I	*IV*	"	290 00	+ 8 22
323	5	1875 I		"	97 20	− 2 53	172	3	1826 I		"	290 03	+ 8 22
370	6	1886 VI		"	96 37	− 2 00	185	4	1832 III		"	290 11	+ 8 46
405	7	1892 IV		"	96 33	− 1 59	209	5	1846 II		"	289 06	+ 8 34
431	8	1898 II		"	94 33	− 1 56	235	6	1852 III		"	289 07	+ 8 34
484	9	1909 II		"	91 51	− 2 25	123	1	1790 II		Tuttle	282 33	+20 26
210	1	1846 III		Brorsen	295 20	− 7 02	257	2	1858 I		"	286 01	+21 31
251	2	1857 II		"	294 34	− 6 54	307	3	1871 III		"	286 07	+21 28
295	3	1868 I		"	294 39	− 7 12	363	4	1885 IV		"	286 19	+21 28
315	4	1873 VI		"	294 36	− 7 13	442	5	1899 III		"	286 06	+21 25
334	5	1879 I		"	294 41	− 7 15	27	1	1366		Tempel$_4$	234 20	− 4 55
359	1	1884 III		Wolf	199 56	− 3 06	290	2	1866 I		"	240 32	− 2 41
398	2	1891 II		"	199 59	− 3 04	151	1	1815		Olbers	322 09	−39 39
433	3	1898 IV		"	200 04	− 3 01	378	2	1887 V		"	321 55	−39 37
426	1	1896 VII		Perrine	231 00	− 3 45	148	1	1812	*III*	Pons	259 17	+18 32
485	2	1909 III		"	229 29	− 3 31	357	2	1884 I	*III*	"	259 47	+18 26
230	1	1851 II		d'Arrest	143 44	− 1 19	4	1	− 12	*I*	Halley	126 24	− 9 30
256	2	1857 VII		"	143 48	− 1 18	5	2	+ 66		"	176 38	−36 55
303	3	1870 III		"	139 23	− 2 05	6	3	141		"	95 19	−14 32
328	4	1877 IV		"	139 44	− 1 54	13	4	837		"	123 44	+10 54
394	5	1890 V		"	139 39	− 1 54	15	5	989		"	96 42	+ 0 00
428	6	1897 II		"	139 44	− 1 52	17	6	1066	*I*	"	95 25	−14 32
200	1	1843 III		Faye	229 59	+ 3 53	23	7	1301		"	140 30	+ 1 21
229	2	1851 I		"	230 01	+ 3 54	28	8	1378		"	125 57	−17 03
261	3	1858 V		"	230 03	+ 3 53	33	9	1456	*I*	"	124 27	−17 01
291	4	1866 II		"	230 02	+ 3 55	42	10	1531		"	125 44	−16 27
312	5	1873 III		"	230 36	+ 3 56	54	11	1607		"	124 07	−16 17
344	6	1881 I		"	230 41	+ 4 05	66	12	1682		"	124 04	−16 44
382	7	1888 IV		"	230 35	+ 4 05	93	13	1759 I	*II*	"	124 12	−16 27
421	8	1896 II		"	—	—	190	14	1835 III	*II*	"	124 30	−16 35
371	1	1886 VII		Finlay	187 49	+ 2 08							

In order to determine the change in latitude per century that the various comets entered in the table have undergone, curves were plotted for each comet with the years as abscissas and the latitudes as ordinates. Generally the aphelia lay along a nearly straight line. In one case, that of d'Arrest's comet, the earlier and later locations were separated by an interval where a marked change of inclination had occurred. This was due to a close approach to Jupiter, and was ignored as being due to what we may call, in a sense, an accidental event, the line being drawn only through the later series of observations. Similarly, Winnecke's comet has been constantly disturbed by approaches to Jupiter, its period being very nearly half that of the planet. Perrine's comet during the short interval of thirteen years which elapsed between the two observed returns also approached very near to Jupiter and was considerably perturbed. Both of these comets have therefore been rejected in the discussion. Aphelia of Halley's comet prior to 1378 were rejected as obviously erroneous. The longitudes were treated in the same manner.

In Table LI are given the longitudes of the aphelia, and their changes per century in longitude and latitude as determined from these curves. Of the thirteen accepted periodic comets of class *A* it will be noticed that the aphelia of seven were moving north with a total motion of 196′, five were moving south with a total motion of 186′, and one was stationary. The mean motion of the group in a north and south direction was therefore about 1′ north per century, or practically zero. Of the five comets of class *B* which have returned at least once, the total northerly motion is 3′, and the total southerly 58′. The mean motion of the aphelia of class *B* is therefore just 11′ south per century. In Figure 34, we have just seen that all of the aphelia save 378 lie along a great circle whose ascending node is in longitude 260° and its inclination 35°. It is a curious coincidence that 378, which lies far to the south of this circle, is the only aphelion in this class which is at present known to be moving northerly.

TABLE LI.

CHANGES IN THE APHELIA OF THE PERIODIC COMETS.

Name.	Cl.	Long.	Δ Long.	Δ Lat.	Name.	Cl.	Long.	Δ Long.	Δ Lat.
		°	′	′			°	′	′
Encke	*A*	339	+ 46	+ 22	Perrine	*A*	229	(−700)	(+105)
$Tempel_2$	*A*	127	+100	+ 20	d' Arrest	*A*	140	+ 80	+ 36
$Tempel_1$	*A*	59	+140	+ 34	Faye	*A*	231	0	+ 28
Holmes	*A*	165	− 20	− 16	Finlay	*A*	188	+ 90	0
De Vico	*A*	165	+250	− 26	Biela	*A*	289	−112	− 96
$Tempel_3$	*A*	223	− 60	+ 20	Tuttle	*B*	286	+ 45	− 15
Brooks	*A*	182	− 35	− 8	$Tempel_4$	*B*	240	+ 75	− 27
Winnecke	*A*	95	(−320)	(+ 73)	Olbers	*B*	322	− 20	+ 3
Brorsen	*A*	295	+ 32	− 40	Pons	*B*	260	+ 42	− 8
Wolf	*A*	200	+ 58	+ 36	Halley	*B*	124	+ 17	− 8

Summarizing the conclusions at which we have arrived hitherto in this chapter, we find that the constant tendency of the planet Jupiter is to bring the orbits of class *A* to coincide with a great circle, practically identical with the planet's orbit. We find that this result has been nearly attained, but that in spite of it, the average aphelion still deviates 0°.95 to the south of the ecliptic. We find that this deviation cannot be satisfactorily explained by chance, and we find further that the orbits seem to have reached a state of equilibrium, where the mean yearly southerly motion of the aphelia is about equal to the mean motion in a northerly direction.

We have obtained similar results for the comets associated with the other major planets, save that in their case, the planets being of much less mass, the associated comets deviate much more widely from the ecliptic, and the position of the mean aphelion deviates as much as 10° to the south of it, instead of only 0°.95. Lastly we find, although this result has less value than the others, being based upon only five comets, that the mean aphelion of class *B* has not yet reached a position of equilibrium, but is still deviating farther and farther to the south, at the rate of 11′ per century.

Although the data from which these various conclusions are drawn are obviously quite inadequate to support any important generalizations that might be built upon them, yet in the light of what is to appear in the following chapters, it will be well now to assume that these results are not due to accident, and to see if we can find a satisfactory explanation for them. Any force tending to draw the comets, when remote from the Sun, towards the south may safely be assumed to be gravitational. It must be due to a remote body, and therefore a massive one, and one located far out of the plane of the ecliptic.

For our present purposes an elementary discussion of the perturbing forces acting between four bodies is needed, and it will be conducted entirely by graphical methods. In either Figure 35 or 36 let *A* represent the Sun, and *B* and *C* two planets revolving about it. Let the center of gravity of *A* and *B* be at *M*, and let *D* be a relatively insignificant mass of matter such as a comet, passing near the planet *B* in its course about the Sun. Let it be required to find what effect will be produced upon *D* by the two planets *B* and *C*. From *C* draw the arc *AW* passing through *M*. Let *A* and *B* be combined at *M*, and let *MC* represent the force of attraction between their unit of mass and the planet *C*. On *CD*, prolonged if necessary, lay off the distance *UC* so that $UC : MC = \overline{MC}^2 : \overline{DC}^2$. Then *UC* will equal the attractive force between a unit of mass of the comet D and the same planet, and the line UM will indicate the direction and amount of the perturbing force due to C acting on the comet D *relatively to* M. This force may be decomposed into two, one acting at right angles, and the other, UV, parallel to the line MC. The lines of both figures representing the forces are drawn accurately to the same scale.

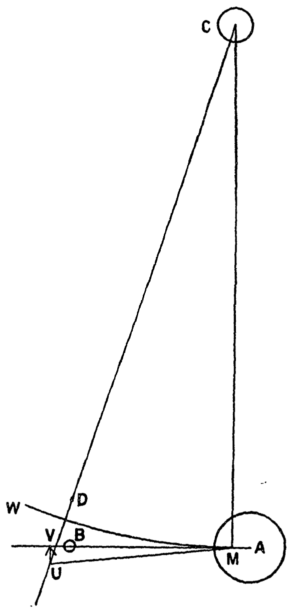

FIG. 35.

We shall notice first, on examining these drawings, that for two comets situated at equal distances relatively to A and B, but on opposite sides of B, the portion of the perturbing force UV, acts in opposite directions, and is also very unequal. In these directions the remote comet is much more perturbed by C than is the nearer one. The general effect of C upon such comets therefore is not an attraction but a repulsion, relatively to its effect upon A. In the first figure, as D approaches the arc MW, UV soon becomes zero, and is then converted into a repulsion before the arc is reached. As D recedes from the line MC, and as it recedes from C, the attraction UV in the first figure diminishes, and the repulsion UV in the second figure increases. As C recedes from A and B the attractive and repulsive forces in the two cases approach one another in magnitude, but do not become equal until the distance of C becomes infinite. As D recedes from C in the second figure the repulsive force UV at first rapidly increases in intensity, then more slowly, until when D is at an infinite distance the force is equal and opposite to MC.

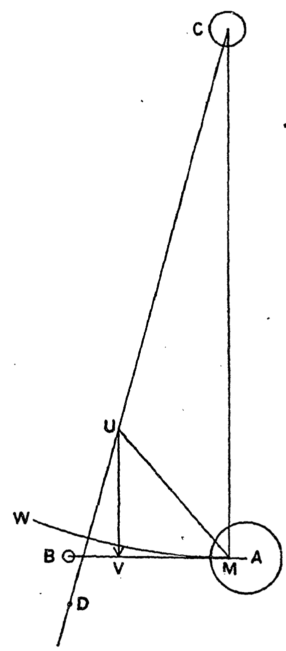

FIG. 36.

The same graphical method may be applied to determine the perturbing forces produced by planet B. On account of its proximity, these are in general much greater than those produced by C, and they are in both figures directed nearly towards the centre of B. The component parallel to MC in this case becomes zero when the line AB is reached. In the first figure the force UV is generally neutralized by the attraction of B, and as a result most of the comets associated with *B* tend to recede from *C* beyond the line *AB*, until the attraction of *B* equals the repulsion of *C*.

While the direction of the motion of these bodies has no effect on the forces developed, yet it does have an important influence on the results produced by them. The ordinary case considered is that where all of the bodies are moving in the plane of the paper, but let us now assume that the axis of revolution of planet *B* lies in this plane and coincides with the line *MC*. The most important effect that will be produced, as long as *C* revolves in the plane of the paper, is that while it is in its present position it will tend to repel the aphelia of all of the comets associated with *B* out of the plane of that planet's orbit.

If the planet *C* is large and very remote, so that the attractive and repulsive forces in the two cases are not very unequal, a ring will develop about *A* parallel to the orbit of *B*, and lying not far from it, towards *C*. Within this ring, which we may describe as a gap, very few cometary aphelia will be found, since the tendency will be for the comets to be either attracted or repelled from it by *C*. If this repulsive force becomes very large, so as to exceed the attraction due to *B*, the comets may pass entirely out of that planet's control. If the angle *BMC* is less than 90°, *UV* in the first figure will be increased, and may equal or exceed the attractive force of *B*. In the second figure *UV* will be diminished. If *BMC* exceeds 90° the reverse effect will be produced.

Let us assume now that the Sun has a large remote companion planet located at some time in the past not far from the north pole of the ecliptic. In class *B* we found the aphelia coincided more or less with a circle whose inclination was 35°, and whose ascending node was in longitude 260°. On Figure 34 such a circle would be represented by a sinusoidal curve with nodes at 80° and 260°, dipping farthest to the south in longitude 170°. As measured from the orbit of Jupiter represented by the curve on Figure 33 it is noticeable that the southerly deviation of the aphelia is most marked near the same longitude. We found in both drawings that as *D* receded from the line *MC* the repelling influence of *C* would become more marked. We have already noted that the remote aphelia of class *B* were pushed much farther to the south than the nearer aphelia of class *A*, but since this difference might be attributed wholly to the smaller mass of the controlling planets of class *B*, let us divide both classes in halves. The number of aphelia in the resulting divisions will then be 17, 17, 6, and 6. The mean deviations towards the south as deduced from the fifth columns of Tables XLV and XLVI will be $-0^\circ.7$, $-1.^\circ 2$, $-7^\circ.2$, and $-12^\circ.9$. The deviation increases very rapidly at the end, that of the last two comets in the last set being $-25^\circ.8$.

This repulsive force is also greater in the case of the comets of type *O* than in those of type *N* owing to the fact that the orbits of the former are more eccentric. Consequently the counteracting repulsion on the portion of the orbit near perihelion is less effective. There is a slight tendency also, as indicated by the theory, to form a gap or ring containing comparatively few aphelia among the eight outer cometary orbits of class *B*, none being found between latitudes $+18^\circ\ 26'$ and $-12^\circ\ 50'$.

CHAPTER XIV.

Class *C*. Comets of Moderate Eccentricity.

In this chapter and the next we shall deal chiefly with the orbits belonging to types *P*, *Q*, and *R*. These types appear to be well defined, and it is thought that the orbits associated with them are retained in that connection by the attraction of three large planets at present unknown. The three types therefore correspond to three different aphelion distances, and what these aphelion distances are we shall later endeavor to ascertain. As is well known, while the location of a comet's aphelion on the celestial sphere can be determined with great precision, its aphelion distance, if beyond the orbit of Neptune, is always a matter of considerable uncertainty. Moreover, comets occasionally change from one type to another by a succession of intermediate steps, these steps occurring when the comet chances to pass near the controlling planet of some other type. Owing chiefly to these two causes, the types and classes of the comets considered in these two chapters are not strictly identical. Doubtless at the present time a species of equilibrium has been established, and the number of comets entering and leaving a given type per century is about equal. In each class we must therefore expect to find a certain large percentage of the controlling type of orbit. In addition we shall also find certain other comets not belonging to this type, which have been introduced into the class by an error in the determination of their aphelion distance. Still other comets are rightly entitled to belong to the class on account of their aphelion distance, but do not belong to the characteristic type because their orbits have been perturbed either in direction, in velocity, or in both, by other planets.

Among the comets of long period we find that class *C* forms a well marked group of twenty orbits. In Table LII, the first column gives the number of the orbit, the second its certainty, the third the grade of brightness of the associated comet, and the fourth and fifth the longitude and latitude of its aphelion. These aphelia were plotted on the chart Figure 37, and it was at once seen, with the exception of numbers 362, 430, 198, and 147, which are indicated by dots, that the others lay approximately along the course of a sinusoid. After a preliminary trial number 369 was also rejected, although it lies so near the curve that it makes little difference in the final result whether it is included or not. The remaining fifteen, or three-quarters of the total number, appear to belong to one type which we shall designate as type *P*. These aphelia are indicated by circles. A curve of sines was now fitted to the observations, and after several different trials it was concluded that for the longitude of its ascending node we could not do better than adopt

TABLE LII.

LOCATION OF THE APHELIA OF CLASS *C*, AND OF CERTAIN MEMBERS OF CLASSES *D* TO *G*.

No.	Cert.	Br.	Long.	Lat.	S.	Dist.	Dev.	Wt.	ϕ	Cos.	Inc.	Node.	Type.
						Class *C*.							
			° ′	° ′					°				
253	2		202 14	+ 0 31	*n*	−.401	+.349	1	23.6	+.916	−.140	+.320	*P*
362	2		50 29	−36 15	*f*	—	—	0	—	—	—	—	*q*
463	3		336 01	+ 1 10	*f*	−.180	+.169	1	10.4	−.984	−.030	−.166	*P*
320	2		190 19	−16 30	*n*	−.443	−.009	1	26.3	+.896	+.004	−.008	*P*
195	2		220 39	−37 52	*n*	−.869	−.079	1	60.3	+.496	+.069	−.039	*P*
128	3		243 01	−47 19	*n*	−.966	−.141	1	75.0	+.259	+.136	−.037	*P*
271	2	*I*	96 55	+29 48	*n*	+.944	−.159	1	70.7	+.330	−.150	−.052	*P*
430	2		100 29	−44 31	*n*	—	—	0	—	—	—	—	*Q*
270	2	*III*	37 10	+32 50	*f*	+.815	+.010	1	54.6	−.579	+.008	−.006	*P*
246	3		35 28	+27 59	*f*	+.780	−.048	1	51.3	−.625	−.037	+.030	*P*
214	3	*IV*	341 27	−28 51	*f*	−.408	−.286	1	24.1	−.913	+.117	+.261	*P*
198	2	*I*	101 09	−35 20	*n*	—	—	0	—	—	—	—	*Q*
351	3		238 37	−30 34	*n*	−.941	+.121	1	70.2	+.339	−.114	+.041	*P*
147	3	*IV*	233 11	+21 46	*n*	—	—	0	—	—	—	—	*R*
369	3		193 46	+21 12	*n*	—	—	0	—	—	—	—	*p*
238	2	*III*	31 06	+16 10	*f*	+.658	−.192	1	41.1	−.754	−.126	+.145	*P*
375	2		285 24	−19 55	*f*	−.863	+.282	1	59.7	−.504	−.243	−.142	*P*
245	3		346 18	+13 31	*f*	+.096	+.219	1	5.5	−.995	+.021	−.218	*P*
244	2		283 04	−30 09	*f*	−.929	+.118	1	68.3	−.370	−.110	−.044	*P*
413	3	*IV*	24 20	+35 40	*f*	+.725	+.140	1	46.5	−.688	+.101	−.096	*P*
						Class *D*.							
193	2		250 09	−19 57	*n*	−.932	+.332	0.5	68.8	+.362	−.155	+.060	*P*
171	2	*II*	142 19	+32 34	*n*	+.645	+.151	0.5	40.2	+.764	+.048	+.058	*P*
255	3		223 30	−37 39	*n*	−.879	−.059	0.5	61.5	+.477	+.026	−.014	*P*
226	3		61 57	+50 09	*f*	+.959	+.190	0.5	73.5	−.284	+.091	−.027	*P*

No.	Cert.	Br.	Long.	Lat.	S.	Dist.	Dev.	Wt.	φ	Cos.	Inc.	Node.	Type.
						Class *D* (continued).							
			° ′	° ′					°				
453	2	*IV*	320 19	−21 49	*f*	−.590	+.009	0.5	36.2	−.807	−.003	−.004	*P*
314	3	*III*	141 38	+43 27	*n*	+.700	+.302	0.5	44.4	+.714	+.106	+.108	*P*
						Class *F*.							
88	3		136 37	+49 02	*n*	+.759	+.341	0.25	49.4	+.651	+.065	+.056	*P*
267	3		219 42	−29 26	*n*	−.820	+.045	0.25	55.1	+.572	−.009	+.006	*P*
287	3		342 49	− 0 26	*f*	−.106	+.075	0.25	6.1	−.994	−.002	−.019	*P*
301	2		132 20	+15 24	*n*	+.620	−.171	0.25	38.3	+.785	−.027	−.034	*P*
318	3		119 05	+14 21	*n*	+.728	−.295	0.25	46.7	+.686	−.054	−.051	*P*
464	3		320 56	− 1 33	*f*	−.391	+.290	0.25	23.0	−.920	−.028	−.067	*P*
						Class *G*.							
422	2		359 17	− 1 26	*f*	+.100	−.120	0.25	5.7	−.995	−.003	+.030	*P*

350°, and for its inclination 40°. Since the surface is a sphere and not a cylinder, the curve of sines is only an approximate solution. Corrections at 30° and 60° from the node were computed, and the curve drawn as represented in Figure 37.

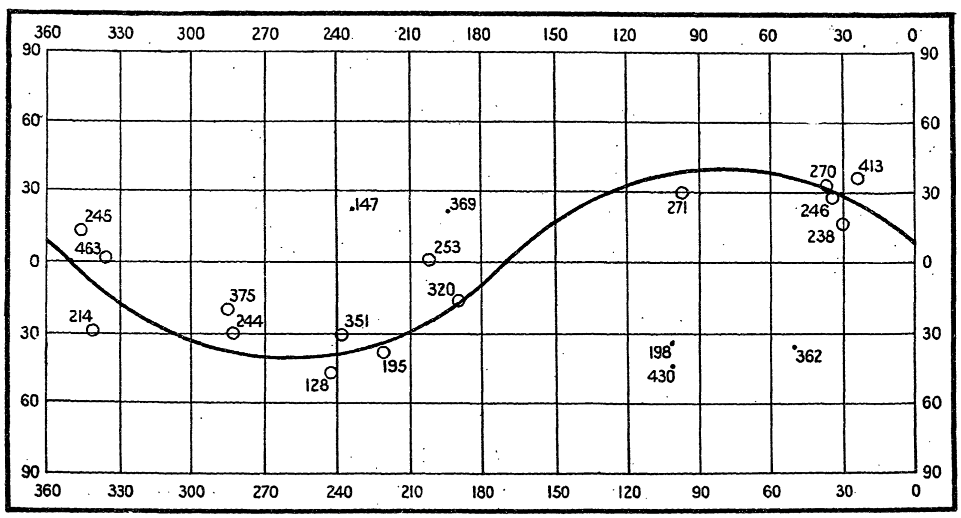

Fig. 37.

Although this form of graphical solution was found to meet the requirements of the case fairly well, and was most convenient for a preliminary determination, yet it was seen that much more accurate results might be obtained by constructing an orthographic projection of a sphere, and projecting the observations upon it. Such a construction is shown in Figure 38, the point of view being located in the plane of the ecliptic, and the central meridian passing through the adopted node. Under these circumstances the reference circle of the aphelia should appear as a straight line. As shown in

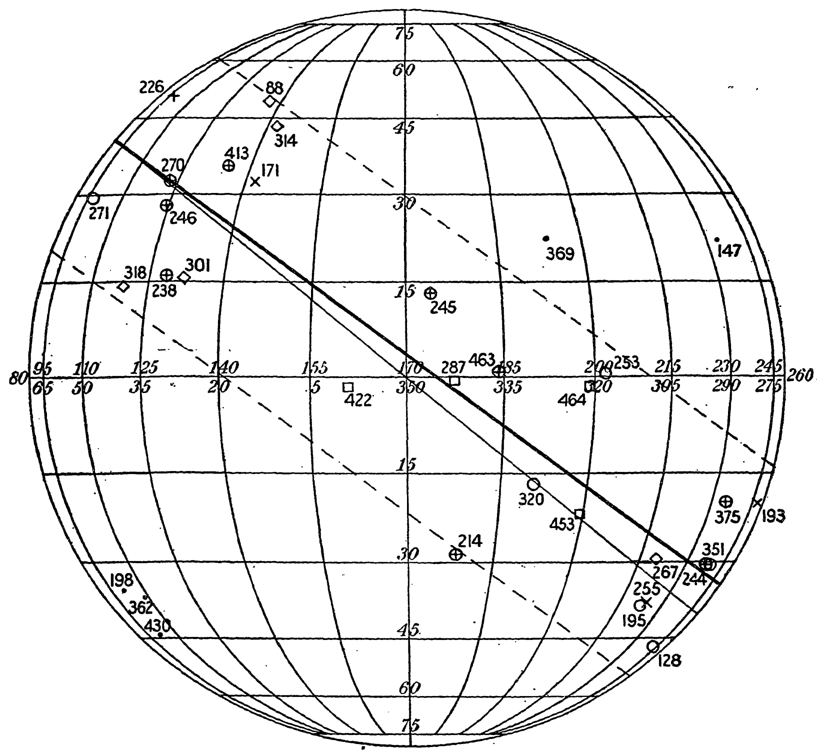

FIG. 38.

the drawing its ascending node is on the further side of the sphere. The upper row of figures along the ecliptic refers to the nearer, and the lower to the farther side. Six aphelia from class *D*, six from *F*, and one from *G* are also shown in this figure. An aphelion on the nearer side, if in class *C* is designated by a circle, if in class *D* by an X, and if in *F* by diamond. An aphelion on the further side, if in class *C* is designated by a superposed cross and circle, if in class *D* by a cross, and if in *F* or *G* by a square.

All these positions are now used to obtain a more accurate location of the reference circle of the type, the aphelia from class *C* being given greater weight than those from the other classes. Rejected observations of class *C* numbers 362, 430, 198, 369, and 147 are indicated as before by dots, and have no weight. They are all classified later under types *Q* and *R*, and subtypes *p* and *q*.

The weighting of the observations has been made to depend exclusively on the uncertainty that we feel with regard to the value of the aphelion distances. If, for instance, in examining the aphelia of the other classes we find some which are highly divergent from the reference circles that are properly associated with them, and if, further, these same aphelia agree fairly well with the reference circle proper to class *C*, it is a fair presumption that there may have been an error made in determining the aphelion distances of these orbits, and that they should perhaps really be associated with class *C*. Nevertheless, to show that they are of doubtful standing, their positions are indicated by marks other than circles, and less weight is given to them in locating the reference circle of the class.

To determine their weight, let us first suppose that the aphelion distances of all these comets are of no value whatever. In this case the assigned weight for all the other classes should be unity, the same as that of the regular members of class *C*. Let us now suppose that all the aphelion distances are known with absolute precision. In this case the weight assigned to the members of the other classes should be zero. The more reliance we place on the aphelion distances, the less should be the weight assigned to the other classes. It is clear however that some weight should be assigned to them, since our knowledge of these distances is admittedly inaccurate. It has been decided therefore to weight the aphelia of class *D* one-half, and those of classes *F* and *G* one-quarter.

It will be noted that the approximate values of the node and inclination of the reference circle are determined from the sinusoid Figure 37, which is based exclusively on the accepted aphelia of class *C*. Certain aphelia approximating to this circle are then selected from the other classes by the method of choice above described. They are then combined with the aphelia of class *C* with the expectation of improving the approximate results. Their effect is necessarily slight, first because aphelia only are selected that lie near the approximate reference circle, and second on account of their weighting. The predominating effect of the accepted aphelia of class *C* is thereby assured.

Finally, two reference circles are located, one using only the aphelia of class *C*, and the other all the aphelia.

With regard to the five rejected orbits, number 369 is 28° distant from the finally adopted reference circle of type *P*. It lies nearer to that reference circle than to any other, save that of type *N*, with which it clearly is not associated. Its connection with *P* is indicated therefore by its insertion in subtype *p*. Very possibly it originally belonged under *P*, but had its orbit shifted by Jupiter, or on the other hand it may be approaching type *P*, but has not as yet been acted upon by the controlling unknown planet for a sufficient number of conjunctions to properly assimilate its orbit to that type.

Orbit number 198 belongs to the great comet of 1843, which is one of a remarkable group of comets including also numbers 61, 339, 353, and 374. Of these, number 353, the great comet of 1882, was very extensively observed, and its orbit very carefully reduced by Kreutz. It is the innermost known member of class *D*. The aphelion distances of the three other members of the group are unknown. These comets all have extremely small perihelion distances, and very eccentric orbits. These are the very characteristics, which render an aphelion distance uncertain, and also permit it to be readily influenced by slight planetary perturbations. In Chapter XVII we shall find reasons for believing that the period of 353 is steadily becoming shorter, and without doubt the same causes are acting on 198, and have done so in the past. It has therefore come into class *C* from class *D*, with whose reference circle it is appropriately allied.

Comet number 430 is known to have passed after perihelion within 1.3 units of the planet Jupiter. It is possible that a similar early encounter changed its aphelion distance and threw it into its present orbit. It also is associated with the comets of class *D* under type *Q*. Numbers 362 and 147 may have been similarly perturbed. Number 362 is unaffiliated with any of the chief types unless it belongs under *O*, and is associated with that planet. It lies nearer to *Q* than to either of the others. It has therefore been entered in the sub-type *q* rather than *p*, in spite of its well determined short aphelion distance.

Turning now to the appended comets, Table LII, of classes *D*, *F*, and *G*, number 171 was observed for twelve months, and its aphelion distance 542, certainly cannot come within the 220 units which is the limit of class *C*. It is probable that it, as well as the five other members of class *D*, really belong in type *P*. It is possible that owing to planetary perturbations, perhaps due in part to planet *P* itself, their orbital speed near perihelion has been increased, so that their apparent aphelion distances have been lengthened. This same explanation can hardly apply to the appended comets of class *F*, but as none of them was observed for over a month and a half, and several of their orbits are consequently not well determined, it is quite possible that some of them were really following elliptical paths whose aphelion distances would bring them properly under class *C*. Under class *G*, comet number 422 was observed for two months. Its coincidence with type *P* is very likely accidental, the more so as it does not fall far beyond the adopted limits of type *R*, with which it would be more naturally affiliated.

Returning now to the explanation of Table LII, if the aphelion is on the nearer side of the sphere, it is marked in the sixth column with an *n*, if on the farther side with an *f*. A straight line passing through the center and representing the assumed reference circle is now drawn by inspection upon the projection of the sphere. Its angle in the present instance is 40°, the same as that of the first approximation, made in connection with the sinusoid. A perpendicular to this line is dropped upon it from each aphelion, and the distance in terms of the radius, from the foot of these perpendiculars to the center is given in the seventh column of the table, distances to the left of the center being reckoned as positive. The deviation from the assumed reference circle is given in the same unit in the eighth column, distances above the circle being reckoned positive. The assigned weight is given in the ninth.

The tenth column gives the angle ϕ whose sine is given in the seventh column. The eleventh column gives the cosine of this angle. If the aphelion is on the nearer side of the sphere the cosine is reckoned as positive. The twelfth column gives the continued product of the deviation and weight into sine ϕ. If the assumed inclination is correct, the algebraic sum of these products should equal zero. If it is positive, the assumed angle should be increased, if negative diminished. This sum may be considered as the moment of a couple tending to change the plane of the orbit about the prolongation of the line of sight passing through the node as an axis. The thirteenth column gives the continued product of the deviation and weight into cosine ϕ. If the assumed node is properly selected the algebraic sum of these products should equal zero. If it is positive the longitude of the node should be increased, if negative diminished. This sum may be considered to be the moment of a couple tending to change the plane of the orbit about the line of its intersection with the plane of the paper. The last column of the table indicates the type or subtype to which the various orbits belong.

An examination of the third column of the table shows that about the same proportion of the comets of this type are visible to the naked eye as was the case with type *O*. By far the brightest comet of the type, and the second brightest of the class, was number 271, the great comet of 1861, described by Chambers as "one of the most magnificent comets on record." Comparing the three great comets of 1858, 1861, and 1882, Professor A. Searle has said that the first came nearest to the ideal form and appearance, the second was the most startling, because unexpected, and had the longest tail, while the third was by far the brightest of the three.

In Table LIII, Part I, the first column indicates whether the accepted aphelia in class *C* alone were considered, or whether the aphelia of classes *D*, *F*, and *G* were also included. The second column gives the weighted number of aphelia included. The third gives the algebraic sum of the products of the deviations multiplied by their respective weights, taken from the eighth and ninth columns of Table LII. If the aphelia were so distributed on either side of the reference circle that this sum equalled zero, the reference circle would be a great circle of the sphere. Since it is a positive quantity in this instance, this indicates that the displacement is towards the north. The quantities in the third column divided by those in the second give the sine of the angle of the displacement. This angle is given in the fourth column.

In Part II of the table the first two columns give as before the classes considered and the number of aphelia. The third column gives the arithmetical sum of the products of the sine of ϕ multiplied by its weight taken from the seventh and ninth columns of Table LII. This quantity divided by the weighted number of observations gives the mean distance of the projections of the aphelia on the plane of the assumed reference circle from the line of sight through the centre of the sphere. This quantity we will call the mean sine. The fourth column gives the alegebraic sum of the quantities given in the twelfth column of Table LII. This quantity divided by the weighted number of observations gives what we may consider as the mean moment tending to change the assumed inclination. This mean moment divided by the mean sine gives the mean force tending to change the inclination. Dividing this again by the mean sine gives the tangent of the angle by which the inclination of the plane should be corrected. To express this algebraically let us indicate the quantities in the second column of Part II by n, those in the third column by w, and those in the fourth by d. Let I indicate the correction to the inclination. Then $\tan I = \frac{dn}{w^2}$. The angle I is given in the fifth column and the corrected inclination in the sixth.

TABLE LIII.

DISPLACEMENT. INCLINATION. NODE.

Part I.					
Class.	No.	Σ Wt. Dev.	Dis.		
C.	15.	+0.494	+1.9°		
C, D, F, G.	19.75	+0.996	+2.8		
Part II.					
Class.	No.	Σ Wt. sin φ.	Σ Dev. sin φ.	Corr. inc.	Inc.
C.	15.	10.018	−0.494	−4.2°	35.8°
C, D, F, G.	19.75	13.253	−0.439	−2.8	37.2
Part III.					
Class.	No.	Σ Wt. cos. φ.	Σ Dev. cos. φ.	Corr. node.	Node.
C.	15.	9.648	−0.011	−0.2°	349.8°
C, D, F, G.	19.75	12.752	+0.091	+1.0	351.0

In Part III of the table the node is corrected in a similar manner by means of the quantities given in the ninth, eleventh, and thirteenth columns of Table LII. The first two columns are identical with those in Part II. The third gives the sum of the products of cos ϕ multiplied by its weight. The fourth column gives the sum of the quantities taken from the thirteenth column of Table LII. To convert the change in the node into difference of longitude we must divide by the sine of the angle of the inclination. Indicating the correction to the node by N, the inclination by i, and the quantities in the second, third, and fourth columns of Part III as before, we have $\tan N = \frac{dn}{w^2 \sin i}$. The angle N is given in the fifth column and the corrected value of the node in the sixth. It will be noticed that whether we use only the orbits of class C, or all the orbits combined, the results differ but little. It should be pointed out in this connection that the word "node" as here used may be defined as the intersection of the great circle parallel to the reference circle and the ecliptic.

The mean deviation from the assumed reference circle of the fifteen orbits of type P, class C, obtained from the eighth column of Table LII is ± 0.155, which is the sine of $\pm 8.°9$. The mean deviation for type N we found to be $\pm 3°6$, and for type O $\pm 16.°4$. In what follows we shall accept as final the second values given in Table LIII, — those which take account of all the aphelia. A heavy line has been drawn in Figure 38 to coincide with the major axis of the ellipse which would represent the reference circle as finally corrected. For convenience of comparison with the types presently to be described, and to serve as definite limits to the zone containing the accepted aphelia of type P, two small circles are established on the sphere parallel to the adopted reference circle, and located 20° to the north and south of it. Dotted lines are drawn in Figure 38 coinciding with the major axes of these two ellipses. If the southern ellipse were drawn in full it would be found to pass to the south of aphelion 214, which is therefore included with the other accepted aphelia of type P. The zone included between these small circles contains approximately 0.34 of the total area of the sphere, and the two other zones each 0.33. Confining our attention exclusively to class C, and dividing the total number of comets within these three zones, 15, 2, and 3, by these numbers, we find that the density of the aphelia within the middle zone may be represented by the number 44, in the northern zone by the number 6, and in the southern zone by the number 9.

As in the case of types N and O, we see by Table LIII that the reference circle of type P differs somewhat, 2°.8, from a great circle. The sum of the weighted positive deviations is $+2.087$, the sum of the weighted negative deviations -1.091, and their difference $+0.996$, as given in the third column of Part I. The ratio is 1.91, and the difference is greater than the sum of the three largest positive deviations, when properly weighted, given in the eighth column of Table LII. Unlike the two earlier types, however, there is no evidence of a systematic increase in the deviation with the increase in aphelion distance. This is shown by an examination of the results given in the eighth column. The mean value of the first eight figures is +.012, of the remaining seven forming class C is +.058, of the six forming class D +.154, and of the seven forming classes F and G +.026. The corresponding arcs are +0°.7, +3°.3, +8°.9, and +2°.7. The variation is not systematic, and during the very long intervals which must elapse between the conjunctions of planet P with its various associated comets, the pull of the outer planet, owing to its own revolution about the Sun, must be sometimes to the north and sometimes to the south. The writer is therefore inclined to attribute this displacement largely to accident.

The effect of the known planets on the aphelia of the comets of type P would be to pull the southern ones towards the north, and the northern ones towards the south, thereby diminishing the inclination of their reference circle, but these planets could not by any means shift the reference circle of type P as a whole either north or south. Similarly the unknown northern planet would tend to shift the northern aphelia of type P farther north, and the southern ones farther south, thereby increasing the angle of their reference circle.

As far as producing a change in the plane of the reference circle is concerned, it will be seen that the known planets work in direct opposition to the assumed northern one, and which would produce the greater effect we have no means of knowing. It is certain, however, that the result would be a tendency to scatter the aphelia, and pull them away from their reference circle. The fact that they remain as close to it as they do, itself implies a concentrating force such as we have already seen would be furnished by a rather massive planet revolving in their plane, which would tend to draw all the comets near which it passed into this circle, and thus replenish the constant losses that its family must experience through the attractions of the other planets near which the members occasionally pass.

There is indeed in the greater planetary world, much as there is on our own terrestrial sphere, a constant struggle for accumulation, each planet trying to collect as many dependants as it can, and at the same time endeavoring to protect itself against loss from the attractions of the other planets. It certainly seems remarkable, when we consider the constant changes of the cometary orbits during the countless ages of the past, that of the twenty known comets of this aphelion distance, so large a proportion as 75 per cent should cling so closely to one circle in the heavens, and that so nearly a great circle.

The likelihood that this distribution is due merely to chance is in fact very remote. If we imagine the area of a sphere divided into three equal parts by two small circles drawn parallel to the equator, then it is possible to calculate the chance that fifteen out of twenty comets shall fall in the middle zone. This computation is complicated by the fact, in the case under consideration, that the parallel circles are not drawn parallel to the equator, but were laid out after the comets had appeared, in such a manner as to best fit the recorded results. Since two points are required to define a great circle upon a sphere, two comets must be reserved for this purpose, leaving only thirteen out of eighteen which were found to fall within the middle zone. The required formula is a complicated one to derive, and the likelihood of an error large. It was concluded therefore that the result could be derived more quickly and with more certainty by means of tables.

The three zones were represented by the three letters *a*, *b*, and *c*, and a table which we will call Table *A* constructed, consisting of 729 horizontal rows, showing all the possible combinations of these three letters taken six at a time, that is to say in six vertical columns. Let t represent the number of trials, or in other words the number of vertical columns employed, and n the number of possible combinations. Then we may construct a table for, let us say, $t-3$, that is for three failures out of the total number of trials. In Table LIV the first column gives the number of trials, the second the total number of possible combinations, the third the number of times that any letter such as *a* must occur in any of the combinations in order to have three failures. Thus in the next to the last line, where there were five trials, in order to have three failures *a* must occur twice in the combination. The fourth column *C* gives the number of times that three failures occurred by actual count in Table *A*. The remaining columns give the first, second, and third differences. For three failures the third differences will be constant and equal to 8. The table may therefore be readily continued by computation for any larger number of trials.

From the last numbers in the second and fourth columns we derive that out of 729 possible combinations of the three letters in sets of six, that three failures will be found 160 times. By similar tables we find that two failures will occur 60 times, one failure 12 times, and a perfect score but once. A

TABLE LIV.

CONSTRUCTION TABLE FOR t—3.

t	n	a	C	1	2	3
1	3	−2	0	0		
2	9	−1	0	8	8	
3	27	0	8	24	16	8
4	81	1	32	48	24	8
5	243	2	80	80	32	8
6	729	3	160			

similar table constructed for five failures, $t-5$, showed that the fifth differences were constant and equalled 32.

By means of Table A, therefore, it was possible to compute tables for eighteen trials from one up to five failures. In eighteen trials there are 387,420,489 possible combinations, with but one perfect score. One failure will occur 36 times, two failures 612 times, three failures 6,492 times, four failures 48,960 times, and five failures 274,176 times. Adding these results, we find the likelihood that as many as thirteen out of eighteen comets should fall by chance within the middle zone is 330,277 out of 387,420,489 or only one chance in 1173. We therefore conclude that this distribution of the aphelia cannot be due to accident, but must have some real cause, and this cause we believe to be the presence of a massive planet which we shall call planet P.

Comparing the number of aphelia in types N, O, and P, we find that the latter must form a group of considerable importance. The longest known period of a comet of type N, orbit 389, is 8.91 years. During the last ten years of the nineteenth century practically all of the comets of type N must therefore have passed perihelion. Not counting duplicate apparitions of the same comet there were 17 comets recognized in all. Of these 12, or 71 per cent, came in the first five years. During the whole of the century nearly all of the comets of type O must have passed perihelion. There were 12 of these comets recognized, of which one half came in the first fifty years. On account of their high eccentricity, the ellipticity of the orbits of the comets of type P is much more difficult to recognize and determine than the ellipticity of the orbits of the other two types. Prior to the last century but one such comet was detected. During the last half of the last century 11 such comets were found in class C alone. In the course of a thousand years nearly all the comets of this type would have passed perihelion. At the same rate of detection 220 comets belonging to this type would have been recognized. Theoretically a lower rate should be found in the latter part of the period, since the comets of shorter period would by that time have been already discovered. If we divide this number by two, however, the number of comets in the three types will still stand in the ratio of 17, 12, and 110.

Let us now turn for a moment to the early history of the idea of searching for an invisible planet by means of cometary orbits. As we have already seen, p. 193, Professor G. Forbes was the pioneer in this work as far as concerns planet *P*. Proc. Roy. Soc., Edinburgh, 1878–80, **10**, 426, 1880–82 **11**, 89, also Observatory, 1880, **3**, 439. He computed the positions and distances of the aphelia of comets numbers 214, 128, 195, 198, 271, 270, and 246. See Figure 37. He then plotted them on the surface of a globe, and rejected numbers 214, 128, and 271, passing a reference circle through the other four. Had he rejected 198 and retained the other six, a portion of the present work would have been superfluous. From these orbits he concluded that the ascending node of the reference circle was located in longitude 250°, and that the inclination was 53°. He also determined the eccentricity and longitude of perihelion. Later he revised his views, and concluded that the reference circle, which he considered to be the orbit of an unknown planet lay nearly in the plane of the ecliptic. He located the planet in 1880 in longitude 174°, latitude 0°.

Quite recently he has again revised his views, M. N., 1909, **59**, 152, and gone back practically to his original elements. Of the twenty comets of class *C* he makes use only of numbers 195, 198, 246, and 270. He also includes in his list number 353 of class *D*, 247 of class *H*, and 339 of class *I*. The last two were observed for but 27 and 15 days respectively, for which reason the writer did not consider their aphelion distances sufficiently well known to be included in any of the better known classes. It may be pointed out that of the seven comets selected by Professor Forbes, numbers 198, 339, and 353 belong to the remarkable group of five orbits of extremely short perihelion distance and high eccentricity whose aphelia lie close together upon the celestial sphere. The aphelia of the three used by him are practically identical. The aphelia of numbers 246 and 270 differ by but 5°. See Figure 37. These five aphelia may therefore be considered as furnishing two points on the reference circle, and number 195 and the somewhat doubtful 247 as furnishing two more. From these four points the orbit of the assumed planet was derived. The elements as given by him are as follows: — ☊ 247° 34′, i 52° 00′ 30″, e 0.1665, π 114° 57′, mean distance 105.1, date of perihelion passage 1702. The period of revolution is 1076 years. By means of assumed conjunctions with these various comets he locates the planet in 1908 in longitude 215° 31′, latitude −33° 53′. It will be noticed that the node and inclination differ materially from those shown in Figure 37.

Monck, in the Journ. Liverpool Astr. Soc., 1889, 7, 210, by combining the aphelia of numbers 195, 320, 271, 270, 136, and a comet which appeared in 1746, but which was rejected by Galle as uncertain, finds for the longitude of the node 330°, and 35° for the inclination. These last results do not differ very greatly from those given in Table LIII, and shown approximately on the figure. Monck also tried to locate in longitude the supposed planet associated with these orbits, by means of assumed conjunctions with the various comets. He placed it in 1889 in longitude 35° to 40°, latitude +30° to +35°.

In conclusion we may remark, whether we accept or reject the theory that the concentrating force above noticed is due to an unknown planet, the interesting fact remains that comets whose aphelion distances lie within the bounds of class C, that is between the limits of 60 and 220 units from the Sun, tend to concentrate their orbits about the reference circle whose elements we have just determined. This fact becomes the more striking when we recollect that all of these orbits, unlike those of type N, are on account of their high eccentricity, extremely narrow in proportion to their length. In the next chapter we shall endeavor to show that other comets whose orbits have still greater aphelion distances favor other reference circles.

CHAPTER XV

Classes *D* to *I*. Comets of High, and of Unknown Eccentricity.

Summary.

After Chapter XV had been put in type a method was discovered of estimating the eccentricity of the orbits of planets *Q* and *R*. This at once led to the belief that the mean distance of these bodies was considerably greater than had previously been supposed, when their orbits were assumed to be circular. In accordance with this change of view some alteration of the preceding tables was necessary. It was found, however, that the changes in both the tables and figures of Chapter XV would be so great, while the change in the resulting conclusions would be so small, that with the exception of Table LXII it would scarcely be worth while to alter them. Table LXII and its description has accordingly been made conformable to the rest of the volume. This statement will account for some inconsistencies that may be noted between the contents of this chapter and the rest of the work. In this chapter the seventeen orbits numbers 109, 203, 218, 227, 279, 280, 285, 314, 319, 321, 326, 395, 400, 402, 409, 448, and 453, are included under class *E*, whereas in the rest of the volume they are entered under *D*. The six orbits 227, 280, 319, 395, 402, and 409, are included in this chapter under type *QR*, whereas in the rest of the volume they are entered under *Q*. While orbit 385 is here entered under *QR*, it is elsewhere included under type *R*.

Class *D* consists of 31 orbits. Their numbers, certainty, the brightness of their associated comets, and the longitudes and latitudes of their aphelia are given in the first five columns of Table LV. The brilliancy of these comets, already noted, is brought out in the third column of the table, the brighter members seeming to have the shorter aphelion distances. Five comets of the very brightest grade belong to this class, three of them, as shown by the last column of the table, belonging to type *Q*. Mentioned in the order of aphelion distance they appeared in the years 1882, 1858, 1769, 1811, and 1680. Of these, the first and the last were the most remarkable, largely because their perihelion distances were both notably small. There are 26 comets belonging to class *G*. While none of them were brilliant, over one-third were visible to the naked eye.

TABLE LV.

POSITIONS OF THE APHELIA OF CLASSES *D* AND *G* AND OF CERTAIN MEMBERS OF CLASSES *C*, *E*, AND *F*.

No.	Cert.	Br.	Long.	Lat.	S.	Dist.	Dev.	Wt.	φ	Cos.	Inc.	Node.	Type.
						CLASS *D*.							
			° ′	° ′					°				
353	2	*I*	101 33	−35 14	*f*	−.576	−.161	1	35.2	− .817	+.093	+.132	*Q*
116	3	*II*	69 26	−52 51	*f*	−.797	+.210	1	52.8	− .605	−.167	−.127	*Q*
142	2	*II*	89 56	− 3 40	*f*	−.064	+.004	1	3.7	− .998	.000	−.004	*Q*
262	2	*I*	195 03	−43 45	*n*	—	—	0	—	—		—	*p*
308	2		131 42	+61 43	*f*	+.881	−.313	1	61.7	− .474	−.276	+.148	*Q*
101	3	*I*	332 30	+19 34	*n*	—	—	0	—	—	—	—	*R*
379	3	*III*	65 30	+ 0 03	*f*	—	—	0	—	—	—	—	*R*
383	2		262 07	+51 06	*n*	+.778	−.085	1	51.1	+ .628	−.066	−.053	*Q*
346	2	*III*	88 39	+ 5 08	*f*	+.089	+.028	1	5.1	− .996	+.002	−.028	*Q*
254	3	*III*	54 19	−42 55	*f*	—	—	0	—	—	—	—	*q*
179	3		79 29	+52 35	*f*	+.794	+.108	1	52.6	− .607	+.086	−.066	*Q*
146	2	*I*	289 10	−60 26	*n*	−.870	+.165	1	60.4	+ .494	−.144	+.082	*Q*
313	2		229 30	+13 42	*n*	—	—	0	—	—	—	—	*R*
411	2		2 58	+ 9 24	*f*	—	—	0	—	—	—	—	*R*
193	2		250 09	−19 57	*n*	—	—	0	—	—	—	—	*P*
168	2		83 18	−53 22	*f*	−.802	+.074	1	53.4	− .596	−.059	−.044	*Q*
284	2	*III*	124 41	− 0 54	*f*	—	—	0	—	—	—	—	*r*
171	2	*II*	142 19	+32 34	*f*	—	—	0	—	—	—	—	*P*
305	3		281 55	+42 28	*n*	+.675	+.155	1	42.5	+ .737	+.105	+.114	*Q*
164	3	*III*	93 11	+ 0 51	*f*	+.015	−.051	1	0.8	−1.000	−.001	+.051	*Q*
263	3		183 18	− 8 38	*n*	—	—	0	—	—	—	—	*R*
255	3		223 30	−37 39	*n*	—	—	0	—	—	—	—	*P*
175	3		237 30	− 5 57	*n*	—	—	0	—	—	—	—	*R*
377	2		79 52	− 4 31	*f*	−.079	+.180	1	4.5	− .997	−.014	−.179	*Q*
97	3		263 32	−72 27	*n*	−.953	−.030	1	72.4	+ .302	+.028	−.009	*Q*
226	3		61 57	+50 09	*f*	—	—	0	—	—	—	—	*P*
65	3	*I*	90 36	+ 8 08	*f*	+.141	−.009	1	8.1	− .990	−.001	+.009	*Q*
387	2	*IV*	100 28	+12 53	*f*	+.223	−.179	1	12.9	− .975	−.040	+.176	*Q*
216	3	*II*	89 27	+46 17	*f*	+.723	+.010	1	46.3	− .691	+.007	−.007	*Q*
327	2		322 40	−60 32	*n*	−.871	+.388	1	60.5	+ .492	−.338	+.191	*Q*
393	3		291 43	+11 59	*n*	+.208	+.359	1	12.0	+ .978	+.075	+.203	*Q*
						CLASS *G*.							
354	2		228 29	+73 11	*n*	+.957	−.186	1	73.2	+ .289	−.178	−.054	*Q*
277	2		313 38	−73 47	*n*	−.960	+.200	1	73.8	+ .279	−.198	+.056	*Q*
199	3		116 24	−41 09	*f*	−.658	−.331	1	41.2	− .752	+.218	+.249	*Q*

No.	Cert.	Br.	Long.	Lat.	S.	Dist.	Dev.	Wt.	φ	Cos.	Inc.	Node.	Type.
			Class *G* (continued).										
			° ′	° ′					°				
465	3	*IV*	262 47	−27 48	*n*	−.460	−.119	1	27.4	+ .888	+.055	−.106	*Q*
403	2		253 11	−50 41	*n*	−.773	−.187	1	50.7	+ .633	+.145	−.118	*Q*
204	2		281 17	−41 34	*n*	−.663	+.150	1	41.6	+ .748	−.099	+.112	*Q*
391	2		87 38	−53 29	*f*	−.804	+.021	1	59.5	− .508	−.017	−.011	*Q*
221	2	*IV*	81 09	+70 43	*f*	+.944	+.050	1	70.7	− .330	+.047	−.002	*Q*
446	2	*IV*	153 49	−11 01	*f*	—	—	0	—	—	—	—	*R*
366	2	*III*	58 50	−59 56	*f*	−.865	+.264	1	59.9	− .502	−.228	−.133	*Q*
355	2		249 56	−66 05	*n*	−.914	−.128	1	66.1	+ .405	+.118	−.052	*Q*
373	2	*III*	245 13	−77 49	*n*	−.977	−.094	1	77.8	+ .211	+.092	−.020	*Q*
239	3	*III*	136 34	− 8 24	*f*	—	—	0	—	—	—	—	*R*
282	2		314 27	−76 22	*n*	−.972	+.170	1	76.4	+ .235	−.165	+.040	*Q*
440	2	*IV*	197 45	− 4 49	*n*	—	—	0	—	—	—	—	*R*
422	2		359 17	− 1 26	*f*	—	—	0	—	—	—	—	*P*
225	2		37 30	−30 17	*f*	—	—	0	—	—	—	—	*r*
436	2		98 42	+48 49	*f*	+.753	−.097	1	48.8	− .659	−.073	+.064	*Q*
384	2	*IV*	196 35	+ 4 31	*n*	—	—	0	—	—	—	—	*R*
427	2		92 57	− 4 17	*f*	−.075	−.055	1	4.3	− .997	+.004	+.055	*Q*
361	2		92 15	− 1 32	*f*	−.027	−.038	1	1.5	−1.000	+.001	+.038	*Q*
240	3	*IV*	114 53	+60 03	*f*	+.866	−.214	1	60.0	− .500	−.185	+.107	*Q*
141	3		284 04	+24 06	*n*	+.408	+.220	1	24.1	+ .913	+.090	+.201	*Q*
155	3		276 35	+10 29	*n*	+.182	+.109	1	10.5	+ .983	+.020	+.107	*Q*
367	3		99 56	−37 51	*f*	−.618	−.144	1	38.2	− .786	+.089	+.113	*Q*
107	3		176 14	−42 55	*f*	—	—	0	—	—	—	—	*R*
			Class *C*.										
430	2		100 29	−44 31	*f*	−.701	−.132	0.5	44.5	−.713	+.046	+.047	*Q*
198	2	*I*	101 09	−35 20	*f*	−.578	−.146	0.5	35.3	−.816	+.042	+.060	*Q*
			Class *E*.										
319	2	*II*	107 14	−25 08	*f*	−.424	−.272	0.25	25.1	− .906	+.029	+.062	*QR*
279	2	*III*	77 13	−55 22	*f*	−.823	+.120	0.5	55.4	− .568	−.050	−.034	*Q*
280	2	*III*	277 25	+ 2 43	*n*	+.047	+.122	0.25	2.7	+ .999	+.002	+.030	*QR*
326	2	*IV*	91 20	−49 46	*f*	−.763	−.017	0.5	49.8	− .646	+.006	+.006	*Q*
402	3	*III*	80 37	−15 02	*f*	−.259	+.160	0.25	15.0	− .966	−.010	−.039	*QR*
227	2	*III*	93 47	+ 0 30	*f*	+.009	−.066	0.25	0.5	−1.000	.000	+.016	*QR*
448	2	*II*	94 01	+17 09	*f*	+.295	−.069	0.5	17.2	− .955	−.010	+.033	*Q*
218	2		55 51	−83 15	*f*	−.983	+.134	0.5	83.2	− .113	−.066	−.008	*Q*
409	2	*III*	112 05	−14 33	*f*	−.251	−.368	0.25	14.6	− .968	+.023	+.089	*QR*

No.	Cert.	Br.	Long.	Lat.	S.	Dist.	Dev.	Wt.	ϕ	Cos.	Inc.	Node.	Type.
				Class *E* (continued).									
			° ′	° ′					°				
400	2		301 00	+77 38	*n*	+.977	+.127	0.5	77.6	+ .215	+.062	+.014	*Q*
395	2		102 58	−16 45	*f*	−.288	−.218	0.25	16.8	− .957	+.016	+.052	*QR*
109	3		83 21	+43 04	*f*	+.683	+.080	0.5	43.1	− .730	+.027	−.029	*Q*
385	3		255 52	+13 21	*n*	+.231	−.235	0.25	13.4	+ .973	−.014	−.057	*QR*
381	3		306 13	−55 44	*n*	−.826	+.330	0.5	55.7	+ .564	−.136	+.093	*Q*
				Class *F*.									
92	3		68 17	−33 49	*f*	−.556	+.313	0.5	33.8	−.831	−.087	−.130	*Q*
94	3	*III*	250 33	+78 22	*n*	+.979	−.078	0.5	78.4	+.201	−.038	−.008	*Q*
104	2	*II*	285 34	−10 55	*n*	−.189	+.263	0.5	10.9	+.982	−.025	+.129	*Q*
106	3	*IV*	276 26	+39 02	*n*	+.630	+.082	0.25	39.0	+.777	+.013	+.016	*QR*
108	3		264 42	−28 23	*n*	−.476	−.077	0.5	28.4	+.880	+.018	−.034	*Q*
114	3	*III*	253 59	+18 24	*n*	+.316	−.265	0.25	18.4	+.949	−.021	−.063	*QR*
124	3	*III*	72 39	−51 25	*f*	−.782	+.186	0.5	51.4	−.624	−.073	−.058	*Q*
154	3		70 56	−67 41	*f*	−.925	+.120	0.5	67.7	−.380	−.056	−.023	*Q*
157	2	*II*	97 02	−13 15	*f*	−.229	−.116	0.25	13.2	−.973	+.066	+.028	*QR*
176	2		59 52	+80 21	*f*	+.986	+.098	0.5	80.4	−.167	+.097	−.016	*Q*
187	3		97 27	− 4 35	*f*	−.080	−.126	0.25	4.6	−.997	+.002	+.031	*QR*
208	3		276 34	+16 01	*n*	+.276	+.101	0.25	16.0	+.961	+.007	+.024	*QR*
212	3		272 26	−55 54	*n*	−.828	+.021	0.5	55.9	+.561	−.009	+.006	*Q*
215	3		281 33	−49 32	*n*	−.761	+.131	0.5	49.5	+.649	−.050	+.043	*Q*
228	2		263 18	+35 06	*n*	+.575	−.093	0.25	35.1	+.818	−.014	−.019	*QR*
243	3	*III*	119 04	−65 57	*f*	−.913	−.195	0.5	66.0	−.407	+.089	+.040	*Q*
249	3	*IV*	266 11	+ 5 45	*n*	+.100	−.068	0.25	5.8	+.995	−.002	−.017	*QR*
250	2		310 24	−58 21	*n*	−.851	+.343	0.5	58.4	+.524	−.146	+.090	*Q*
268	2	*III*	303 42	−73 08	*n*	−.957	+.161	0.5	73.1	+.291	−.077	+.023	*Q*
281	3	*IV*	263 23	−54 25	*n*	−.813	−.064	0.5	54.4	+.582	+.026	−.019	*Q*
288	3	*III*	259 40	−68 08	*n*	−.928	−.060	0.5	68.1	+.373	+.027	−.011	*Q*
294	3		69 26	−31 09	*f*	−.517	+.308	0.25	31.2	−.855	−.040	−.066	*QR*
296	2		94 27	−36 55	*f*	−.601	−.055	0.25	36.9	−.800	+.008	+.011	*QR*
306	2		274 10	−76 28	*n*	−.972	+.021	0.5	76.5	+.233	−.010	+.002	*Q*
322	3		99 39	−16 04	*f*	−.277	−.161	0.25	16.1	−.961	+.011	+.039	*QR*
330	3		260 19	−35 49	*n*	−.585	−.136	0.5	35.8	+.811	+.040	−.055	*Q*
338	2		62 24	−61 41	*f*	−.880	+.217	0.5	61.7	−.474	−.095	−.051	*Q*
340	3		278 19	−28 34	*n*	−.478	+.132	0.5	28.7	+.877	−.032	+.058	*Q*
341	2	*III*	256 10	+21 44	*n*	+.370	−.225	0.25	21.7	+.929	−.021	−.052	*QR*
343	3	*IV*	75 27	−10 10	*f*	−.177	+.249	0.25	10.2	−.984	−.011	−.061	*QR*
349	3		91 59	− 5 48	*f*	−.101	−.033	0.25	5.8	−.995	+.001	+.008	*QR*
364	2		110 19	−23 10	*f*	−.393	−.312	0.25	23.2	−.919	+.031	+.072	*QR*
372	2		81 09	−31 47	*f*	−.527	+.131	0.25	31.8	−.850	−.018	−.028	*QR*

No.	Cert.	Br.	Long.	Lat.	S.	Dist.	Dev.	Wt.	ϕ	Cos.	Inc.	Node.	Type.
CLASS *F* (continued.)													
			° ′	° ′					°				
376	2		286 08	−22 36	*n*	−.384	+.259	0.5	22.6	+ .923	−.050	+.120	*Q*
408	3		281 39	−35 59	*n*	−.588	+.167	0.5	36.0	+ .809	−.049	+.068	*Q*
429	2		250 26	−59 06	*n*	−.858	−.166	0.5	59.1	+ .514	+.071	−.043	*Q*
434	2		76 26	− 4 59	*f*	−.087	+.240	0.25	5.0	− .996	−.005	−.060	*QR*
435	2		268 25	+23 58	*n*	+.406	−.020	0.25	24.0	+ .914	−.002	−.005	*QR*
437	3		280 09	− 1 46	*n*	−.031	+.172	0.5	1.8	+1.000	−.002	+.086	*Q*
452	3	*IV*	74 22	−10 30	*f*	−.182	+.269	0.25	10.5	− .983	−.012	−.066	*QR*
467	3	*IV*	106 11	+13 09	*f*	+.228	−.271	0.5	13.2	− .974	−.031	+.132	*Q*
473	3		269 41	− 7 16	*n*	−.122	−.005	0.5	7.0	+ .992	.000	−.002	*Q*
477	3	*III*	257 35	+ 8 10	*n*	+.136	−.217	0.25	7.8	+ .991	−.074	−.054	*QR*
478	3		282 00	+52 21	*n*	+.795	+.120	0.5	52.7	+ .606	+.048	+.036	*Q*

The location of the aphelion of every member of these two classes is shown upon Figure 39. The reference circle is drawn by inspection. When belonging to type *Q*, the elliptic orbits of class *D* are indicated by circles, and the hyperbolic orbits of class *G* by crosses. When belonging to types *P*, *R*, or to subtypes *p*, *q*, or *r* they are indicated by dots. An inspection of the figure

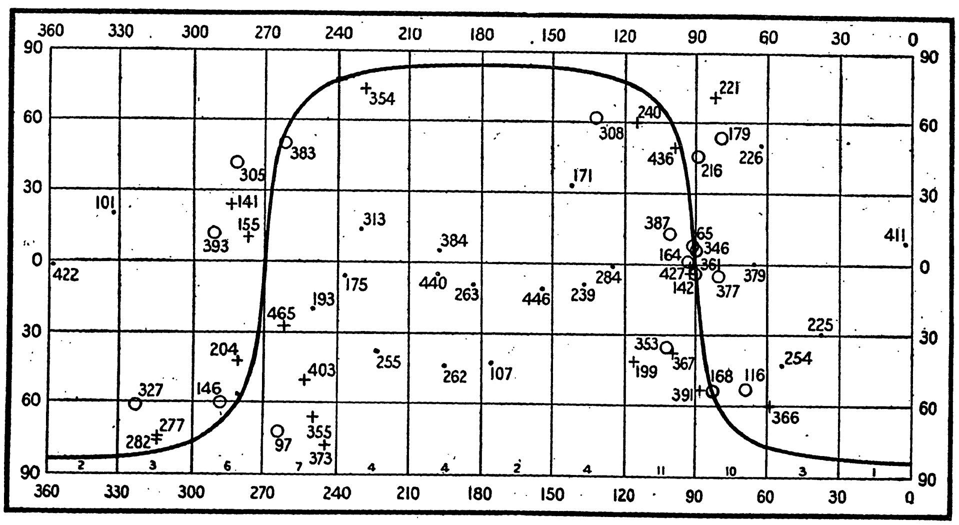

FIG. 39.

shows that these aphelia have a tendency to collect near the meridian of longitude 90°, and a less marked tendency to collect near 270°. The numbers at the bottom of the figure between the meridians indicate the number of aphelia found in each sector of the sphere. The assumed inclination of the reference circle is 83°.

The relation of this very singular reference circle to the various aphelia can be seen to better advantage in the orthographic projection of the sphere in Figure 40, where the concentration of the aphelia near the extremities of the reference circle clearly shows its true inclination, which is here changed

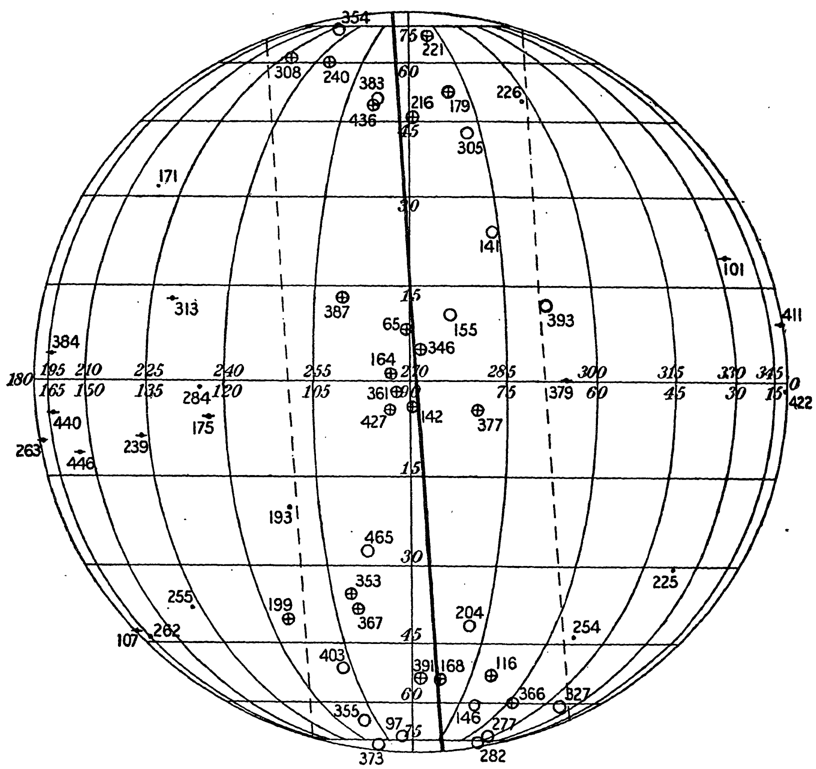

FIG. 40.

to 86°. In neither of these figures are any aphelia included save those belonging to classes *D* and *G*. As we shall see presently it is probable that those dots shown in the latter figure that have a horizontal line drawn through them should be omitted, as belonging to another type.

This figure would be made still more striking had the node been chosen at 95° instead of 90°. This would have shifted the circles marked with a cross farther to the right, and those unmarked farther to the left. If this reference circle really indicates the orbit of a remote unknown planet, we see at once that it would permit that body to pass very near to the pole of the ecliptic, and it would thus be capable of producing the southerly displacements of the reference circles of types *N* and *O* that we have already noted. In Chapters XII and XIII we found that this body must be massive, but in Chapter XIX we shall find that a close approach to the pole is unnecessary. In Chapter XIII we saw that the maximum southern deviation produced in classes *A* and *B* lay near longitude 170°. By Figure 40 it will be seen that the deviation from the pole also lies in this same general direction. In point of fact it lies in longitude 183.°4.

In Chapter XII we found that the orbits of the hyperbolic comets could be explained by the existence of a very massive planet. We now find that three-quarters of the hyperbolic aphelia coincide with this reference circle. The reason for making our computations in that chapter relatively to planet *Q* rather than to planet *P* is now apparent. Since the aphelia of the comets of classes *D* and *G* lie along the same reference circle, it is evident now why they are combined in this investigation, and it is because many of the aphelia of classes *E* and *F* do not lie along this circle that they are omitted. We shall find presently that the majority of their aphelia tend to lie along still another reference circle that we shall call *R*. The planet corresponding to this circle, although more remote than planet *Q*, produces few if any hyperbolic or slow elliptical velocities among its associated comets. From this we conclude that its mass is appreciably smaller than that of *Q*.

Since owing to the great mass of planet *Q* the orbits of its associated comets may be either hyperbolic or strongly elliptical, it is clear that orbits of all intermediate grades of eccentricity may be produced. The comets of classes *E* and *F* are consequently divided between the two planets *Q* and *R*, and there is no way of distinguishing their allegiance save by the positions of their aphelia. Where the two reference circles intersect, this test also fails us, and in appearance the comet may belong to either planet. This may be true also in fact.

Of the 20 aphelia of classes *D* and *G* which do not belong to type *Q*, and are therefore represented on Figure 40 by dots, there are 11 which belong to type *R*. A short horizontal line has been drawn through these dots to indicate that they probably do not properly belong in this figure at all, since a very slight change in their perihelion velocity, due either to perturbations, or to errors, would throw them into classes *E* or *F*, which are associated with type *R*. Even if we include these aphelia the concentration about the reference circle shown in Figure 40 is quite pronounced, but if we omit them it becomes very marked.

The latter portion of Table LV is devoted to the comets of type Q to be found under classes C, E, and F. To the two comets belonging to class C we have already referred in the last chapter. Their orbits are treated precisely like those of classes D and G, excepting that they are weighted only one-half, for the reasons previously explained. Seven comets belonging to type Q are found under class E. They are also weighted one-half. Seven others are found which may belong either to Q or R. These last are weighted therefore only one-quarter each. Under class F, the parabolic comets, 25 are found which belong to type Q and are weighted one-half, and 19 which may belong to either Q or R and are weighted one-quarter. The comets in this class are arranged in chronological order.

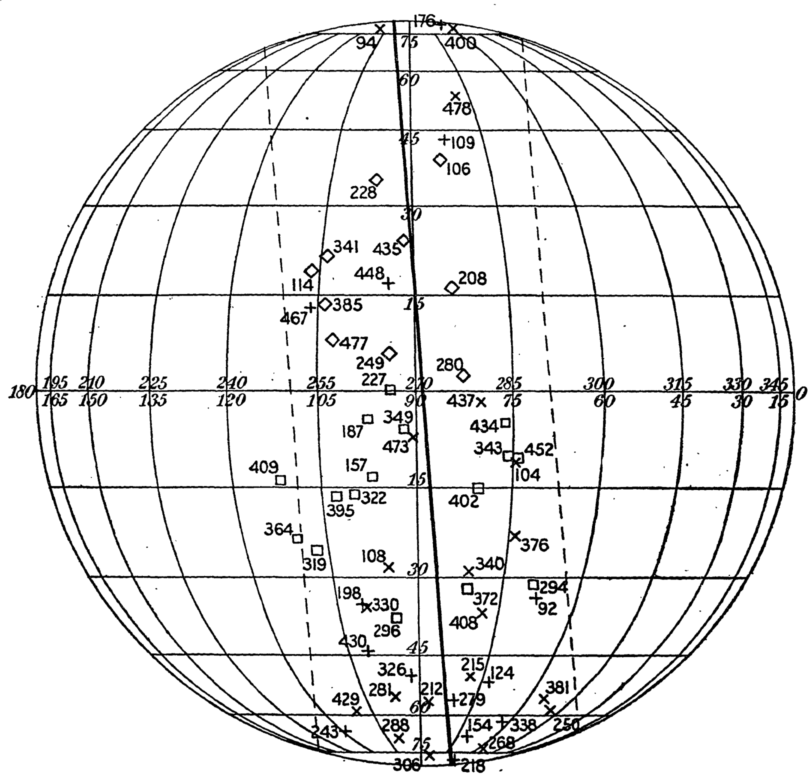

FIG. 41.

The aphelia of all these comets are plotted in Figure 41, those weighted one-half being indicated as before by crosses, and those weighted one-quarter by squares. The difference in the amount of concentration at the north and south poles is at once apparent. It was at first thought that this might be due in part to the fact that comets with southern aphelia would have northern perihelia, and would therefore be more readily visible than the others from northern observatories.

To test this suggestion Table LVI was prepared. This table refers exclusively to the comets of type *Q*. The first column gives the class, the second the total number of aphelia in each class, the third and fourth their distribution north and south of the ecliptic, and the fifth and sixth their percentage distribution. The last five columns are similarly arranged, but refer only to those comets that were visible to the naked eye, and that have appeared since 1850. In the first part of the table the last line shows that 67 per

TABLE LVI.

DISTRIBUTION IN LATITUDE OF THE APHELIA OF TYPE *Q*.

Class.	No.	North.	South.	N. p. c.	S. p. c.	No.	North.	South.	N. p. c.	S. p. c.
C	2	0	2	0	100	0	0	0	—	—
D	18	10	8	56	44	3	2	1	67	33
E	7	3	4	43	57	3	1	2	33	67
F	25	4	21	16	84	5	1	4	20	80
G	18	6	12	33	67	4	1	3	25	75
Total	70	23	47	33	67	15	5	10	33	67

cent of the aphelia had southern latitudes. In the second part we find that since 1850 the southern aphelia also formed 67 per cent of the total. Although the southern heavens are not yet scanned for comets as carefully as the northern ones, yet the difference since 1850 for naked eye comets must be very slight. It would therefore appear that the suggestion that the preponderance of southern aphelia was due to terrestrial conditions must be given little weight.

Why two-thirds of the comets of this type should come from regions south of the ecliptic is not at first evident, but will be explained later. It will be noticed, however, from an examination of the third and fourth columns of the table, that the excess from the south comes chiefly from classes *F* and *G*. This excess then comes from those comets that might really belong in class *D*, but that owing to the northern position of planet *Q* were made to appear to have higher eccentricities than they really possessed.

An inspection of Figure 40 showed that the true inclination of the reference circle was very nearly 90°. It was therefore decided for the second approximation to adopt the central meridian. This is the line from which the measurements were made that are used in the later columns of Table LV. Tables LV and LVII are arranged precisely like Tables LII and LIII, and therefore require no further explanation.

Owing to the correction to the node, the limiting circles 20° on either side of the finally adopted reference circle would appear as rather wide ellipses. It is for this reason that several of the accepted aphelia lie outside the dotted lines which indicate the location of the major axes of these circles. Of the 31 aphelia belonging to class *D*, 18, or 58 per cent, belong to type *Q*. Of the 26 orbits belonging to class *G*, 19, or 73 per cent, belong to this type. The average deviation from the assumed reference circle for classes *D* and *G*, taken from the eighth column of Table LV, is ±0.143, which is the sine of ±8°.2. If the finally adopted reference circle were used instead of the approximate one this value would be appreciably smaller.

TABLE LVII.

DISPLACEMENT. INCLINATION. NODE.

Part I.					
Class.	No.	Σ Wt. Dev.	Dis.		
D, G	37	+0.444	+0.7°		
D, G, C, E, F	60.5	+1.198	+1.1		
Part II.					
Class.	No.	Σ Wt. sin φ.	Σ Dev. sin φ.	Corr. inc.	Inc.
D, G	37	22.415	−0.974	−4.1°	85.9°
D, G, C, E, F	60.5	36.034	−1.502	−4.0	86.0
Part III.					
Class.	No.	Σ Wt. cos φ.	Σ Dev. cos φ.	Corr. node.	Node.
D, G	37	24.995	+1.235	+4.2°	94.2°
D, G, C, E, F	60.5	41.590	+1.691	+3.4	93.4

By Table LVII we find that the displacement of the reference circle from a great circle of the sphere is $+1^\circ.1$. The sum of the weighted positive deviations for the five classes taken from Table LV is +5.037, the sum of the negative deviations is −3.839, and their difference +1.198. The ratio of these deviations to one another is 1.31, and the elimination of the four most positive deviations would be necessary in order to change the displacement of the reference circle to a negative value. It is on the whole uncertain whether this deviation from the great circle is or is not the result of accident.

The number of comets belonging to classes *D* and *G* contained in the middle one of the three zones, Figure 40, is 37. Omitting those of type *R*, we have in the northern zone 4, and in the southern zone 5. Dividing these numbers as before by the area of the zones, we have a comet density for the middle zone that may be represented by the number 109, a density for the northern zone of 12, and for the southern zone of 15.

The aphelia of classes *E* and *F* are entered in Table LVIII. No comets of the highest grade of brilliancy are found in the former class, but the unusual proportion of 14 out of 25 were visible to the naked eye. As in the case of class *D* the brighter comets seemed to have the shorter aphelion distances. Two of the early comets of class *F* were of the highest grade of brilliancy, and had they been more accurately observed might have fallen into class *E*.

The aphelia of all these orbits are plotted in Figure 42. Those of class *E* and types *R* and *QR* are indicated by crosses, those of class *F* and types

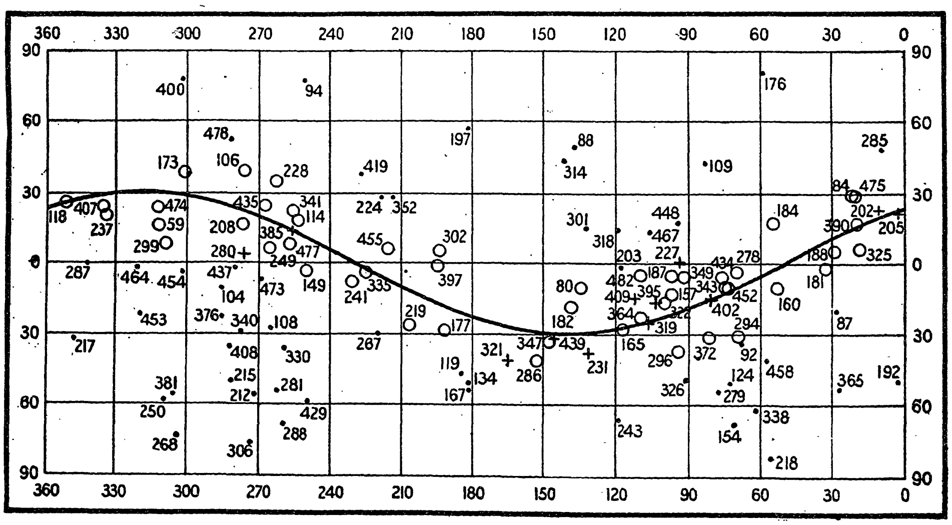

FIG. 42.

TABLE LVIII.

POSITIONS OF THE APHELIA OF CLASSES *E* AND *F*, AND OF CERTAIN MEMBERS OF CLASSES *C*, *D*, AND *G*.

No.	Cert.	Br.	Long.	Lat.	S.	Dist.	Dev.	Wt.	ϕ	Cos.	Inc.	Node.	Type.
			CLASS *E*.										
			° ′	° ′					°				
319	2	*II*	107 14	−25 08	*n*	−.870	+.009	.75	60.5	+.492	−.006	+.003	*QR*
279	2	*III*	77 13	−55 22	*n*	—	—	0	—	—	—	—	*Q*
280	2	*III*	277 25	+ 2 43	*f*	+.658	−.330	.75	41.1	−.754	−.164	+.188	*QR*
326	2	*IV*	91 20	−49 46	*n*	—	—	0	—	—	—	—	*Q*
402	3	*III*	80 37	−15 02	*n*	−.555	+.017	.75	33.7	+.832	−.007	+.011	*QR*
321	3		165 24	−41 47	*f*	−.918	−.237	1	66.6	−.397	+.218	+.094	*R*
203	2	*III*	117 30	− 1 38	*n*	—	—	0	—	—	—	—	*r*
227	2	*III*	93 47	+ 0 30	*n*	−.586	+.351	.75	35.9	+.810	−.154	+.213	*QR*
448	2	*II*	94 01	+17 09	*n*	—	—	0	—	—	—	—	*Q*
453	2	*IV*	320 19	−21 49	*n*	—	—	0	—	—	—	—	*P*
218	2		55 51	−83 15	*n*	—	—	0	—	—	—	—	*Q*
409	2	*III*	112 05	−14 33	*n*	−.870	+.218	.75	60.5	+.492	−1.42	+.080	*QR*
314	3	*III*	141 38	+43 27	*f*	—	—	0	—	—	—	—	*P*
400	2		301 00	+77 38	*f*	—	—	0	—	—	—	—	*Q*
285	2		8 34	+48 17	*n*	—	—	0	—	—	—	—	*r*
395	2		102 58	−16 45	*n*	−.810	+.120	.75	54.1	+.586	−.073	+.052	*QR*
109	3		83 21	+43 04	*n*	—	—	0	—	—	—	—	*Q*
202	2	*IV*	10 33	+22 54	*n*	+.706	+.047	1	44.9	+.708	+.033	+.033	*R*
205	2		2 38	+20 58	*n*	+.773	−.035	1	50.6	+.635	−.027	−.022	*R*
439	2		145 37	−32 08	*f*	−.994	−.050	1	83.7	−.110	+.050	+.005	*R*
385	3		255 52	+13 21	*f*	+.488	−.005	.75	29.2	−.873	−.002	+.003	*QR*
192	3	*IV*	2 43	−49 36	*n*	—	—	0	—	—	—	—	*q*
381	3		306 13	−55 44	*f*	—	—	0	—	—	—	—	*Q*
352	2	*III*	213 58	+27 45	*f*	—	—	0	—	—	—	—	*r*
231	2		130 56	−38 06	*n*	−.976	−.151	1	77.4	+.218	+.147	−.033	*R*
			CLASS *F*.										
59	3	*I*	311 49	+16 02	*f*	+.955	−.230	1	72.7	−.297	−.220	+.068	*R*
80	3	*IV*	135 15	− 9 38	*n*	−.933	+.340	1	68.9	+.360	−.317	+.122	*R*
84	3	*II*	21 32	+29 09	*n*	+.601	+.210	1	36.9	+.800	+.126	+.168	*R*
87	3	*I*	27 36	−20 30	*n*	—	—	0	—	—	—	—	*r*
88	3		136 37	+49 02	*n*	—	—	0	—	—	—	—	*P*
92	3		68 17	−33 49	*n*	—	—	0	—	—	—	—	*Q*
94	3	*III*	250 33	+78 22	*f*	—	—	0	—	—	—	—	*Q*
104	2	*II*	285 34	−10 55	*f*	—	—	0	—	—	—	—	*Q*
106	3	*IV*	276 26	+39 02	*f*	+.805	+.268	.75	53.6	−.593	+.162	−.119	*QR*

No.	Cert.	Br.	Long.	Lat.	S.	Dist.	Dev.	Wt.	φ	Cos.	Inc.	Node.	Type.
				Class *F* (continued).									
			° ′	° ′					°				
108	3		264 42	−28 23	*f*	—	—	0	—	—	—	—	*Q*
114	3	*III*	253 59	+18 24	*f*	+.489	+.080	.75	29.3	−.872	+.030	−.052	*QR*
118	3		352 13	+26 24	*n*	+.877	+.009	1	61.3	+.480	+.008	+.004	*R*
119	3		184 52	−47 27	*f*	—	—	0	—	—	—	—	*r*
124	3	*III*	72 39	−51 25	*n*	—	—	0	—	—	—	—	*Q*
134	3	*III*	181 39	−50 34	*f*	—	—	0	—	—	—	—	*r*
149	3		250 33	− 3 22	*f*	+.270	−.215	1	15.7	−.963	−.058	+.207	*R*
154	3		70 56	−67 41	*n*	—	—	0	—	—	—	—	*Q*
157	2	*II*	97 02	−13 15	*n*	−.735	+.152	.75	47.3	+.678	−.084	+.077	*QR*
160	3	*III*	52 53	−10 21	*n*	−.131	−.132	1	7.5	+.991	+.017	−.131	*R*
165	3	*III*	116 45	−27 35	*n*	−.928	.000	1	68.1	+.373	.000	.000	*R*
167	3		182 11	−54 20	*f*	—	—	0	—	—	—	—	*r*
173	3		300 49	+39 22	*f*	+.941	+.190	1	70.2	−.339	+.179	−.064	*R*
176	3	*IV*	59 52	+80 21	*n*	—	—	0	—	—	—	—	*Q*
177	2		192 17	−28 18	*f*	−.700	−.134	1	44.4	−.714	+.094	+.096	*R*
181	2	*III*	32 47	− 2 07	*n*	+.241	−.176	1	13.9	+.971	−.042	−.171	*R*
182	3	*III*	139 03	−18 35	*n*	−.980	+.190	1	78.5	+.199	−.186	+.038	*R*
184	3		54 59	+16 36	*n*	+.070	+.290	1	4.0	+.998	+.020	+.290	*R*
187	3		97 27	− 4 35	*n*	−.675	+.299	.75	42.5	+.737	−.150	+.165	*QR*
188	2		28 56	+ 4 38	*n*	+.352	−.110	1	20.6	+.936	−.039	−.103	*R*
197	2		182 02	+56 38	*f*	—	—	0	—	—	—	—	*q*
208	3		276 34	+16 01	*f*	+.742	−.108	.75	47.9	−.670	−.060	+.054	*QR*
212	3		272 26	−55 54	*f*	—	—	0	—	—	—	—	*Q*
215	3		281 33	−49 32	*f*	—	—	0	—	—	—	—	*Q*
217	2		348 02	−31 45	*n*	—	—	0	—	—	—	—	*p*
219	2		206 45	−26 23	*f*	−.531	−.211	1	32.1	−.847	+.112	+.179	*R*
224	3		218 34	+27 54	*f*	—	—	0	—	—	—	—	*r*
228	2		263 18	+35 06	*f*	+.675	+.277	.75	42.5	−.737	+.141	−.153	*QR*
237	2		333 59	+20 08	*n*	+.955	−.155	1	72.7	+.297	−.148	−.046	*R*
241	3		231 24	− 8 17	*f*	−.050	−.139	1	2.9	−.999	+.007	+.139	*R*
243	3	*III*	119 04	−65 57	*n*	—	—	0	—	—	—	—	*Q*
249	3	*IV*	266 11	+ 5 45	*f*	+.552	−.200	.75	33.5	−.834	−.082	+.125	*QR*
250	2		310 24	−58 21	*f*	—	—	0	—	—	—	—	*Q*
267	3		219 42	−29 26	*f*	—	—	0	—	—	—	—	*P*
268	2	*III*	303 42	−73 08	*f*	—	—	0	—	—	—	—	*Q*
278	2	*III*	70 15	− 3 41	*n*	−.332	+.120	1	19.4	+.943	−.040	+.113	*R*
281	3	*IV*	263 23	−54 25	*f*	—	—	0	—	—	—	—	*Q*
286	3		153 13	−41 29	*f*	−.970	−.206	1	75.9	−.244	+.200	+.050	*R*
287	3		342 49	− 0 26	*n*	—	—	0	—	—	—	—	*P*
288	3	*III*	259 40	−68 08	*f*	—	—	0	—	—	—	—	*Q*
294	3		69 26	−31 09	*n*	−.510	−.306	.75	30.7	+.860	+.117	−.197	*QR*
296	2		94 27	−36 55	*n*	−.789	−.242	.75	52.1	+.614	+.143	−.112	*QR*

No.	Cert.	Br.	Long.	Lat.	S.	Dist.	Dev.	Wt.	φ	Cos.	Inc.	Node.	Type.
Class *F* (continued).													
			° ′	° ′					°				
299	3		308 53	+ 7 38	*f*	+.899	−.370	1	64.0	−.438	−.333	+.162	*R*
301	2		132 20	+15 24	*n*	—	—	0	—	—	—	—	*P*
302	2		194 10	+ 4 59	*f*	−.462	+.362	1	27.5	−.887	−.167	−.321	*R*
306	2		274 10	−76 28	*f*	—	—	0	—	—	—	—	*Q*
318	3		119 05	+14 21	*n*	—	—	0	—	—	—	—	*P*
322	3		99 39	−16 04	*n*	−.769	+.123	.75	50.3	+.639	−.071	+.059	*QR*
325	2	*IV*	19 02	+ 5 48	*n*	+.488	−.170	1	29.2	+.873	−.083	−.148	*R*
330	3		260 19	−35 49	*f*	—	—	0	—	—	—	—	*Q*
335	2		224 58	− 3 34	*f*	−.100	−.010	1	5.7	−.995	+.001	+.010	*R*
338	2		62 24	−61 41	*n*	—	—	0	—	—	—	—	*Q*
340	3		278 19	−28 34	*f*	—	—	0	—	—	—	—	*Q*
341	2	*III*	256 10	+21 44	*f*	+.540	+.112	.75	32.7	−.842	+.045	−.070	*QR*
343	3	*IV*	75 27	−10 10	*n*	−.449	+.051	.75	26.7	+.893	−.017	+.035	*QR*
347	2	*III*	148 05	−32 48	*f*	−.986	−.062	1	80.4	−.167	+.061	+.010	*R*
349	3		91 59	− 5 48	*n*	−.623	+.241	.75	38.5	+.783	−.112	+.141	*QR*
364	2		110 19	−23 10	*n*	−.885	+.056	.75	62.2	+.465	−.038	+.019	*QR*
365	3	*III*	26 46	−52 47	*n*	—	—	0	—	—	—	—	*q*
372	2		81 09	−31 47	*n*	−.648	−.233	.75	40.4	+.762	+.113	−.134	*QR*
376	2		286 08	−22 36	*f*	—	—	0	—	—	—	—	*Q*
390	2		19 45	+16 33	*n*	+.561	+.004	1	34.1	+.828	+.002	+.003	*R*
397	2		194 36	− 0 55	*f*	−.509	+.268	1	30.6	−.861	−.136	−.231	*R*
407	3		335 44	+23 37	*n*	+.955	−.095	1	72.7	+.297	−.091	−.028	*R*
408	3		281 39	−35 59	*f*	—	—	0	—	—	—	—	*Q*
419	3		227 11	+38 20	*f*	—	—	0	—	—	—	—	*r*
429	2		250 26	−59 06	*f*	—	—	0	—	—	—	—	*Q*
434	2		76 26	− 4 59	*n*	−.426	+.149	.75	25.2	+.905	−.047	+.101	*QR*
435	2		268 25	+23 58	*f*	+.686	+.072	.75	43.3	−.728	+.037	−.039	*QR*
437	3		280 09	− 1 46	*f*	—	—	0	—	—	—	—	*Q*
452	3	*IV*	74 22	−10 30	*n*	−.437	+.037	.75	25.9	+.900	−.012	+.025	*QR*
454	3		301 34	− 3 59	*f*	—	—	0	—	—	—	—	*p*
455	2		215 50	+ 6 08	*f*	−.156	+.215	1	9.0	−.988	−.034	−.212	*R*
458	3		57 51	−41 06	*n*	—	—	0	—	—	—	—	*q*
464	3		320 56	− 1 33	*n*	—	—	0	—	—	—	—	*P*
467	3	*IV*	106 11	+13 09	*n*	—	—	0	—	—	—	—	*Q*
473	3		269 41	− 7 16	*f*	—	—	0	—	—	—	—	*Q*
474	3		313 09	+24 58	*f*	+.990	−.075	1	81.9	−.141	−.074	+.011	*R*
475	3		20 29	+29 13	*n*	+.610	+.205	1	37.6	+.792	+.125	+.162	*R*
477	3	*III*	257 35	+ 8 10	*f*	+.462	−.094	.75	27.5	−.887	−.032	+.062	*QR*
478	3		282 00	+52 21	*f*	—	—	0	—	—	—	—	*Q*
482	3	*III*	109 29	− 5 21	*n*	−.787	+.343	1	51.9	+.617	−.270	+.212	*R*

No.	Cert.	Br.	Long.	Lat.	S.	Dist.	Dev.	Wt.	ϕ	Cos.	Inc.	Node.	Type.
						CLASS C.							
147	3	IV	233° 11′	+21° 46′	f	+.236	+.295	.25	13°.7	−.972	+.017	−.072	R
						CLASS D.							
101	3		332 30	+19 34	n	+.965	−.169	.5	74.8	+.262	−.082	−.022	R
379	3	III	65 30	+ 0 03	n	−.231	+.141	.5	13.4	+.973	−.016	+.069	R
313	2		229 30	+13 42	f	+.106	+.209	.5	6.1	−.994	+.011	−.104	R
411	2		2 58	+ 9 24	n	+.705	−.220	.5	44.8	+.710	−.077	−.078	R
263	3		183 18	− 8 34	f	−.700	+.226	.5	44.4	−.714	−.079	−.080	R
175	3		237 30	− 5 57	f	+.058	−.155	.5	3.3	−.998	−.004	+.078	R
						CLASS G.							
446	2	IV	153 49	−11 01	f	−.926	+.302	.5	67.8	−.378	−.140	−.057	R
239	3	III	136 34	− 8 24	n	−.928	+.360	.5	68.1	+.373	−.167	+.067	R
440	2	IV	197 45	− 4 49	f	−.503	+.190	.5	30.2	−.864	−.048	−.082	R
384	2	IV	196 35	+ 4 31	f	−.437	+.335	.5	25.9	−.900	−.073	−.151	R
107	3		176 14	−42 55	f	−.858	−.292	.5	59.1	−.513	+.251	+.150	R

R and *QR* by circles, and those belonging to these two classes but of types *P* and *Q* and subtypes *p*, *q*, and *r* by dots. The approximate reference circle is located on the figure by inspection. In Figure 43 all the aphelia of types *R* and *QR* are plotted, to whatever class they may belong, also all aphelia of classes *E* and *F* of type *P* and of the subtypes *p*, *q*, and *r*. Aphelia of type *Q* and of classes *E* and *F* however have been omitted, since as already shown they probably depend on planet *Q*, having eccentricities intermediate between those of classes *D* and *G*. Moreover, they have been already plotted where they belong in Figure 41. They would therefore serve no useful purpose in Figure 43, but would tend to confuse the results.

In this figure aphelia of classes *E* and *F* type *R* are weighted unity, and are represented by circles. If of type *QR* they are weighted three-quarters, and are represented by squares. If of type *P* or a subtype they have no weight, and are represented by dots. The aphelia of the other classes, *C*, *D*, and *G*, that belong to type *R* are weighted one-quarter for *C*, and one-half for the two others, and are represented by crosses. The provisional inclination of the reference circle, 30°, adopted in Figure 42 has been retained in Figure 43. Tables LVIII and LIX are arranged exactly like their predecessors and therefore need no further explanation.

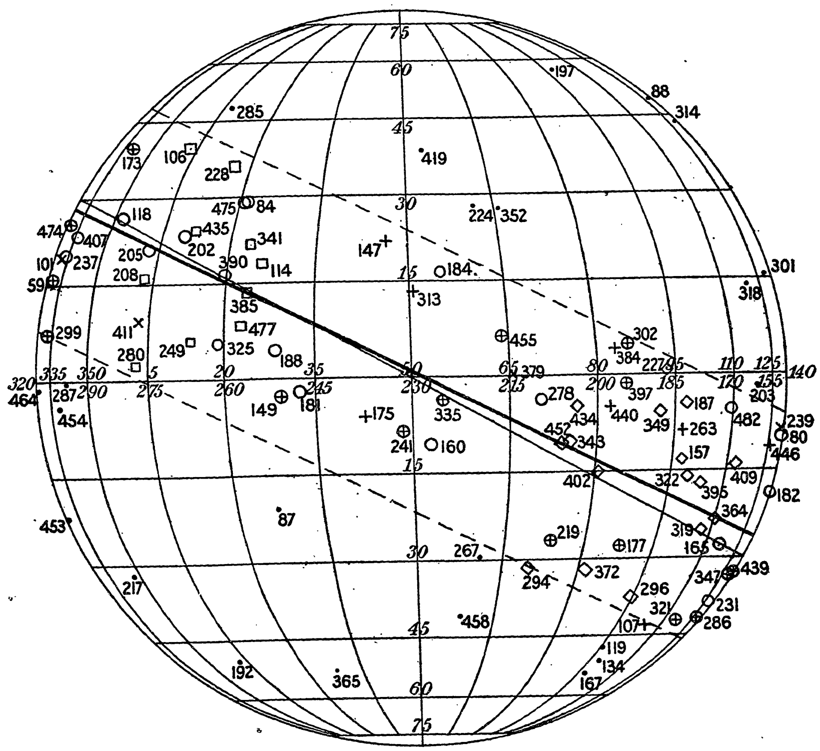

FIG. 43.

Of the 25 aphelia belonging to class *E*, 20 per cent belong to type *R*, 28 per cent to type *Q*, and 28 per cent more to type *QR*. Of the 91 aphelia belonging to class *F*, 33 per cent belong to type *R*, 27 per cent to type *Q*, and 21 per cent to type *QR*. The two types *Q* and *R* may be said to have an equal right to be included in the two classes *E* and *F*, and to be with *QR*, the only types legitimately included in them. They form, as we see from the above, 76 per cent of class *E*, and 81 per cent of class *F*. The average deviation from the assumed reference circle of Figure 43, deduced from the eighth column of Table LVIII, for classes *E* and *F* is ± 0.163 which is the sine of $\pm 9°.4$.

By Table LIX we find that the deviation of the reference circle from a great circle of the sphere is 1°.1. The sum of the weighted positive deviations is +5.724, and of the negative −4.521. The difference is 1.203 and the ratio 1.27. The elimination of four comets out of the seventy-three whose deviations are computed, namely numbers 80, 184, 302, and 482, would change the northern deviation to a southern one. From this fact and the small value of the ratio we conclude that the deviation of the reference circle from a great circle is probably due merely to accidental causes.

TABLE LIX.

DISPLACEMENT. INCLINATION. NODE.

Part I.					
Class.	No.	Σ Wt. Dev.	Dis.		
E, F	54.5	+0.666	+0.7°		
E, F, C, D, G	60.25	+1.203	+1.1		
Part II.					
Class.	No.	Σ Wt. sin ϕ.	Σ Dev. sin ϕ.	Corr. inc.	Inc.
E, F	54.5	35.458	−1.330	−3.3°	26.7°
E, F, C, D, G	60.25	38.725	−1.737	−4.0	26.0
Part III.					
Class.	No.	Σ Wt. cos ϕ.	Σ Dev. cos ϕ.	Cor. node.	Node.
E, F	54.5	36.133	+1.203	+6.4°	236.4°
E, F, C, D, G	60.25	40.212	+0.921	+4.5	234.5

The number of aphelia belonging to classes *E* and *F* contained in the middle zone of Figure 43 is 61, in the northern zone 10, and in the southern 13. Dividing by the areas of these zones we obtain for the density of the middle zone the number 179, for the northern zone 30, and for the southern zone 39.

Turning now to the accurately observed comets which have not been classified under either of the five preceding types, we find they number 21 in all, and constitute a miscellaneous group, Table LX, with no clearly marked affinities. If we applied the rule strictly, that a comet whose aphelion lay within 20° of any reference circle should be classified under that type, numbers 284, 203, 87, and 454 would be omitted from the list, and be classified under types *N* and *O*. Such a classification would be obviously erroneous, but it suggests that a number of the supplementary orbits given at the end of Tables LII, LV, and LVIII owe their inclusion in those tables merely to chance, and that they should really appear in Table LX classified under the subtypes.

TABLE LX.

COMETS CLASSIFIED IN THE SUB-TYPES.

No.	Cert.	Br.	Long.	Lat.	Cl.	Sub.	Dev.	Ang_D.	Perp.	S. diam.	$Corr_N$.	$Corr_D$.	Sin.	Corr.	Ang.
			° ′	° ′				°						°	°
362	2		50 29	−36 15	*C*	*q*	+.485	+29.0	.81	+.058	+.046	−.023	+.023	+1.3	+30.3
369	3		193 46	+21 12	"	*p*	+.517	+31.1	1.00	−.005	−.005	−.048	−.053	−3.0	+28.1
262	2	*I*	195 03	−43 45	*D*	*p*	−.359	−21.0	.80	−.005	−.004	−.048	−.052	−3.0	−24.0
254	3	*III*	54 19	−42 55	"	*q*	+.389	+22.9	.73	+.058	+0.41	−.023	+.018	+1.0	+23.9
284	2	*III*	124 41	− 0 54	"	*r*	+.428	+25.3	.50	+.016	+.008	−.013	−.005	−0.3	+25.0
203	2	*III*	117 30	− 1 38	"	*r*	+.394	+23.2	.55	+.016	+.009	−.013	−.004	−0.2	+23.0
285	2		8 34	+48 17	"	*r*	+.457	+27.2	.70	+.016	+.011	−.013	−.002	−0.1	+27.1
192	3	*IV*	2 43	−49 36	*E*	*q*	+.517	+31.1	.65	+.058	+.037	−.023	+.014	+0.8	+31.9
352	2	*III*	213 58	+27 45	"	*r*	+.520	+31.3	1.00	−.016	−.016	−.013	−.029	−1.7	+29.6
87	3	*I*	27 36	−20 30	*F*	*r*	−.476	−28.4	.98	+.016	+.016	−.013	+.003	+0.2	−28.2
119	3		184 52	−47 27	"	*r*	−.434	−25.7	.63	−.016	−.010	−.013	−.023	−1.3	−27.0
134	3	*III*	181 39	−50 34	"	*r*	−.471	−28.1	.62	−.016	−.010	−.013	−.023	−1.3	−29.4
167	3		182 11	−54 20	"	*r*	−.518	−31.2	.65	−.016	−.010	−.013	−.023	−1.3	−32.5
197	2		182 02	+56 38	"	*q*	−.510	−30.7	.55	−.058	−.031	−.023	−.054	−3.1	−33.8
217	2		348 02	−31 45	"	*p*	−.395	−23.3	.90	+.005	+.004	−.048	−.044	−2.5	−25.8
224	3		218 34	+27 54	"	*r*	+.498	+29.9	1.00	−.016	−.016	−.013	−.029	−1.7	+28.2
365	3	*III*	26 46	−52 47	"	*q*	+.500	+30.0	.60	+.058	+.034	−.023	+.011	+0.6	+30.6
419	3		227 11	+38 20	"	*r*	+.574	+35.0	.78	−.016	−.012	−.013	−.025	−1.4	+33.6
454	3		301 34	− 3 59	"	*p*	+.395	+23.3	.80	+.005	+.004	−.048	−.044	−2.5	+20.8
458	3		57 51	−41 06	"	*q*	+.360	+21.1	.75	+.058	+.042	−.023	+.019	+1.1	+22.2
225	2		37 30	−30 17	*G*	*r*	−.480	−28.7	1.00	+.016	+.016	−.013	+.003	+0.2	−28.5

The first five columns of the table give the number of the orbit, its certainty, the brightness of the associated comet, and the longitude and latitude of its aphelion. The sixth column gives the class to which it belongs, and the seventh the sub-type under which it has been classified. The subtype is determined by the reference circle to which the aphelion lies nearest.

The different aphelia are plotted on square projection in Figure 44. If belonging to class *C*, the aphelion is indicated by a vertical cross, if to classes *D* or *G* by an X, and if to classes *E* or *F* by a circle. The three adopted reference circles of types *P*, *Q*, and *R* are also shown. They differ slightly from the assumed reference circles given in Figures 37, 39, and 42, the deviation in the case of type *Q* being most conspicuous.

To determine the distance of any aphelion from an adopted reference circle, the former is plotted on the corresponding spherical projection of Figures 38, 40, or 43. A diameter is then drawn parallel to the major axis of the ellipse which represents the adopted circle, and the perpendicular distance of the aphelion from this line is measured and entered in the eighth column of Table LX. The angle of which this deviation is the sine is given in the ninth column. The height of the perpendicular passing through the aphelion measured from the diameter to the circumference of the sphere is given in the tenth, and the minimum semi-diameter of the ellipse representing the adopted reference circle in the eleventh column. This latter quantity is found by the formula for the correction of the node, p. 230, omitting sin i. Its sign depends on whether the longitude of the aphelion is given in the upper or lower row of figures along the ecliptic as shown in the figure. The correction of the deviation of the aphelion for the inclination of the adopted reference circle to the line of sight is given in the twelfth column, and is the product of the quantities in the tenth and eleventh. The thirteenth column gives the correction for the displacement of the reference circle. It is the natural sine of the displacement given in the preceding tables. Since the displacement of all three reference circles is towards the north, the sign of this correction is always negative. The sum of the quantities given in the twelfth and thirteenth columns gives the sine of the correcting angle, and is given in the fourteenth column. The angle itself is given in the fifteenth column, and is added to the angle given in the ninth to give the result in the last column, which is the angular distance of the aphelion from the corrected reference circle.

The minimum deviation is 20°.8 for orbit 454 and the maximum is 33°.8 for orbit 197. The portion of the sky left uncovered by the middle zones of types *P*, *Q*, and *R* is about one-quarter of the total area of the sphere, so that the mean density of aphelia in this region is about 84. The average density in the remaining three-quarters, omitting all the orbits of types *N* and *O*, is 219.

Turning now to the comets whose orbits are less accurately known, either because of their appearance before accurate observations were possible, or because of the short duration of their visibility, we shall find them classified

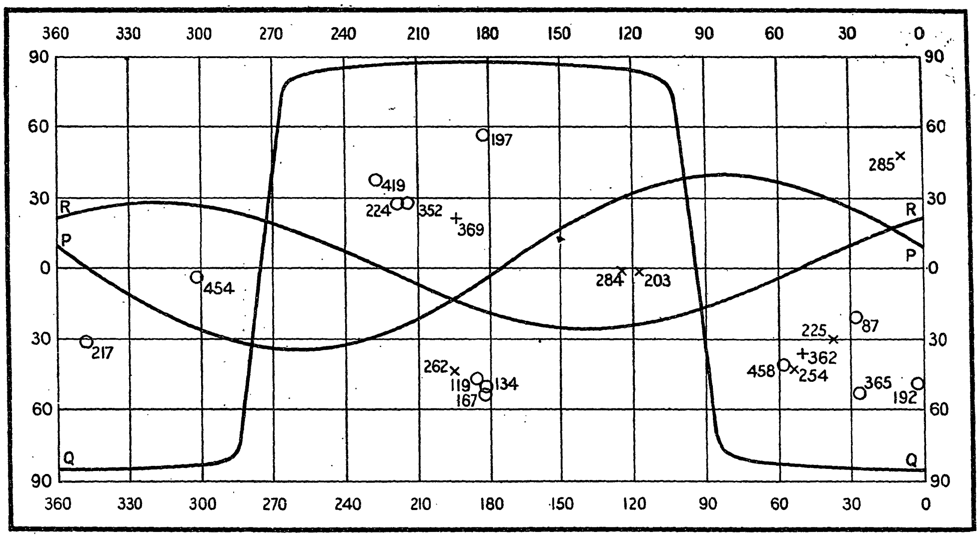

FIG. 44.

in Table LXI. Class *H* consists in general of faint comets and those of medium brightness, while class *I* consists chiefly of very bright ones. It is thought that few, if any, of the former class could have reached the conspicuousness of grade II, while most of the latter, save perhaps eight to ten of medium brightness appearing after 1650, belonged to the first two grades. Few of the comets of class *H* were observed through a period exceeding thirty days. The aphelia of this class are plotted in Figure 45. The chart shows a notably uniform distribution, the most so that has yet been found. Dividing it into four zones of equal area, according to the latitude, we find a slightly less density towards the extreme north, the number of aphelia in each zone beginning at the north being 17, 22, 27, and 21. Dividing it into twelve equal sectors of 30° of longitude each, the number of aphelia in each sector beginning at 0° are as follows:—9, 9, 7, 8, 6, 2, 11, 8, 5, 7, 7, and 8. These numbers are entered at the bottom of the figure. While we cannot definitely assign to their respective types the 87 orbits of class *H*, since their aphelion distances are unknown, we may note that the large proportional number of 20 of them necessarily fall into the subtypes, and that as many as 32 might belong to type *P*, 29 to type *Q*, and 39 to type *R*.

TABLE LXI.

POSITIONS OF THE APHELIA OF CLASSES *H* AND *I*.

No.	Year.	Cert.	Br.	Long.		Lat.		No.	Year.	Cert.	Br.	Long.		Lat.	
Class *H*.								Class *H* (continued).							
				°	′	°	′					°	′	°	′
74	1701	4		312	46	− 9	54	152	1816	5		97	31	+34	20
75	1702	5	*IV*	321	36	+ 3	23	153	1818 I	5		257	24	+ 0	10
76	1706	4		239	43	−44	56	161	1822 I	4	*IV*	7	44	+12	15
79	1723	4	*III*	216	02	+21	32	163	" III	4		46	52	+30	03
81	1737 I	4	*III*	148	41	−18	04	166	1824 I	4		71	09	+20	54
82	" II	5		91	56	−24	39	169	1825 II	4		193	58	− 2	42
83	1739	4		274	33	−53	02	174	1826 III	5		216	51	− 0	26
86	1743 II	5	*IV*	59	47	+38	40	178	1827 II	4		123	56	−14	05
89	1748 I	4	*III*	53	36	−17	24	186	1833	4		45	13	+ 7	12
90	" II	4	*IV*	76	00	+57	02	194	1840 III	4	*III*	177	53	−41	09
91	1757	4	*IV*	304	55	+12	50	206	1845 III	4	*III*	89	36	−46	44
95	1759 III	4	*III*	320	18	+ 4	08	222	1848 I	4		180	38	+79	22
96	1762	4	*IV*	341	23	−64	10	232	1851 IV	4		193	46	+61	04
98	1764	4	*III*	188	15	−50	26	234	1852 II	4		111	21	−27	06
99	1766 I	4		321	44	−39	57	242	1854 II	4	*III*	348	22	−76	12
103	1770 II	4		31	47	+30	55	247	1855 II	4		60	00	− 8	41
110	1780 II	4		95	20	+66	14	252	1857 III	4		52	21	−38	01
111	1781 I	4	*III*	81	00	−23	33	260	1858 IV	4		13	28	−76	42
112	" II	4	*III*	200	38	−23	39	265	1859	4		202	03	+76	50
115	1785 I	4		275	01	+24	03	266	1860 I	4		330	30	+29	10
120	1788 I	4	*III*	281	18	−10	31	269	" IV	5		182	09	+23	21
121	" II	4		188	08	−27	12	272	1861 III	4		347	34	+18	34
122	1790 I	4		237	46	−28	22	274	1862 II	4	*IV*	120	05	− 3	37
125	1792 I	4		212	35	−16	07	276	" IV	4		314	24	+31	27
126	" II	4		307	36	−24	05	283	1864 I	4		5	25	+ 9	47
127	1793 I	4		69	38	+48	41	304	1870 IV	4		184	27	−32	44
130	1796	4		16	36	+ 3	54	316	1873 VII	4		264	29	+ 7	44
131	1797	4	*IV*	225	27	+49	40	317	1874 I	4		119	42	+58	53
132	1798 I	4		291	01	+11	48	329	1877 V	4		246	09	−61	15
133	" II	4		223	33	+22	48	331	1878 I	4		102	04	− 2	23
135	1799 II	4	*III*	340	16	−42	08	337	1879 IV	4		141	01	−71	22
136	1801	4		5	58	+13	06	345	1881 II	4		125	22	− 6	03
137	1802	4		143	58	−18	13	356	1883 II	4	*IV*	285	02	−36	47
138	1804	4		341	44	+23	05	392	1890 III	4		274	51	−63	01
143	1808 I	4		256	54	+43	25	418	1895 III	4		239	44	+58	22
144	" II	5		66	36	−28	19	420	1896 I	4		30	23	+ 0	41
145	1810	4		265	43	−53	51	423	" IV	4		332	27	−41	01
150	1813 II	4	*IV*	39	45	+24	44	438	1898 IX	4	*IV*	19	21	− 8	25

CLASS *H* (continued).

No.	Year.	Cert.	Br.	Long. °	′	Lat. °	′
444	1899 V	4		94	38	−10	14
445	1900 I	4		199	43	−13	10
450	1902 I	4		76	23	+43	17
451	" II	4		340	23	+14	09
456	1903 IV	4	*III*	286	59	−52	24
459	1904 II	4		30	14	−40	02
466	1905 VI	4		16	44	−53	32
468	1906 II	4		206	46	+80	50
483	1909 I	4		128	35	− 3	57

CLASS *I*.

No.	Year.	Cert.	Br.	Long. °	′	Lat. °	′
1	−372	5	*I*	—		—	
2	−137	5		50	06	+ 3	24
3	− 69	5		154	32	−28	01
7	+240	5		111	06	−43	28
8	539	5		—		—	
9	565	5		287	54	−57	26
10	568	5		157	06	− 1	42
11	574	5		337	29	−11	05
12	770	5		188	35	−61	34
14	961	4		129	32	−77	11
16	1006	5		136	16	−17	27
18	1092	4		344	22	−14	19
19	1097	5		196	37	−51	46
20	1231	5		324	11	− 5	12
21	1264	4	*I*	130	07	− 5	41
22	1299	5		171	13	−65	00
24	1337	5	*I*	189	58	−40	28
25	1351	5		—		—	
26	1362	5		56	00	− 5	17
29	1385	4		283	55	−10	27
30	1402	5	*I*	35	42	−54	59
31	1433	4		100	35	+ 9	02
32	1449	4		90	26	+ 1	17
34	1457 I	5		270	21	+ 3	23

CLASS *I* (continued).

No.	Year.	Cert.	Br.	Long. °	′	Lat. °	′
35	1457 II	5		195	38	+ 0	53
36	1468	5		192	15	−35	17
37	1472	4	*I*	226	00	+ 8	23
38	1490	5		257	53	−36	57
39	1499	5		183	48	−11	25
40	1500	5		130	10	−19	17
41	1506	5		85	01	+38	44
43	1532	5	*I*	293	27	−12	52
44	1533	5		43	54	+27	54
45	1556	4	*I*	100	22	−29	48
46	1558	5		11	54	−54	19
47	1577	4		344	49	+69	27
48	1580	4		292	17	−64	32
49	1582	5		67	40	+23	18
50	1585	4		193	41	+ 2	54
51	1590	4		38	22	+22	56
52	1593	4		348	58	−12	03
53	1596	4		108	21	−42	42
55	1618 I	5		140	44	− 8	52
56	" II	4	*I*	190	57	+35	14
57	1652	5		254	08	+58	15
58	1661	4	*I*	294	15	−17	26
60	1665	4		237	22	−23	07
61	1668	4		102	30	−35	22
62	1672	4		282	37	−69	27
63	1677	4		289	28	−75	44
67	1683	4		284	19	−82	52
68	1684	4		78	15	+26	35
69	1686	4		259	23	−31	17
70	1689	5		167	27	−60	53
71	1695	5		241	17	+ 8	46
72	1698	5		97	05	− 5	14
73	1699	4		9	52	−62	04
77	1707	4	*II*	236	10	−27	07
78	1718	4	*II*	306	03	− 3	14
339	1880 I	4	*II*	100	59	−35	15
374	1887 I	4	*II*	101	39	−37	47

The bright comets of class *I* are plotted in Figure 46 as circles. In addition to them the bright comets of grades *I* and *II* of all the other classes are plotted as crosses. Their classes and numbers are as follows: — class *A*

102, class *B* 190, 275, class *C* 198, 271, class *D* 65, 101, 116, 142, 146, 171, 216, 262, 353, class *E* 319, 448, class *F* 59, 84, 87, 104, 157, class *G* none, class *H* none, subclasses *a* and *b* none. Of the bright comets 58 belong to class *I*, and 21 to the other classes. The cross lying in the enclosing circle in longitude 102° latitude −35° represents two different aphelia, numbers 198 and 353, the circle itself represents three, numbers 61, 339, and 374. No comet that has returned is entered more than once.

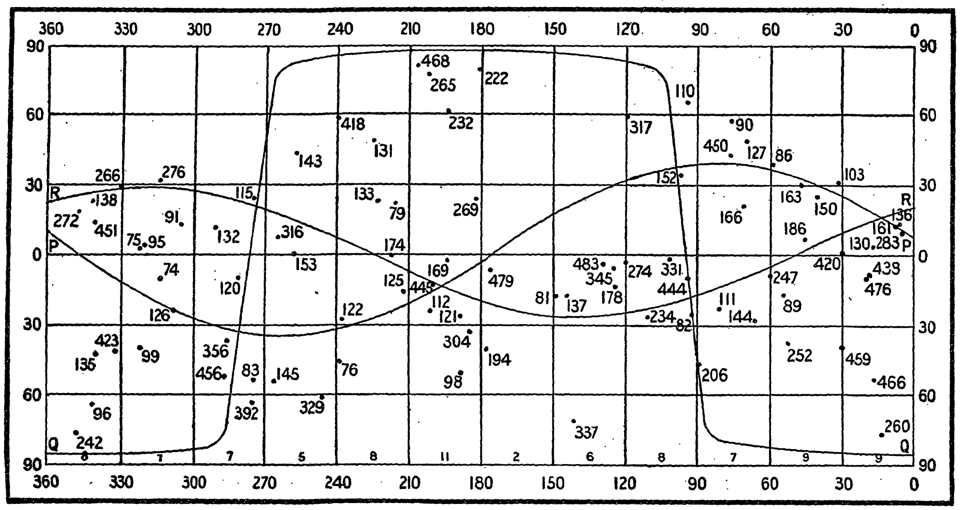

FIG. 45.

The locations of the earlier aphelia of class *I* are somewhat uncertain, and it is likely that some of them are quite erroneous, differing from their true positions by 30° or more. Thus, numbers 4, 5, 6, 13, 15, 17 and 23 have recently been identified by Messrs. Cowell and Crommelin as certainly early appearances of Halley's comet. Their aphelia should all agree practically with that of the appearance in 1835, number 190, longitude 124°, latitude −17°. Such, however, we find is not the case. An examination of the sixth column of Table LXIX, p. 292, shows that the early deviations ranged from 8° to 56°, averaging 30°.

Considering that the errors of the aphelia of the few earlier comets will more or less balance one another, and dividing the chart into four zones of latitude as before, we find the number of aphelia that they contain as follows: 6, 19, 26, 28. The small number in the northern zones is undoubtedly due partly to the fact that their perihelia would fall far to the south, and these comets would therefore be likely to be overlooked by the early observers. Dividing the chart into twelve sectors of longitude as before, we find the distribution as follows: — 4, 5, 6, 16, 8, 5, 9, 3, 4, 11, 3, and 5. The distribution is much more uneven than in the other case. Of the 58 comets belonging to class *I*, 9 fall into the subtypes. Of the remainder as many as 22 might be associated with type *P*, 24 with type *Q*, and 23 with type *R*. In both classes *H* and *I* rather more comets might belong to type *P* and rather less to type *Q* than we should have expected, judged by our investigations of the other classes. Nevertheless, in spite of this fact there is a notable proportion of the bright comets of class *I* whose aphelia are found

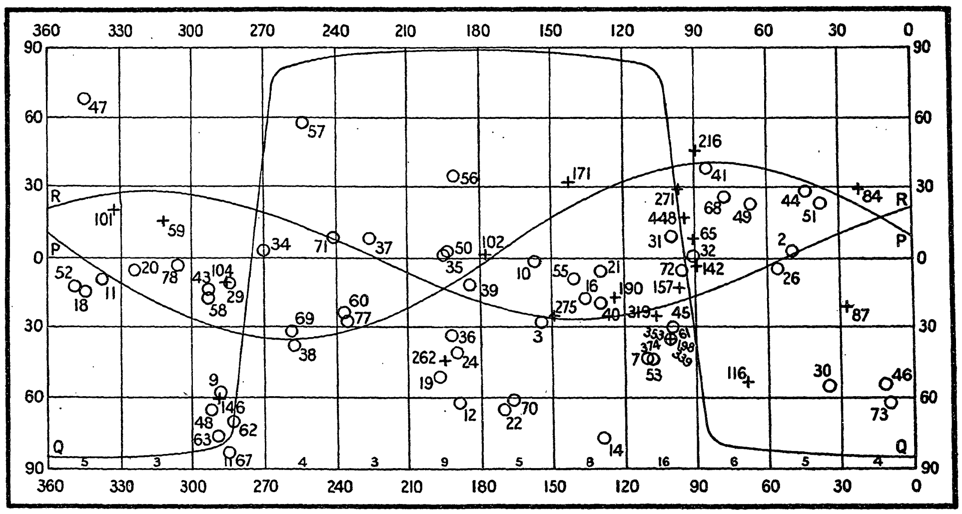

FIG. 46.

clustered about longitudes 100° and 290°, and are therefore clearly associated with type *Q*. A comparison with the reference circle shows that their longitudes are situated a little further to the east, as though a slow shifting of the node had occurred. This can hardly be the case, however, as any real motion would be much slower than that indicated. Besides the well known group of comets near 102°, lat. −35° surrounding the aphelion of the great comet of 1882, number 353, there is a more widely separated group of bright comets near long. 289°, lat. −60° surrounding the aphelion of number 146, the great comet of 1811. Although coming from the same part of the sky, the elements of this latter group do not particularly resemble one another. Most of them are of early date, and their elements are therefore rather uncertain.

Summarizing the results contained in this and the two previous chapters, we may represent the distribution of the various orbits by classes and types in Table LXII. The first part of the table gives the actual number of orbits belonging to each class and type. If of the type *QR* one-third of the orbits are associated with type *Q*, and two-thirds with type *R*. As shown by the

TABLE LXII.

SUMMARY OF THE CLASSES AND TYPES.

Type.	*A.*	*B.*	*C.*	*D.*	*E.*	*F.*	*G.*	Total.
Distribution.								
N	34	0	0	0	0	0	0	34
O	0	12	0	0	0	0	0	12
P	0	0	15	6	0	6	1	28
Q	0	0	2	30	1	31.7	19	83.7
R	0	0	1	7	5	42.3	5	60.3
Sub.	0	0	2	5	2	11	1	21
Total	34	12	20	48	8	91	26	239
Percent of Classes.								
N	100	0	0	0	0	0	0	14
O	0	100	0	0	0	0	0	5
P	0	0	75	12	0	7	4	12
Q	0	0	10	63	12	35 } 81	73	35
R	0	0	5	15	63	46 }	19	25
Sub.	0	0	10	10	25	12	4	9
Total	100	100	100	100	100	100	100	100
Percent of Types.								
N	100	0	0	0	0	0	0	100
O	0	100	0	0	0	0	0	100
P	0	0	54	21	0	21	4	100
Q	0	0	2	36	1	38	23	100
R	0	0	2	12	8	70	8	100
Sub.	0	0	9	24	9	53	5	100
Total	14	5	9	20	3	38	11	100

lower row of figures, the total number of orbits in class *D* is 48. The total number in type *Q*, as shown by the last column, is 83.7. The total number of type *Q* in class *D* is 30. The second part of the table gives the percentage by classes. Thus, 63 per cent of the orbits of class *D* belong to type *Q*. Of all the orbits 35 per cent, as shown by the last column, belong to type *Q*. The third part of the table gives the percentage by types. Thus 36 per cent of the orbits of type *Q* belong to class *D*, while 20 per cent of all the orbits, as shown by the lower row of figures, belong to that class.

The proportions of the dominating type of each class are clearly shown in the second part of the table. If the aphelia were distributed upon the sphere merely by chance, we should find that 34 per cent of any class would belong to any given type. Of class *A* 100 per cent belong to type *N*. Of class *B* 100 per cent belong to type *O*. Of class *C* 75 per cent belong to type *P*. Of class *D* 63 per cent belong to type *Q*. Of class *E* 63 per cent belong to type *R*. For class *F* as before noted, types *Q* and *R* should be combined, since comets having nearly parabolic orbits belong to both of these types. Accidental distribution would give 60 per cent, we actually find 81. Of class *G* 73 per cent, instead of 34, belong to type *Q*. But 9 per cent of all the orbits, as is shown by the last column, are consigned to the subtypes. These orbits seem to be in a state of transition from one of the five main types to another. This must necessarily occur if a comet of one type chances to pass near to some planet other than that to which it properly owes allegiance.

In Table LXIII the comets are classified by types with regard to their brilliancy, the arrangement being similar to that of Table XXXVIII, p. 187. But little additional description is therefore necessary. The table refers solely to the comets of the nineteenth century. Both appearances and reappearances are included in types *N* and *O*. Thus Encke's comet was visible to the naked eye seven times, twice with brilliancy *III*, and five times with brilliancy *IV*. Under type *N*, Biela's, De Vico's and Holmes' comets were each visible once, under brilliancy *IV*, making 10 in all as shown in the last column but two. Under type *O*, Pons' was the only comet visible twice to the naked eye.

Of the 5 brightest comets, 3, or 60 per cent, as shown by the third and fourth columns, belonged to type *Q*. This, as shown by the fifth column, is 4 per cent of the 67 comets belonging to this type. As the aphelion distances of the types recede from the Sun, and the returns consequently occur at larger intervals, the percentage of bright comets steadily increases. Thus, as shown in the last column, while only 11 per cent of the comets belonging to type *N* were visible to the naked eye, 30 per cent of types *O* and *P* were visible, 35 per cent of type *Q*, and 39 per cent of type *R*.

TABLE LXIII.

TYPES OF COMETS ACCORDING TO THEIR BRILLIANCY.

Type.	No.	I.	Pc.	Pc.	II.	Pc.	Pc.	III.	Pc.	Pc.	IV.	Pc.	Pc.	Total.	Pc.	Pc.
N	91							2	6	2	8	26	9	10	12	11
O	17				2	22	12	2	6	12	1	3	6	5	6	30
P	23	1	20	4	1	11	4	3	8	13	2	6	9	7	9	30
Q	67	3	60	4	3	34	4	10.5	29	16	7	23	11	23.5	29	35
R	48				1	11	2	10.5	29	22	7	23	15	18.5	23	39
—	50				2	22	4	3	8	6	5	16	10	10	12	20
Sub.	16	1	20	6				5	14	31	1	3	6	7	9	43
	312	5	100	2	9	100	3	36	100	11	31	100	10	81	100	26

The information regarding the comets of classes *H* and *I* is so indefinite that they could not safely be classified in any type, and they are therefore indicated by a dash in the first column. The two of them entered under grade *II*, belonging to class *I*, undoubtedly belong to type *Q*, however, since their orbits practically coincide with that of the great comet of 1882. The remaining 48 belong to class *H*, consisting chiefly of faint comets, whose orbits for this reason are not well known. But 20 per cent of them, as shown by the last column, were visible to the naked eye. Perhaps the most unexpected fact brought out by the table is the high average brilliancy of the comets belonging to the subtypes. A little less than one half of them were naked eye comets.

In Table LXIV a summary is given of the main facts deduced hitherto relating to the five types. Except in the last four columns the data derived for each type are based on all the orbits associated with it, every class being included according to its weight. The first two columns give the type considered and the total weighted number of orbits associated with it. These figures are taken from Tables LIII, LVII, and LIX. The third column gives the displacement of the reference circle in a direction perpendicular to its plane. The fourth column gives the ratio to one another of the sums of the deviations on the two sides of the assumed reference plane, from which the displacement of the finally adopted reference plane is computed. The fifth column indicates the minimum number of orbits that would have to be rejected in order to change the sign of the displacement. This number happens to be 4 in each case. The sixth column gives the ratio obtained by dividing the numbers in the fifth column by those in the second. The seventh column gives the average deviation of the aphelia from the assumed reference circle. A consideration of the results given in these seven columns leads us to believe that the displacements found for types *N* and *O* are real, but that the displacements found for types *P*, *Q*, and *R*, may be accidental. The test of the accuracy of this conclusion will be, that as time passes, and more and more orbits are computed, we shall find that the displacement for the first two types remains nearly constant, while the displacement for the last three approaches zero. The eighth and ninth columns give the inclinations and nodes of the adopted reference circles.

TABLE LXIV.

DISPLACEMENTS AND DENSITIES.

Type.	No.	Disp.	Ratio.	Com.	Ratio.	Av. Dev.	Inc.	Node.	Class.	Within.	Without.	Ratio.
		°				°	°	°				
N	34	− 1.0	1.70	4	.12	3.5	0.0	—	*A*	200	0	∞
O	12	−10.0	3.37	4	.33	16.4	0.0	—	*B*	21	8	2.6
P	19.75	+ 2.8	1.91	4	.20	8.9	37.2	351.0	*C*	44	8	5.5
Q	60.5	+ 1.1	1.31	4	.07	8.2	86.0	93.4	*D, G*	109	14, (28)	7.8, (3.9)
R	60.25	+ 1.1	1.27	4	.07	9.4	26.0	234.5	*E, F*	179	34	5.0

The tenth column gives the classes which are exclusively considered in the last three columns. These columns give the density of the aphelia within and without the middle zone of the sphere, and their ratio. In the case of type *N* the boundaries of this zone lie but 10° on either side of the reference circle. In the case of all the other types the distance adopted is 20°. The densities may be looked upon as representing the total number of comets of the given aphelion distance, corresponding to the type considered, that would be known were the whole sphere covered with aphelia as densely as is the zone in question. These densities are therefore increasing every year as more and more orbits are recognized.

The small value of the ratio given in the last column for type *O* is due to the small masses of the controlling planets, and the powerful interference of planet *Q*. Two values are given in the last two columns for type *Q*. The first of these values excludes all the orbits which probably properly belong to type *R*, and are indicated by dots crossed by lines in Figure 40. The second figure includes all the orbits belonging to classes *D* and *G* found outside the middle zone. The truth undoubtedly lies between these two results, though probably nearer the first. For all three types, *P*, *Q*, and *R*, the ratio in the last column should lie between 5 and 6. To express this matter verbally we may say that *for a given class of orbits, as determined by their aphelion distance, there are at least five comets within the appropriate cometary belt per unit area of sky, for every one outside of it.* In Chapter XIV we found that for class *C*, type *P*, the chance that the distribution was due merely to accident was about as 1 to 1200. On account of the larger number of aphelia involved in the other types these figures would attain a still larger ratio. The chance of its occurring three times in succession by mere accident therefore is quite out of the question.

TABLE LXV.

DISTRIBUTION WITHIN THE COMETARY BELTS.

Dist.	Lat.	*A.*	Dist.	Lat.	*B.*	Dist.	*C.*	Total.	*D.G.*	Total.	*E.F.*	Total.
° ′	° ′		°	°		°						
+11 15	+10 15					+22.5						
		1					0	1	2	2	2	2
+ 8 45	+ 7 45		+35	+25		+17.5						
		0			2		1	3	1	7	6	10
+ 6 15	+ 5 15		+25	+15		+12.5						
		6			1		2	4	5	12	7	10
+ 3 45	+ 2 45		+15	+ 5		+ 7.5						
		7			2		2	3	6	18	10	11
+ 1 15	+ 0 15		+ 5	− 5		+ 2.5						
		6			2		3	4	5	11	9	9
− 1 15	− 2 15		− 5	−15		− 2.5						
		7			2		2	4	7	16	10	10
− 3 45	− 4 45		−15	−25		− 7.5						
		5			2		2	5	8	17	8	11
− 6 15	− 7 15		−25	−35		−12.5						
		2			1		2	2	2	10	5	5
− 8 45	− 9 45		−35	−45		−17.5						
		0					1	2	1	4	4	5
−11 15	−12 15					−22.5						

We will now consider briefly the distribution of the aphelia within the cometary belts. In Table LXV the five types are considered separately. The first and fourth columns give distances on either side of the reference circles, the second and fifth the corresponding latitudes for types *N* and *O*, and the third and sixth the number of aphelia found between them. The seventh column, which applies to the last three types, gives the angular distances on either side of the major axes of the adopted reference circles represented in Figures 38, 40, 41, and 43. The eighth, tenth, and twelfth columns give the number of aphelia found between them for each type, in the classes indicated at the heads of these columns. The ninth, eleventh, and last columns give the total number of aphelia of each type for all classes.

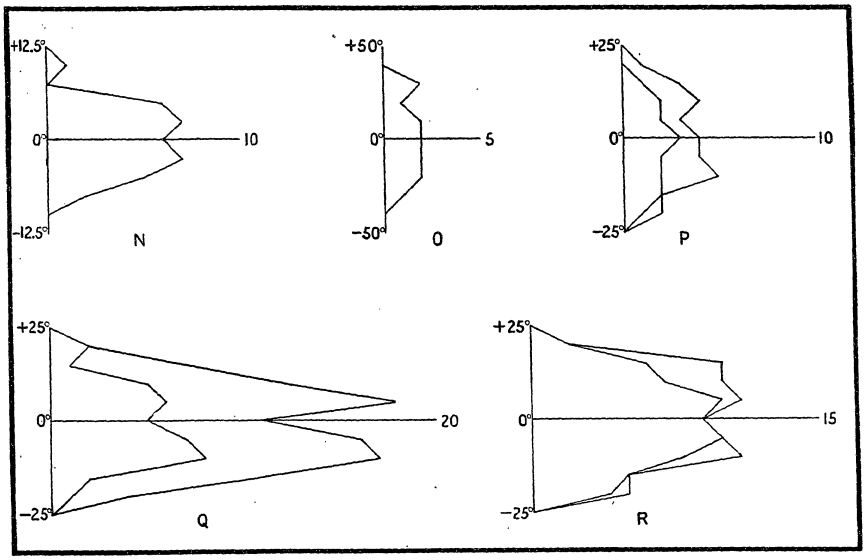

FIG. 47.

These results are represented graphically in Figure 47, where the ordinates give the distances measured from the reference circles, and the abscissas give the number of comets taken from the table. It will be noticed that the ordinates of type *N* are on double the scale, and of type *O* on half the scale, of the ordinates of the remaining types. The most marked condensation towards the reference circle therefore, occurs in the case of type *N*. This is due in part to the great mass of Jupiter, and in part to the small linear distances separating it from its associated aphelia. As might have been expected, the next most marked condensation, as is indicated by its conical form, is found in the case of the massive planet *Q*. Next follows *R*. This planet is doubtless many times more massive than Jupiter, but the linear distances separating the aphelia in its vicinity are so great, that the process of condensation must be slow, and moreover the aphelia are constantly perturbed by planet *Q*. Planet *P* has somewhat less control over its associated comets, indicating a lesser mass, and the planets of type *O* least of all. The deep notch shown at ordinate zero in the curves for type *Q* is well marked. It shows also in type *N* and again in type *R*, but does not appear in the case of the other types. It is perhaps due merely to accident, but we have already seen, p. 215, that such a notch might be produced in the case of type *N* by the action of planet *Q*, and similarly the notch in type *Q* might have been produced by planet *R*, although there is no reason to suppose that the latter is located near the pole of the reference circle of type *Q* at the present time. In closing it may be remarked, had the approximate value of the node of type *Q* been more accurately chosen, the concentration of the aphelia of this type about the reference circle would have been still further increased.

CHAPTER XVI

Cometary Groups. Identification of Comets.

All the aphelia of all the classes, except *A*, *B*, *a*, and *b*, have been plotted on the four charts, Figures 48, 49, 50, and 51. For convenience of reference the polar and ecliptic charts are made to overlap between latitudes 40° and 50°. The bright comets of grades *I* and *II* are indicated by circles, those of grades *III* and *IV* by crosses, while the telescopic comets are indicated by dots.

The reason for omitting all the periodic comets is that on account of their large perturbations by their controlling planets, due to their frequent close approach to them, they are peculiarly liable to suffer a change in their elements and in the positions of their aphelia. Thus, if we follow Clausen, Olbers, and Winnecke and identify the moderately bright comet of 1766, number 100, with Winnecke's comet, whose appearance in 1909 is indicated by number 484, we find that in the mean time its aphelion had moved in longitude through 19°, the most of this change having taken place prior to 1819. Similarly, if we follow Le Verrier and Brünnow, and identify number 64, of 1678, with De Vico's comet number 201, of 1844, we find that in the mean time its aphelion had also shifted 19° in longitude. The next largest changes found are those of d'Arrest's and Tuttle's comets, each of which has shifted about 4°. The positions of all the aphelia of all the periodic comets which have returned are given in Table L, p. 214, and their latest positions are plotted in Figures 33 and 34.

A glance at the two ecliptic charts, Figures 50 and 51, enables us to trace a faintly marked line of maximum density inclined slightly less than 20° to the ecliptic, whose descending node is in longitude 50° and its ascending in longitude 230°. It follows approximately the reference circle of type *R*. Similarly an examination of the polar charts shows a condensation of the aphelia corresponding to type *Q* lying approximately along the meridian of 90° and 270°. It passes to the left of the northern pole, and to the right of the southern one.

The aphelia show a tendency sometimes to associate themselves in groups, the most striking instance being where the circles representing the four bright comets, numbers 61, 198, 339, and 353, overlap near longitude 101°, latitude $-35°$. The elements of the orbits of these four comets are all similar, and it is generally agreed that they are related to one another, the four having formed originally a single body. Comets having identical aphelia may or may not have similar orbits. Thus the aphelia of orbits 72 and 187 located in longitude 97°, latitude $-5°$, Figure 50, are nearly coincident in position, but for the former we find $\omega = 151°$, $\Omega = 66°$, $i = 169°$, and for the latter $\omega = 50°$, $\Omega = 227°$, and $i = 6°$. A mere coincidence of the aphelia

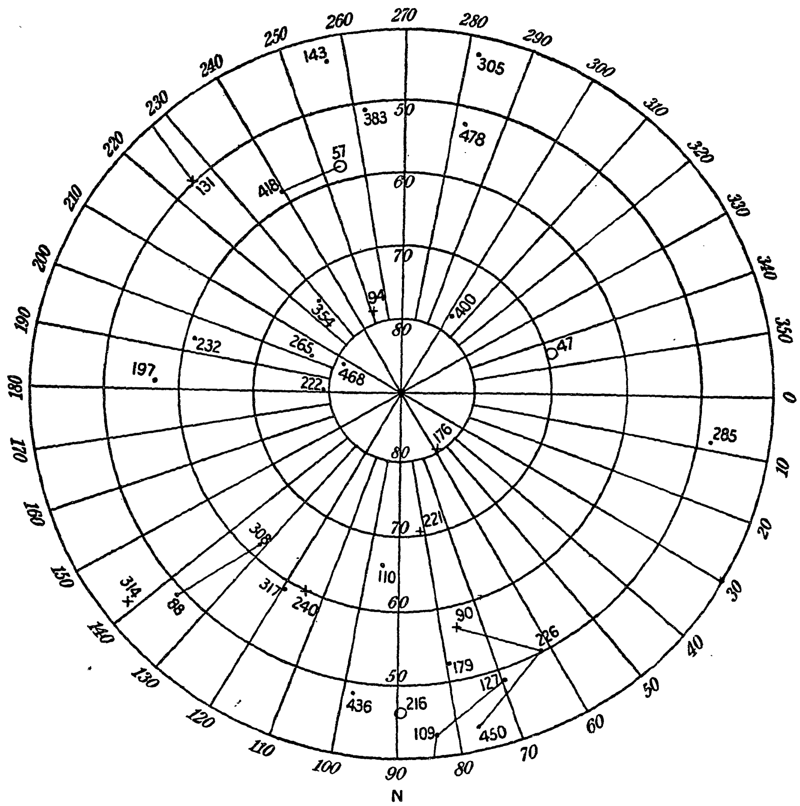

FIG. 48.

where the orbital elements are unlike cannot be taken as evidence of original association of the two bodies. On the other hand, as we have already seen, there are at least two cases among the periodic comets where the position of a cometary aphelion seems to have been shifted by planetary action through an angle of nearly 20° within less than 200 years. When, however, we find both the aphelia and the elements in close coincidence, we may feel reasonably assured that the two comets were originally one, and have only separated within a comparatively recent time astronomically speaking, that is to say, for instance, within a period of perhaps a million years.

In order to facilitate the selection of those comets belonging to the same group, a preliminary table was prepared, in which they were arranged more or less in the order of the longitudes of their aphelia, attention being paid at the same time, however, to their latitudes, so that those aphelia lying near one another in the sky should also be near one another in the table. The columns following the comet's number gave its year and Roman numeral, the class to which it belonged, a mark indicating the brightness of the comet,

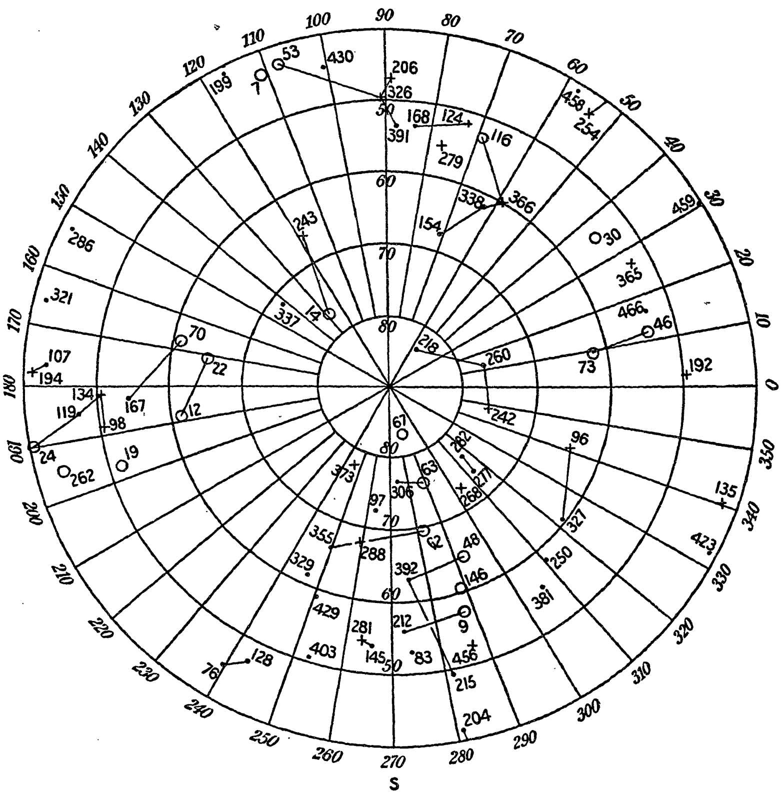

FIG. 49.

the longitude and latitude of its aphelion, the argument of its perihelion, the longitude of its node, its inclination, the logarithm of its perihelion distance, and, finally, a number indicating the group of comets with which it seemed to be associated. Casting one's eye down the two columns containing the argument of the perihelion and the longitude of the node, it was soon found to be a frequent occurrence for two or three orbits whose aphelia lay in the same part of the sky to have similar values in these columns, while all the rest of the orbits in the vicinity were quite unlike them. If an examination of the other columns likewise showed a similarity of the elements, these orbits were grouped together.

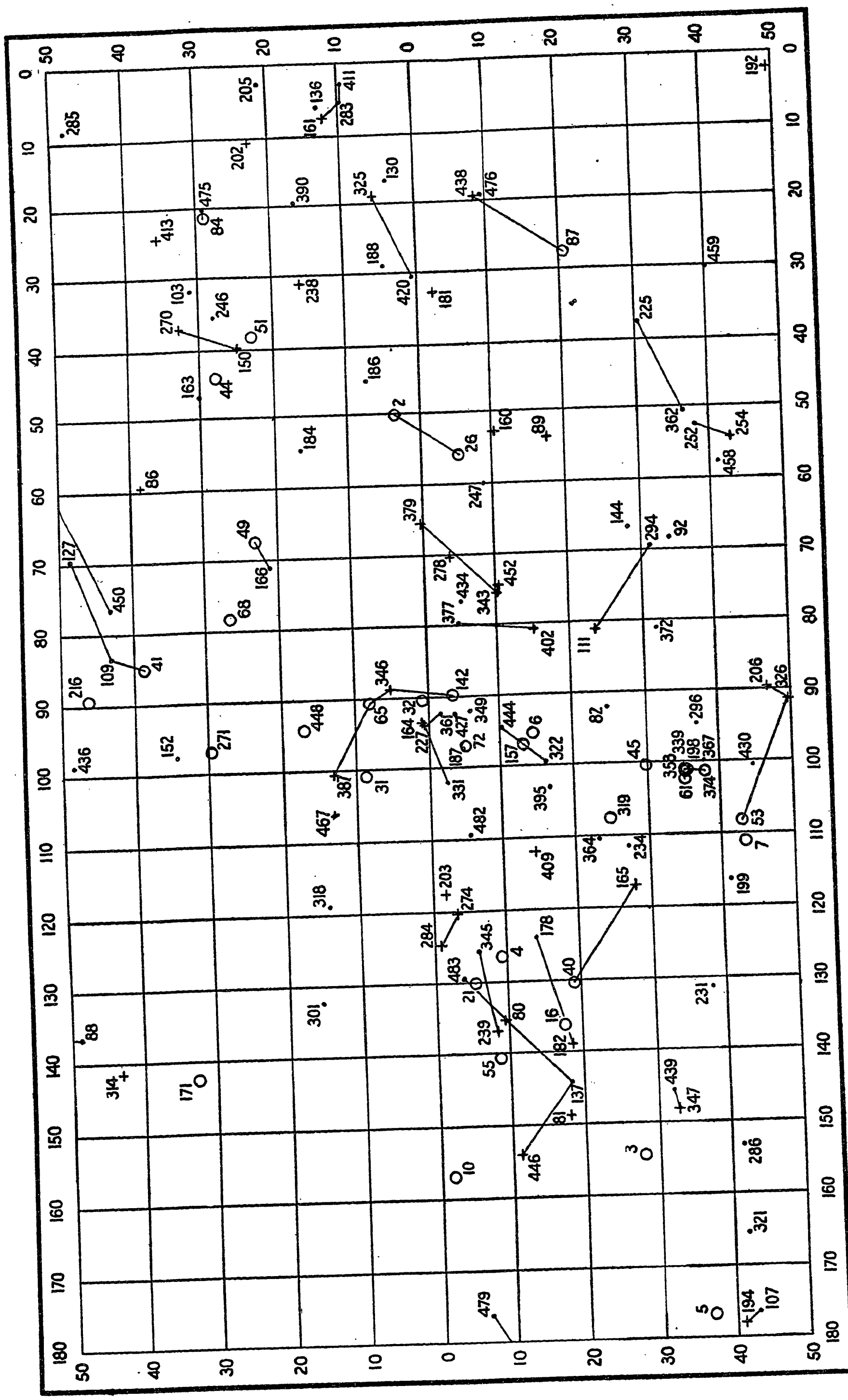

FIG. 50.

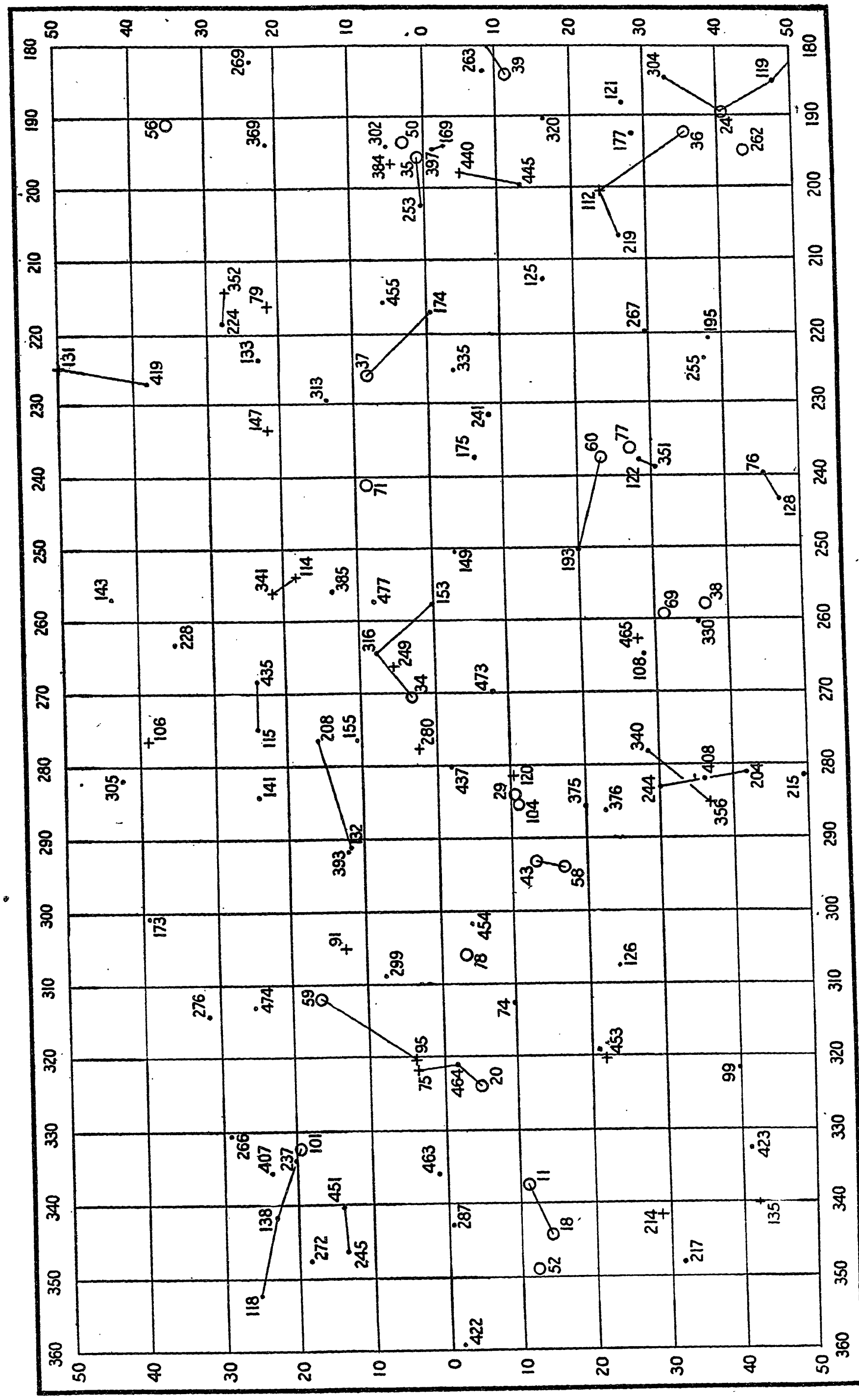

FIG. 51.

These groups are compiled in Table LXVI, the first four columns giving the number of the comet, its year and designation, its class, and its brightness. The fifth column gives the longitude of its aphelion, and beneath each group the mean value. The sixth column gives the deviation from the mean, and the seventh and eighth the latitude of its aphelion and deviation. The next eight columns give the elements of the comet's orbit and their deviations from the mean values. The last column gives the number assigned to the group of orbits, the numbers being arranged in the order of the mean longitudes of the aphelia. The comets in each group are arranged in chronological order.

In order to decide which comets should be grouped together, it was necessary to set some arbitrary bounds to the range of their aphelia, and it was concluded to include in the table only those groups the distance of the individual aphelia of which did not exceed 15° from their nearest companion. Orbits whose aphelia were farther apart than this figure seldom resembled one another, which indicated that the distance adopted should not be increased. On the other hand, when two such distant aphelia did occur, an aphelion of a later comet with similar elements would sometimes fall between them reducing the distance between the individuals of the group, thus indicating their relation to one another, and showing that the adopted value of 15° should not be diminished. A case in point was orbit number 138, of group 65, lying in longitude 342°, latitude +23°. See Figure 51. Such related aphelia are joined on the charts by straight lines. Owing to the impossibility of accurately representing a spherical surface on a plane, all of these lines are slightly too long. On the ecliptic charts the ratio of these lengths in longitude, for latitude 0° is 1.000; for latitude 10°, it is 1.015; for latitude 20°, 1.064; for 30°, 1.155; for 40°, 1.306; and for 50°, 1.556. On the polar charts the ratios are smaller. For latitude 40°, it is 1.139; for 50°, 1.086; for 60°, 1.047; for 70°, 1.020; for 80°, 1.005; and for 90°, 1.000.

In order to decide whether the elements of two orbits resembled one another, it was proposed to set another arbitrary limit to the deviations that their elements might exhibit from their mean value. At first it was decided to make this limit 10°, but it was soon found that while this figure might properly serve a general purpose, there were so many instances where it was exceeded among comets obviously related, that a larger number should be chosen. It was finally concluded that it was impossible to adopt any definite reasonable figure to include all cases, without making the number of exceptions nearly equal to the number of cases that came within the rule.

Thus in group 23 which includes the bright comets of 1668, 1843, 1880, and 1882, which are now universally attributed to one group, an examination of the table shows that the difference between the longitudes of the perihelia for the last two comets amounts to 16°, and between the longitudes of their nodes to 20°. If we include in the group, as is also customary, the bright southern comet of 1887, the deviations from the mean in these cases will largely exceed 10°. These five comets all have phenominally short perihelion distances which makes the resemblance between their orbits conspicuous.

Sometimes considerations besides similarity of their elements confirm the relationship between comets. Thus the four naked-eye comets of group 19, longitude 92°, latitude +6°, all belong to class *D*. Their orbits, therefore, resemble one another at their computed aphelia to an unusual extent. The first of them, that of 1680, had an extremely short perihelion distance, while in the case of the others this quantity was of normal length. The other elements of this comet strongly resemble those of the third member of the group. A comparatively slight perturbation at a considerable distance from the Sun will materially affect log q. Thus a deviation of 10° at the distance of Neptune would send a comet, whose original perihelion distance was unity, directly into the Sun. At greater distances the required deviation would be much smaller. In selecting the comets of a particular group considerable deviations in log q may therefore be permitted, especially if q is appreciably less than unity.

A very suggestive indication of relationship between the different members of a group exists when the comets reach perihelion within a year or two of one another, thus indicating that they have not yet had time to separate very far in their orbits. This implies that they probably belong to the same class and have about the same aphelion distance. Examples of this occur in groups 10 and 60.

When the aphelia are situated near the pole, as in the case of group 3, large differences in their longitudes are of little consequence, as is well exhibited on the chart. Groups in which the aphelia are close together and the inclination small may have very similar orbits even when the other elements are unlike. Groups 24, 42, and 62 illustrate this statement. Grouping of this kind can be studied most readily by applying graphical methods to each particular case.

It will be noticed that some groups are much more prolific than others, thus groups 23 and 36 consist of five members each, groups 11, 19, 20, and

30 each contain four members, while two or three members constitute the great majority of the groups. Omitting the first member of group 36, which arrived as early as 1337, we find that the mean interval between arrivals in group 11 was 34 years, in group 36, 35 years, in group 23, 55 years, in group 30, 60 years, in group 19, 70 years, and in group 20, 98 years. The most remarkable of these groups is number 23, already mentioned. All five of its members belonged to the brightest of the three grades. The next most remarkable is group 19, two of whose members were of the brightest grade, and two intermediate. As far as can be determined from their members group 30 belongs to type *R*, and group 36 to subtype *r*, but the other four groups all belong to type *Q*.

TABLE LXVI.

COMETARY GROUPS.

No.	Year.	Cl.	Br.	Long.	Dev.	Lat.	Dev.	ω	Dev.	☊	Dev.	*i*.	Dev.	log *q*.	Dev.	Gr.
				°	°	°	°	°	°	°	°	°	°			
161	1822 I	*H*	.	8	+ 3	+12	+2	345	−1	177	+1	126	−4	9.70	−0.10	
283	1864 I	*H*	.	5	+ 0	+10	0	346	0	175	−1	135	+5	9.80	0.00	
411	1893 IV	*D*	.	3	− 2	+ 9	−1	347	+1	175	−1	130	0	9.91	+0.11	
				5		+10		346		176		130		9.80		1
46	1558	*I*	0	12	+ 1	−54	+4	120	+5	335	+7	111	+1	9.45	−0.21	
73	1699	*I*	0	10	− 1	−62	−4	110	−5	322	−6	109	−1	9.87	+0.21	
				11		−58		115		328		110		9.66		2
218	1847 III	*E*	.	56	+37	−83	−4	92	−6	338	+12	97	−1	0.25	+0.44	
242	1854 II	*H*	+	348	−31	−76	+3	102	+4	315	−11	97	−1	9.44	−0.37	
260	1858 IV	*H*	.	13	− 6	−77	+2	99	+1	325	− 1	100	+2	9.74	−0.07	
				19		−79		98		326		98		9.81		3
84	1742	*F*	0	22	+ 1	+29	0	328	0	186	−2	113	+1	9.88	−0.04	
475	1907 II	*F*	.	20	− 1	+29	0	329	+1	189	+1	110	−2	9.97	+0.05	
				21		+29		328		188		112		9.92		4
87	1744	*F*	0	28	+ 4	−20	−8	152	−5	46	+6	47	+9	9.35	−0.13	
438	1898 IX	*H*	+	19	− 5	− 8	+6	162	+5	34	−6	29	−9	9.62	+0.14	
				24		−14		157		40		38		9.48		5

No.	Year.	Cl.	Br.	Long.	Dev.	Lat.	Dev.	ω	Dev.	☊	Dev.	*i*.	Dev.	log *q*.	Dev.	Gr.
				°	°	°	°	°	°	°	°	°	°			
325	1877 I	*F*	+	19	−5	+ 6	+2	347	−5	187	−11	153	−1	9.91	+0.07	
420	1896 I	*H*	.	30	+6	+ 1	−3	358	+6	208	+10	155	+1	9.77	−0.07	
				24		+ 4		352		198		154		9.84		6
150	1813 II	*H*	+	40	+2	+25	−4	205	−4	43	+7	99	+ 9	0.08	+0.06	
270	1861 I	*C*	+	37	−1	+33	+4	213	+4	30	−6	80	−10	9.96	−0.06	
				38		+29		209		36		90		0.02		7
225	1849 II	*G*	.	38	−6	−30	+3	33	−5	203	−1	67	+4	0.06	+0.10	
362	1885 III	*C*	.	50	+6	−36	−3	43	+5	205	+1	59	−4	9.87	−0.09	
				44		−33		38		204		63		9.96		8
2	−137	*I*	0	50	−3	+ 3	+4	350	−10	220	−8	160	+6	0.00	+0.16	
26	1362	*I*	0	56	+3	− 5	−4	10	+10	237	+9	148	−6	9.67	−0.17	
				53		− 1		0		228		154		9.84		9
252	1857 III	*H*	.	52	−1	−38	+2	134	+4	24	+4	121	−1	9.57	−0.09	
254	1857 V	*D*	+	54	+1	−43	−3	125	−5	15	−5	124	+2	9.75	+0.09	
				53		−40		130		20		122		9.66		10
116	1785 II	*D*	0	69	+4	−53	+8	127	+9	65	− 7	93	+7	9.63	−0.22	
154	1818 II	*F*	.	71	+6	−68	−7	112	−6	70	− 2	90	+4	0.08	+0.23	
338	1879 V	*F*	.	62	−3	−62	−1	115	−3	87	+15	77	−9	0.00	+0.15	
366	1886 II	*G*	+	59	−6	−60	+1	120	+2	68	− 4	84	−2	9.68	−0.17	
				65		−61		118		72		86		9.85		11
49	1582	*I*	0	68	−2	+23	+1	332	−1	227	−3	119	−3	9.23	−0.27	
166	1824 I	*H*	.	71	+1	+21	−1	334	+1	234	+4	125	+3	9.77	+0.27	
				70		+22		333		230		122		9.50		12
343	1880 V	*F*	+	75	+5	−10	−5	12	+6	249	+2	61	+ 9	9.82	−0.01	
379	1888 I	*D*	+	66	−4	0	+5	0	−6	245	−2	42	−10	9.84	+0.01	
				70		− 5		6		247		52		9.83		13
90	1748 II	*H*	+	76	+5	+57	+7	246	+9	33	− 6	67	0	9.80	0.00	
226	1849 III	*D*	.	62	−9	+50	0	237	0	31	− 8	67	0	9.95	+0.15	
450	1902 I	*H*	.	76	+5	+43	−7	228	−9	52	+13	67	0	9.65	−0.15	
				71		+50		237		39		67		9.80		14

No.	Year.	Cl.	Br.	Long.	Dev.	Lat.	Dev.	ω	Dev.	☊	Dev.	i.	Dev.	log q.	Dev.	Gr.
				°	°	°	°	°	°	°	°	°	°			
111	1781 I	*H*	+	81	+ 6	−24	+4	156	+ 4	83	+ 9	82	− 8	9.89	+0.19	
294	1867 III	*F*	.	69	− 6	−31	−3	149	− 3	65	− 9	97	+ 7	9.52	−0.18	
				75		−28		152		74		90		9.70		15
124	1790 III	*F*	+	73	− 5	−51	+1	119	+ 6	33	+ 7	116	− 4	9.90	−0.02	
168	1825 I	*D*	.	83	+ 5	−53	−1	107	− 6	20	− 6	123	+ 3	9.95	+0.03	
				78		−52		113		26		120		9.92		16
41	1506	*I*	0	85	+ 6	+39	−5	242	+ 2	133	+11	135	+ 8	9.59	+0.20	
109	1780 I	*E*	.	83	+ 4	+43	−1	237	− 3	124	+ 2	126	− 1	8.98	−0.41	
127	1793 I	*H*	.	70	− 9	+49	+5	240	0	108	−14	120	− 7	9.60	+0.21	
				79		+44		240		122		127		9.39		17
377	1887 IV	*D*	.	80	0	− 5	+5	15	− 5	245	+ 2	18	−10	0.14	+0.06	
402	1892 I	*E*	+	81	+ 1	−15	−5	25	+ 5	241	− 2	39	+11	0.01	−0.07	
				80		−10		20		243		28		0.08		18
65	1680	*D*	0	91	− 1	+ 8	+ 2	351	− 3	272	− 2	61	− 2	7.79	−1.58	
142	1807	*D*	0	90	− 2	− 4	−10	4	+10	267	− 7	63	0	9.81	+0.44	
346	1881 III	*D*	+	89	− 3	+ 5	− 1	354	0	271	− 3	63	0	9.87	+0.50	
387	1889 IV	*D*	+	100	+ 8	+13	+ 7	346	− 8	286	+12	66	+ 3	0.02	+0.65	
				92		+ 6		354		274		63		9.37		19
53	1596	*I*	0	108	+14	−43	+5	59	− 8	330	+ 4	128	+ 3	9.75	−0.15	
206	1845 III	*H*	+	90	− 4	−47	+1	76	+ 9	338	+12	131	+ 6	9.60	−0.30	
326	1877 II	*E*	+	91	− 3	−50	−2	63	− 4	317	− 9	121	− 4	9.98	+0.08	
391	1890 II	*G*	.	88	− 6	−53	−5	69	+ 2	320	− 6	121	− 4	0.28	+0.38	
				94		−48		67		326		125		9.90		20
227	1850 I	*E*	+	94	− 2	3	+1	181	+ 2	93	− 3	68	− 8	0.03	−0.16	
331	1878 I	*H*	.	102	+ 6	− 2	−1	178	− 1	102	+ 6	78	+ 2	0.14	−0.05	
361	1885 II	*G*	.	92	− 4	− 2	−1	178	− 1	92	− 4	81	+ 5	0.40	+0.21	
				96		− 1		179		96		76		0.19		21
157	1819 II	*F*	0	97	0	−13	0	13	0	274	− 2	81	− 5	9.53	−0.30	
322	1874 VI	*F*	.	100	+ 3	−16	−3	16	+ 3	282	+ 6	99	+13	9.71	−0.12	
444	1899 V	*H*	.	95	− 2	−10	+3	11	− 2	272	− 4	77	− 9	0.25	+0.42	
				97		−13		13		276		86		9.83		22

No.	Year.	Cl.	Br.	Long.	Dev.	Lat.	Dev.	ω	Dev.	☊	Dev.	i.	Dev.	log q.	Dev.	Gr.
				°	°	°	°	°	°	°	°	°	°			
61	1668	I	0	102	0	−35	+1	80	+ 5	357	+ 6	144	+ 3	7.68	−0.13	
198	1843 I	C	0	101	− 1	−35	+1	83	+ 8	1	+ 10	144	+ 3	7.74	−0.07	
339	1880 I	I	0	101	− 1	−35	+1	86	+11	6	+ 15	145	+ 4	7.74	−0.07	
353	1882 II	D	0	102	0	−35	+1	70	− 5	346	− 5	142	+ 1	7.89	+0.08	
374	1887 I	I	0	102	0	−38	−2	58	−17	325	− 26	128	−13	7.99	+0.18	
				102		−36		75		351		141		7.81		23
274	1862 II	H	+	120	− 2	− 4	−2	27	−62	327	+116	172	− 3	9.99	+0.01	
284	1864 II	D	+	125	+ 3	− 1	+1	151	+62	95	−116	178	+ 3	9.96	−0.02	
				122		− 2		89		211		175		9.98		24
40	1500	I	0	130	+ 6	−19	+5	20	− 4	310	+ 4	105	+ 1	0.15	+0.39	
165	1823	F	+	117	− 7	−28	−4	28	+ 4	303	− 3	104	0	9.36	−0.40	
				124		−24		24		306		104		9.76		25
14	961	I	0	130	+ 6	−77	−5	82	+ 4	356	+ 4	95	− 7	9.73	−0.04	
243	1854 III	F	+	119	− 5	−66	+6	75	− 3	348	− 4	109	+ 7	9.81	+0.04	
				124		−72		78		352		102		9.77		26
239	1853 III	G	+	137	+ 6	− 8	−1	170	− 2	141	+ 7	62	− 8	9.49	−0.14	
345	1881 II	H	.	125	− 6	− 6	+1	174	+ 2	126	− 8	78	+ 8	9.77	+0.14	
				131		− 7		172		134		70		9.63		27
178	1827 II	H	.	124	− 8	−14	+2	21	− 3	318	−10	136	0	9.91	+0.41	
182	1830 II	F	+	139	+ 7	−19	−3	27	+ 3	338	+10	135	− 1	9.10	−0.40	
				132		−16		24		328		136		9.50		28
88	1747	F	.	137	+ 3	+49	−7	230	− 6	147	0	101	+ 1	0.34	+0.25	
308	1871 IV	D	.	132	− 2	+62	+6	243	+ 7	147	0	98	− 2	9.84	−0.25	
				134		+56		236		147		100		0.09		29
80	1729	F	+	135	− 6	−10	+1	10	− 2	311	− 3	77	+15	0.61	+0.46	
137	1802	H	.	144	+ 3	−18	−7	22	+10	310	− 4	57	− 5	0.04	−0.11	
446	1900 II	G	+	154	+13	−11	0	12	0	328	+14	63	+ 1	0.01	−0.14	
483	1909 I	H	.	130	−11	− 4	+7	5	− 7	307	− 7	53	− 9	9.93	−0.22	
				141		−11		12		314		62		0.15		30

No.	Year.	Cl.	Br.	Long.	Dev.	Lat.	Dev.	ω	Dev.	☊	Dev.	i.	Dev.	log q.	Dev.	Gr.
				°	°	°	°	°	°	°	°	°	°			
347	1881 IV	*F*	+	148	+1	−33	− 1	122	−1	97	+ 1	140	0	9.80	−0.04	
439	1898 X	*E*	.	146	−1	−32	0	124	+1	96	0	140	0	9.88	+0.04	
				147		−32		123		96		140		9.84		31
70	1689	*I*	0	167	−8	−61	− 3	78	−4	279	0	63	+ 4	8.81	−0.61	
167	1824 II	*F*	.	183	+8	−54	+ 4	85	+3	279	0	55	− 4	0.02	+0.60	
				175		−58		82		279		59		9.42		32
107	1774	*G*	.	176	−1	−43	− 1	137	−1	181	− 3	83	+ 1	0.16	+0.14	
194	1840 III	*H*	+	178	+1	−41	+ 1	138	0	186	+ 2	80	− 2	9.87	−0.15	
				177		−42		138		184		82		0.02		33
12	770	*I*	0	189	+9	−62	+ 2	87	−9	89	− 9	120	+ 4	9.78	+0.14	
221	299	*I*	0	171	−9	−65	− 1	104	+8	107	+ 9	111	− 5	9.50	−0.14	
				180		−64		96		98		116		9.64		34
39	1499	*I*	0	184	+4	−11	− 2	34	−2	326	+ 4	21	+ 5	9.98	−0.30	
479	1907 VI	*H*	.	176	−4	− 7	+ 2	39	+3	317	− 5	10	− 6	0.58	+0.30	
				180		− 9		36		322		16		0.28		35
24	1337	*I*	0	190	+4	−40	+ 4	91	−5	93	−10	140	+ 5	9.92	+0.18	
98	1764	*H*	+	188	+2	−50	− 6	105	+9	120	+17	127	− 8	9.74	0.00	
119	1787	*F*	.	185	−1	−47	− 3	99	+3	107	+ 4	132	− 3	9.54	−0.20	
134	1799 I	*F*	+	182	−4	−51	− 7	96	0	99	− 4	129	− 6	9.92	+0.18	
304	1870 IV	*H*	.	184	−2	−33	+11	91	−5	95	− 8	147	+12	9.59	−0.15	
				186		−44		96		103		135		9.74		36
169	1825 II	*H*	.	194	0	− 3	− 1	177	−1	193	− 1	90	−15	9.95	+0.17	
397	1891 I	*F*	.	195	+1	− 1	+ 1	179	+1	194	0	121	+16	9.60	−0.18	
				194		− 2		178		194		105		9.78		37
35	1457 II	*I*	0	196	−3	+ 1	0	185	+2	184	− 8	10	−12	9.88	0.00	
253	1857 IV	*C*	.	202	+3	+ 1	0	181	−2	201	+ 9	33	+11	9.87	−0.01	
				199		+ 1		183		192		22		9.88		38
440	1899 I	*G*	+	198	−1	− 5	+ 4	9	−7	25	− 7	146	0	9.51	−0.31	
445	1900 I	*H*	.	200	+1	−13	− 4	24	+8	40	+ 8	146	0	0.12	+0.30	
				199		− 9		16		32		146		9.82		39

No.	Year.	Cl.	Br.	Long.	Dev.	Lat.	Dev.	ω	Dev.	☊	Dev.	i.	Dev.	log q.	Dev.	Gr.
				°	°	°	°	°	°	°	°	°	°			
36	1468	I	0	192	−8	−35	−7	70	+ 8	71	−4	142	−5	9.92	−0.10	
112	1781 II	H	+	201	+1	−24	+4	61	− 1	77	+2	153	+6	9.98	−0.04	
219	1847 IV	F	.	207	+7	−26	+2	55	− 7	77	+2	147	0	0.17	+0.15	
				200		−28		62		75		147		0.02		40
224	1849 I	F	.	219	+3	+28	0	208	0	215	+5	85	+5	9.98	+0.60	
352	1882 I	E	+	214	−2	+28	0	209	+ 1	205	−5	74	−6	8.78	−0.60	
				216		+28		208		210		80		9.38		41
37	1472	I	0	226	+4	+ 8	+4	246	−61	286	−57	171	−2	9.69	+0.21	
174	1826 III	H	.	217	−5	0	−4	5	+60	40	+57	175	+2	9.27	−0.21	
				222		+ 4		305		343		173		9.48		42
131	1797	H	+	225	−1	+50	+6	280	+ 4	329	+4	129	−7	9.72	+0.22	
419	1895 IV	F	.	227	+1	+38	−6	273	− 3	321	−4	142	+6	9.28	−0.22	
				226		+44		276		325		136		9.50		43
122	1790 I	H	.	238	0	−28	+2	114	− 2	173	−4	150	+2	9.87	−0.20	
351	1881 VIII	C	.	239	+1	−31	−1	118	+ 2	181	+4	145	−3	0.28	+0.21	
				238		−30		116		177		148		0.07		44
76	1706	H	.	240	−2	−45	−1	59	− 5	13	+5	55	+1	9.63	−0.27	
128	1793 II	C	.	243	+1	−47	+1	70	+ 6	2	−6	52	−2	0.17	+0.27	
				242		−46		64		8		54		9.90		45
60	1665	I	0	237	−7	−23	−1	156	0	228	−4	104	−8	9.03	−0.53	
193	1840 II	D	.	250	+6	−20	+2	157	+ 1	237	+5	121	+9	0.09	+0.53	
				244		−22		156		232		112		9.56		46
57	1652	I	0	254	+7	+58	0	300	0	88	+2	79	+1	9.93	0.00	
418	1895 III	H	.	240	−7	+58	0	299	− 1	83	−3	76	−2	9.93	0.00	
				247		+58		300		86		78		9.93		47
114	1784	F	+	254	−1	+18	−2	336	+ 6	57	+6	129	−7	9.85	+0.15	
341	1880 III	F	+	256	+1	+22	+2	323	− 7	45	−6	142	+6	9.55	−0.15	
				255		+20		330		51		136		9.70		48

No.	Year.	Cl.	Br.	Long.	Dev.	Lat.	Dev.	ω	Dev.	☊	Dev.	i.	Dev.	log q.	Dev.	Gr.
				°	°	°	°	°	°	°	°	°	°			
34	1457 I	*I*	0	270	+ 6	+ 3	−1	195	+ 5	250	− 2	13	−13	9.85	0.00	
153	1818 I	*H*	.	257	− 7	0	−4	180	−10	256	+ 4	34	+ 8	9.84	−0.01	
316	1873 VII	*H*	.	264	0	+ 8	+4	196	+ 6	250	− 2	30	+ 4	9.86	+0.01	
				264		+ 4		190		252		26		9.85		49
145	1810	*H*	.	266	+ 2	−54	0	115	− 1	309	+ 2	63	− 1	9.99	+0.05	
281	1863 V	*F*	+	263	− 1	−54	0	116	0	305	− 2	64	0	9.89	−0.05	
				264		−54		116		307		64		9.94		50
62	1672	*I*	0	283	+17	−69	−1	110	0	298	+10	83	+ 3	9.84	−0.02	
355	1883 I	*G*	.	250	−16	−66	+2	111	+ 1	278	−10	78	− 2	9.88	+0.02	
				266		−68		110		288		80		9.86		51
115	1785 I	*H*	.	275	+ 3	+24	0	204	− 1	264	+ 2	70	0	0.06	+0.13	
435	1898 VI	*F*	.	268	− 4	+24	0	206	+ 1	259	− 3	70	0	9.80	−0.13	
				272		+24		205		262		70		9.93		52
9	565	*I*	0	288	+ 8	−57	−1	80	0	160	0	121	− 1	9.92	−0.11	
212	1846 V	*F*	.	272	− 8	−56	0	79	− 1	161	+ 1	122	0	0.14	+0.11	
				280		−56		80		160		122		0.03		53
63	1677	*I*	0	289	+ 7	−76	0	99	+ 1	237	+13	101	− 1	9.49	−0.27	
306	1871 II	*F*	.	274	− 8	−76	0	96	− 2	212	−12	102	0	0.03	+0.27	
				282		−76		98		224		102		9.76		54
340	1880 II	*F*	.	278	− 4	−29	+4	145	+ 3	257	− 3	123	+ 4	0.26	+0.38	
356	1883 II	*H*	+	285	+ 3	−37	−4	139	− 3	264	+ 4	115	− 4	9.49	−0.39	
				282		−33		142		260		119		9.88		55
204	1845 I	*G*	.	281	− 1	−42	−6	115	− 7	337	+ 7	47	+ 3	9.96	+0.03	
244	1854 IV	*C*	.	283	+ 1	−30	+6	130	+ 8	324	− 6	41	− 3	9.90	−0.03	
				282		−36		122		330		44		9.93		56
48	1580	*I*	0	292	+ 9	−65	−6	89	− 1	19	+ 6	65	+ 6	9.78	−0.08	
215	1846 VIII	*F*	.	282	− 1	−50	+9	94	+ 4	5	− 8	50	− 9	9.92	+0.06	
392	1890 III	*H*	.	275	− 8	−63	−4	86	− 4	14	+ 1	63	+ 4	9.88	+0.02	
				283		−59		90		13		59		9.86		57

No.	Year.	Cl.	Br.	Long.	Dev.	Lat.	Dev.	ω	Dev.	☊	Dev.	i.	Dev.	log q.	Dev.	Gr.
				°	°	°	°	°	°	°	°	°	°			
132	1798 I	*H*	.	291	+ 7	+12	−2	343	+ 3	122	+ 6	44	−2	9.69	−0.24	
208	1846 I	*F*	.	277	− 7	+16	+2	338	− 2	111	− 5	47	+1	0.17	+0.24	
				284		+14		340		116		46		9.93		58
43	1532	*I*	0	293	− 1	−13	+2	24	− 4	87	+ 3	33	0	9.72	+0.04	
58	1661	*I*	0	294	0	−17	−2	33	+ 5	82	− 2	33	0	9.65	−0.03	
				294		−15		28		84		33		9.68		59
277	1863 I	*G*	.	314	0	−74	+1	74	− 2	117	+ 6	85	+1	9.90	−0.11	
282	1863 VI	*G*	.	314	0	−76	−1	78	+ 2	105	− 6	83	−1	0.12	+0.11	
				314		−75		76		111		84		0.01		60
59	1664	*F*	0	312	− 4	+16	+6	311	+ 5	81	+ 1	159	−8	0.01	+0.01	
95	1759 III	*H*	+	320	+ 4	+ 4	−6	301	− 5	80	0	175	+8	9.98	−0.02	
				316		+10		306		80		167		0.00		61
20	1231	*I*	0	324	+ 2	− 5	−4	121	− 76	14	− 48	6	+1	9.98	−0.12	
75	1702	*H*	+	322	0	+ 3	+4	310	+113	189	+127	4	−1	9.81	−0.29	
464	1905 IV	*F*	.	321	− 1	− 2	−1	159	− 38	342	− 80	4	−1	0.52	+0.42	
				322		− 1		197		62		5		0.10		62
96	1762	*H*	+	341	+ 9	−64	−2	115	− 1	349	+ 1	86	+4	0.00	0.00	
327	1877 III	*D*	.	323	− 9	−61	+1	117	+ 1	346	− 2	77	−5	0.00	0.00	
				332		−62		116		348		82		0.00		63
11	574	*I*	0	337	− 3	−11	+1	15	− 8	128	+ 1	47	+9	9.98	0.00	
18	1092	*I*	0	344	+ 4	−14	−2	31	+ 8	126	− 1	29	−9	9.97	−0.01	
				340		−12		23		127		38		9.98		64
101	1769	*D*	0	332	−10	+20	−3	329	+ 1	175	− 7	41	−8	9.09	−0.48	
118	1786 II	*F*	.	352	+10	+26	+3	323	− 5	195	+ 13	51	+2	9.60	+0.03	
138	1804	*H*	.	342	0	+23	0	332	+ 4	177	− 5	56	+7	0.03	+0.46	
				342		+23		328		182		49		9.57		65
245	1854 V	*C*	.	346	+ 3	+14	0	287	− 8	238	+ 10	14	−2	0.13	+0.18	
451	1902 II	*H*	.	340	− 3	+14	0	302	+ 7	218	− 10	17	+1	9.77	−0.18	
				343		+14		295		228		16		9.95		66

The relationship between the different cometary orbits was early recognized by astronomers, but it was generally assumed to be due to the return of the same comet. The idea that several comets might be travelling in nearly the same orbit, although suggested by Clausen in 1831, was not generally accepted until the end of the last century. Consequently we find that for nearly two centuries after Halley had successfully demonstrated that the brilliant comet known by his name had appeared on several different occasions at intervals of seventy-five to eighty years, other astronomers continued to endeavor to emulate his example, and find other brilliant comets of long period. Sometimes they were satisfied with only two appearances, an early brilliant comet, and a later faint one. Sometimes they insisted on many more. Thus Laugier and Mauvais identified fourteen comets prior to 1843 as being possibly identical with the great comet that appeared in that year. Of the fourteen there is only one that could by any possibility, according to our present knowledge, be identical with it, and for that the evidence is not satisfactory.

In all, besides the regularly recognized periodic comets, at least 188 different ones have been identified by different astronomers, either as moving in similar orbits, or as having appeared more than once. In addition to Galle's Cometenbahnen and the Vierteljahrschrift, Cooper's and Chambers' catalogues of comets have been consulted by the author for identifications. Since these writers had already consulted Pingré, Olbers, and other early authorities, it seemed unnecessary for the author to do so also. Of these 188 comets we may at once reject 40, because the elements of their orbits are unknown, and their identification depends merely on the year of their perihelion passage or their brightness. One possible exception to this summary treatment may however be mentioned.

Cooper, Markree Catalogue, p. 61, and Miss Clerke, History of Astronomy, 4th ed., p. 350, discuss the possible identity of the comets of −372 and 1843 III. Miss Clerke decides against it, because Aristotle describes the earlier comet as setting in the clear skies of winter "due west nearly at the same time as the Sun." The path of the comet of 1843 was such that save for a couple of hours near perihelion, the whole of its revolution was described in an orbit lying to the south of the ecliptic. It could not therefore set *due* west in winter, when near the Sun. A reference to Pingré, however, shows that Aristotle did not himself use the expression "due west." What he did say was "The first day it (the comet) was not seen, it set before the

Sun. It was observed the second day, the Sun having left it behind him, though at a very small distance." This is exactly what we should have expected had the comet moved in an orbit like that of 1843 III. Pingré is puzzled to know what Aristotle meant by the remark "the first day it was not seen." It would seem fairly obvious that he meant that he did not see it himself, but that some one who was up before sunrise told him of it.

Aristotle then says "But its light (that is its tail) extended like a sort of lane between two rows of trees, and reached a third way across the heavens, so that it was given the name of road. It reached as far as the belt of Orion, where it vanished." An extremely long narrow tail with parallel sides, sweeping around the Sun in a single day, exactly describes the tail of the comet of 1843 III. If the tail was 60° long, the head must have been in the vicinity of 14 *Piscii*, which the Sun would have reached about the middle of February. Allowing for precession this would have been about 15° south of due west. Although the elements of the earlier comet are extremely uncertain, it is worthy of note that assuming them to be correct, its aphelia lies within 30° of that of 1843 III.

An interesting feature of Aristotle's comet is that it is said to have broken in two, and several astronomers have suggested that it was the original not only of the comet of 1843, but of the great comet of 1882 as well. If so, the period of the latter previous to 1882 must have been about 1127 years, — a not impossible value. Kreutz makes it 1245 years. There should have been an intermediate apparition therefore about the year 755. Now, it is recorded in Chambers' second Catalogue of Comets that in 762 "a comet was seen in the east like unto a beam." This description, although inadequate, is characteristic of the comets belonging to this group, whose tails are long, narrow, and comparatively straight.

Similarly the comet of 1843 must have appeared on three previous occasions, at intervals of about 550 years. According to the catalogue we find that in the autumn of 192 "a grand comet 100 cubits long was seen to the south of the sidereal divisions of α and κ Virginis." This description would seem to imply that the length of its tail was its impressive feature, and also that it was a southern comet. In 744 "a great comet was seen in Syria." Since it was great, and was not recorded elsewhere, it would seem to have been visible only in the south. In 1305 "from April 15 to April 21, a long tail was seen." This would indicate a very small perihelion distance, since the head was not visible, and the duration was so short. In fact it was precisely the appearance presented in Europe in March by the great comet of 1843. The invisibility of the head also implies that the comet came from the south. Assuming these three identifications to be correct, the four periods since −372 are 563, 552, 561, and 538 years. The period computed by Kreutz is 515 years. This indicates a gradually shortening period, which is what we should expect.

There is less to be said in favor of the identification of the brilliant comet of 1264 number 21, with that of 1556 number 45, which created so much discussion in the early part of the last century. That they were identical was first suggested by Dunthorne in 1751. Later, Hind investigated the matter, and recently Forbes has identified them with the brilliant comets of 1843, 1880, and 1882. These last three have practically identical elements, so that we shall merely consider the last and best known of them, and compare its elements ω, ☊, i, and log q with those of the great comets of 1264 and 1556.

1264	160°,	141°,	16°,	9.92
1556	101°,	175°,	32°,	9.69
1882	70°,	346°,	142°,	7.89

It is true that the aphelia of 1556 and 1882 lie within 5° of one another, both being distant about 40° from the aphelion of 1264. This fact and the notable brightness of the three are apparently the only reasons for assuming their identity.

Of the 148 comets the elements of whose orbits are known, that were selected by the earlier astronomers, either as identical reappearances, or as comets belonging to the same groups, only 36 are included in Table LXVI. In the other cases, either the elements of their orbits were too widely divergent, or else their aphelia were situated in different parts of the sky. Thus, one well known astronomer considered the comet of 1822 I as identical with the comets of 1590, 1780, and 1797. There were two comets in 1780, but it was hoped to determine the one to which he referred by a comparison of their elements with those of the other comets in the group. Below is given the designations of these five comets and the elements of their orbits, ω, ☊, i, and log q.

1590	308°,	166°,	151°,	9.75
1797	280°,	329°,	129°,	9.72
1822 I	345°,	177°,	126°,	9.70
1780 I	237°,	124°,	126°,	8.98
1780 II	254°,	141°,	108°,	9.71

Except for a general resemblance of the perihelion distance and inclination in some cases, these comets resemble one another about as closely as they would any others that might have been selected at random. Which of the comets of 1780 he intended to include in the group it is now impossible to determine. Similarly, another astronomer conjectured the identity of the comets of 1582 and 1826 V. Investigation shows that the aphelion of the first lies in latitude +23°, that of the second in latitude +80°. They therefore approach the Sun from regions separated by over 57°.

Most of these earlier identifications are omitted by Galle, although even his catalogue contains some that appear to be of very doubtful value. The 36 comets taken from previous identifications that have been accepted in this chapter as perhaps related have been entered in the following 16 groups in the preceding table:—

Group	1, numbers	161, 283, 411	Group	31, numbers	347, 439
"	11 "	116, 366	"	47 "	57, 418
"	14 "	90, 226	"	49 "	153, 316
"	16 "	124, 168	"	50 "	145, 281
"	19 "	142, 346	"	52 "	115, 435
"	20 "	53, 206	"	59 "	43, 58
"	23 "	61, 198, 339, 353, 374	"	61 "	59, 95
"	26 "	14, 243	"	63 "	96, 327

Of these accepted groupings, that of group 20 is probably most open to question, on account of the differences in the longitude of the aphelia between number 53 and the other members of the group. Of the rejected groupings perhaps the most probable relation is that of numbers 82 and 319, whose elements will be found in Table LXVIII at longitudes 92° and 107°. Other rejected groupings, which were less improbable than many, were numbers 163 and 184, number 254 with the group containing numbers 124 and 168, numbers 109 and 179, numbers 40 and 178, numbers 19 and 194, number 38 with the group containing numbers 145 and 281, numbers 59 and 237, and numbers 99 and 214. These rejected groupings are named in the order of the longitudes of their aphelia.

Table LXVII contains only the latest orbits of the periodic comets together with three in sub-class *a*. It has been given the same arrangement as the provisional table described on p. 271.

The majority of the orbits in the preliminary table were found to bear no apparent relation to one another. When this was the case they were

TABLE LXVII.

ELEMENTS OF THE ORBITS OF THE PERIODIC COMETS.

No.	Year.	Cl.	Br.	Long.	Lat.	ω	☊	i.	log q.	Name.
				°	°	°	°	°		
259	1858 III	*A*	.	20	− 8	26	175	20	0.06	
368	1886 IV	*A*	.	51	− 1	177	53	13	0.12	
336	1879 III	*A*	.	59	− 3	159	79	10	0.25	$Tempel_1$
213	1846 VI	*B*	.	63	+10	340	260	31	0.18	
100	1766 II	*a*	+	73	0	177	74	8	9.60	Winnecke
484	1909 II	*A*	.	92	− 2	172	99	18	9.99	"
470	1906 IV	*A*	.	103	− 3	20	264	9	0.23	
190	1835 III	*B*	0	124	−17	111	55	162	9.77	Halley
358	1884 II	*A*	.	127	+ 5	301	5	5	0.11	
460	1904 III	*A*	.	127	+ 1	185	121	13	0.14	$Tempel_2$
428	1897 II	*A*	.	140	− 2	173	146	16	0.07	d'Arrest
64	1678	*a*	.	146	− 1	159	163	3	0.06	De Vico
386	1889 III	*B*	.	147	−27	60	271	31	0.04	
275	1862 III	*B*	0	150	−25	153	137	114	9.98	
424	1896 V	*A*	.	154	− 7	140	193	11	0.16	
417	1895 II	*A*	.	158	− 1	168	170	3	0.11	
201	1844 I	*a*	+	163	+ 3	279	64	3	0.07	De Vico
415	1894 IV	*A*	.	165	+ 3	297	49	3	0.14	"
469	1906 III	*A*	.	165	− 5	14	332	21	0.33	Holmes
102	1770 I	*A*	0	178	+ 1	224	132	2	9.83	
457	1903 V	*A*	.	182	+ 2	344	18	6	0.29	Brooks
471	1906 V	*A*	.	188	+ 2	316	52	3	9.98	Finlay
447	1900 III	*A*	.	189	− 4	171	197	30	9.97	
348	1881 V	*A*	.	199	+ 5	313	66	7	9.86	
406	1892 V	*A*	.	199	− 5	170	207	31	0.16	
433	1898 IV	*A*	.	200	− 3	173	206	25	0.20	Wolf
472	1906 VI	*A*	.	211	+ 4	196	195	15	0.21	
236	1852 IV	*B*	+	216	−33	57	346	41	0.10	
389	1889 VI	*A*	.	220	−10	70	330	10	0.13	
481	1908 II	*A*	.	224	− 5	114	290	5	0.06	$Tempel_3$
485	1909 III	*A*	.	230	− 4	167	242	16	0.07	Perrine
382	1888 IV	*A*	.	231	+ 4	201	210	11	0.24	
113	1783	*A*	.	233	+ 4	355	56	45	0.16	
396	1890 VII	*A*	.	238	− 3	13	45	13	0.26	
290	1866 I	*B*	.	241	− 3	171	231	163	9.99	$Tempel_4$
159	1819 IV	*A*	.	249	+ 2	350	77	9	9.95	
462	1905 II	*A*	.	250	+ 4	352	77	31	0.15	
486	1909 IV	*A*	.	254	− 1	4	71	19	0.14	
292	1867 I	*B*	.	257	+ 1	358	78	18	0.20	
211	1846 IV	*B*	.	259	−13	13	78	85	9.82	
357	1884 I	*B*	+	260	+18	199	254	74	9.89	Pons.

No.	Year.	Cl.	Br.	Long.	Lat.	ω	☊	i	log g.	Name.
				°	°	°	°	°		
220	1847 V	*B*	.	261	−15	129	310	19	9.69	
85	1743 I	*A*	.	276	0	6	87	2	9.94	
442	1899 III	*B*	.	286	+21	207	270	54	0.01	Tuttle
235	1852 III	*A*	.	289	+ 9	223	246	13	9.93	Biela
334	1879 I	*A*	.	295	− 7	15	101	29	9.77	Brorsen
412	1894 I	*A*	.	311	− 4	46	84	6	0.05	
378	1887 V	*B*	.	322	−40	65	85	45	0.08	Olbers
480	1908 I	*A*	.	339	+ 1	185	334	13	9.53	Encke

copied down in the order of the longitudes of their aphelia, and inserted in Table LXVIII, which is arranged precisely like Table LXVII. This arrangement presents them in a convenient form for comparison with the orbits of future comets, enabling the computer to see at a glance which, if any, of the earlier orbits the new one resembles. In order to make this comparison he must first obtain the longitude and latitude of the aphelion of the new orbit. To do this let L' be the longitude measured from the

TABLE LXVIII.

ELEMENTS OF THE REMAINING COMETS OF LONG PERIOD.

No.	Year.	Cl.	Br.	Long.	Lat.	ω	☊	i	log q.
				°	°	°	°	°	
205	1845 II	*E*	.	3	+21	205	347	56	0.10
192	1840 I	*E*	+	3	−50	72	120	53	9.79
136	1801	*H*	.	6	+13	220	42	159	9.41
285	1864 III	*E*	.	9	+48	232	32	110	9.97
202	1844 II	*E*	+	11	+23	211	32	131	9.93
130	1796	*H*	.	17	+ 4	184	17	115	0.20
466	1905 VI	*H*	.	17	−54	90	286	126	0.11
476	1907 III	*H*	.	19	− 9	40	161	15	0.09
390	1890 I	*F*	.	20	+17	200	8	57	9.43
413	1894 II	*C*	+	24	+36	324	206	87	9.99
365	1886 I	*F*	+	27	−53	127	36	83	9.81
188	1835 I	*F*	.	29	+ 5	210	58	171	0.31
459	1904 II	*H*	.	30	−40	41	218	100	0.27
238	1853 II	*C*	+	31	+16	199	41	122	9.96
103	1770 II	*H*	.	32	+31	260	109	149	9.72
181	1830 I	*F*	+	33	− 2	6	206	21	9.96
246	1855 I	*C*	.	35	+28	323	190	129	0.34

No.	Year.	Cl.	Br.	Long.	Lat.	ω	☊	i.	log q.
				°	°	°	°	°	
30	1402	*I*	0	36	−55	91	117	55	9.58
51	1590	*I*	0	38	+23	308	166	151	9.75
44	1533	*I*	0	44	+28	278	299	28	9.51
186	1833	*H*	.	45	+ 7	261	323	7	9.67
163	1822 III	*H*	.	47	+30	238	98	144	9.93
160	1821	*F*	+	53	−10	169	49	106	8.96
89	1748 I	*H*	+	54	−17	17	233	95	9.92
184	1832 II	*F*	.	55	+17	205	72	137	0.07
458	1904 I	*F*	.	58	−41	53	276	125	0.43
86	1743 II	*H*	+	60	+39	119	6	134	9.72
176	1826 V	*F*	+	60	+80	280	235	91	8.43
247	1855 II	*H*	.	60	− 9	23	260	157	9.75
144	1808 II	*H*	.	67	−28	132	24	141	9.78
92	1758	*F*	.	68	−34	37	231	68	9.33
278	1863 II	*F*	+	70	− 4	4	251	113	0.03
452	1902 III	*F*	+	74	−10	153	49	156	9.60
434	1898 V	*F*	.	76	− 5	22	278	167	0.18
279	1863 III	*E*	+	77	−55	56	250	86	9.80
68	1684	*I*	0	78	+27	330	268	65	9.98
179	1827 III	*D*	.	79	+53	259	150	126	9.14
221	1847 VI	*G*	+	81	+71	277	191	108	9.52
372	1886 VIII	*F*	.	81	−32	32	258	86	0.17
216	1847 I	*D*	0	89	+46	254	22	49	8.63
32	1449	*I*	0	90	+ 1	357	261	156	9.52
82	1837 II	*H*	.	92	−25	130	132	62	9.92
349	1881 VI	F	.	92	− 6	6	274	113	9.65
427	1897 I	*G*	.	93	− 4	127	86	146	0.03
164	1822 IV	*D*	+	93	+ 1	181	93	127	0.06
448	1901 I	*E*	0	94	+17	203	110	131	9.39
296	1868 II	*F*	.	94	−37	127	52	131	9.76
6	141	*I*	0	95	−15	121	13	163	9.86
110	1780 II	*H*	.	95	+66	254	141	108	9.71
271	1861 II	*C*	0	97	+30	330	279	85	9.92
72	1698	*I*	0	97	− 5	151	66	169	9.86
187	1834	*F*	.	97	− 5	50	227	6	9.71
152	1816	*H*	.	98	+34	304	323	43	8.69
436	1898 VII	*G*	.	99	+49	233	74	70	0.23
367	1886 III	*G*	.	100	−38	39	288	100	9.93
45	1556	*I*	0	100	−30	101	175	32	9.69
430	1898 I	*C*	.	100	−45	47	262	73	0.04
31	1433	*I*	0	101	+ 9	189	96	104	9.69
395	1890 VI	*E*	.	103	−17	163	100	99	0.10
467	1906 I	*F*	+	106	+13	199	92	44	9.33
319	1874 III	*E*	0	107	−25	152	119	66	9.83
482	1908 III	*F*	+	110	− 5	172	103	140	9.98

No.	Year.	Cl.	Br.	Long.	Lat.	ω	☊	*i*	log *q*.
				°	°	°	°	°	
364	1885 V	*F*	.	110	−23	36	262	42	0.03
7	240	*I*	0	111	−43	82	189	44	9.57
234	1852 II	*H*	.	111	−27	37	317	131	9.96
409	1893 II	*E*	+	112	−15	47	337	160	9.83
240	1853 IV	*G*	+	115	+60	278	220	119	9.24
199	1843 II	*G*	.	116	−41	124	157	53	0.21
203	1844 III	*E*	+	118	− 2	178	118	46	9.40
318	1874 II	*F*	.	119	+14	332	274	148	9.95
317	1874 I	*H*	.	120	+59	269	30	59	8.65
4	−12	*I*	0	126	−10	35	35	113	9.95
21	1264	*I*	0	130	− 6	160	141	16	9.92
231	1851 III	*E*	.	131	−38	87	224	38	9.99
301	1870 I	*F*	.	132	+15	198	142	122	0.00
16	1006	*I*	0	136	−17	94	38	162	9.77
55	1618 I	*I*	0	141	− 9	25	293	21	9.71
337	1879 IV	*H*	.	141	−71	84	32	108	0.00
314	1873 V	*E*	+	142	+43	234	177	121	9.59
171	1825 IV	*D*	0	142	+32	257	216	146	0.09
81	1737 I	*H*	+	149	−18	100	226	18	9.35
286	1864 IV	*F*	.	153	−41	118	203	49	9.89
3	−69	*I*	0	155	−28	150	165	70	9.90
10	568	*I*	0	157	− 2	24	294	4	9.96
321	1874 V	*E*	.	165	−42	93	252	42	9.99
5	66	*I*	0	177	−37	68	33	140	9.65
222	1848 I	*H*	.	181	+79	261	212	96	9.51
197	1842 II	*F*	.	182	+57	241	208	106	9.70
269	1860 IV	*H*	.	182	+23	312	45	32	9.83
263	1858 VII	*D*	.	183	− 9	156	160	159	0.15
121	1788 II	*H*	.	188	−27	30	352	65	9.88
320	1874 IV	*C*	.	190	−16	150	216	34	0.23
56	1618 II	*I*	0	191	+35	287	76	37	9.59
177	1827 I	*F*	.	192	−28	151	185	102	9.70
50	1585	*I*	0	194	+ 3	331	38	6	0.04
232	1851 IV	*H*	.	194	+61	294	44	74	9.15
369	1886 V	*C*	.	194	+21	201	193	88	9.41
302	1870 II	*F*	.	194	+ 5	355	13	99	0.26
262	1858 VI	*D*	0	195	−44	129	165	117	9.76
384	1889 I	*G*	+	197	+ 5	340	357	166	0.26
19	1097	*I*	0	197	−52	125	208	74	9.86
265	1859	*H*	.	202	+77	282	357	95	9.30
468	1906 II	*H*	.	207	+81	276	72	83	9.86
125	1792 I	*H*	.	213	−16	154	191	140	0.11
455	1903 III	*F*	.	216	+ 6	185	213	66	9.69
79	1723	*H*	+	216	+22	331	14	130	0.00
267	1860 II	*F*	.	220	−29	41	9	48	0.12

No.	Year.	Cl.	Br.	Long.	Lat.	ω	☊	i	log q.
				°	°	°	°	°	
195	1840 IV	*C*	.	221	−38	134	249	58	0.17
255	1857 VI	*D*	.	224	−38	95	139	142	0.00
133	1798 II	*H*	.	224	+23	215	250	138	9.89
335	1879 II	*F*	.	225	− 4	4	46	107	9.95
354	1882 III	*G*	.	228	+73	254	249	96	9.98
313	1873 IV	*D*	.	230	+14	194	231	96	9.90
241	1854 I	*F*	.	231	− 8	171	227	114	0.31
147	1811 II	*C*	+	233	+22	314	93	31	0.20
77	1707	*I*	0	236	−27	27	53	89	9.93
175	1826 IV	*D*	.	238	− 6	14	44	26	9.93
71	1695	*I*	0	241	+ 9	204	216	22	9.93
373	1886 IX	*G*	+	245	−78	86	137	102	9.82
329	1877 V	*H*	.	246	−61	103	184	116	0.03
429	1897 III	*F*	.	250	−59	66	32	70	0.13
149	1813 I	*F*	.	251	− 3	171	241	159	9.84
94	1759 II	*F*	+	251	+78	274	140	79	9.90
403	1892 II	*G*	.	253	−51	129	253	90	0.29
385	1889 II	*E*	.	256	+13	236	311	164	0.35
143	1808 I	*H*	.	257	+43	254	323	134	9.59
477	1907 IV	*F*	+	258	+ 8	294	143	9	9.71
38	1490	*I*	0	258	−37	130	289	52	9.87
69	1686	*I*	0	259	−31	82	354	35	9.53
288	1865 I	*F*	+	260	−68	112	253	92	8.41
330	1877 VI	*F*	.	260	−36	143	251	102	0.20
383	1888 V	*D*	.	262	+51	291	138	56	0.18
465	1905 V	*G*	+	263	−28	133	223	141	0.02
228	1850 II	*F*	.	263	+35	243	206	40	9.75
97	1763	*D*	.	264	−72	89	356	73	9.70
108	1779	*F*	.	265	−28	62	25	33	9.85
249	1855 IV	*F*	+	266	+ 6	326	52	170	0.09
473	1906 VII	*F*	.	270	− 7	9	85	57	0.08
83	1739	*H*	.	275	−53	105	207	124	9.83
106	1773	*F*	+	276	+39	314	121	61	0.05
155	1818 III	*G*	.	277	+10	348	90	117	9.93
280	1863 IV	*E*	+	277	+ 3	357	97	78	9.85
437	1898 VIII	*F*	.	280	− 2	5	96	23	0.36
120	1788 I	*H*	+	281	−11	58	157	168	0.03
408	1893 I	*F*	.	282	−36	85	186	144	0.08
305	1871 I	*D*	.	282	+42	223	279	88	9.82
478	1907 V	*F*	.	282	+52	294	55	120	9.99
29	1385	*I*	0	284	−10	167	269	128	9.89
141	1806 II	*G*	.	284	+24	225	322	145	0.03
67	1683	*I*	0	284	−83	88	173	97	9.75
375	1887 II	*C*	.	285	−20	159	280	104	0.21
104	1771	*F*	0	286	−11	76	28	11	9.96

No.	Year.	Cl.	Br.	Long.	Lat.	ω	☊	i	log q.
				°	°	°	°	°	
376	1887 III	F	.	286	−23	37	135	140	0.00
456	1903 IV	H	+	287	−52	127	294	85	9.52
146	1811 I	D	0	289	−60	65	140	107	0.02
393	1890 IV	D	.	292	+12	331	85	154	0.31
173	1826 II	F	.	301	+39	279	198	40	0.30
400	1891 IV	E	.	301	+78	270	218	78	9.99
454	1903 II	F	.	302	− 4	6	117	44	0.44
268	1860 III	F	+	304	−73	77	85	79	9.47
91	1757	H	+	305	+13	269	214	13	9.53
78	1718	I	0	306	− 3	6	128	149	0.01
381	1888 III	E	.	306	−56	59	101	74	9.96
126	1792 II	H	.	308	−24	147	283	131	9.99
299	1869 II	F	.	309	+ 8	188	312	112	0.09
250	1857 I	F	.	310	−58	122	313	88	9.89
74	1701	H	.	313	−10	165	299	138	9.77
474	1907 I	F	.	313	+25	317	97	142	0.31
276	1862 IV	H	.	314	+31	231	356	138	9.90
453	1903 I	E	+	320	−22	134	2	31	9.61
99	1766 I	H	.	322	−40	101	244	139	9.70
266	1860 I	H	.	330	+29	210	324	80	0.08
423	1896 IV	H	.	332	−41	41	151	88	0.06
237	1853 I	F	.	334	+20	276	70	160	0.04
407	1892 VI	F	.	336	+24	253	264	25	9.99
463	1905 III	C	.	336	+ 1	358	157	40	0.05
135	1799 II	H	+	340	−42	136	327	103	9.80
214	1846 VII	C	+	341	−29	100	262	151	9.80
287	1864 V	F	.	343	0	179	341	163	0.05
47	1577	I	0	345	+69	256	25	105	9.25
272	1861 III	H	.	348	+19	332	145	138	9.92
217	1847 II	F	.	348	−32	32	174	100	0.33
52	1593	I	0	349	−12	12	164	88	8.95
422	1896 III	G	.	359	− 1	2	178	56	9.75

node, L the true longitude reduced to the epoch of 1900.0, B the latitude, and t the epoch employed for the new orbit.

$$\text{Then} \quad \sin B = -\sin\omega \sin i$$
$$\tan L' = \tan\omega \cos i$$
$$\text{and} \quad L = L' + \Omega - 50''.2\,(t - 1900.0)$$

There are two possible values of L′, differing by 180°, and care must be taken to select the correct one. This may be most simply done graphically, projecting the plane of the orbit upon a drawing of the celestial sphere. If the comet is not of the periodic class, and the elements of its orbit do not resemble semble those of any of the comets contained in Tables LXVI or LXVIII, it may be set down as a representative of a new type of orbit. A series of tables to aid in the identification of comets, extending to the year 1896, was prepared by the late W. C. Winlock, and will be found in the Pub. Astron. Soc. Pac., 1896, 8, 141.

The total number of groups contained in Table LXVI is 66, the total number of comets 157. The number of periodic comets, classes *A* and *B*, contained in Table LXVII is 46, and the number of comets of long period, not grouped, contained in Table LXVIII is 184.

TABLE LXIX.

ELEMENTS OF HALLEY'S COMET.

No.	Year.	Int.	Long.	Lat.	Dist.	Δ ω	Δ ☊	Δ *i*.	Δ log *q*.
			°	°	°	°	°	°	
4	−12		126.4	− 9.5	8	3	27	8	0.01
5	+66	78	176.6	−36.9	56	43	22	22	.12
6	141	75	95.3	−14.5	29	10	42	1	.09
13	837	(77)	123.7	+10.9	28	166	152	7	.01
15	989	(76)	96.7	0.0	32	69	29	1	.02
17	1066	77	95.4	−14.5	29	10	29	1	.09
23	1301	(78)	140.5	+ 1.4	25	75	83	5	.04
28	1378	77	126.0	−17.0	1	3	8	0	.00
33	1456	78	124.4	−17.0	0	6	11	0	.01
42	1531	75	125.7	−16.4	1	7	10	1	.01
54	1607	76	124.1	−16.3	0	4	7	1	.00
66	1682	75	124.0	−16.7	0	2	4	0	.00
93	1759 I	77	124.2	−16.4	0	0	1	0	.00
190	1835 III	76	124.5	−16.6	0	0	0	0	.00

In order to obtain an idea of the amount of inaccuracy that we might expect to find in the accepted values of the elements of the earlier orbits, Table LXIX has been prepared. This table relates to Halley's comet, and has been summarized from the elements given in Galle's catalogue. It is not probable that the orbit has changed materially during the period considered. The nearest possible approach of the comet to the Earth is 0.06 units, and to Jupiter 0.7. The first column gives the number of the orbit, the second the year of perihelion passage, the third the interval or mean interval that has elapsed between passages, the fourth and fifth the longitude and latitude of the aphelion, and the sixth the distance between the position of the aphelion as determined from the last passage, number 190, and the position as determined from the passage in question, located upon the celestial sphere. The last four columns give in a similar manner the differences for ω, ☊, i, and log q. It will be noticed that many of the differences prior to 1378 are very large but that the last two columns are more reliable than the others.

We will now see if among all the comets grouped together in Table LXVI, or in the other tables, there are any cases that may be considered identical. The earlier astronomers in their studies of this subject labored under certain disadvantages, and seem to have made their investigations in a rather haphazard fashion. Our present investigation will be more systematic. Comets have been observed so carefully during the past century, that any members of classes *A* or *B* that appeared, and were well seen, would almost certainly have been detected, and identified by the elliptical forms of their orbits. Nevertheless it has been a matter of interest to compare the elements of the various recognized members of classes *A* and *B* in Table LXVII with the elements of the comets having similar aphelia in Tables LXVI and LXVIII. In Table LXX the first column gives the numbers of the two comets compared. When one of them is generally recognized as having returned more than once, its earliest appearance is the one selected for comparison, and its name appears in the second column. An exception to this rule is made in the case of Halley's comet, where the last appearance that we consider in this volume, that of 1835, has been selected. The reason for including Halley's comet in this list will appear later. The third column gives the years and designations of the comets, the fourth the classes to which they belong, and the fifth the last period of the periodic comet. The sixth column gives their brightness, the seventh the inclination of their orbits, and the eighth the angular distance between their aphelia. The inclination is given because if near 0° or 180° and the aphelia are near together, a large variation in ω and ☊ is not significant. The last four columns give the differences between their elements. The last four identifications are taken from Table LXVI, the others from Table LXVIII.

Admitting the identity of the second group as accepted by Clausen, Olbers, and Winnecke, and of the seventh group as shown by Le Verrier and Brünnow, we should have little difficulty in accepting most of the other identifications. The most doubtful feature of the second group, and this applies also to

TABLE LXX.

POSSIBLE IDENTIFICATIONS OF THE PERIODIC COMETS.

Nos.	Name.	Years.	Cl.	Per.	Br.	Inc.	Dist.	$\Delta\omega$	$\Delta\Omega$	Δi.	$\Delta \log q$.
						° °	°	°	°	°	
259, 476		1858 III, 1907 III	*A, H*	6.61	. .	20, 15	1	14	14	5	0.03
100, 158	Winnecke	1766 II, 1819 III	*a, a*	5.89	+ .	8, 11	23	15	39	3	0.29
16, 190	Halley	1006, 1835 III	*I, B*	76.3	0 0	162, 162	12	17	17	0	0.00
21, 230	d'Arrest	1264, 1851 II	*I, a*	6.68	0 .	16, 14	14	15	7	2	0.15
3, 275		−69, 1862 III	*I, B*	119.9	0 0	70, 114	6	3	28	44	0.08
10, 417		568, 1895 II	*I, A*	7.22	0 .	4, 3	1	144	124	1	0.15
64, 201	De Vico	1678, 1844 I	*a, a*	5.86	+ +	3, 3	17	120	99	0	0.01
50, 371	Finlay	1585, 1886 VII	*I, a*	6.54	0 .	6, 3	6	16	14	3	0.04
71, 200	Faye	1695, 1843 III	*I, A*	7.56	0 .	22, 11	12	5	7	11	0.30
34, 105	Biela	1457 I, 1772	*I, a*	6.62	0 .	13, 17	21	18	7	4	0.14
43, 210	Brorsen	1532, 1846 III	*I, a*	5.46	0 .	33, 31	6	10	16	2	0.09
58, 210	"	1661, 1846 III	*I, a*	5.46	0 .	33, 31	10	19	21	2	0.16
20, 412		1231, 1894 I	*I, A*	7.42	0 .	6, 6	13	75	70	0	0.07

the ninth, is the large difference in log q, these comets having at one apparition apparently double the perihelion distance that they had at the other. Possibly both these groups should be omitted from the table. The next most doubtful case is probably the fifth group, which shows a large deviation in the inclination. The other elements are generally, sufficiently accordant, however, and the distance between the aphelia small. If genuine, the identification would be of interest, since comet 275 is associated with the well-known Perseid meteoric shower. Assuming the identity, and that sixteen revolutions had occurred in the mean time, the period comes out 120.7 years, according well with the value given in the table. Comet number 386 is properly associated with number 275, as we have already seen, since both their aphelion position and aphelion distance are in close agreement. Their inclinations, however, differ by 83°, and it so happens, curiously enough, that the inclination of number 3 lies nearly half-way between that of the other two.

We have, however, already seen by Table LXIX how uncertain are the elements of some of the earlier orbits, and it is possible that the small deviations found in several of these groups are due merely to accident. This may be the case with the third group containing Halley's comet, or it may be that comet number 16 still revolves in the original path from which Halley's comet was torn, and made to follow its present small periodic orbit.

The first group differs from all the others in that both comets are modern. Both were also extremely faint. The interval between the observations secured of number 476 extended through only 7 days, of 259 through 27 days. We find by the table that the period of number 259 is 6.61 years. Schulhof sets as probable limits 5.8 and 7.5 years. Assuming eight revolutions to have been made, the period would be 6.13 years, and the next perihelion passage should occur July 19, 1913. If seven revolutions have passed, the period would be 7.01 years, and the next perihelion passage occur June 4, 1914. An examination of the periodic times of Jupiter's family shows that from 5.60 to 5.99 years there are seven comets, from 6.00 to 6.39 there are none, from 6.40 to 6.79 there are ten, and from 6.80 to 7.19 there are five. The gap from 6.00 to 6.39 is clearly marked, and we may therefore eliminate the assumed period of 6.13 years for comet 259, and adopt that of 7.01. It may be pointed out in this connection that half the period of Jupiter is 5.93 years. The period of Winnecke's comet is 5.89 years, and that of $Tempel_1$ 5.98. The gap is therefore not due to simple commensurability with the period of Jupiter, but depends also on the eccentricity of the orbit, which affects the aphelion distance.

As far as closeness of aphelia and identity of elements are concerned, the groups containing Finlay's comet and the first identification of Brorsen are among the most accordant, and both lie far within the limits of probable error and probable deviation through perturbations. Finally an examination was made of the groups in Table LXVI to see if in any case there were two comets of type *H*, the insufficiently observed type, whose elements resembled one another in a striking manner. No such case was found. It was therefore concluded that we had at present no evidence of any other unrecognized comet of short period.

A search for identity among the comets not generally recognized as periodic seemed promising, because the elliptical forms of their orbits are much less readily determined, and it was therefore probable that many of them that had been observed, had passed without their periodic character having been discovered. Provided the intrinsic brilliancy, which is independent of the distance of the Earth, is not too small, a comet of comparatively insignificant size may become a most magnificent object by the mere chance of passing in our vicinity. This difference of distance may in fact be more effective than any other cause in varying the conspicuousness of a comet at its various perihelion passages.

By Table XLI, p. 191, we find that the shortest period of any of the twenty comets of class *C* is 235 years. The longest period is 1140 years. The shortest period of class *D* is 1240 years. It is not possible therefore that we should find two accurate observations of any comet in class *D*. There is a chance for class *C*, but we need not look for the earlier observation after the year 1700. An examination was made of Table LXVI, and every group recorded which had a member appearing prior to that year. Twenty-four of these groups were found, and it was not impossible therefore that any one of these early comets might have been observed a second time. On the other hand none of the comets entered in Table LXVIII could possibly have been accurately observed twice, or the two appearances would have been entered as another group. Let us now examine these twenty-four groups in turn. We shall find that certain of them can be at once rejected, it being very improbable that any of their members could have been reappearances of the same object. In other groups the reappearance of the same body is possible, while in a few others it is perhaps probable.

The rejected groups and comets are as follows: —

Group 2, 1558, 1699.	Group 26, 961, 1854 III.	Group 42, 1472, 1826 III.
Group 9, −137, 1362.	Group 32, 1689, 1824 II.	Group 46, 1665, 1840 II.
Group 12, 1582, 1824 I.	Group 34, 770, 1299.	Group 54, 565, 1846 V.
Group 17, 1506, 1793 I.	Group 36, 1337, 1870 IV.	Group 55, 1677, 1871 II.
Group 19, 1680, 1881 III.	Group 38, 1457 II, 1857 IV.	Group 61, 1664, 1759 III.
Group 20, 1596, 1845 III.		

The reasons for rejection were either because of a lack of sufficient resemblance in the elements, especially in i and log q, or else because the orbit of the later comet was so well known as to make it certain that its period must have been much longer than that indicated by the observed interval of time.

In the first column of Table LXXI are entered the numbers of those groups which have not been rejected. The second column gives the numbers of the two comets whose identity is in question, and the third their years and designating numbers. The fourth column gives the classes to which they belong, the fifth their brightness, and the sixth the inclinations of their orbits. The seventh column gives the distance of their mean aphelion from the reference circle of class *C*, near which in case of their identity we should expect most of them to be found. The eighth column gives the interval of time that has elapsed between their appearances, and the ninth the distance between their aphelia. The next four columns give the differences between their elements, and the last a number indicating the probability of their identity, 1 indicating a higher degree than 2. In the following detailed description of these groups we shall give the main reasons for and against the acceptance of their identity.

Group 23. The elements of these two comets closely resemble one another, but those of the first are rather uncertain. Two other comets of the group have periods of 515 and 1240 years. The high eccentricity of their orbits and their small perihelion distances permit slight perturbations to produce large changes in their periods. It is not impossible, therefore, that the period of 212 years may be correct, and the two comets identical, as was first suggested by Klinkerfues.

Group 40. Both comets were small although provided with tails. Their orbits were well observed. Their mean aphelion is located only 14° from the reference circle of type *P*, and their elements differ but little more than we might expect as the result of mere planetary perturbations. Identity plausible.

Group 47. Their aphelia, but 8° apart, lie far from the reference circle. Their elements are extremely accordant as was noticed by Deichmüller and Berberich. The earlier comet, according to Hevelius, almost equalled the Moon in magnitude. This doubtless refers to dimensions rather than to light. The later comet was very faint, and although observed through a period of 27 days gave no indication of departure from a parabolic orbit, Vierteljahrschrift 1898, 33, 327. Identity possible.

Group 49. The elements of the earlier comet depend on observations extending through only four days. They are therefore very uncertain. The inclinations differ markedly, but the other elements are very accordant. Identity possible.

Group 57. The earlier comet was well observed by Tycho. Their aphelia, 8° apart, are somewhat remote from the reference circle, but their elements are in close agreement. Identity plausible.

Group 59. If identical, as suggested by Olbers, the period of these comets is too short to permit them to be classified under class *C*, but about right to be associated with planet *O*, and be placed in class *B*. If identical, however, the comet should have reappeared in 1790, especially as both its predecessors were bright. Although looked for, it was not seen. It is due again in

1919. By the last table we have seen that both these comets may be identical with that of Brorsen. Identity plausible.

Group 62. On account of their small inclinations these orbits are very similar in spite of the discrepancy in the values of ω and Ω. If we assume that the inclination of the earlier comet was 0° instead of 6°, $\Delta\ \omega$ and $\Delta\ \Omega$ will be reduced to 0° for practical purposes, and $\Delta\ i$ will be increased to 4°. The difference between their perihelion distances is large, however. Their distance from the reference circle is but 13°. Observations fairly accurate. Both were visible to the naked eye. Identity possible.

Group 66. This is a rather interesting case since there are three perihelia involved, those of 574, 1092, and 1593. Intervals 518 and 501 years. All that we know of the first of these bodies, unfortunately, is that its elements are very uncertain, and that it was the size of a peach, which presumably means that it was large enough to be fairly conspicuous. The comet of 1092 approached pretty close to the Earth, and its elements are satisfactory. Of the comet of 1593 we know but little save that the observations by Ripensis, a pupil of Tycho, were rather crude. On account of its small perihelion distance this latter comet must apparently be rejected. Its inclination also is quite different from that of the others. The elements of the others differ among themselves to a considerable extent, though perhaps not more than might be accounted for by the inaccuracies of the early observations. Identity possible.

It will be noticed by the fourth column of the table that of all the comets included in it, in not a single instance do we definitely know the eccentricity of the orbit. We are consequently in the position of not being able to disprove identity, rather than of being able to prove it. Of the 341 comets of high eccentricity hitherto observed with sufficient accuracy to determine the elements of their orbits, it appears by Table LXXI that from our present observations there are only eight pairs that can possibly be identical. Of these there are only four whose identity is plausible.

It is a curious fact that although there were nine accurately observed comets from 1551 to 1600, and seventeen from 1651 to 1700, yet, with the exception of Halley's comet, and the two very brilliant ones that appeared in 1618, there were no comets observed between 1601 and 1650 with sufficient accuracy to determine the elements of their orbits. From the present time, however, the comets belonging to class *C* that appeared since 1650 will begin to return. During the hundred years prior to 1750, excluding the

TABLE LXXI.

COMETS WHOSE IDENTITY IS NOT IMPOSSIBLE.

Gr.	Nos.	Years.	Cl.	Br.	Inc.	R. C.	Int.	Dist.	Δω	ΔΩ	Δi	Δ log q	Prob.
					° °	°		°	°	°	°		
23	61,339	1668, 1880 I	*I, I*	00	144, 145	69	212	1	6	9	1	0.06	1
40	36,112	1468, 1781 II	*I, H*	0+	142, 153	14	313	14	9	6	11	0.06	1
47	57,418	1652, 1895 III	*I, H*	0 .	79, 76	88	243	8	1	5	3	0.00	2
49	34,316	1457, I 1873 VII	*I, H*	0 .	13, 30	39	416	8	1	0	17	0.01	2
57	48,392	1580, 1890 III	*I, H*	0 .	65, 63	32	310	8	3	5	2	0.10	1
59	43, 58	1532, 1661	*I, I*	00	33, 33	13	129	4	9	5	0	0.07	1
62	20, 75	1231, 1702	*I, H*	0+	6, 4	13	471	9	171	175	2	0.17	2
66	11, 18	574, 1092	*I, I*	00	47, 29	7	518	7	16	2	18	0.01	2

periodic comets, observations necessary to locate 31 orbits were obtained. Of these ten per cent, as we have seen by p. 186, should belong to class *C*. One or more of them may be expected during the present century. After 1750 the number of comets observed multiplied rapidly, so that in the next century doubtless identifications will be much more numerous. Of the comets which have been recognized by their eccentricities as belonging to class *C*, the earliest return, that of number 253, cannot be expected in less than about 180 years, and the others generally much later.

CHAPTER XVII

Elements. Asteroids. Resisting Medium. Miscellaneous Characteristics.

In this chapter we shall first discuss statistically some of the elements of the cometary orbits, and will begin with the nodes and inclinations. Since the orbits of the members of class *A* are all more or less parallel to the orbit of Jupiter, it seemed possible that a study of the orbits of the other comets might similarly indicate the planes of the orbits of the unknown outer planets. The poles of the different orbits of class *A* were accordingly plotted on polar coördinate paper, their mean position obtained, and as a result it was found that the longitude of the node and inclination of the mean orbit were approximately 120° and 5° respectively. The maximum inclination was 45° for comet 113, and the next largest 31° for comets 406 and 462.

The orbits of class *B* were next tried. The resulting node and inclination were 270° and 15° respectively, agreeing approximately with those of the great circle mentioned on p. 216. The planes of the orbits were widely scattered, three of them being retrograde. Two of these were of the comets associated with the meteor showers of August 10 and November 14, the Perseids and the Leonids, and the third was that of Halley's comet. These retrograde orbits were associated one with Uranus, one with Neptune, and one with planet *O*. The largest inclinations found, next to those that were retrograde, were number 211, 85°, and number 357, 74°. Obviously such large figures strongly affected the result where only twelve orbits were concerned.

The planes of the orbits of classes *C* and *D* were still more widely scattered in all directions. The node and inclination for the former being 210° and 20°, and for the latter 205° and 85°. While the inclinations were not far out of the way, the nodes were entirely erroneous. It was clear that nothing of value could be obtained by continuing this investigation. A count was next made of the direct and retrograde orbits in each class. In class *B* twenty-five per cent were retrograde, and in class *C* this proportion had risen to thirty per cent, but in the more remote classes the division was about equal.

A study was next undertaken of the perihelion distances. For class *A* the mean value of q was 1.18, for class *B* 0.96, for *C* 0.81, for *D* 0.50, for *E* 0.65, for *F* 0.79, and for *G* 0.76. At first sight these results seem more or less systematic. Further investigation, however, shows that the variations are due mainly to the existence of a few orbits having very small values of q. Class *A* is larger than the others because any comets belonging to it of small perihelion distance would soon lose their gaseous atmospheres owing to their frequent approaches to the Sun. Therefore only those of large perihelion distance would remain visible. Encke's comet is an exception, which however proves the rule, for even in as short an interval of time as the one hundred and twenty years which have elapsed since its discovery, it shows clear evidence of diminution in brilliancy. It seems probable that it is a comparatively recent addition to Jupiter's family, and that it will soon have joined the great body of invisible comets.

In Table LXXII is given a list of all the orbits of small and of large perihelion distance. The successive columns give the number of the orbit, the year and designation, the class, the type, the logarithm of the perihelion distance, and the distance itself. Comets which closely approach the Sun are naturally brilliant, which explains why five out of the sixteen of small perihelion distance fall into class *I*, the class of the bright and inaccurately known comets. Where the approach is very close, the nucleus is often ill-defined, which farther complicates the question of classifying them otherwise than under *I*. Not all of these comets were brilliant, however, numbers 317, 152, and 109 being invisible to the naked eye. On the other hand, of those comets of long perihelion distance, only number 80 could be seen without a telescope. Were the inaccurately known orbits of classes *H* and *I* classified under types, we should find that there were only four orbits of short perihelion distance, numbers 352, 70, 52, and 160, which did not belong to type *Q*. Of the seventeen comets of large perihelion distance, eleven belong in class *F*. These comets can be traced generally through only a short arc of their orbits, which renders it difficult to differentiate their paths from parabolas. Only one comet, 469, Holmes, is recognized as periodic. Of the 167 different comets that appeared in the last half of the last century 94, or 56 per cent of the perihelia, fell within the Earth's orbit.

TABLE LXXII.

COMETS OF SMALL AND OF LARGE PERIHELION DISTANCE.

No.	Year.	Cl.	Tp.	log *q*.	*q*.	No.	Year.	Cl.	Tp.	log *q*.	*q*.
61	1668	*I*	—	7.6800	0.0048	80	1729	*F*	*R*	0.6075	4.050
339	1880 I	*I*	—	7.7399	.0055	479	1907 VI	*H*	—	0.5845	3.842
198	1843 I	*C*	*Q*	7.7425	.0055	464	1905 IV	*F*	*P*	0.5236	3.339
65	1680	*D*	*Q*	7.7940	.0062	454	1903 II	*F*	*p*	0.4432	2.775
353	1882 II	*D*	*Q*	7.8890	.0077	458	1904 I	*F*	*q*	0.4325	2.707
374	1887 I	*I*	—	7.9852	.0097	361	1885 II	*G*	*Q*	0.3993	2.508
288	1865 I	*F*	*Q*	8.4124	.0258	437	1898 VIII	*F*	*Q*	0.3588	2.285
176	1826 V	*G*	*Q*	8.4296	.0269	385	1889 II	*E*	*QR*	0.3532	2.255
216	1847 I	*D*	*Q*	8.6293	.0426	88	1747	*F*	*P*	0.3421	2.198
317	1874 I	*H*	—	8.6490	.0446	246	1855 I	*C*	*P*	0.3411	2.193
152	1816	*H*	—	8.6858	.0485	469	1906 III	*A*	*N*	0.3266	2.121
352	1882 I	*E*	*r*	8.7836	.0608	217	1847 II	*F*	*p*	0.3253	2.115
70	1689	*I*	—	8.8091	.0644	474	1907 I	*F*	*R*	0.3121	2.052
52	1593	*I*	—	8.9499	.0891	393	1890 IV	*D*	*Q*	0.3111	2.047
160	1821	*F*	*R*	8.9630	.0918	241	1854 I	*F*	*R*	0.3106	2.045
109	1780 I	*E*	*Q*	8.9836	.0963	188	1835 I	*F*	*R*	0.3099	2.041
						173	1826 II	*F*	*R*	0.3027	2.008

Table LXXIII gives the eccentricity, and the ratio of the major and minor axes, of the orbits of several of the comets belonging to classes *A*, *B*, and *C*. The successive columns give the catalogue number, the year and designation, the class, the eccentricity, the ratio, and the name, where the comet has been recognized at more than one perihelion passage. The first four orbits are the only ones known in which the eccentricity is less than 0.5. The next three are the only orbits in Jupiter's family where it exceeds 0.8. The mean eccentricity for class *A* is 0.64. The next three orbits are associated with Saturn and Uranus, and are the only ones in class *B* where the eccentricity is less than 0.9. The next two are associated with Neptune, and are the only ones in this class where it exceeds 0.95. The next two are the only ones in class *C* where it is less than 0.97, and the last two the only ones where it exceeds 0.995.

It will be noticed that in all the orbits of class *A*, and in three of those of class *B*, the length of the minor axis is more than half that of the major. In the most eccentric orbit, the last one, the ratio is about as one to eighty. While many orbits have been assumed to be parabolic, few elliptical ones have

TABLE LXXIII.

ECCENTRICITY OF CERTAIN COMETS.

No.	Year.	Cl.	e.	$\frac{b}{a}$	Name.
469	1906 III	*A*	0.4121	0.911	Holmes
336	1879 III	"	.4626	.887	$Tempel_1$
457	1903 V	"	.4698	.883	Brooks
396	1890 VII	"	.4727	.881	——
334	1879 I	"	.8098	.586	Brorsen
348	1881 V	"	.8284	.560	——
461	1905 I	"	.8474	.531	Encke
213	1846 VI	*B*	.7286	.685	——
442	1899 III	"	.7571	.653	Tuttle
292	1867 I	"	.8654	.501	——
190	1835 III	"	.9675	.253	Halley
220	1847 V	"	.9739	.227	——
320	1874 IV	*C*	.9628	.270	——
246	1855 I	"	.9652	.262	——
369	1886 V	"	.9967	.081	——
198	1843 I	"	.999914	.013	——

been shown by actual computation to be more eccentric than this. But even in this case the length of the minor axis, though small, is still between six and seven units.

There is a certain resemblance in some respects between the asteroids and the comets of class *A*. Their masses, although unknown, must both extend through a very wide range, and must in many cases be similar. The eccentricity of the most eccentric asteroid Occlo [475], 0.381, is not far inferior to that of comet Holmes [469], 0.412, the least eccentric comet. The mean inclination of the orbits of the asteroids is 8°, that of the comets 14°. The mean distance of the asteroids is 2.65, that of the comets 3.44. For both of these classes of bodies the values of these two elements are frequently identical.

If we imagine a series of large comets furnished with massive nuclei to have formerly revolved in class *A*, and in the presence of a resisting medium, either continuous, or composed of freely distributed meteoric masses moving in all directions, we should in the process of time, as these comets lost their meteoric and gaseous surroundings, have obtained a series of small planets, possessing very much the characteristics of the asteroids as we now know them. The resisting medium would have reduced both their eccentricity and their mean distance as shown by La Place, while the long continued action of Jupiter would have reduced the inclination of their orbits. The meteoric material, lost and encountered, would have been in part absorbed by collisions with the nuclei, and in part have adopted their general direction of motion. On account of its small mass in proportion to its surface it would have suffered greater resistance to its motion, and therefore have descended more rapidly towards the Sun until stopped by the pressure of light. That portion of the meteoric matter which revolves in the same direction as the present asteroids would have ceased to offer resistance to their motion, but would appear to us in the form of the zodiacal light, which upon this hypothesis would owe its existence in large measure to the planet Jupiter, and not merely to the Sun, as has been heretofore generally supposed. Whatever portion of the original resisting material was gaseous must long since have been absorbed into the Sun or lost, like the material of a comet's tail, into interstellar space. The origin by capture of both the periodic comets and asteroids has been discussed in the past by S. Alexander, Callendreau, H. A. Newton, and especially by See, A. N. 1909, 180, 185. Also "Researches on the Evolution of the Stellar Systems" Vol. II, 376, 699, 701.

Since it might naturally be suggested that the influence of Jupiter was too slight to produce in this manner the almost countless asteroids that we know to be circulating about the Sun, together with those invisible bodies now in the intermediate state between comets and asteroids, the following investigation has been undertaken, in which a study has been made of the inclination of the cometary orbits exclusive of classes *A* and *B* and the corresponding sub-classes. In Table LXXIV the first column gives the limiting inclinations of the orbits, which are here considered as either direct or retrograde. The second the number of direct orbits between these inclinations, the third the number of retrograde orbits, the fourth the differences, and the fifth the totals. The sixth and seventh columns give the relative percentage of direct and retrograde orbits, and the last the difference between them.

Since comets arrive from all parts of the sky, and at all inclinations, it is fair to assume that normally there should be an equal number of direct and retrograde orbits. In fact, as is shown by the fourth column, among those orbits whose inclinations exceed 60°, and which therefore are little affected by Jupiter and the other known planets of our system, the direct revolutions exceed in number those that are retrograde. At inclinations below 60°

on the other hand, we find that comets with a direct rotation are decidedly in the minority. As is well known, it is only under unusual circumstances that Jupiter is able to capture or materially alter the orbit of a comet with a retrograde revolution.

The total number of retrograde comets is 179, of direct 155. If their numbers were originally equal, then Jupiter, and the other known planets to a less degree, must have captured 24 comets with direct orbits, and forced them into classes *A* and *B*. That is to say, these planets have hitherto captured about seven per cent of the total supply of comets. These all originally had direct orbits, but a certain small number of retrograde comets have also been captured.

TABLE LXXIV.

INCLINATION OF COMETARY ORBITS OTHER THAN CLASSES *A* AND *B*.

Inclination.	Dir.	Ret.	Diff.	Total.	Dir.	Ret.	Diff.
° °							
0–10	7	7	0	14	50	50	0
10–20	9	11	– 2	20	45	55	–10
20–30	10	15	– 5	25	40	60	–20
30–40	15	30	–15	45	33	67	–34
40–50	21	21	0	42	50	50	0
50–60	19	31	–12	50	38	62	–24
60–70	24	17	+ 7	41	59	41	+18
70–80	24	26	– 2	50	48	52	– 4
80–90	26	21	+ 5	47	55	45	+10
	155	179	–24	334	46	54	– 8

If we assume that the total number of comets in our system that under favorable circumstances would be visible in our telescopes numbers about 300,000, we shall probably, as will be shown later in Chapter XX, be not far from the truth. To this figure we must add the enormous number of comets whose perihelion distances greatly exceed unity, and are therefore not visible to us at all. If Jupiter has captured seven per cent of these comets, which seems a plausible figure, then 21,000 visible comets have already been captured by that planet, besides many more invisible ones. Since some of these must have split in several pieces it is quite obvious that Jupiter is fully competent to have produced all the asteroids hitherto discovered and many more besides.

Of all the periodic comets at present known, there is but one whose motion exhibits a distinct deviation from what would be expected from an application of the theory of gravitation. This deviation, occurring in the case of Encke's comet, produces a slight retardation in its motion, and was during the past century attributed to the existence of a resisting medium in space. The fact that the effect was in 1868 suddenly reduced to half its former value, and that no other comet exhibited this retardation, has led to general doubt as to the accuracy of this explanation. A second suggestion was that the retardation was due, not to a resisting medium, but to the then recently discovered pressure of light. The same objections held in this case however as in the other, so that no real advance was made. A third suggestion was that the comet encountered a ring of meteors crossing its orbit. This avoids the difficulty that other comets are not similarly retarded, and, moreover, if we suppose that prior to 1868 it encountered the ring twice in its revolution, and since then only once, owing to a change in the orbit of the meteors, we escape the other difficulty also.

The trouble with this rather plausible hypothesis is that the comet, as far as its effective momentum is concerned, itself consists merely of a very widely distributed group of meteors. If these two groups were to cross one another, very few of the individual members would collide, and the attraction of these small bodies for one another at the enormous distances at which they would pass would be entirely negligible.

The writer would suggest a cause which does undoubtedly tend to produce the effect observed, and is perhaps sufficient to wholly explain it, and that is the internal work of the comet itself. A small portion of this work is expended in propelling luminous gases away from the Sun, but this occurs in all visible comets to a greater or less degree. The feature, however, for which Encke's comet is especially noted is the extraordinary change that takes place in its apparent volume as it approaches the Sun. In 1838 its volume at perihelion was to its volume when first discovered, in the ratio of 1 to 800,000. Clerke's History of Astronomy, 1902, 92. After perihelion it again expanded. Other comets have shown the same peculiarity, but none to anything like the same extent. This change in volume and change in period are both so extraordinary, and so striking, in the case of this comet, that we cannot but feel that they are in some way related to one another.

The change in apparent volume cannot be due to an actual contraction of the swarm of meteors. What really happens seems to be that when remote from the Sun all the meteors in the swarm are surrounded by luminous gas, and when near the Sun only those meteors near the centre of the swarm are illuminated. The illumination, and change of illumination, which are undoubtedly of electric origin, use up energy, which must be derived from the orbital momentum, hence the retardation that we observe. Backlund has noted that the changes in acceleration were most marked at sun-spot maxima. Journ. Brit. Astron. Assoc., 1909, 20, 125. Such a maximum occurred in 1837, just before the marked contraction above noted. The distance of this comet from the Sun is subject to the greatest and most rapid changes of any member of its class. In the early part of the last century Encke's comet was frequently visible to the naked eye. It is now only seen with difficulty in a large telescope. The change in luminosity is therefore very great, and would thus perhaps account for the diminution in the rate of retardation observed.

Although it is usually stated that Encke's comet is the only one that exhibits a retardation, yet this statement from one point of view is not strictly correct. Among the non-periodic comets, the one of all others in which we should expect to find some such effect is the great comet of 1882 II. This comet passed very near the Sun, and was well observed both before and after perihelion passage. For two and a half weeks before and for the same interval after perihelion the nucleus was recorded as single, although it was sometimes described as granular. After that it appeared double, and four weeks after perihelion four distinct nuclei lying along the axis of the tail, were clearly seen. These remained visible until the comet disappeared. Kreutz, with great labor and infinite pains, worked out fourteen different orbits for this comet, based on different hypotheses with regard to it. The one which we have adopted in this volume is based on those observations made before and after perihelion and extending until the nucleus divided. No change in velocity was detected, the observations were accurate, and the period, 1245 years, doubtless well represents the interval that had elapsed since the previous perihelion passage.

Other orbits were deduced from observations made on each of the four nuclei. These observations extended through seven months. The period which was deduced for the nucleus nearest the Sun, and which therefore followed the others in its orbit, was 666 years, or but little more than half that of the comet before perihelion passage. The period of the second nucleus was 775 years, that of the third 885 years, and that of the fourth 967 years. Every one of the nuclei, therefore, was retarded, or pushed backwards from the tail, the first mentioned nucleus most of all. These nuclei at their next appearance will therefore probably be seen as four separate comets, separated at intervals of about 100 years from one another. This arrangement certainly makes for safety, in the case of a possible collision between the comet and the Sun. The retardation of these comets, unlike that of Encke's, seems to be due to the repulsion of the tail, the action being not unlike that of a rocket, except that the momentum of the tail here takes the place of the resistance of the air. As in the case of Encke's comet, however, if these explanations are correct, the retardation is due to a consumption of energy, caused by physical changes taking place within the comet itself, and having no similarity to the planetary perturbations due to Jupiter which are believed to have caused the division in two of Biela's comet. As we saw in Chapter XIV, the great comet of 1843, number 198, belonging to the same group as that of 1882, has had its period so shortened that it is included in class *C*, although its orbit belongs to type *Q*. The period of the comet of 1882 is the shortest of class *D*.

A considerable number of the comets which we have been studying have exhibited interesting individual peculiarities. These, together with a few statistics, we shall summarize as far as possible in the following four tables. Together with the year and designation, each column contains a reference to the publication where the information is given. Much of this information is taken from the Markree Catalogue by Cooper (Co.), Chambers' Handbook of Astronomy, 1889 (Ch.), and Clerke's History of Astronomy, 1902 (Cl.), all of which give not only convenient summaries of information, but also generally references from which the original observations can be found. Other data are taken from the American Journal of Science, 1886, **131**, (A), Galle (G), the Harvard Annals, **3** (H), Astronomische Nachrichten, **176**, (N), Popular Astronomy 1898, **6**, (P), the Vierteljahrschrift 1901, **36**, (Va.), 1906, **41** (Vb.) and 1909, **44** (Vc.), and Young's General Astronomy, 1899 (Y).

In the first column of the upper part of Table LXXV is given a list of those comets whose duration of visibility exceeded two years, together with their duration in months. The first of these was traced to a distance of 8.2 units, the longest distance on record. The second column gives those comets which are known to have transited the Sun. Under these circumstances they were always invisible. The third column gives those comets through whose tails the Earth has passed. The first of these is a little uncertain, but the second is well attested. In the lower part of the table are recorded those comets whose heads are known to have passed near the Earth, together, in some cases, with the distance of nearest approach. No comet is included in the list whose distance when given, exceeds 0.1. Possibly some of the other comets should be excluded, and it is very likely that some omissions have occurred.

TABLE LXXV.

ORBITAL STATISTICS.

Visibility.				Transited Sun.				Earth Traversed Tail.			
1889 I	32	G	302	1819 II		Co.	138	1819 II		Co.	138
1904 I	25	N	25	1826 V		Co.	153	1861 II		Cl.	326
"	"	Vb.	73	1882 II		Cl.	358				
1905 IV	42	Vc.	157								
Head passed near the Earth.											
837	0.04	Ch.	400	1499	—	Ch.	515	1781 I	—	Ch.	522
1092	—	Co.	67	1746	—	Ch.	520	1797	0.09	Co.	124
1351	0.10	Co.	69	1759 III	0.07	Co.	100	1798 II	0.03	Co.	125
1472	0.03	Co.	72	1770 I	0.015	Cl.	106				

In the first column of Table LXXVI is given a list of comets having unusually large nuclei, together with their diameters in miles. It is obvious, on account of their small mass, that these nuclei cannot be solid bodies, and the same conclusion doubtless applies to other cometary nuclei. The second column gives a list of comets having unusually large heads, together with their diameters in miles. The third of these is Halley's comet, and the fourth Holmes'. The third column gives those comets the length of whose tails has equalled or exceeded 100,000,000 miles. Chambers also mentions the comet of 1811 II, but this must be a mistake as it was not a conspicuous object, and its tail was short. In the last column are recorded those comets whose tails were visible over an arc exceeding 60° in length. It will be noticed that the majority of them have exceeded 90°.

The first column of Table LXXVII gives three comets whose nuclei have, according to several observers, exhibited a "singular scintillation." The last of these was Encke's comet. It seems to be a rather unusual phenomenon, but is perhaps dependent in part upon the condition of the observer's atmosphere. Presumably it was something different from what was seen in the star disks at the time, since it was pronounced "singular."

TABLE LXXVI.

PHYSICAL STATISTICS.

Nucleus.			Head.			Tail.			Angular Length.		
										°	
1815	5000	Ch. 482	1680	600,000	Y 439	1680	100,000,000	Ch. 484	837	80	Co. 66
1825 IV	5000	" "	1811 I	1,200,000	"	1811 I	100,000,000	" "	1264	100	" 68
1845 III	8000	" "	1835 III (H)	400,000	Ch. 483	1843 I	200,000,000	" "	1618 II	70–104	" 77
1858 VI	6000	" "	1892 III (Hm)	700,000	Y 439	1882 II	200,000,000	Cl. 363	1680	70– 90	Ch. 518
									1769	90– 97	Co. 104
									1843 I	65	Ch. 484
									1861 II	118	Cl. 326

TABLE LXXVII.

MISCELLANEOUS PECULIARITIES.

Scintillation.		Obliteration.		Condensation.		Disappearance.		Sheath.	
1824 II	Co. 147	1807	Cl. 106	1835 III (H)	Cl. 102	1769	Ch. 413	1680	Cl. 364
1843 I	" 162	1890 VI	" "	1884 I (P)	" 365	1825 IV	Co. 150	1858 VI	H. 361
1848 II (E)	" 185			1892 III (Hm.)	" 369	1835 III (H)	Cl. 102	1882 II	Cl. 364
								1889 V	" 367

Frequently the head of a comet has been observed to transit a small star, and in nearly every case the light of the star has remained unaltered. Perhaps the most satisfactory observation of this sort was made by Professor Wendell on the comet 1902 III with the polarizing photometer attached to the 15-inch refractor. Harvard Circular 68. The nucleus passed within 1′.1 of a seventh magnitude star, the diameter of the coma being 6′. Although measured to hundredths of a magnitude, the absorption was pronounced insensible. On the other hand, in the second column of the table are given two instances where the light of the star was nearly or quite obliterated. The first was due to a transit of the tail near the head, the second to a transit of the head itself.

We have already noticed in this chapter the great change in the volume of Encke's comet as it approached the Sun. Similar but less marked changes in other comets are not infrequent. In the third column of the table are given three instances, all of periodic comets, where the bulk was so far condensed that the light was nearly or quite indistinguishable from that of an ordinary star. Instances of the disappearance of the tail just before or during perihelion passage are not unknown. Sometimes the tail is recovered later, sometimes not. Three instances of this sort are given in the fourth column. The last column of the table refers to a curious faint light which is sometimes seen to envelop the whole comet. It is illustrated in Chambers' Handbook, p. 479.

In the first column of Table LXXVIII are included those comets which have separated into two or more parts. The first of these was the great comet described by Aristotle, as mentioned in the last chapter. The second broke into many parts, as did also the third. The fourth and fifth are Biela's comet and the one observed by Liais. The others are well known modern instances where companion comets have been seen. The second column records instances where the nucleus was observed to have separated into two or more parts. This phenomenon was also noted by Douglass at Tucson in the case of Halley's comet, when during its last apparition on May 24, 1910, at 17^h 45^m,

TABLE LXXVIII.

MISCELLANEOUS PECULIARITIES.

Divided.		Compound Nucleus			Colored.		Visible in the daytime				Beard.	
−372	Cl. 96	1618 II	granular	Cl. 362	1556	Co. 74	−136		—	Co. 62	1744	Co. 95
−11	Co. 63	1652	"	" "	1577	" "	−43		—	Ch. 407	1823	" 146
1618 II	Ch. 408	1744	double	Co. 95	1593	" 76	575(?)		—	" "	1825 I	" 149
1846 II(B)	" "	1846 II(B)	multiple	A 85	1618 II	Ch. 517	1106		—	" 575	1825 IV	" 150
1860 I	Cl. 339	1850 I	cluster	Co. 188	1680	Co. 85	1402 I	8	0.350	" 581	1848 II(E)	" 185
1881 II	Ch. 409	1882 II	quadruple	Ch. 475	1759 III	" 100	1402 II		—	" "	1850 I	" 188
1882 II	Cl. 363	1888 I	triple	" 481	1811 I	" 133	1472		0.486	" 407	1851 IV	" 193
1889 V	" 366	1898 I	granular	P 120	1835 III(H)	Cl. 102	1532		0.519	" "	1871 V(E)	Ch. 421
					1843 I(?)	Co. 161	1577		0.178	" "	1877 II	" 541
					1882 I	Cl. 357	1618 II		0.390	" "	1880 V	Y. 451
					1884 I	" 366	1843 I	2	0.006	Co. 159	1882 II	Cl. 363
					1886 I	Ch. 399	1853 III	5	0.307	Ch. 533		
							1882 II	3	0.008	Cl. 359		

G. M. T., he saw a temporary division of the nucleus, the two centres being separated by an interval of 30″.

Comets are usually of a pale bluish or greenish white, but those recorded in the third column of the table all exhibited, in whole or in part, a reddish or orange yellow tint. Cooper questions the color of 1843 I. The fourth column gives a list of those comets that were bright enough to be visible with the naked eye when the Sun was above the horizon. In certain cases, where known, this is followed by the number of days of visibility and by the perihelion distance. The largest perihelion distance recorded is that of the comet of 1532. Chambers states that this comet was visible in the day time, but does not give the authority. The comet of —136 is recorded as being "brighter than the Sun." For this reason it seemed safe to insert it in the list. This comet is identical with that of −137, as given in Table XXXIV, and the comet of 575 is probably so with that of 574. The perihelion distances of both are surprisingly large, however, that of the former being 1.010, and of the latter 0.963.

Comets are sometimes said to be furnished with a beard, that is to say a tail projecting towards the Sun. The name arose presumably because it is located on the side opposite to the hair, as the tail was formerly sometimes called. A beard would be somewhat difficult to explain on the theory of the repulsion of light, but on the electrical theory it is obviously simply due to the positively electrified molecules, and is similar to the so-called canal rays. As the tail is often not directly opposed to the Sun, so the beard sometimes does not project directly towards it. The comet of 1532 was said to be furnished with a beard two fathoms long, but this statement is so obviously inaccurate that it is thought that it probably refers to the tail. Since the days of careful telescopic study it is found that a beard is by no means an unusual feature. A list of the comets furnished with this appendage is given in the last column. In the comet of 1823 the beard was longer than the tail proper. Encke's comet is twice recorded, 1848 II and 1871 V, as being furnished with a beard. Bond refers to the outer faint veil of 1858 VI, H. A. 3, 361. This feature might possibly be classed as a beard.

The following comets were more or less remarkable for one reason or another:—

1264 was one of the finest comets on record since accurate descriptions of these bodies have been kept. Ch. 514.

1577. The tail deviated 21° from the line of the radius vector. Ch. 411.

1707. The nucleus was not round, but indented. Co. 90.

1744. This was the finest comet of the eighteenth century. It was noted for having six tails. Ch. 411. Most fine comets have two tails, one straight and the other curved. Since photography has been applied to comets, it has been found that the tail usually consists of a bundle of straight rays, and it would not be surprising for a bright comet to show six of them.

1769. A very fine comet. Its tail exhibited double curvature, that is to say was S-shaped. Ch. 411.

1770 I. Lexell's comet. It approached so near the earth, 1,400,000 miles, that its head measured 2.°3 in diameter. Cl. 106, Co. 107. In recent times it has been associated with the comets of 1889 V and 1895 II. Cl. 367.

1825 IV. The nucleus was not round, but as if formed of three points. The tail separated into three, and later into five divisions, like that of 1744. Co. 150. Ch. 411.

1843 I. The second tail of this fine comet was reported by De Moussy to be separated from the first on two days early in March. Co. 160. A somewhat analogous discontinuity of the tail was exhibited in Brooks' comet 1893, IV. See lower figure in Knowledge 1894, 17, 36.

1863 IV, 1863 V. In both of these comets the tail deviated from the plane of the orbit. Ch. 411. This fact shows plainly the futility of the attempt to compute the rate of recession of the tail by the angle which it makes with the radius vector. This fact is further shown by the three photographs of comet 1893 IV, published in the Astrophysical Journal, 1905, 22, 251, and by the fact that for a time the tail of this comet was actually ahead of the prolongation of the radius vector. Ibid 1905, 21, 326.

1877 I. This comet gave an anomalous spectrum unlike that of any other. Y 444.

1877 II. Winnecke's comet threw out a secondary tail making an angle of 60° with its primary one, and having about the same length, 1°. Y. 451.

1881 II. The secondary tail of this comet reached a length of 55°, the longest secondary on record. Ch. 412.

1881 III. A star seen through one of the nuclear jets was spread out into a nebulous patch of light. Cl. 353.

1882 I. When near the Sun its spectrum was monochromatic, that of the sodium D line. Cl. 357.

1882 II. At perihelion the sodium and iron lines were seen in its spectrum. Cl. 361, 364.

1884 I. Pons. Although its perihelion distance was 0.776, yet during its condensation to a stellar disk it gave out a sodium spectrum, implying a high temperature. Cl. 366. This was also shown by Halley's comet at unit distance from the Sun, during its recent appearance.

1892 III. Holmes' comet, unlike any other, gave a continuous spectrum with neither bright nor dark lines. Y. 444. On condensation later to a stellar disk it became yellow. It was very remote from the Sun at this time. Cl. 369.

CHAPTER XVIII.

APHELION DISTANCES. PLANET P. PERIHELION. ECCENTRICITY.

IN Figure 32, p. 193, a curve is shown representing the distribution of all the accurately known elliptical cometary orbits with regard to the logarithms of their aphelion distances. On this curve the approximate distances of the planets, including *O*, *P*, and *Q*, are also indicated. The result with regard to the last two was merely a preliminary determination, which could not be accurate since no distinction was made in that chapter between the orbits belonging to the different cometary types.

The aphelion distances of the orbits of classes *A* and *B* are given in Table XL, p. 189. It will be recalled that precisely the same comets make up these classes as go to form types *N* and *O*. No such complete identity however, exists for the other types, and in Table LXXIX the orbits of classes *C*, *D*, and *E* are grouped according to the types to which they belong. The four columns give in succession the number of the orbit, its type, its class, and the logarithm of its aphelion distance.

TABLE LXXIX.

APHELION DISTANCES.

No.	Type.	Class.	Log. dist.	No.	Type.	Class.	Log. dist.	No.	Type.	Class.	Log. dist.	No.	Type.	Class.	Log dist.
253	*P*	*C*	1.8779	375	*P*	*C*	2.2971	198	*Q*	*C*	2.1089	164	*Q*	*D*	2.7910
463	"	"	1.9450	245	"	"	2.2973	353	"	*D*	2.3636	377	"	"	2.8525
320	"	"	1.9494	244	"	"	2.3239	116	"	"	2.3820	97	"	"	2.8774
195	"	"	2.0043	413	"	"	2.3377	142	"	"	2.4564	65	"	"	2.9304
128	"	"	2.0224	193	"	*D*	2.6857	308	"	"	2.5092	387	"	"	2.9595
271	"	"	2.0398	171	"	"	2.7340	383	"	"	2.5490	216	"	"	2.9741
270	"	"	2.0441	255	"	"	2.8254	346	"	"	2.5575	327	"	"	2.9863
246	"	"	2.0931	226	"	"	2.9154	179	"	"	2.5786	393	"	"	2.9956
214	"	"	2.0966	453	"	*E*	3.3905	146	"	"	2.6263	319	"	*E*	3.0569
351	"	"	2.1529	314	"	"	3.4548	168	"	"	2.6917	279	"	"	3.1335
238	"	"	2.2276	430	*Q*	*C*	2.0434	305	"	"	2.7760	280	"	"	3.1399

No.	Type.	Class.	Log. dist.	No.	Type.	Class.	Log. dist.	No.	Type.	Class.	Log. dist.	No.	Type.	Class.	Log. dist.
326	*Q*	*E*	3.1644	109	*Q*	*E*	3.5527	263	*R*	*D*	2.8189	395	*R*	*E*	3.4742
402	"	"	3.1703	385	"	"	3.9731	175	"	"	2.8317	202	"	"	3.6294
227	"	"	3.2742	381	"	"	4.2923	319	"	*E*	3.0569	205	"	"	3.6767
448	"	"	3.3617	147	*R*	*C*	2.2154	280	"	"	3.1399	439	"	"	3.7679
218	"	"	3.3979	101	"	*D*	2.5145	402	"	"	3.1703	385	"	"	3.9731
409	"	"	3.3997	379	"	"	2.5263	321	"	"	3.2253	231	"	"	4.3655
400	"	"	3.4579	313	"	"	2.6522	227	"	"	3.2742				
395	"	"	3.4742	411	"	"	2.6646	409	"	"	3.3997				

Table LXXX is arranged precisely like Table XLII, p. 192, and is deduced from Tables XL and LXXIX. The first column gives the logarithms of aphelion distances taken at regular intervals. A count was made of all the comets the logarithms of whose aphelia lay between 0.58 and 0.62, and the result was entered in the second column of the table against 0.60. The number of comets lying between 0.60 and 0.64 was similarly entered against 0.62, and so on. These results are plotted in Figure 52,

TABLE LXXX.

ENUMERATION OF ORBITS.

Type *N*.

Log.	Orb.	Log.	Orb.
0.58	0	0.76	13
0.60	1	0.78	10
0.62	1	0.80	5
0.64	0	0.82	2
0.66	1	0.84	1
0.68	4	0.86	1
0.70	9	0.88	1
0.72	9	0.90	1
0.74	9	0.92	0

Type *O*.

Log.	Orb.	Log.	Orb.
0.90	0	0.95	1

Type *O*.

Log.	Orb.	Log.	Orb.
1.00	2	1.40	0
1.05	1	1.45	1
1.10	0	1.50	5
1.15	0	1.55	5
1.20	0	1.60	1
1.25	1	1.65	2
1.30	2	1.70	2
1.35	1	1.75	0

Type *P*.

Log.	Orb.	Log.	Orb.
1.80	0	1.90	3
1.85	1	1.95	2

Type *P*.

Log.	Orb.	Log.	Orb.
2.00	4	2.70	2
2.05	6	2.75	1
2.10	2	2.80	1
2.15	1	2.85	1
2.20	2	2.90	1
2.25	3	2.95	1
2.30	4	3.00	0
2.35	2	—	—
2.40	0	3.30	0
2.45	0	3.35	1
2.50	0	3.40	1
2.55	0	3.45	1
2.60	0	3.50	1
2.65	1	3.55	0

Type *Q*.

Log.	Orb.	Log.	Orb.
1.9	0	3.2	5
2.0	1	3.3	4
2.1	2	3.4	5
2.2	1	3.5	3
2.3	2	3.6	1
2.4	3	3.7	0
2.5	5	3.8	0
2.6	6	3.9	1
2.7	4	4.0	1
2.8	4	4.1	0
2.9	7	4.2	1
3.0	6	4.3	1
3.1	5	4.4	0

Type *R*.

Log.	Orb.	Log.	Orb.
2.1	0	3.4	2
2.2	1	3.5	1
2.3	1	3.6	2
2.4	0	3.7	3
2.5	2	3.8	1
2.6	4	3.9	1
2.7	2	4.0	1
2.8	2	4.1	0
2.9	2	4.2	0
3.0	1	4.3	1
3.1	3	4.4	1
3.2	4	4.5	0
3 3	3		

where the abscissas are taken from the first column of the table and the ordinates from the second. In the third curve under type P, the four most remote points, which are due to comets numbers, 453 and 314, do not come within the limits of the figure.

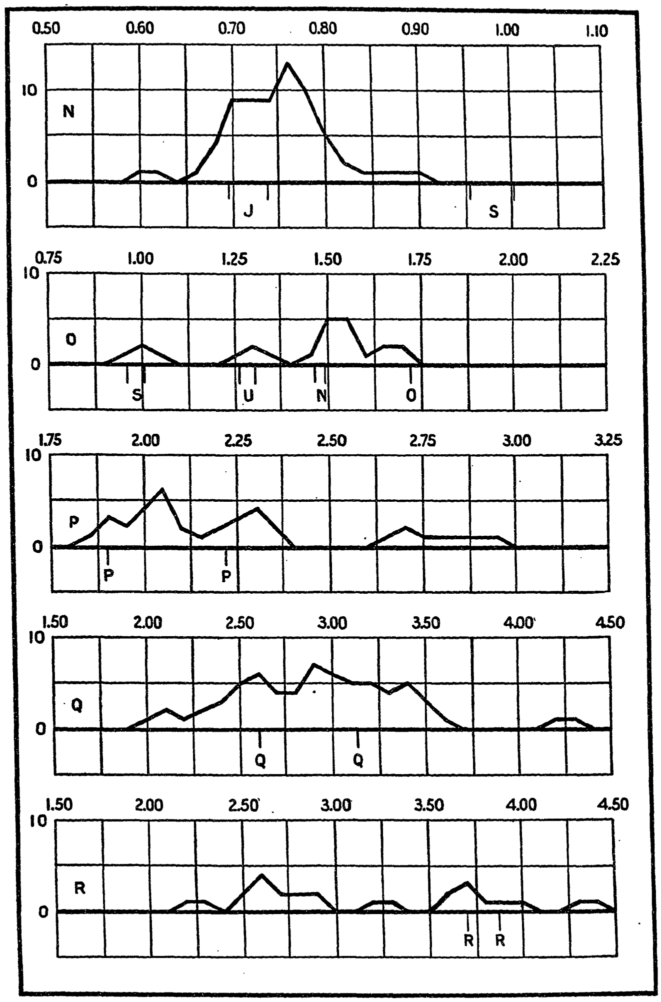

FIG. 52.

Devoting our attention now for the present to this type, it is natural to endeavor to explain the considerable deviation among themselves of its aphelion distances in a manner analogous to that adopted for the parabolic and hyperbolic orbits of type Q in Chapter XII. Let us assume that planet P, like the planets nearer the Sun, moves in an approximately circular orbit, with which its related cometary aphelia nearly coincide. The apparently great aphelion distance of some of these orbits then is not real, but is due to the position of planet P. If the planet is on the opposite side of its orbit to the comet, it will attract the latter as it approaches the Sun, and give it a higher perihelion velocity than it would otherwise possess. On the other hand, if the planet and comet are on the same side of the planet's orbit, the comet will be delayed in its approach to the Sun, and its computed aphelion distance will be less than is really the case.

In Table LXXXI the first column gives a list of all the comets of type P that belong to class C, the second column gives the values of ϕ taken from Table LII, p. 222, and the third these angles reduced so as to read continuously around the reference circle, beginning at the ascending node. These angles give approximately the argument of the comet's aphelion upon the planet's orbit. The fourth and fifth columns give the logarithms of the perihelion distance and mean semi-diameter. The sixth column gives the elliptic minus the parabolic perihelion velocity, computed by the formula $k = -\frac{10.525\sqrt{q}}{a}$ taken from p. 195. The seventh column gives the same quantity, assuming that the major semi-diameter of the comet's orbit was in every case 120 units instead of its computed value. The last column gives the difference between these two velocities, and indicates the change in velocity that has been produced upon the comet according to this hypothesis by the attraction of planet *P*.

TABLE LXXXI.

ACCELERATION DUE TO PLANET *P*.

No.	ϕ	A'	log *q*.	log *a*.	*k*	*k'*	*k—k'*
	°	°					
245	6	6	0.1327	1.9992	−0.123	−0.204	+0.081
238	41	41	9.9584	1.9290	.118	.167	+0.049
413	46	46	9.9926	2.0386	.096	.174	+0.078
246	51	51	0.3411	1.7995	.247	.260	+0.013
270	55	55	9.9641	1.7466	.181	.168	−0.013
271	71	109	9.9151	1.7419	.173	.159	−0.014
253	24	204	9.8732	1.5809	.239	.152	−0.087
320	26	206	0.2274	1.6569	.301	.228	−0.073
195	60	240	0.1706	1.7097	.250	.213	−0.037
351	70	250	0.2840	1.8575	.203	.243	+0.040
128	75	255	0.1744	1.7272	.241	.214	−0.027
244	68	292	9.9023	2.0247	.089	.157	+0.068
375	60	300	0.2122	1.9997	.134	.224	+0.090
214	24	336	9.8019	1.7976	.134	.140	+0.006
463	10	350	0.0473	1.6494	.249	.185	−0.064

It will be noticed that the comets are arranged in the order of their arguments and that the last column shows a corresponding systematic change in the deviations. This is more clearly seen in Figure 53, which is plotted in polar coordinates with the angles taken from the third column of the table, and the radii from the last. The heavy line, which should be approximately but not strictly a circle, is drawn so as to intersect as many points as possible. Its radius is that of the zero vector, and its centre is indicated by the cross.

It will be seen that with two exceptions all the comets lie near the curve, and that the higher velocities lie on the side of the Sun towards argument 340°. This angle minus 180° therefore represents the present argument of planet *P*. With regard to its mass, it appears by the last column that the planet is capable of exciting a maximum increase of velocity of 0.09 kilometres per second. But the velocity varies as the acceleration, which varies as the mass. Constructing a table similar to Table XLIV, p. 199, and treating the results in the same way, we find that even if planet *P* were as remote as 120 units, its mass would be as much as 0.082 in terms of that of the Sun. This would be nearly ninety times that of Jupiter. Assuming *P* to have the same density and albedo as Neptune we find it would shine with one-half the light of that planet, and present a disk 8″ in diameter.

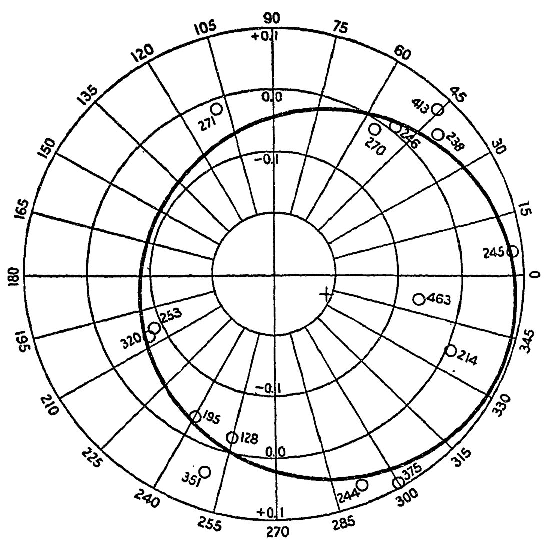

FIG. 53.

Clearly the existence of such a body is highly improbable. It would certainly have been discovered already by photography, if not by visual observations, and, moreover, its perturbations of the known planets could not have escaped notice. The curve, Figure 53, leads us to believe that some connection between the arguments and the aphelion distances exists, but it is pretty certain that the hypothesis that this relation is due mainly to the acceleration of planet *P* is erroneous.

Let us now return to the curves represented in Figure 52. We notice that in the case of Jupiter the aphelia rise at first on both sides symmetrically, but that there is a notch at the summit of the curve. Another and more prominent notch occurs in the case of type *P*, and another in the case of *Q*. The aphelia of type *R* appear to be more irregularly distributed. This will be discussed in the next chapter. At first sight we should consider these notches merely an accidental feature, but on further consideration it would seem that there is a possible explanation for them. At those distances from the Sun where any given planet was rapidly changing its distance we should expect to find fewer comets than where the planet was for a considerable portion of its orbit at a nearly uniform distance.

To test the accuracy of this hypothesis it was decided to determine the mean distance, eccentricity, and longitude of the perihelion of the orbit of

TABLE LXXXII.

TYPE *N*. DISTRIBUTION OF THE APHELIA ON THE REFERENCE PLANE.

No.	Long.	Aph. dist.	M.l.	M.d.	S	H	Dev.	V	Eccent.	Peri.
	°		°							
259	20	5.89			*n*	− .848	+ .75	+.400	− .636	+ .300
368	51	4.97			*f*	− .920	− .15	−.120	+ .138	+ .018
336	59	4.83	43	5.23	*f*	− .901	− .29	−.220	+ .261	+ .064
484	92	5.56			*f*	− .648	+ .34	−.700	− .220	− .238
470	103	5.36	98	5.46	*f*	− .511	+ .09	−.811	− .046	− .073
358	126	4.87			*f*	− .152	− .53	−.970	+ .081	+ .514
460	126	4.67			*f*	− .152	− .73	−.970	+ .111	+ .708
428	140	5.90			*f*	+ .090	+ .42	−.998	+ .038	− .419
424	154	5.76			*f*	+ .337	+ .19	−.961	+ .064	− .183
417	158	6.17			*f*	+ .400	+ .57	−.940	+ .228	− .536
469	165	5.10			*f*	+ .512	− .54	−.895	− .276	+ .483
415	165	5.11			*f*	+ .512	− .53	−.895	− .271	+ .474
102	178	5.63	152	5.40	*f*	+ .713	− .08	−.760	− .057	+ .061
457	182	5.43			*f*	+ .767	− .30	−.715	− .230	+ .214
471	188	6.02			*f*	+ .841	+ .26	−.630	+ .219	− .164
447	189	6.21			*f*	+ .855	+ .44	−.618	+ .376	− .272
406	199	5.55			*f*	+ .954	− .25	−.452	− .238	+ .113
348	199	7.73			*f*	+ .958	+1.93	−.450	+1.849	− .868
433	200	5.61			*f*	+ .970	− .19	−.430	− .184	+ .082
472	211	6.54			*f*	+1.040	+ .71	−.225	+ .738	− .160
389	220	7.24			*f*	+1.070	+1.40	−.000	+1.498	.000
481	224	5.22			*f*	+1.070	− .63	−.050	− .674	+ .032
485	229	5.76			*n*	+1.063	− .08	+.100	− .085	− .008
382	231	5.96			*n*	+1.070	+ .12	+.130	+ .128	+ .016
113	233	5.06			*n*	+1.063	− .78	+.150	− .829	− .117
396	238	5.08	211	5.95	*n*	+1.042	− .75	+.218	− .782	− .164
159	249	4.80			*n*	+ .981	−1.00	+.410	− .981	− .410
462	250	5.94			*n*	+ .966	+ .14	+.442	+ .135	+ .062
486	254	5.52			*n*	+ .930	− .28	+.505	− .260	− .141
85	276	5.32			*n*	+ .656	− .38	+.806	− .249	− .306
235	289	6.19			*n*	+ .652	+ .58	+.920	+ .378	+ .534
334	295	5.61	269	5.56	*n*	+ .350	+ .03	+.954	+ .010	+ .029
412	311	6.48			*n*	+ .063	+1.00	+.999	+ .063	+ .999
480	339	4.09	325	5.28	*n*	− .389	−1.20	+.886	+ .467	−1.063

Jupiter by means of the aphelia of its associated comets. To this end Table LXXXII was prepared. The first three columns give the number of the orbit, the longitude of its aphelion, and its aphelion distance. The numbers in the last of these columns varied so irregularly that it was concluded to divide the orbit into six sections of 60° each, and determine the mean distance of the aphelia in each section. The next two columns give the mean values of the longitudes and aphelion distance for each of these sections. A plot was then made, Figure 54, with these longitudes as abscissas and the aphelion distances as ordinates. By the side of each observation is recorded the number of aphelia from which it is derived. A sinusoid was then drawn coinciding as nearly as might be with these results. Half the sum of the maximum and minimum ordinates gives the mean distance of the planet's orbit, their difference divided by their sum gives its eccentricity, and the abscissa of the minimum gives the longitude of perihelion. These three quantities as determined from the curve are 5.48, 0.0657 and 45°.

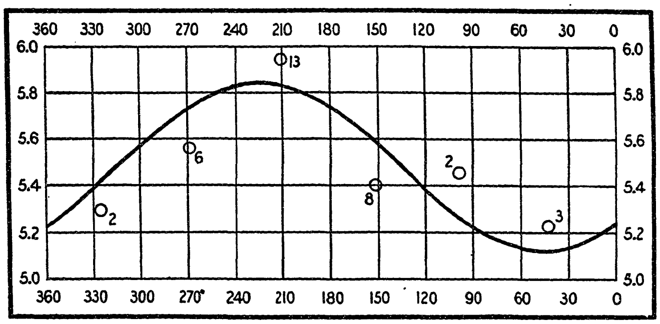

FIG 54.

To obtain more accurate results Figure 55 was constructed. A semi-ellipse was first drawn with an eccentricity of 0.0657. The horizontal line marked by the cross representing its major axis, while the cross itself is at the focus. From the focus as a centre short lines were drawn intersecting the curve at angles of ten degrees. Through these intersections vertical lines were passed which may be considered as lying upon the surface of an ellipsoidal cylinder. Other horizontal lines were drawn uniformly spaced. The vertical line representing the right hand side of the cylinder was given the longitude of perihelion, 45°, and the other vertical lines marked with their longitudes accordingly. As in Figure 54 the horizontal lines represent distances from the Sun. The thirty-four aphelia were then plotted, the plane circles referring to the longitudes marked at the top of the figure, while the circles superposed upon crosses refer to the longitudes marked at the bottom. An inclined line AB was now drawn, the ordinate at A being made equal to the maximum ordinate in Figure 54, and that at B to the minimum. This line may be considered as the intersection of a plane cutting the cylinder.

It corresponds in part to the assumed reference circle in the earlier figures, representing the orthographic projection of the sphere.

This graphical solution of the eccentricity and perihelion for an elliptic orbit depends on the proposition that if the ordinates at A and B are proportional to the aphelion and perihelion distances, then the radius vector at any point C will be proportional to the ordinate measured on the perpen-

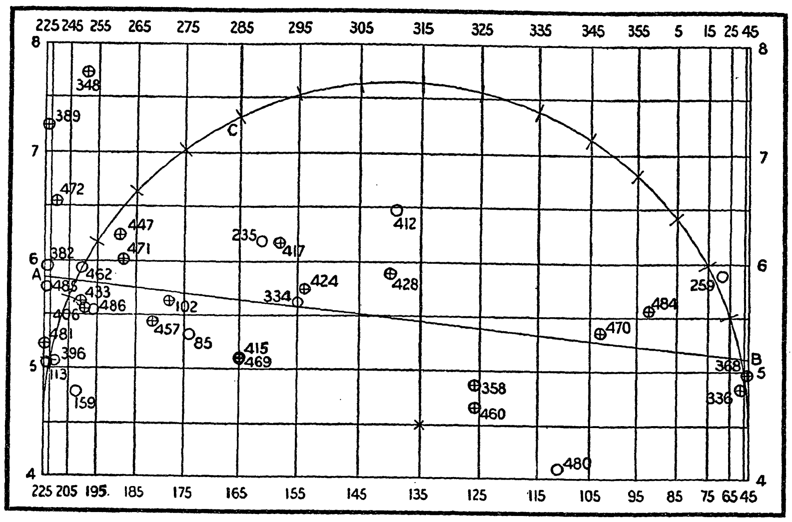

FIG. 55.

dicular passing through C, from the line of zero ordinates to the intersection with the line AB. The unit used in measuring the ellipse is the length of the semi-major diameter. We find this length is equal to 3.15 of the units used to measure the ordinates.

Returning now to Table LXXXII, the sixth column indicates whether the aphelion lies on the nearer or on the farther side of the cylinder. An n indicates that the corresponding longitude is given at the top of the figure, and that the aphelion lies on the nearer side. The seventh column gives the distances of the aphelia in terms of the mean semi-diameter from the vertical line intersecting the focus, distances to the left being reckoned as positive. These distances are horizontal, and not like those in the earlier tables measured along the reference circle. The eighth column gives the deviations in terms of the scale of ordinates, measured in a vertical direction from the inclined plane AB, distances above the plane being reckoned as positive. The ninth column gives the vertical distances in terms of the mean semi-diameter, measured along the perpendiculars passing through the aphelia, from the horizontal line intersecting the focus to the circumference of the ellipse. If the aphelion is on the nearer side of the cylinder this distance is reckoned as positive. The tenth column gives the product of the deviations into the horizontal distances. If the assumed inclination of AB is correct the algebraic sum of those products will be zero, if it is positive the inclination should be increased. The last column gives the product of the deviations into the vertical distances. If the longitude of the perihelion is correctly selected the algebraic sum of these products will be zero. If it is positive the longitude of the perihelion should be increased.

In Table LXXXIII, Part I, the first column gives the number of observations considered, the second the algebraic sum of the deviations above and below the reference plane taken from the eighth column of Table LXXXII, the third the correction to the assumed mean distance of the planet obtained by dividing this quantity by 34, the fourth the resulting mean distance, the fifth the true mean distance, and the last the ratio between these two.

In Part II the first column gives the number of observations, the second the arithmetical sum of the horizontal distances, and the third the algebraic sum of the deviations multiplied by the horizontal distances, taken from the next to last column of Table LXXXII. This quantity divided by the ratio of the units, 3.15, and multiplied by the number of observations divided by the square of ΣH, as explained under the similar case in chapter XIV, gives the tangent of the correction to the angle of the reference plane, given in the fourth column. The assumed eccentricity 0.0657 is the tangent of the assumed angle of the reference plane, or angle of the eccentricity 3°.76. This angle, plus the correction, is given in the fifth column, and the corrected eccentricity, which is the tangent of this angle, is given in the sixth. The true eccentricity is given in the seventh column and the observed minus the true value, in the last.

The first three columns in Part III give the number of observations, the arithmetical sum of the vertical distances, and the algebraic sum of the quantities taken from the last column of Table LXXXII. The correction to the assumed longitude of perihelion given in the fourth column is obtained by multiplying the quantity in the third by that in the first, and dividing the product by the square of that in the second, times the ratio 3.15, times the sine of the corrected angle of the eccentricity, taken from the fifth column of Part II. The fifth column gives the resulting longitude of perihelion, the sixth the true longitude, and the last the observed minus the true value.

TABLE LXXXIII.

TYPE *N*. MEAN DISTANCE. ECCENTRICITY. PERIHELION.

PART I.

No.	Deviation.	Corr.	Distance.	True.	Ratio.
34	+0.28	+0.01	5.49	5.20	1.056

PART II.

No.	Σ H.	Σ Dev. H.	Corr. eccent.	Ang. eccent.	Eccent.	True.	O—C.
34	24.446	+0.764	+0.79°	4.55°	0.080	0.048	0.032

PART III.

No.	Σ V.	Σ Dec. V.	Corr. peri.	Perihelion.	True.	O—C.
34	19.730	−0.419	−8.3°	36.7°	11.9°	24.8°

Comparing the true values of these elements of the orbit of Jupiter with those deduced by the cometary method, we notice first that the deduced value of the mean distance of the planet is 5.6 per cent too large. For Saturn the excess is 5.5 per cent, for Uranus 8.0, and for Neptune 13.1 per cent. Since all these comets are drawn in by the planet from the outside, their aphelia being in all cases gradually shortened, it is only what we should expect, to find them slightly in excess of the planet's mean distance. We also notice that the excesses are greater for the two planets that are less massive, and that, having longer periods, meet their associated comets less frequently.

The deviation of the deduced eccentricity .032 is about what we might expect, judged from the size of the quantities shown in the tenth column of

Table LXXXII. This deviation is therefore due to accidental causes, and the result indicates roughly the size of the error in the eccentricity that we may expect to find when it is computed by this method.

In order, however, that the observed value of the eccentricity should be reduced to zero, the value of Σ Dev. H in the third column of Part II of the table would have to be changed from +0.764 to −3.033. To accomplish this it would be necessary to reject the three comets giving the largest positive residuals in the tenth column of Table LXXXII, namely numbers 348, 389, and 472. The positive value of Σ Dev. H can therefore hardly be due to accident, and we accordingly conclude that this method of computing the eccentricity and perihelion of a planet's orbit has some value, and may be used to advantage when other methods are inapplicable, namely when the planet itself is still undiscovered. And this is true even when the eccentricity of the orbit is small, as in the case of the planet Jupiter.

In the case of planet *P*, its associated comets are comparatively few, numbering only 15, but an examination of their aphelion distances indicated that this disadvantage was more than compensated by the fact that the planet's orbit possessed apparently a very high eccentricity. Indeed it could hardly be otherwise for a body moving in an orbit located at not very

TABLE LXXXIV.

TYPE *P*. DISTRIBUTION OF THE APHELIA ON THE REFERENCE PLANE.

No.	A′	Aph. dist.	M. (A′)	M. dist.	S.	H.	Dev.	V.	Eccent.	Peri.
245	6°	198.3			*n*	+1.351	+31	+.110	+ 41.9	+ 3.4
238	41	168.9			*n*	+0.910	+20	+.789	+ 18.2	+15.8
413	46	217.6			*n*	+0.817	+75	+.840	+ 61.3	+63.0
246	51	123.9			*n*	+0.721	−15	+.880	− 10.8	−13.2
270	55	110.7	40°	163.7	*n*	+0.630	−25	+.909	− 15.8	−22.7
271	109	109.6	109	109.6	*n*	−0.251	+13	+.744	− 3.3	+ 9.7
253	204	75.5	—	—	*f*	−0.602	− 6	−.270	+ 3.6	+ 1.6
320	206	89.0	205	82.2	*f*	−0.595	+ 7	−.282	− 4.2	− 2.0
195	240	101.0			*f*	−0.370	+ 9	−.641	− 3.3	− 5.8
351	250	142.2			*f*	−0.268	+46	−.732	− 12.3	−33.7
128	255	105.3			*f*	−0.213	+ 6	−.771	− 1.3	− 4.6
244	292	210.8	259	139.8	*f*	+0.382	+86	−.945	+ 32.9	−81.3
375	300	198.2			*f*	+0.540	+67	−.930	+ 36.2	−62.3
214	336	124.9			*f*	+1.171	−35	−.541	− 41.0	+18.9
463	350	88.1	329	137.1	*f*	+1.329	−78	−.230	−103.6	+17.9

unequal distances between the Sun and the massive planet Q. The case is, in fact, analogous to that of Mars and Jupiter, the orbit of the former being as is well known quite eccentric.

In Table LXXXIV the first column gives the number of the comet's orbit, the second the argument of its aphelion, taken from the third column of Table LXXXI, and the third the aphelion distance taken from Table XLI. The orbit is divided as before into six sections. The fourth and fifth columns contain the mean values of A′ and of the aphelion distances for each section. No aphelia are found in the third section.

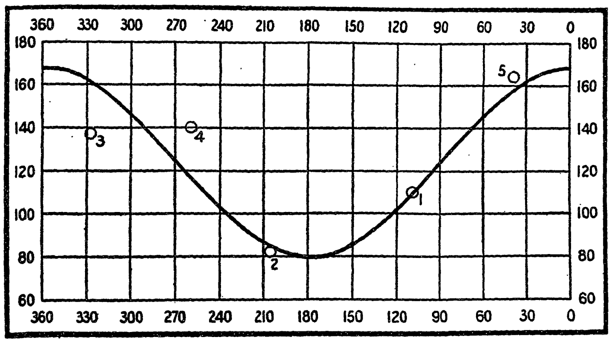

Fig. 56.

Figure 56 is plotted with the quantities in the fourth column as abscissas and those in the fifth as ordinates. The sinusoid was then fitted to them, and from it we obtain for the mean distance 124 units, for the eccentricity 0.355, and for the argument of perihelion 180°. Apparently the aphelion of the planet's orbit is very near its ascending node.

Figure 57 is now drawn in the same manner as Figure 55. The unit used in measuring the ellipse, its semi-major diameter, is equal to 126.5 of the

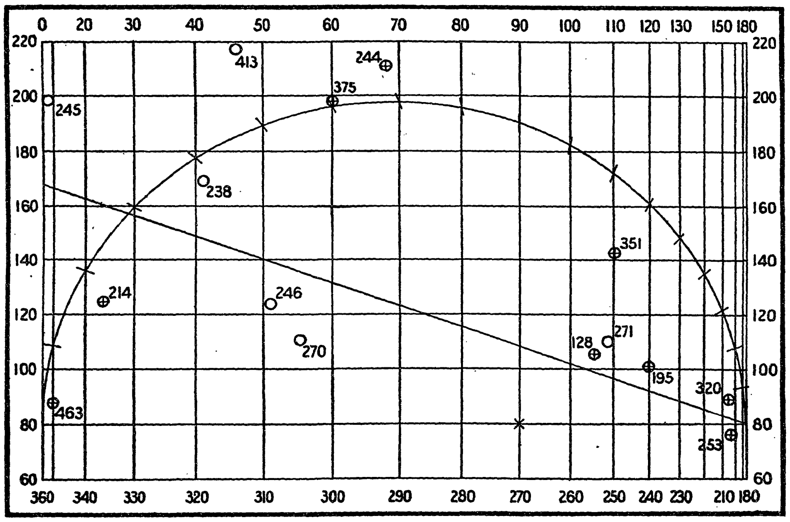

Fig. 57.

ordinate units. By means of measures made on this figure the remaining columns of Table LXXXIV are computed, and from it the results shown in Table LXXXV are deduced in precisely the same manner as was done for the two preceding tables. It should be noted that the last column of Part III gives the argument of the perihelion, instead of its longitude, as before.

TABLE LXXXV.

TYPE *P*. MEAN DISTANCE. ECCENTRICITY. PERIHELION.

PART I.

No.	Deviation.	Correction.	Distance.
15	+201	+13	137

PART II.

No.	Σ H	Σ Dev. H	Corr. eccent.	Ang. eccent.	Eccent.
15	10.150	−1.5	−0.10	19.44	0.353

PART III.

No.	Σ V	Σ Dec. V	Corr. peri.	Perihelion.
15	9.614	−95.3	−20.2	159.8

The resulting distance of the planet is 137 units, but as we have just seen a certain percentage must be subtracted from this figure. The more massive we assume the planet to be, the smaller percentage should we subtract. On the other hand, on account of its long period, its associated comets seldom pass near it, which would induce us to subtract a larger percentage than in the case of the known planets. If we subtract 10 per cent, the distance of the planet will be 123 units. The corresponding period would be 1360 years.

An inspection of Figure 57 shows that if we wished to make the inclined line horizontal, so that the derived value of the eccentricity should be reduced to zero, it would be necessary to reject the five orbits giving the largest positive values in the tenth column of Table LXXXIV. This is one third of the total number. Such a procedure would obviously be quite inadmissible, and it therefore appears highly probable that the planet's orbit is really very eccentric, and that our result, 0.35, is not due merely to accident. The orbits are too few in number to enable us to fix satisfactory limits to the eccentricity, but it is believed to be not less than 0.20 nor more than 0.45. It is not thought that the error in the argument of the perihelion exceeds 20°. The logarithms of the perihelion and aphelion distances of each of the known planets is indicated in Figure 52 by the short vertical lines beneath the upper two curves. These distances were computed for planet *P*, and prove to be 80 and 166 units. Their logarithms are indicated beneath the third curve.

It will be remembered that on the discarded hypothesis that the computed aphelion distances were not real, but only apparent, their true value in all cases being 120 units, we found by Figure 53 that the velocities would lie along a curve the argument of whose maximum was 340°. The argument of its minimum would therefore be 160°, agreeing accurately with the argument of the perihelion found in Table LXXXV, although the result was obtained by quite a different method. Figure 53 is therefore an indirect illustration of the reliance that we may place on the value of the argument of the perihelion found in Table LXXXV, and the latter explains the coincidences of the observations shown in the figure with the heavy line of the curve.

Although this earlier hypothesis was discarded, it undoubtedly contains elements of truth, and the values and distribution of the different aphelia are certainly affected more or less by the present location of the planet. The more remote the planet may be from the Sun, the greater will be its influence on the aphelia of its associated comets. No effect of this sort that could be attributed to Jupiter has been detected among the comets of type *N* that have returned more than once. On the other hand, it is clearly influential in the case of type *Q*, and must be so also in the case of type *R*. Until we learn the actual mass of each planet, it will be impossible to determine the relative influence of the two causes upon the orbits of the associated comets, but in the case of *P* the ellipticity of its orbit is certainly by far the more influential of the two.

We noted above that assuming the existence of *Q* we should expect to find the orbit of *P* very eccentric. We may now make this statement in the inverse form, having found the orbit of *P* very eccentric, this furnishes an additional argument in favor of the existence of *Q*. Regarding its mass we unfortunately have very little information. Its distance from the Sun, being six times that of Uranus, bears about the same relation to it as the distance of Neptune does to the distance of Jupiter. By Chapters IX and X we saw how inadequate the method of planetary perturbations would be in such a case. Here its employment would be even more hopeless, since accurate observations of Uranus do not extend through even a single revolution of that planet. Doubtless, in the course of centuries the longitude and mass of *P* might be determined from its perturbations of Neptune, and especially of *O*, but at present we have only two facts to guide our judgment. One is the large number of comets associated with it, which on page 234 we estimated at not less than 110. The other is the concentration of the aphelia towards the reference circle exhibited in Figure 47, page 265. Both of these facts lead us to believe that it is a body of considerable mass, equal to or greater than any of our so-called major planets.

CHAPTER XIX

Distribution of the Aphelia. Planets Q and R.

In this chapter we shall begin by considering the distribution of the aphelia along the reference circles of the various cometary types. The longitudes of the aphelia of types N and O have already been given in Tables XLV and XLVI, pp. 202 and 204. Table LXXXVI shows the distribution for types P, Q, and R. The first column gives the number of the orbit, the second the class to which it belongs, the third its deviation from the assumed reference circle copied from the eighth columns of Tables LII, LV, and LVIII. The fourth column gives the angle of which the deviation is the sine, and the fifth its corresponding cosine. The sixth column gives the distance of the aphelion copied from the seventh columns of the three tables above mentioned. The seventh column gives the ratio of the numbers in the sixth column divided by those in the fifth. This ratio is the sine of the angle measured from the centre of Figures 38, 40, 41, and 43 along the assumed reference circles to the intersection of the reference circle with the great circle perpendicular to it which passes through the aphelion in question. This angle γ is given in the eighth column. In the last column this angle is reduced so as to read continuously around the reference circle, starting from the ascending node. It is in fact the argument of the comet's aphelion measured on the planet's orbit. The values given for this angle for type P in the last chapter were only approximate. In this table the 19 comets of type QR are all entered under type Q, and again under type R.

These results are plotted in Figures 58 and 59. The figures on the outside give the longitudes in each case. The second, that is to say, the next to the inner circle of short lines in Figure 58 indicates the longitudes of the various aphelia of type N. Similarly the third circle of lines indicates the longitudes of type O. The fourth circle gives the arguments of type P, taken from the last column of Table LXXXVI, the corresponding figures being given in the circle within it. If the aphelia belong to class C they are inserted within the circle, if they belong to the other classes they are entered outside of it. The figures are so oriented that the ascending node of zero argument shall be located correctly in longitude 350°. The correct orientation of the two circles of figures can of course only occur in four places, but excepting in the case of type Q in Figure 59, the deviation between them is never large.

TABLE LXXXVI.

LOCATION OF THE APHELIA OF TYPES P, Q AND R.

No.	Cl.	Dev.	β	Cos.	Dist.	Ratio.	γ	A
			Type *P*.					
			°				°	°
245	*C*	.219	12.7	.976	.096	.098	5.6	5.6
422	*G*	.120	6.9	.993	.100	.101	5.8	5.8
238	*C*	.192	11.1	.981	.658	.671	42.2	42.2
413	*C*	.140	8.0	.990	.725	.733	47.1	47.1
246	*C*	.048	2.8	.999	.780	.781	51.4	51.4
270	*C*	.010	0.6	1.000	.815	.815	54.6	54.6
226	*D*	.190	11.0	.982	.959	.977	77.7	77.7
271	*C*	.159	9.2	.987	.944	.956	72.9	107.1
88	*F*	.341	19.9	.940	.759	.808	53.9	126.1
318	*F*	.295	17.2	.955	.728	.762	49.6	130.4
314	*E*	.302	17.6	.953	.700	.735	47.3	132.7
171	*D*	.151	8.7	.988	.645	.653	40.8	139.2
301	*F*	.171	9.8	.985	.620	.629	39.0	141.0
253	*C*	.349	20.4	.937	.401	.428	25.3	205.3
320	*C*	.009	0.5	1.000	.443	.443	26.3	206.3
267	*F*	.045	2.6	.999	.820	.821	55.2	235.2
195	*C*	.079	4.5	.997	.869	.871	60.6	240.6
255	*D*	.059	3.4	.998	.879	.881	61.8	241.8
351	*C*	.121	7.0	.992	.941	.949	71.6	251.6
128	*C*	.141	8.1	.990	.966	.976	77.4	257.4
193	*D*	.332	19.4	.943	.932	.988	81.1	261.1
244	*C*	.118	6.8	.993	.929	.936	69.4	290.6
375	*C*	.282	16.4	.959	.863	.900	64.2	295.8
453	*E*	.009	0.5	1.000	.590	.590	36.2	323.8
214	*C*	.286	16.6	.958	.408	.426	25.2	334.8
464	*F*	.290	16.9	.957	.391	.409	24.1	335.9
463	*C*	.169	9.7	.986	.180	.183	10.5	349.5
287	*F*	.075	4.3	.997	.106	.106	6.1	353.9
			Type *Q*.					
			°				°	°
227	*E*	.066	3.8	.998	.009	.009	0.5	0.5
164	*D*	.051	2.9	.999	.015	.015	0.9	0.9
346	*D*	.028	1.6	1.000	.089	.089	5.1	5.1

Type Q (continued).

No.	Cl.	Dev.	β	Cos.	Dist.	Ratio.	γ	A
			°				°	°
65	*D*	.009	0.5	1.000	.141	.141	8.1	8.1
387	*D*	.179	10.3	.984	.223	.227	13.1	13.1
467	*F*	.271	15.7	.963	.228	.237	13.7	13.7
448	*E*	.069	4.0	.998	.295	.295	17.2	17.2
109	*E*	.080	4.6	.997	.683	.685	43.2	43.2
216	*D*	.010	0.6	1.000	.723	.723	46.3	46.3
436	*G*	.097	5.6	.995	.753	.757	49.2	49.2
124	*F*	.186	10.7	.983	.782	.796	52.8	52.8
179	*D*	.108	6.2	.994	.794	.799	53.0	53.0
240	*G*	.214	12.4	.977	.866	.886	62.4	62.4
308	*D*	.313	18.2	.950	.881	.928	68.1	68.1
221	*G*	.050	2.9	.999	.944	.945	70.9	70.9
176	*F*	.098	5.6	.995	.986	.991	82.3	82.3
400	*E*	.127	7.3	.992	.977	.985	80.1	99.9
94	*F*	.078	4.5	.997	.979	.982	79.1	100.9
354	*G*	.186	10.7	.983	.957	.974	76.9	103.1
478	*F*	.120	6.9	.993	.795	.801	53.2	126.8
383	*D*	.085	4.9	.996	.778	.781	51.4	128.6
305	*D*	.155	8.9	.988	.675	.683	43.1	136.9
106	*F*	.082	4.7	.997	.630	.632	39.2	140.8
228	*F*	.093	5.3	.996	.575	.577	35.2	144.8
141	*G*	.220	12.7	.976	.408	.418	24.7	155.3
435	*F*	.020	1.1	1.000	.406	.406	24.0	156.0
341	*F*	.225	13.0	.974	.370	.380	22.3	157.7
114	*F*	.265	15.4	.964	.316	.328	19.1	160.9
208	*F*	.101	5.8	.995	.276	.277	16.1	163.9
385	*E*	.235	13.6	.972	.231	.238	13.8	166.2
393	*D*	.359	21.0	.934	.208	.223	12.9	167.1
155	*G*	.109	6.3	.994	.182	.183	10.5	169.5
477	*F*	.217	12.5	.976	.136	.139	8.0	172.0
249	*F*	.068	3.9	.998	.100	.100	5.7	174.3
280	*E*	.122	7.0	.992	.047	.047	2.7	177.3
437	*F*	.172	9.9	.985	.031	.031	1.8	181.8
473	*F*	.005	0.3	1.000	.122	.122	7.0	187.0
104	*F*	.263	15.2	.965	.189	.196	11.3	191.3
376	*F*	.259	15.0	.966	.384	.398	23.5	203.5
465	*G*	.119	6.8	.993	.460	.463	27.5	207.5
108	*F*	.077	4.4	.997	.476	.477	28.5	208.5
340	*F*	.132	7.6	.991	.478	.483	28.9	208.9
330	*F*	.136	7.8	.991	.585	.590	36.2	216.2
408	*F*	.167	9.6	.986	.588	.596	36.6	216.6
204	*G*	.150	8.6	.989	.663	.670	42.1	222.1
215	*F*	.131	7.5	.991	.761	.768	50.2	230.2

No.	Cl.	Dev.	β	Cos.	Dist.	Ratio.	γ	A
Type *Q* (continued).								
			°				°	°
403	*G*	.187	10.8	.982	.773	.787	51.9	231.9
281	*F*	.064	3.7	.998	.813	.815	54.6	234.6
212	*F*	.021	1.2	1.000	.828	.828	55.9	235.9
429	*F*	.166	9.6	.986	.858	.870	60.5	240.5
381	*E*	.330	19.3	.944	.826	.875	61.0	241.0
146	*D*	.165	9.5	.986	.870	.882	61.9	241.9
250	*F*	.343	20.1	.939	.851	.906	65.0	245.0
355	*G*	.128	7.4	.992	.914	.921	67.1	247.1
288	*F*	.060	3.4	.998	.928	.930	68.4	248.4
327	*D*	.388	22.8	.922	.871	.945	70.9	250.9
97	*D*	.030	1.7	1.000	.953	.953	72.4	252.4
268	*F*	.161	9.3	.987	.957	.970	75.9	255.9
306	*F*	.021	1.2	1.000	.972	.972	76.4	256.4
277	*G*	.200	11.5	.980	.960	.980	78.5	258.5
373	*G*	.094	5.4	.996	.977	.981	78.8	258.8
282	*G*	.170	9.8	.985	.972	.987	80.8	260.8
218	*E*	.134	7.7	.991	.983	.992	82.8	277.2
154	*F*	.120	6.9	.993	.925	.932	68.8	291.2
243	*F*	.195	11.2	.981	.913	.931	68.6	291.4
338	*F*	.217	12.5	.976	.880	.902	64.4	295.6
366	*G*	.264	15.3	.965	.865	.896	63.6	296.4
279	*E*	.120	6.9	.993	.823	.829	56.0	304.0
116	*D*	.210	12.1	.978	.797	.815	54.6	305.4
168	*D*	.074	4.2	.997	.802	.805	53.6	306.4
391	*G*	.021	1.2	1.000	.804	.804	53.5	306.5
326	*E*	.017	1.0	1.000	.763	.763	49.7	310.3
430	*C*	.132	7.6	.991	.701	.707	45.0	315.0
199	*G*	.331	19.3	.944	.658	.697	44.2	315.8
367	*G*	.144	8.3	.990	.618	.624	38.6	321.4
296	*F*	.055	3.2	.998	.601	.603	37.1	322.9
92	*F*	.313	18.2	.950	.556	.585	35.8	324.2
198	*C*	.146	8.4	.989	.578	.584	35.7	324.3
353	*D*	.161	9.3	.987	.576	.584	35.7	324.3
294	*F*	.308	17.9	.952	.517	.543	32.9	327.1
372	*F*	.131	7.5	.991	.527	.532	32.1	327.9
319	*E*	.272	15.8	.962	.424	.441	26.2	333.8
364	*F*	.312	18.2	.950	.393	.414	24.5	335.5
395	*E*	.218	12.6	.976	.288	.295	17.2	342.8
322	*F*	.161	9.3	.987	.277	.281	16.3	343.7
409	*E*	.368	21.6	.930	.251	.270	15.7	344.3
402	*E*	.160	9.2	.987	.259	.262	15.2	344.8
157	*F*	.116	6.7	.993	.229	.231	13.4	346.6
452	*F*	.269	15.6	.963	.182	.189	10.9	349.1

No.	Cl.	Dev.	β	Cos.	Dist.	Ratio.	γ	A
				Type *Q* (continued).				
			°				°	°
343	*F*	.249	14.4	.969	.177	.183	10.5	349.5
349	*F*	.033	1.9	1.000	.101	.101	5.8	354.2
434	*F*	.240	13.9	.971	.087	.090	5.2	354.8
187	*F*	.126	7.2	.992	.080	.080	4.6	355.4
377	*D*	.180	10.4	.984	.079	.079	4.5	355.5
427	*G*	.055	3.2	.998	.075	.075	4.3	355.7
142	*D*	.004	0.2	1.000	.064	.064	3.7	356.3
361	*G*	.038	2.2	.999	.027	.027	1.6	358.4
				Type *R*.				
			°				°	°
175	*D*	.155	8.9	.988	.058	.059	3.4	3.4
313	*D*	.209	12.1	.978	.106	.108	6.2	6.2
147	*C*	.295	17.2	.955	.236	.247	14.3	14.3
149	*F*	.215	12.4	.977	.270	.276	16.0	16.0
477	*F*	.094	5.4	.996	.462	.464	27.6	27.6
385	*E*	.005	0.3	1.000	.488	.488	29.2	29.2
114	*F*	.080	4.6	.997	.489	.490	29.3	29.3
341	*F*	.112	6.4	.994	.540	.543	32.9	32.9
249	*F*	.200	11.5	.980	.552	.562	34.2	34.2
280	*E*	.330	19.3	.994	.658	.662	41.5	41.5
435	*F*	.072	4.1	.997	.686	.688	43.5	43.5
228	*F*	.277	16.1	.961	.675	.702	44.6	44.6
208	*F*	.108	6.2	.994	.742	.746	48.2	48.2
106	*F*	.268	15.5	.964	.805	.835	56.6	56.6
173	*F*	.190	11.0	.982	.941	.958	73.3	73.3
299	*F*	.370	21.7	.929	.899	.968	75.5	75.5
59	*F*	.230	13.3	.973	.955	.982	79.1	79.1
474	*F*	.075	4.3	.997	.990	.993	83.2	83.2
101	*D*	.169	9.7	.986	.965	.979	78.2	101.8
237	*F*	.155	8.9	.988	.955	.967	75.2	104.8
407	*F*	.095	5.5	.995	.955	.960	73.7	106.3
118	*F*	.009	0.5	1.000	.877	.877	61.3	118.7
205	*E*	.035	2.0	.999	.773	.774	50.7	129.3
411	*D*	.220	12.7	.976	.705	.722	46.2	133.8
202	*E*	.047	2.7	.999	.706	.707	45.0	135.0
475	*F*	.205	11.8	.979	.610	.623	38.5	141.5
84	*F*	.210	12.1	.978	.601	.615	38.0	142.0
390	*F*	.004	0.2	1.000	.561	.561	34.1	145.9
325	*F*	.170	9.8	.985	.488	.491	29.4	150.6
188	*F*	.110	6.3	.994	.352	.354	20.7	159.3
181	*F*	.176	10.1	.984	.241	.245	14.2	165.8

No.	Cl.	Dev.	β	Cos.	Dist.	Ratio.	γ	A
				TYPE *R* (continued).				
			°				°	°
184	*F*	.290	16.9	.957	.070	.073	4.2	175.8
160	*F*	.132	7.6	.991	.131	.133	7.6	187.6
379	*D*	.141	8.1	.990	.231	.234	13.5	193.5
278	*F*	.120	6.9	.993	.332	.334	19.5	199.5
434	*F*	.149	8.6	.989	.426	.431	25.5	205.5
452	*F*	.037	2.1	.999	.437	.437	25.9	205.9
343	*F*	.051	2.9	.999	.449	.449	26.7	206.7
294	*F*	.306	17.8	.952	.510	.536	32.4	212.4
402	*E*	.017	1.0	1.000	.555	.555	33.7	213.7
227	*E*	.351	20.6	.936	.586	.626	38.8	218.8
349	*F*	.241	13.9	.971	.623	.642	39.9	219.9
372	*F*	.233	13.5	.972	.648	.667	41.8	221.8
187	*F*	.299	17.4	.954	.675	.708	45.1	225.1
157	*F*	.152	8.7	.988	.735	.744	48.1	228.1
322	*F*	.123	7.1	.992	.769	.775	50.8	230.8
296	*F*	.242	14.0	.970	.789	.814	54.5	234.5
395	*E*	.120	6.9	.993	.810	.816	54.7	234.7
482	*F*	.343	20.1	.939	.787	.838	56.9	236.9
319	*E*	.009	0.5	1.000	.870	.870	60.5	240.5
364	*F*	.056	3.2	.998	.885	.887	62.5	242.5
409	*E*	.218	12.6	.976	.870	.891	63.0	243.0
165	*F*	.000	0.0	1.000	.928	.928	68.1	248.1
231	*E*	.151	8.7	.988	.976	.988	81.1	261.1
80	*F*	.340	19.9	.940	.933	.993	83.2	263.2
239	*G*	.360	21.1	.933	.928	.995	84.2	264.2
182	*F*	.190	11.0	.982	.980	.998	86.4	266.4
439	*E*	.050	2.9	.999	.994	.995	84.2	275.8
286	*F*	.206	11.9	.978	.970	.992	82.8	277.2
347	*F*	.062	3.6	.998	.986	.988	81.1	278.9
446	*G*	.302	17.6	.953	.926	.972	76.4	283.6
321	*E*	.237	13.7	.972	.918	.944	70.7	289.3
107	*G*	.292	17.0	.956	.858	.898	63.9	296.1
263	*D*	.226	13.1	.974	.700	.719	46.0	314.0
177	*F*	.134	7.7	.991	.700	.706	44.9	315.1
219	*F*	.211	12.2	.977	.531	.544	33.0	327.0
397	*F*	.268	15.5	.964	.509	.528	31.9	328.1
440	*G*	.190	11.0	.982	.503	.512	30.8	329.2
302	*F*	.362	21.2	.932	.462	.496	29.7	330.3
384	*G*	.335	19.6	.942	.437	.464	27.6	332.4
455	*F*	.215	12.4	.977	.156	.159	9.1	350.9
335	*F*	.010	0.6	1.000	.100	.100	5.7	354.3
241	*F*	.139	8.0	.990	.050	.050	2.9	357.1

In the latter figure the inner circle gives the location of the aphelia of the orbits of type *Q* that belong to classes *C*, *D*, and *E*. The two aphelia of class *C* lie close together within the circle near longitude 45°, one having the same orientation as an aphelion belonging to class *D*. The aphelia of class *D*, when at a less distance than 1,000 units, also lie within the circle, but outside of those of class *C*. Those that lie at a greater distance than this, together with the single aphelion of class *E* are entered outside the circle,

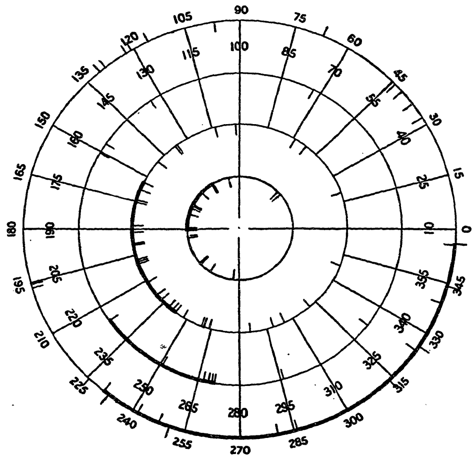

FIG. 58.

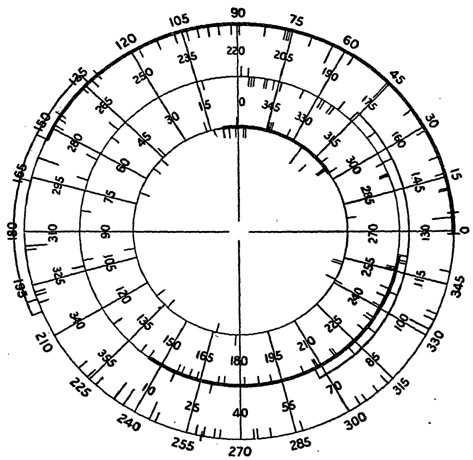

FIG. 59.

and the corresponding numbers of the argument outside of that. In the second circle are entered the aphelia of type *Q* that belong to classes *F* and *G*, the former being within, and the latter without the circle. The third circle contains the aphelia of type *R*. The nine long lines within the circle belong to classes *C* and *D*. The other lines within it belong to class *F*. The five lines partly within it and partly outside belong to class *E*, and the five lines wholly outside to class *G*. The corresponding arguments lie within this circle.

For each type the aphelia are divided in halves, and a heavy line drawn connecting all the aphelia in the half that lies more closely together. Thus 17 of the 34 aphelia of type *N* lie within 79° of one another, 6 of the 12 aphelia of type *O* lie within 45°, and 17 of the 33 aphelia of type *Q*, classes *C*, *D*, and *E*, lie within 64° of one another. The other types are much more widely scattered. The maximum scattering necessarily cannot reach 180°, and that of type *R* where 34 out of 67 aphelia extend through an arc of 154° represents the most nearly uniform distribution shown. Attention may be called to the fact that the 5 aphelia of class *G* type *R*, hyperbolic orbits, all lie within 68° of one another, and that 10 of the 19 aphelia of class *G* type *Q* lie within 108°, having their middle point within 8° of the south pole of the ecliptic. These facts are indicated by light lines joining the aphelia in question.

Although no such concentration of aphelia occurs in the other types, yet we see that they are not always as uniformly scattered as we might expect them to be if their disposition were due merely to chance. Thus in type *P* in the 90° extending from argument 145° to 235° only 2 aphelia are found, while in the remaining 270° their are 26, the relative ratios of density being as 1 to 4.33. If we consider only class *C*, we find 12 aphelia in the 180° extending from argument 235° to 55°, and but 3 in the remaining semi-circumference, the ratio of the densities being as 4 to 1. Similarly for type *Q*, counting the aphelia of all classes in the 120° extending from argument 20° to 140° there are but 15, while in the remaining 240° there are 81, the relative densities being as 1 to 2.7.

This concentration of the aphelia of the inner types, such as *N* and *O*, we shall find later is probably due to the attraction of the great outer planets. In the case of the aphelia of the outermost planet *R*, however, no such concentrating force exists, and it is therefore reasonable to assume that the aphelia are on the whole distributed uniformly per unit length along the orbit of this planet, their observed concentration being only apparent. The same is true to a certain extent with planet *Q*. Making this assumption, moreover, we shall find that the reasons for certain facts hitherto unexplained became perfectly obvious.

Thus in Figure 41, p. 243, we found that the aphelia of the parabolic and hyperbolic comets associated with type *Q* were much more densely distributed in the vicinity of the south pole than they were in the opposite portion of the sky. The unusual proportion of comets coming from the south, although often previously noted, has never hereforeto been satisfactorily explained. If, however, we assume that the aphelion of planet *Q* lies near the south pole, and that its orbit is rather eccentric, a uniform distribution of the aphelia along the orbit would make the whole matter perfectly clear.

Again, referring back to Figure 52, p. 317, we notice that the logarithms of the aphelia of type *Q* rise sharply to a maximum at 2.60, they then remain comparatively level as far as 3.40, from which point they again fall sharply. The level region extends through a range of 0.80. For Jupiter the range is only 0.06. The smaller number of orbits concerned in the case of type P produces a more irregular curve, but as in the case of Q we find that it is lengthened between the maxima, and we have found independently in Chapter XVIII that this corresponds to a considerable eccentricity of its orbit.

In the case of widely separated binary stars, orbits of small eccentricity are absolutely unknown, we shall therefore assume a uniform distribution of the cometary aphelia, and see to what result it leads us. In Table LXXXVI under type Q the arguments of the cometary aphelia contained in the last column are divided into twelve groups of 30° each, and their means taken, those aphelia which belong in type QR being weighted only one-third. The number of comets in each group is recorded, again weighting those of type QR but one-third, and the results are then plotted in polar coordinates in Figure 60. The ellipse passed through these twelve points will represent the orbit of Q, based on the assumption that the aphelia are uniformly distributed along its circumference per unit of length. With one exception all the points lie fairly near the adopted curve. This one is for the arc lying between arguments 270° and 300°, where only 5 aphelia are found instead of 14, as we might have expected. The exception is so striking that it seems as if there must be some explanation for it.

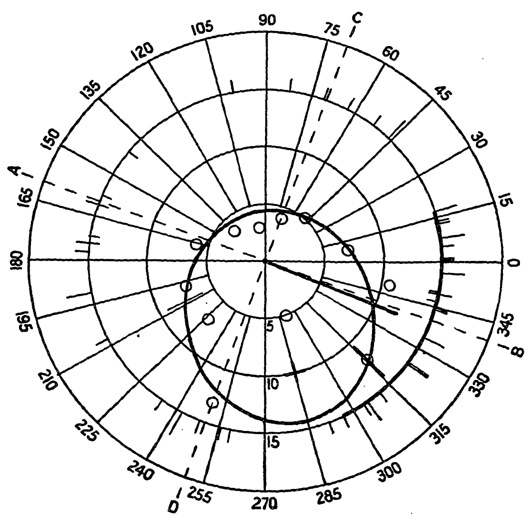

FIG. 60.

To put the matter in another form we may ask, is there any cause which should produce a scarcity of aphelia in this portion of the orbit? It is very certain that there is only one body capable of doing it, and that is planet R. The aphelion of the orbit of Q we find is located in longitude 100° latitude −75°. The orbits of Q and R are inclined to one another at an angle of 73°.9, and when R is remote from the plane of Q, it can have little effect on the distribution of the cometary aphelia in that plane. When however, R passes through this plane, it will, as we have seen by p 219, tend to pull certain of the aphelia towards it, and to repel certain of the others, leaving two gaps on opposite sides of the orbit. In Figure 60 the line *AB* joins the two positions that *R* will take when it passes through the plane of the orbit of *Q*. The line *CD* is drawn perpendicular to this line, and it will readily be seen that the position where the effect of the differential pull of *R* on the Sun and on any comet temporarily lying in the orbit of *Q* will be greatest, will be in the vicinity of argument 285°, *R* being at this time near perihelion, and in the direction of *A*. These aphelia will therefore tend to be repelled in the direction of *B*. We notice also that a similar though much slighter effect is produced in arguments 225° and 105°, which is what we should expect.

The orbit of planet *R*, is represented in Figure 61. It is deduced from Table LXXXVI in precisely the same manner as the curve of Figure 60, except that the aphelia of type *QR* are weighted two-thirds instead of one-third. The largest deviation found, that in argument 17°, depends on the existence of six and one-third comets where we should have expected only four and one-third. This deviation may probably be attributed to accidental causes, and the same is true of the similar large deviation in argument 138°. The construction of these two ellipses is analagous to the graphical construction of the apparent orbit of a double star, save that in this case we are limited by an additional equation of condition, namely that the Sun must lie in one of the foci of the ellipse.

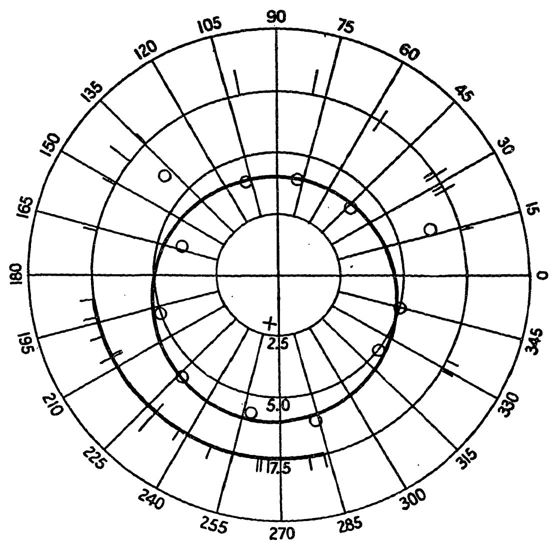

FIG. 61.

The method employed by the writer in drawing these ellipses was first described by him in Popular Astronomy 1908, 16, 147, but as he believes it is not generally known, is very convenient, and has many applications, it may be well to refer to it briefly again. In the ordinary method of construction, that based on the employment of a thread and two needles, the difficulty that always arises is to vary accurately and by small amounts the length of the thread. The object of the present method is to overcome this difficulty. Let us suppose that it is desired to draw an ellipse whose semi-major axis and eccentricity are known. We first lay off the two axes, where $b = a\sqrt{1-e^2}$

and mark the position of the two foci. We next insert the two needles, so inclined to one another that their heads shall be slightly more distant than their points. The thread is then tied in a loop, which is thrown over the needles and given a single turn about each one. If we wish to draw the upper half of an ellipse, the major axis being supposed horizontal, a pin is placed inside the loop between the needles, and slightly below them. By simply varying the distance between the pin and the line joining the needles we can vary the major axis of the ellipse by as small or as large a quantity as we desire. This arrangement is very convenient, as it enables us to allow for the slight stretching of the thread, which always occurs in practice. In constructing the ellipse it is more accurate to determine the position of the pin, by the minor axis as marked on the paper, rather than by the major axis itself. The writer usually uses one pencil to draw the ellipse, and another held in the other hand to push the thread down to the bottom of the needles. By holding the drawing pencil at the proper angle it is possible to maintain the thread upon it at a constant height of about two millimetres.

The eccentricity of the orbit of planet Q as derived from Figure 60 is 0.54, and the argument of its perihelion 105°. Taking the perihelion distance by Figure 52 at 400 units we find the aphelion distance will be 1350, and the mean distance 875. This would correspond to a period of 25900 years. By selecting this value for the perihelion distance, we shall find that it and the corresponding aphelion distance bear about the same relation to their curve in Figure 52, as the perihelion and aphelion distances of Jupiter and planet P do to theirs.

We notice also in this figure that each of the three more remote types possesses a few comets whose aphelion distances correspond closely to the maximum intensity of the other types. These comets are separated from the main bulk of the type by considerable intervals, within which no comets exist. Thus in type P there are four comets the logarithms of whose aphelia extend from 2.69 to 2.92, which we see lie well between the perihelion and aphelion distances of planet Q. There are also two comets whose aphelia lie between 3.39 and 3.46, not shown upon the figure, as previously noted, which still do not extend beyond the main mass of the curve of Q. But the most striking case is under type R, where we find seven comets whose aphelia extend from 2.51 to 3.22, which all fall within the limits of the main mass of curve Q. None of their orbits could possibly be classified under type Q, however, but all belong clearly to type R. Indeed they make up the main bulk of the R curve, while the comets which probably really indicate the distance of planet R from a small maximum beyond them. That this maximum is so small is due to the enormous distance of the planet, few astronomers caring to compute the eccentricity of an orbit, however well it may be observed, that differs so little from unity. This part of the curve indeed is based upon only five orbits, but fortunately four of them are ranked under certainty 2. The fact that the available orbits are so few, makes it particularly desirable that when a well observed comet of this type has an orbit whose eccentricity closely approaches unity, that it should be computed with all possible care. Twenty-two such orbits already exist, and are classified in Table XXXIV p 170 as of certainty 2, class F, and of types R or QR. They are numbers 157, 165, 177, 181, 188, 219, 228, 237, 278, 296, 302, 325, 335, 341, 347, 364, 372, 390, 397, 434, 435, and 455. What is needed in each case is a statement of the most probable value of the eccentricity, together with the probable limiting values, and not merely the indefinite remark that the deviation from a parabolic orbit is slight, or insignificant. The fact is the deviation from a parabolic orbit is always significant. The eccentricity of the orbit of planet R as determined by the curve of Figure 61 is 0.20, and the argument of its perihelion 80°. Taking the perihelion distance by Figure 52 at 5000 units, its aphelion proves to be 7500, and its mean distance 6250. This corresponds to a period of 494000 years.

In the Introduction to Lewis's "Measures of Double Stars by F. G. W. Struve," Mem. Roy. Astron. Soc. 1906, 56, xvii, is given a list of 38 visual binaries whose orbits are well known. Classifying them according to their eccentricity, we find that from 0.00 to 0.29 there are 4 orbits, from 0.30 to 0.49 there are 15, from 0.50 to 0.69 there are 11, and from 0.70 to 0.99 there are 8. Just half the eccentricities are less than 0.50. The maximum is 0.95 and the minimum 0.14. The eccentricity of Q, 0.54, is therefore a little greater than the average, while that of R, 0.20, is rather small, there being but two orbits out of the 38 whose eccentricity is less.

It might be thought that the simplest way to construct the orbit of one of these planets would be merely to plot the aphelion distances of its associated comets in polar coordinates. Unfortunately this plan does not yield satisfactory results. It is most successful in the case of planet P, giving an orbit whose eccentricity and argument of perihelion agree in general with the values shown in Figure 53, p. 319. It is not thought, however, that the method is as satisfactory or reliable as the one employed in the latter part of the last chapter. For planets Q and R all the parabolic and hyperbolic orbits would have to be ignored, and this leads naturally to an erroneous result.

Turning back now to Table XLIII, p. 196, we find that the last five hyperbolic orbits of class G, those having the highest hyperbolic excess, are all marked 3 in the column of certainty, but of the remaining 21 orbits all but three are marked 2, — an extraordinarily high percentage of accurately known orbits. Excepting in classes A and B there is no other group so accurately determined. In the first 14 orbits, which we may associate in one group, there is no case where the hyperbolic excess exceeds 0.006 kilometres per second. Two of this group belong to type R, and the remaining 12 to type Q. Of these latter the arguments of 10 lie within a range of 108°, between 208° and 316°, mean value 262°, the other two lying on the opposite side of the sphere, at 71° and 103°.

Of the remaining 12 hyperbolic orbits, those of high hyperbolic excess, seven belong to type Q. Four would be the number allotted by chance. Regarding their arguments, they are distributed irregularly upon the sphere, though it so happens that none of them falls within the above mentioned range of 108°. Their hyperbolic excesses are many of them extremely high, although the highest of all belongs to type R. As in the first group, as we proceed down the list, the excess increases but slowly in the first six orbits. In the last six, however, the excess is sometimes double and sometimes treble that of its predecessor. This is not what we should expect if the excess were due to the attraction of a remote planet, but is what might be expected if due to the entry into our system of comets expelled from other regions of space. The fact that the higher excesses generally belong to comets of early date casts some doubt upon these figures, but this does not apply to numbers 361 and 367.

It is often said that if comets came to us directly from the stars we should expect to find hyperbolic excesses similar to the velocity in space of our Sun. This would be about seventy times as high as anything hitherto recorded. Let us see, however, if this is really true. Since, owing to the attraction of our larger planets, some of our comets are certainly ejected into the stellar universe, we should expect to receive a corresponding supply from the outside, to keep up the cometary equilibrium. If we divide all the comets approaching us from without into two equal groups, one containing those approaching at the higher speeds, and the other those approaching at the lower, we should at their start expect these two groups to point on the average equally near to the Sun. As they got nearer us, however, and began to feel the influence of the Sun's attraction, the change in direction of their motion, due to this influence, would be more marked in the case of the slowly moving comets on account of the longer time during which they were subjected to it. Near the Sun, therefore, low hyperbolic velocities would be in excess, especially so where the perihelion distance had such an extremely small value in comparison with the stellar distances as unity.

It has also been suggested that if comets reached our system from without, that they would generally approach us from the direction of the "Sun's way" or "solar apex." This would be true for high speeds, but we should not expect it to be noticeable for speeds but little exceeding one per cent of that of the Sun's own motion. A curious coincidence may be noted in this connection. Just outside the circle surrounding the orbits of *Q* in Figure 60, and of *R* in Figure 61 a number of lines have been drawn representing the arguments of all the comets of these types visible to the naked eye. The longer lines represent comets of brilliancy *I* and *II*, and the shorter lines comets of brilliancy *III* and *IV*. Eight naked-eye comets belong to type *QR*, and are represented by short lines within the circles in both figures. Three of these comets have been arbitrarily assigned to type *Q*, and five to type *R*. The corresponding lines being prolonged outside the circles in these cases. A heavy line, as before, indicates the arguments where the aphelia are especially concentrated. The mean argument of the eighteen aphelia in the case of type *Q* is 342°.9, and of the twelve aphelia in the case of type *R* is 228°.2. Reducing these two positions to longitude and latitude, we find for the bright comets of type *Q* $\lambda = 92°.2$, $\beta = -17°.1$ and for *R* $\lambda = 99°.6$, $\beta = -19°.1$. For both types therefore the brightest comets come from the same part of the sky. The orbits of *Q* and *R* intersect at present in λ 92°.2, β – 16°.6, which is also curious, but in the course of a few million years this position will change appreciably owing to the revolution of the nodes.

If any known comets have reached us from interstellar space, they must, as we have just seen, after correcting for their orbital motion, possess a proper motion nearly identical with that of the Sun. Using Kapteyn's two stellar radiants corrected for the motion of the solar system, so that they lie in diametrically opposite portions of the sky, for one of them we shall have $\lambda = 93°$, $\beta = -11°.5$, again closely coinciding with the positions of the cometary aphelia. The solar anti-apex according to Campbell, Pub. Astron. Soc. Pac. 1910, 22, 237, when reduced to longitude and latitude, lies in $\lambda = 93°$, $\beta = +4°.5$. If these bright comets originally came from interstellar space, therefore, it seems that they generally overtook the Sun, and this in spite of the fact that the motion of the Sun according to Campbell is appreciable greater than that of the average star.

If instead of selecting only the bright comets of type *Q*, we use all the comets of this type belonging to classes *C*, *D*, and *E*, we shall obtain a position in the same longitude, but about 6° farther south. The mean argument is 336°.6, and its position may be checked by refering back to the inner circle of Figure 59. It will be noticed that according to the second circle a number of parabolic orbits are also concentrated in this same argument, but if we take the parabolic orbits of type *Q* as a whole, the concentration will lie in an entirely different portion of the sky. The location of the point of maximum concentration of the hyperbolic orbits also bears no obvious relation to this argument. We shall presently see that there is another and a more consistent explanation of the location of the aphelia of type *Q*, classes *C*, *D*, and *E*, depending on the present position of the planet *Q*.

If instead of selecting only the bright comets of type *R*, we use all the comets of this type, we obtain a position in argument 210°.2, lying about 18° to the west of the former one, or near longitude 82°. Here again the hyperbolic comets seem to bear no relation to the argument. While the writer believes that we may sometimes receive comets from interstellar space, he is not prepared to express a definite opinion as to whether the close agreement between the radiants of the bright comets and of the stellar system is causal, or merely a coincidence. It might be that visiting comets were able to collect a certain amount of gaseous material during their long interstellar journeys, and thus appear to us phenomenally bright.

Let us now return to the first group of 14 hyperbolic orbits and see what light our results will throw on the direction of revolution of planet *Q*. In Figure 62 let *S* represent the Sun and *Q* a planet, revolving in a circular orbit *Q G*. We have seen in Chapter XII that if a comet have its aphelion near *Q*, its speed will be accelerated in the portion of its orbit lying between *S* and *Q*, and retarded in the other half of its orbit between *Q* and *S*. Its total periodic time will not be affected, but it will reach *S* with a diminished speed. The principal portion of the retardation will occur near *Q*. If an equal number of comets possess direct and retrograde revolutions, the point Q will be the longitude of their maximum retardation. Turning to Figure 59, we find by the inner circle, which contains the elliptic orbits of this type, that the more crowded section extends between arguments 304° and 8°, mean value 336°. This then must have been the argument of planet Q when these comets were in aphelion.

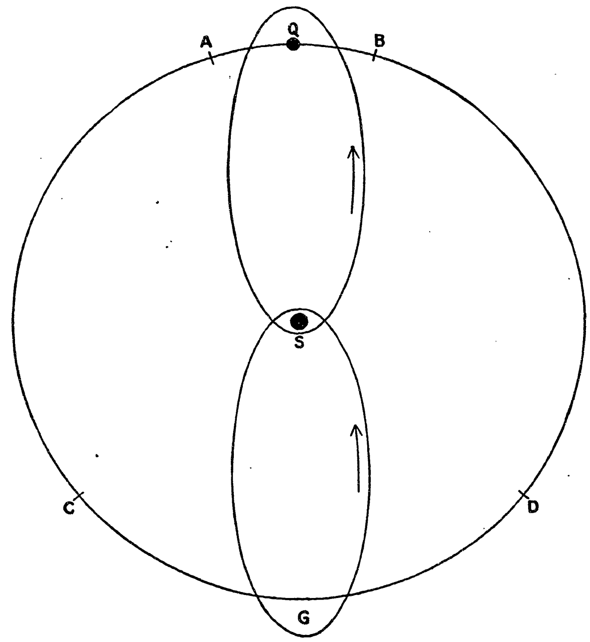

FIG. 62.

We have already seen that the mean argument of the ten hyperbolic comets was 262°. As shown by Figure 62, to locate the planet we must subtract 180° from this, leaving 82°. This then is the argument which the planet had when the hyperbolic comets were on their way to the Sun between G and S. Since the elliptic and hyperbolic comets arrived at about the same time, the latter must have received their acceleration later, and consequently the planet must have moved from argument 336° to argument 82°. It therefore either traversed an arc of 106° or 254°, depending on the direction of its revolution in its orbit. Since less than half of the comet's orbit was described from aphelion, and since the period of the associated comet cannot be longer than that of its controlling planet, it is obvious that Q must have described the shorter arc, and that its motion consequently was direct and not retrograde.

Let us now see what confirmatory facts we possess bearing on this relation of the planet to the comets. We notice by Figure 62 that the range of argument of the elliptical comets on account of their proximity to the planet should be smaller than the range for those that are hyperbolic, also that the range of acceleration, on account of this same proximity, should be much greater. An examination of Table XLIII, p. 196, shows that for the hyperbolic comets the range of acceleration is only 0.00597 kilometers per second, while for those that are elliptical the comets are found in two different classes, C and D, and have, therefore, a very much greater range. With regard to the range of argument we have already found that while for the hyperbolic comets it is 108°, for those that are elliptical it is only 64°. From both of these ranges a constant quantity should be subtracted due to the motion of Q in its orbit. The ratio of these angles is therefore greater than it appears.

The ten hyperbolic comets above mentioned may be divided into two groups, those whose ascending nodes lie between 68° and 157°, and which therefore revolve in the same general direction as the planet, and those whose ascending nodes lie between 223° and 336°, and which therefore revolve in the opposite direction. Named in the order of their arguments, the first group consists of numbers 277, 373, 282, 366, and 199, and the second group of numbers 465, 204, 403, 355, and 391. If their hyperbolic excesses are due to the planet, and the motion of the latter is direct, it may be shown that the arguments of the first group should exceed those of the second. These arguments are as follows: — First group, 258°, 259°, 261°, 296°, and 316°, mean 278°. Second group, 208°, 222°, 232°, 247°, and 306°, mean 243°.

The mean value of k should also be highest for the first group, and the middle values of each group should exceed those at the ends. The values of k for the first group are .00084, .00494, .00597, .00348, and .00149, mean 00334. Second group, .00194, .00273, .00259, 00415, and .00312, mean .00290. The mean value of k for the second group is smaller than for the first, but the excess of the middle values is not marked, although the mean of the three, 315, slightly exceeds the two end values. For the first group the excess of the middle values is striking.

It will be noted that planet Q does not itself tend to concentrate the aphelia of type Q in the positions of the elliptic and hyperbolic maxima. No planet can concentrate the aphelia of its own type. What the planet does do is to render certain orbits whose aphelia have already been distributed by other causes, either elliptical or hyperbolic as the case may be. Thus, of all the aphelia included in the two inner circles of Figure 59, planet Q renders a certain number between arguments 208° and 316° hyperbolic. It also between arguments 304° and 8° throws a certain proportion into classes C and D, but the majority of the orbits, which lie in other arguments are not at present affected by it.

Having now determined the elements of the orbit of Q, it only remains to locate the planet. There are five different methods of doing this, all depending on the position, or change of position, of the cometary aphelia. Three of these are employed in dealing with the comets of class *A*, and the other two with classes *D* and *G*. Eight different results have been obtained, lettered from *a* to *h*, as it has been found convenient to separate the comets of class *B* from these of class *A*, even when similarly treated. These results are as follows: —

(*a*) Class *A*. Weight 2. Mean position of the aphelia.
(*b*) Class *A*. Weight 2. Mean motion of the aphelia.
(*c*) Class *A*. Weight 2. Relative motion of the aphelia.
(*d*) Class *B*. Weight 1. Mean position of the aphelia.
(*e*) Class *B*. Weight 1. Stationary position of the aphelion of Halley's comet.
(*f*) Class *B*. Weight 1. Direction of motion of the aphelion of Tuttle's comet.
(*g*) Class *G*. Weight 3. Argument of the aphelia of the hyperbolic orbits.
(*h*) Class *D*. Weight 5. Argument of the aphelia of the long elliptical orbits.

Taking these up in order, in Figure 63 let the heavy line represent the plane of the ecliptic, and let the circles be drawn in a plane perpendicular to it, 0° being located in longitude 90°. Let a large planet at a considerable distance from the Sun be located in argument 0° at the point *s*. As the planet moves northerly, it will tend to drag the aphelia of the comets of class *A* as a whole to the north of the ecliptic, until the planet reaches latitude 63°.6. At this point its mean attraction for the nearer aphelia in argument 0°, and for the more remote aphelia in argument 180° will exactly equal its attraction for the Sun in the centre. Including those aphelia lying in the ecliptic, but in other longitudes, a somewhat smaller figure than 63.6° would be obtained, but this is immaterial for our present purposes. The aphelia will therefore cease to move northerly as a whole, and as the planet approaches the pole, it will, as we have already seen in Chapter XIII, repel the aphelia towards the south. The character of this varying force is represented symbolically by the curved line, *sn* indicating a northward pull,

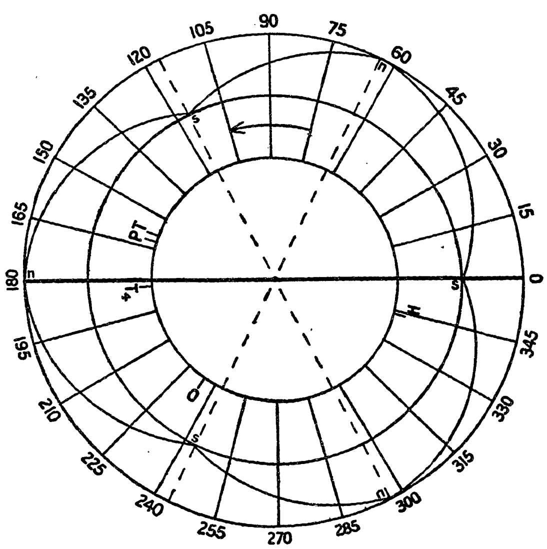

FIG. 63.

and the prolongation of the curve across the pole to *s* indicating a southerly one. The effect of the distance of the aphelia is of little consequence. For class *B* the neutral point would be in latitude 63°.5.

In Chapter XIII we found that the aphelia of class *A* deviated on the whole to the south of the ecliptic, and that the deviation seemed to be actual, and not due to accidental causes. The three-lobed curve *sn* is not drawn to scale as far as its radius vector is concerned, but we can see that according to it, in order to produce a southerly deviation of the cometary aphelia, planet *Q* must lie within about 30° of one of the three points marked *s*. This fact we shall indicate upon another curve, Figure 64.

This curve represents the same section of the sphere as Figure 63. The inner broken circle indicates our results with method (*a*) the open spaces representing the three sections of its orbit where it is possible that the planet may be located. The extremeties of the arcs are placed in arguments lying just midway between the arguments of the points *s* and *n*. The three lines being light but continuous indicate the assigned weight of this method is 2.

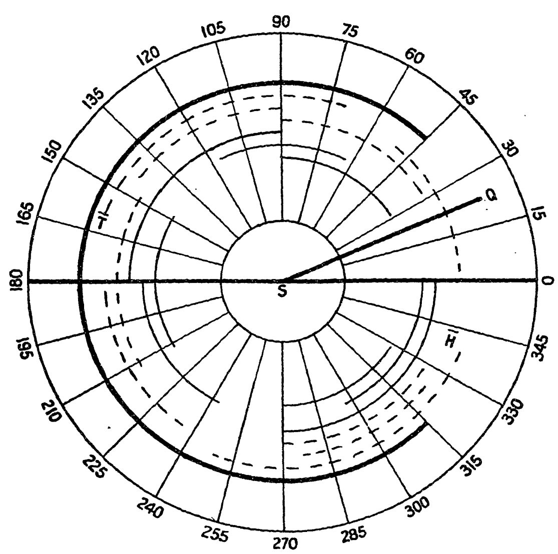

FIG. 64.

In the chapter above mentioned we found that the mean direction of motion of the various aphelia of class *A* was slightly towards the north. By the curve *sn* in Figure 63 we find that there are three sections of its orbit in which the planet might lie. These are indicated by the spaces between the three lines in the second broken circle of Figure 64.

If we consider the aphelia of class *A* as forming a ring around the heavens, more or less parallel to the ecliptic, and as being nearly balanced in their positions by the varying attractions of the known planets, we see that they should furnish a moderately sensitive test for the present position of planet *Q*. The ring indeed will act as if pivotted in longitudes at right angles to the nodes, that is in approximately 0° and 180°, and will vibrate back and forth twice with every revolution of the planet, the nearer aphelia being always drawn towards the planet and the further ones being repelled from it.

In Figure 65, based on Table LI, p. 216, the abscissas represent the longitudes, taken from the third column of the table, and the ordinates the centennial motion in latitude, taken from the last column. Where the aphelia are fairly numerous we can see that ordinate 0 will lie near longitude 200°. In longitudes 90° from this point the aphelia seem on one side to be rising, and on the other to be falling. A curve representing the observations as closely as possible has been drawn with its maximum and minimum points lying in longitudes 90° and 270°. We see that near longitude 90° of this figure, which would be represented on the right hand side of Figure 64, the aphelia are moving towards the north. Therefore Q would have to be to the north of the ecliptic if in this longitude. An arc of the third circle of Figure 64 is therefore drawn from the ecliptic to the south pole. A similar arc is drawn from argument 90° to 180°, because if near longitude 270°, Q must be south of the ecliptic. Assuming these three curves to be correct, we see that there are only two short arcs left, one in the first, and one in the third quadrant, where it is possible that Q could be found.

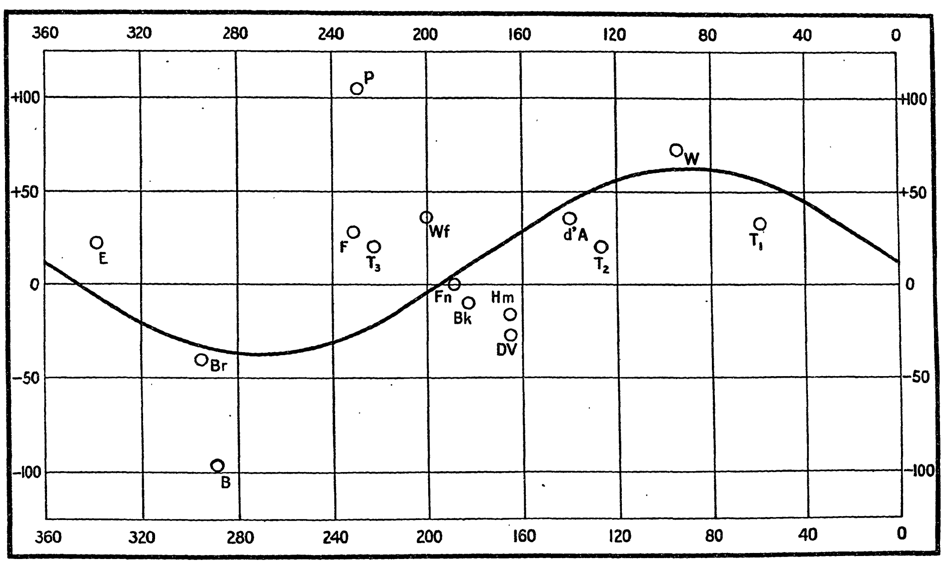

FIG. 65.

Turning now to class B, we recall that the mean latitude of its aphelia lies considerably to the south of the mean latitude of the aphelia of class A. The accuracy of this determination is therefore somewhat diminished since the aphelia do not generally lie very near to the ecliptic. The most northern one is represented by T in Figure 63, and the most southern one by *O*. The same curves will apply in general as for class *A*, and the fourth circle in Figure 64 is therefore drawn like the inner one, excepting that it is dotted to indicate its inferior weight.

Of the twelve periodic comets of class *B* only two have orbits that have been thoroughly investigated and corrected for planetary perturbations. These are Halley's and Tuttle's. A paper by Cowell and Crommelin, A.N. 185, 267, shows that since the return of 1835, after correcting for all known planetary perturbations, the elements are but little changed. The latitude of the aphelion indeed is practically identical with that actually found at the return of 1910. Prior to that time since 1607 the motion of the aphelion had been on the whole towards the south, at the rate of 8′ per century. How much of this motion was due to the attraction of the known planets, however, can not now be determined.

The position of the aphelion of Halley's comet in latitude −16°.6 is indicated by an *H* in the fifth circle of Figure 64. We will assume that the planet might be as much as 20° north or south of this position without producing an appreciable change in the latitude of the aphelion during the last seventy-five years. A similar break in the continuity of the circle will occur in the opposite portion of the orbit. Two other breaks where the planet will produce no affect on the aphelion will occur 86°.2 north and south of *H*. Allowing 20° on either side of them, the remaining four arcs will average 50° each in length, and will cover that portion of the circumference, a little over half, where it is least likely that planet *Q* will be found.

The orbit of Tuttle's comet has been carefully investigated by Rahts, and during the four later appearances, beginning with that of 1858, after allowing for all known planetary perturbations, we find a fairly uniform southerly motion of its aphelion amounting to 15′ per century. Its location in latitude is indicated by a *T* in the sixth circle of Figure 64. The planet cannot occupy a position within 90° to the north of this point and yet permit a southerly motion of the aphelion. This portion of the orbit is therefore eliminated, and also the region opposite it. The motions of the aphelia of the three remaining comets of class *B* that have appeared twice, namely, $Temple_4$, Olbers, and Pons are so uncertain, no knowledge existing of their planetary perturbations, that it has been thought best not to use them at all.

We now come to a study of the hyperbolic orbits. The mean argument of the group of ten already referred to, lying between arguments 207°.5 and

315°.8, and represented in Figure 59, is 260°.5. The maximum hyperbolic excess k equals 0.00597, Table XLIII p. 197. The mean of the excesses of the first three, counting in the order of their arguments, is 0.00242, of the middle four 0.00398, and of the last three 0.00270. We thus see that the excess approaches a maximum in argument 260°.5 and diminishes at both ends. At first sight this would seem to imply, by what we have already seen, that planet Q had recently been 180° distant from this point or, in argument 80°.5, and had now passed it. If we examine Figure 60, however, we shall see that the argument of the aphelion is 285°, and that comets coming from that region could be thrown into hyperbolic orbits more readily than from any other. The two short lines intersecting the orbit at 207°.5 and 315°.8 indicate the limiting arguments of the ten hyperbolic comets. If planet Q had an argument in excess of 45°, hyperbolic comets whose arguments exceeded 315° would be possible, since they would be continually accelerated by the planet, acting under most favorable conditions. If, on the other hand, the argument of Q were less than 315°, hyperbolic comets whose arguments exceeded 270°, would be impossible, since they would be constantly pulled backward by the planet. A continuous arc has therefore been drawn in Figure 64, stretching through three-quarters of a circumference, and reaching from 45° to 315°. While the ends of this arc are no more definitely defined than those of many of the others, yet the presence of the hyperbolic comets is so distinctly prohibitive to the possibility of the existence of the planet in other portions of its orbit, that this result has been weighted 3, and is so indicated by the heavy line in the seventh circle.

An examination of this figure now shows that there is only one short arc of 3° that is not eliminated from our consideration of the orbit of Q by one or another of these seven determinations. This arc extends northerly from argument 0°, and might readily be cut off by the uncertain ending of any one of the three arcs which bound it on the north and south. The only portions of the orbit that are not eliminated by more than one determination, are the arc extending 32° north from argument 0°, and a short arc of 3° in the third quadrant. This latter we need not consider, since the investigation of the hyperbolic comets would certainly eliminate that. The one result which interferes with the larger arc is based on the lack of motion in latitude of the aphelion of the orbit of Halley's comet during the last 75 years. There can hardly be any question of this lack of motion, but the reason that this investigation, and that of the orbit of Tuttle's comet were each given so little weight was that only a single comet was involved in each case, and with two assumed planets, *O* and *P* either of them capable of disturbing these orbits, a displacement in latitude of a few minutes would be quite possible. In Cowell and Crommelin's "Essay on the Return of Halley's Comet," Pub. Astron. Gesell. 1910, 23, 11, they imply that such a perturbation might actually have taken place, accounting for the deviation between their computed and the observed dates of perihelion passage. In 1873 when the comet was in aphelion, in longitude 124°.5, planet *O* according to p. 162 of this volume, would have been near longitude 80°. In 1900 it would have been in longitude 106°, and the comet in 113°, latitude −14°, and beyond the orbit of Uranus. The action of the assumed planet would have been to delay the observed date of perihelion, since the comet would have been moving almost directly away from it. In point of fact the perihelion passage was delayed for three days. If planet *Q* had been within the specified arc, it would have tended to pull the comet's aphelion to the north, — and to neutralize this action planet *O* must have been to the south of latitude −14°. This is not impossible as far as any information is concerned that we now possess with regard to planet *O*, and it is thought to justify the small weight given to the arc in the fifth circle of Figure 64.

We will now take up the last, and most definite method of determining the position of the planet. This depends on the use of the elliptical orbits of type *Q*. Their aphelia we have already found were very strongly concentrated between arguments 304°.0 and 8°.1, as is shown in the inner circle of Figure 59. Their mean argument is 338°.8, and their mean distance 1107 units. This would correspond to a period of 13038 years. The radius vector of the planet's orbit in this argument, as measured from a large scale representation of Figure 60, proves to be 906 units. The orbit of the mean comet is shown as a heavy straight line in Figure 60, and it will be seen that it extends beyond the orbit of the planet about as far as we might expect, judged by the orbit of Jupiter and its associated comets.

To compute the time required by the comet to drop from aphelion to a radius vector of 906 units, we must construct a table similar to the first four columns of Table XLIV, p. 199. We shall assume $q=1$ and $e=0.995$, since moderate variations in these quantities will make no appreciable difference in the result. It appears that the time required will be 4075 years, and the time required by the comet to reach the Sun from the orbit of *Q* will therefore be 2444 years. Our problem then becomes extremely simple. The mean date of perihelion of these fifteen comets was 1847, and it appears that 2444 years before that time the argument of planet Q was 338°.8. To determine its position in 1910, after an interval of 2507 years, we measure off an equivalent area on the large scale drawing already referred to, which is constructed on coordinate paper. From this we find that the present distance of the planet from the Sun is 575 units, that it is situated in latitude +22°.9, equivalent to δ+46°.2, and that it is moving northward at the rate of 106″ per year.

This latitude is indicated in Figure 64 by the heavy line *SQ*, and it is an interesting circumstance to find that it falls well within the arc which is left as the most probable one by the seven other methods of investigation. Were the orbit of Q retrograde, it will be noted that its probable position as indicated by this last method would be in the vicinity of argument 315°, a region prohibited by nearly every one of the other methods. We thus have an independent indication of the direction of revolution of the planet.

Adopting the latitude and declination above given, we find that its corresponding longitude is 95°.1 and its right ascension $6^h\ 27^m$. To determine its mass we shall assume, following Figure 59, that it is unable to generate a hyperbolic orbit in a comet whose argument is less than 207.5°. This, by measurement on the large scale drawing already mentioned, implies a radius vector of 685 units. But the comet's aphelion probably lies some distance beyond the orbit of Q, and the distance of the planet at this time is not accurately known and is constantly diminishing. The problem therefore is much more indefinite than that which was solved in Table XLIV, p. 199. Fortunately wide variations in the premises will lead to conclusions which are not very dissimilar. We shall therefore assume that our earlier result gives the maximum mass of Q, and shall now further assume that the distance of the comet and also that of Q, are equal to 800 units. This will give a smaller and perhaps more probable value. By means of a table similar to the one above mentioned we derive a mass of $Q = 0.062\,\mu$ where μ represents the mass of the Sun. The former value that we found was $0.100\,\mu$. Adopting this new mass, and also a density and albedo equal to that of Neptune, we find for the angular diameter 1″.6, and 15.4 for the magnitude.

It appeared possible that a body having the mass of Q, and located so far from the plane of the ecliptic, would have some perceptible perturbing effect in latitude upon the outer planets, and especially upon Neptune. The general nature of this effect would be to force the planet to move in a small circle of the sphere. With the assumed value of the mass and position of Q, the deviation from a great circle would probably not exceed 1″.

At first it was thought that such a deviation had been detected, based on a comparison of Le Verrier's orbit, deduced from his tables, given in the Annals of the Paris Observatory, Mem. **14** Part II, and Newcombs' orbit, as derived from his tables, Astron. Papers Amer. Ephem. **7** Part 4. The maximum deviation in latitude between the two orbits was about 2″.7 in the year 1830. By 1850 it had been reduced to 2″.3. See Figure 66, where the heavy horizontal line represents the orbit according to Le Verrier, and the heavy sinusoidal line the orbit according to Newcomb. The dots represent the Greenwich observations. If we assumed that Le Verrier's tables were correct for the earlier observations, extending through 1873, the last date given by him being marked by a short vertical line, and if Newcomb's tables were correct for the later observations, it was clear that the planet was really moving in a small circle, lying between 1″.0 and 2″.3 to the south of the ecliptic. This is indicated by the two lighter sinusoids, whose axes are located south of Le Verrier's orbit.

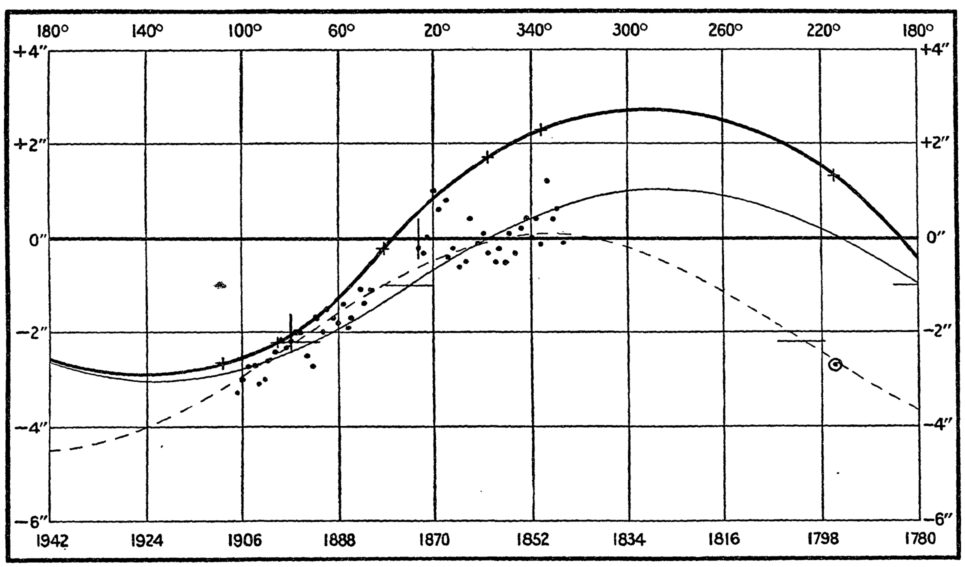

Fig. 66.

This was such a startling result that it seemed desirable to confirm it in more detail. There seemed to be no question of the accuracy of Newcomb's tables for the later observations, as they were confirmed at several different observatories. The agreement of Le Verrier's orbit with the earlier observations was based only on his own statement to that effect, which however seemed sufficient to the writer. It was finally concluded that it would be well to compute the position of Neptune. for one of the earlier dates by each set of elements, and then compare both with the observations made at that time.

The position of Neptune according to Newcomb for Oct. 15, 1850, was kindly furnished the writer by Professor Eichelberger. The work was carefully gone over and checked by the writer, who then made the same computation by means of the tables of Le Verrier. According to Le Verrier the heliocentric position of Neptune was $l = 335° \; 55' \; 12''.81$ $b = -0° \; 46' \; 36''.94$, according to Newcomb $l = 335° \; 55' \; 13.25$ $b = -0° \; 46' \; 34''.63$, $\Delta b = 2''.31$, Le Verrier placing the planet this amount farther to the south. This compared with the deviation $2''.3$ shown by the writer's earlier results.

The deviation thus having been verified, the interesting question arose as to which of the two astronomers was correct, for obviously one of them was seriously in error. Both places were reduced to their geocentric positions β according to Le Verrier being $-0° \; 47' \; 42''.06$, and according to Newcomb $-0° \; 47' \; 40''.01$. By Walkers' ephemeris, Smithsonian Contributions to Knowledge, 1851, 2 Appendix 2 p. 9, we have for Oct. 15, 1850, $\alpha = 336° \; 39' \; 56''.79$ and $\delta = -10° \; 36' \; 20''.12$. Reducing this position to longitude and latitude, we obtain $\beta = -0° \; 47' \; 40''.33$. The error of the ephemeris according to the Greenwich Observations between the dates Oct. 2 and Nov. 4, was $-0''.03$, and the mean error for the year $-0''.40$, from which we obtain the two observed positions $-0° \; 47' \; 40''.36$ and $-0° \; 47' \; 40''.73$. The first of these depends on 13 observations, the second on 33. Comparing the observed and computed positions we have for Le Verrier $O-C = +1''.70$ and $+1''.33$, and for Newcomb $-0''.35$ and $-0''.72$. It thus appears that Newcomb's tables give the more accurate results of the two, and that it is not safe to rely on the table of deviations given by Le Verrier from which the earlier conclusions had been drawn. In this place I wish to express my thanks to Professor Russell who measured for me some photographs taken at this observatory, from which it was concluded that the present deviation of Neptune from Newcomb's orbit was negligible. Since the deviation in the past appears to have been very small, it did not seem worth while to continue this investigation any further at present.

CHAPTER XX.

PLANET R. LONGITUDE OF P. INCLINATION OF THE AXIS OF THE SUN. FACTS INDICATING THE EXISTENCE OF Q. SUMMARY.

IT might be thought that little information could be obtained about so difficult and remote an object as planet R, — a planet which no one now living can hope to see, unless indeed it be self-luminous, and which is far too remote to perturb appreciably any of the planets with which we are at present familiar. We are literally groping in the dark with regard to it, yet there are certain facts which, if we can properly interpret them, should give us some light upon this difficult subject. We shall begin with a study of the orbits of some of the more eccentric asteroids. A list of those whose eccentricity exceeds 15° was accordingly made from the Jahrbuch for 1910. This eccentricity corresponds to a decimal of 0.259. From this list all those asteroids were selected that had been under observation through a period of not less than eight years, in order to determine the rate of motion of their aphelia. This determination was made by a comparison of their elements as given in the various Jahrbuchs extending back to 1860. From these elements the rate of motion was determined graphically by the same method that was employed for the comets, as described on p. 215.

In Table LXXXVII the first two columns give the number and name of the asteroid, the third its eccentricity, the fourth the interval in years between its first and last observation, the fifth and sixth the longitude and latitude of its aphelion, and the last the rate of motion of its aphelion in longitude per century. The longitudes are plotted on the inner circle of Figure 58, p. 336, and a heavy line drawn through the nine showing the greatest concentration. This line extends from longitude 118°.7 to 180°.0, mean value 149°.4, total range 61°.3. In spite of the smaller eccentricity of their orbits, the concentration is therefore somewhat greater than for the comets of type N, shown in the second circle, but not as great as for the comets of type O. Reducing the arguments of the points of maximum condensation of the orbits of the different types of comets to longitude, we find that they follow one another around the circle so that the aphelia form a sort of spiral. The longitudes are approximately as follows:—Asteroids 149°, comets of type *N* 194°, type *O* 239°, type *P* 290°, types *Q* and *R* 85°. The differences are 45°, 45°, 51°, and 155°. The degree of condensation becomes less and less marked as we proceed outwards. For the asteroids the range we found was 61°.3, for the orbits of type *N* it is 78°.8, for *O* 45°.1, for *P* 125°.2, for *Q* 117°.9, and for *R* 154.°3.

TABLE LXXXVII.

ASTEROIDS OF HIGH ECCENTRICITY.

No.	Name.	ϕ	Year.	Longitude.	Latitude.	Advance.
		°		°	°	°
41	Daphne	15.4	29	39.9	−10.5	+2.4
265	Anna	15.3	19	44.4	+24.3	+4.0
344	Desiderata	18.3	9	100.9	+15.0	+4.2
225	Henrietta	15.3	19	118.7	−20.5	−4.0
393	Lampetia	19.2	10	120.8	−14.9	+6.4
217	Eudora	17.6	13	135.0	− 5.0	+2.0
132	Aethra	19.4	17	151.2	+22.3	−1.6
253	Mathilda	15.5	13	153.7	− 2.9	+0.6
75	Euridice	17.8	32	156.0	+ 2.1	+2.0
33	Polyhymnia	19.7	45	163.5	+ 0.8	+3.6
391	Ingeborg	18.0	11	179.6	−12.9	−1.0
164	Eva	20.3	30	180.0	+23.8	+0.6
324	Bamberga	19.8	9	188.3	− 7.3	+0.8
50	Virginia	16.8	24	191.5	+ 0.8	+2.2
36	Atalanta	17.4	43	222.1	−13.0	+2.4
183	Istria	20.5	23	224.3	+26.2	+1.0
109	Felicitas	17.2	28	237.0	− 6.3	+2.4
372	Palma	15.6	8	263.0	−21.7	−7.2

Owing to the attraction of the known planets it is certain that the aphelia of the asteroids and of the comets of types *N* and *O* must slowly revolve about the Sun. An examination of the last column of the table shows that all but four of the numbers are positive, the average advance amounting to 70′ per century. In general we may say, if a perturbing planet, such as Jupiter, approaches a cometary orbit most nearly at the aphelion of the comet, and if the aphelion lies within the orbit of the planet, the comet's apsides should advance. If, on the other hand, the aphelion lies outside the orbit of the planet, the apsides should recede. Herschel's

Outlines of Astronomy, p 482. Turning back to Table LI, p. 216, we find that the first fifteen comets are associated with Jupiter. The aphelia of the first seven of these lie within the orbit of that planet, and the aphelia of the remainder without it. In the fourth column of the table are given the changes per century in the longitudes of the aphelia.

The total change in longitude of the first seven comets is + 424′, of the remaining eight − 872′. At first sight this seems just what we should expect, but on closer examination we find that most of the recession of the last eight is due to comet Perrine, which happened to pass very close to Jupiter between the two observations. Similarly, the very large recession of Winnecke's comet is due to the same cause. If, instead of using the last perihelion of this comet, we had used the previous one, we should find instead of a recession of 320′, an advance of 60′. On account of the close conjunctions with the planet it would seem proper in a general study of the motions of the aphelia to reject these two orbits as affected by exceptional circumstances. Similarly the large advance of 250′ for De Vico's comet in the first seven orbits has been rejected, since it is nearly twice as great as any other, and as it is known to have recently passed close to Jupiter. Adding the remaining figures, we find that the first six comets show a total advance of the aphelia amounting to 174′, or a mean advance of 29′ per century, and that the six outer comets show a total advance of 148′, or a mean advance of 25′ per century. We thus see that instead of these two figures balancing one another, as we might have expected, there is a decided positive revolution of the apsides, and the same result would have been obtained had Perrine's comet alone been rejected. The result in the latter case would have been a mean advance of the fourteen aphelia at the rate of 18′ per century.

The figures in the table were deduced from 80 determinations of longitude, recorded at different appearances of the comets. Of these 80, 44 were advances, 8 showed no change, and 28 were recessions. From this we conclude that there is some force or influence other than that of Jupiter acting upon these aphelia, and that it is causing them slowly to advance in longitude. According to Backlund, M.N., 1910, **70**, 440, the advance of the aphelion of Encke's comet, owing to the perturbations of Jupiter, should be 65′. The observed advance as shown by Table LI, is only 46′. This motion was extremely uniform as indicated by the construction curve from which it was obtained. The difference between these figures is therefore probably real, and due to a holding back of the orbit by some unknown force. The mean advance of the five comets of class *B*, as shown by Table LI, is 32′. The mean advance of the last three of these comets is +13′, and this is in spite of the perturbing action of all the known planets, which would tend to produce a retrograde revolution. Doubtless some of the advance is due to planets *O* and *P*, but the close grouping of the aphelia can not be due to them, since the mean advance even of *P* amounts to over 25° per century.

If the spiral arrangement above described is to be maintained, the mean revolution of these three sets of aphelia must be identical. Dividing the asteroids into three groups of six each, according to the longitude of their aphelia, we find the mean motion of the first group is 150′ per century, of the second group 42′, and of the third group 16′. It therefore appears that a concentrating process is now going on, and the aphelia in the smaller longitudes are gradually overtaking the others.

Combining all the comets in the table excepting Winnecke, De Vico, and Perrine, and arranging them in the order of the longitudes of their aphelia, we find the mean advance of the first six is 47′ per century, of the next five it is 33′, and of the last six 6′. The concentrating process therefore appears to be going on among them also, although somewhat more slowly than among the asteroids. The mean advance for the asteroids of the twelve aphelia lying between longitudes 100° and 200° is 80′. The mean advance for the comets of the six aphelia lying between the same longitudes is 46′. It therefore appears that even in the same longitudes the aphelia of the asteroids are overtaking those of the comets at the present time. On the other hand the aphelia of the comets of types *N* and *O* appear to be advancing at about the same speed.

It is probable that the additional speed of the asteroids is due to the planet Jupiter. If the orbit that we have already determined for *R* is correct, its maximum speed would be but 6′.36 per century, so that the excess above this figure must be due to the other planets. It appears therefore that the aphelia when concentrated gradually pass the planet *R*, which then holds them back by its attraction. As the advancing forces finally overcome this retardation, the aphelia will advance in spite of the planet, till after a revolution of about 180° *R* will again concentrate and retard them on the opposite side of its orbit. Most of the aphelia at a given distance from the Sun would then be found in one group, as we see is now the case. These facts therefore confirm the existence of planet *R*, and perhaps indicate that its revolution is direct, but do not give us any clew as to its present longitude.

Let us now see what evidence we have bearing on the mass of *R*. Since many of its associated orbits have been very carefully determined, and have turned out to be parabolic, we shall assume that this characteristic is due to the attraction of the planet. This will give us at all events a maximum value for the mass. We shall refer to the hyperbolic comets of this type presently. Adopting the method employed in Chapter XII we find that *R* has a mass about thirty times that of Jupiter, and if of the same density as the Sun, has about one third its diameter. The disparity between them would therefore be somewhat less than that between the Earth and Moon.

If we compare it with the planet Neptune, and assume for it the same density and albedo, its magnitude will be 26 and its angular diameter $0''.12$. While these computations can furnish only very rough approximations to the truth, yet they make it fairly clear that unless the planet is self-luminous, it must be far beyond the reach of our present means of observation.

We shall now endeavor to determine the longitude of planet *P*. This object is too remote to locate by planetary perturbations such as we employed for *O*, and its orbit is not sufficiently inclined so as to enable us to locate it by means of perturbations in latitude, such as were employed for *Q*. There seems to be but one method left us, and that not a very good one, but it is perhaps better than none at all. As we noted in Chapter XII there are two ways in which a cometary orbit may become hyperbolic. First, a large planet on the opposite side of its orbit may so attract a comet approaching the Sun as to give it a hyperbolic velocity. Second, if a comet approaches closely even a small planet, it may leave it with a considerably accelerated or retarded motion, according to the circumstances of the conjunction. In this second case the approach must be a comparatively close one. There is a possibility that such an approach may have occurred in the present instance. If we consider only the hyperbolic orbits, the comet must come directly from the planet.

The twenty-six hyperbolic aphelia have accordingly been plotted on a chart containing the orbit of *P*. On this chart were also located the intersections of their orbits with the surface of an ellipsoid generated by the revolution of the orbit of *P* about its major axis. It was found that in only four cases did the intersection with this ellipsoid, immediately preceding the comet's perihelion, lie within 15° of the orbit of P. None of the other comets therefore could have passed near the planet, the distance in general exceeding 30 units, or the distance of Neptune from the Sun. These orbits, therefore, could not owe their hyperbolic character to planet P.

The location of the aphelia of the four remaining hyperbolic comets are shown in Figure 39, p. 240. The intersection of the orbit of the first, number 204, longitude 281°, lies 8° or about 20 units from the orbit of P. It is also, therefore, too remote to be appreciably affected by the planet. The distance of the second comet, number 403, longitude 254°, was but 1° from the orbit of of the planet. The hyperbolic character of the orbits of both of these comets, however, is fully accounted for by the situation of planet Q, as we have already seen. There is, therefore, no reason for attributing their hyperbolic velocities to planet P.

The two remaining comets, numbers 384 and 440, appeared within ten years of one another, and came from the same part of the sky, the longitudes of their aphelia being 197° and 198°. Their orbits approached within 5°.3 and 3°.4 or 6.8 and 4.6 units respectively of that of P. They belong to type R, being two of the five hyperbolic comets associated with that planet. They approached the orbit of P most nearly in longitudes 177° and 190°. To determine the interval of time that has elapsed since these conjunctions, we may make use of a very convenient approximate formula for a parabolic orbit derived by Professor Comstock, Popular Astronomy, 1902, 10, 171, $r^3 = 177.64\ T^2$, where r is the distance of the comet, and T the interval in years. The value of r in these two cases is 73.3 and 77.0 units, and the intervals of time that elapsed between the conjunctions and perihelion were 47.1 and 50.7 years respectively.

Assuming that the planet lay in that portion of its orbit nearest the comet at the time of conjunction, the argument of its position in the year 1900.0 was 224°.8 and 231°.1 in the two cases, with a difference of only 6°.3. Reducing these arguments to longitude, gives us 208°.9 and 215°.3 respectively, mean 212°.1. This would correspond to a latitude of −26°. On its approach to the Sun comet 440 passed within something over 2 units of Jupiter, and behind it. The two bodies did not stay long near one another, as they were moving in opposite directions and very rapidly, as compared with the speeds at the earlier conjunction. Nevertheless, Jupiter must have retarded the comet's motion somewhat, so that had it not been for this conjunction its orbit would have been still more hyperbolic. Comet 384 passed near Uranus, and was undoubtedly somewhat perturbed by that planet. Only a complete mathematical investigation could definitely determine whether the hyperbolic character of this orbit was fully accounted for by this conjunction.

The method, therefore, seems to give a plausible although not a conclusive value for the longitude of the body that we are now seeking, and it is quite possible that any day another comet may appear, the location of whose aphelion, together with its hyperbolic velocity, will give our result a much higher degree of probability.

Of the five hyperbolic orbits of class *R*, two are accounted for by planet *P*, or if not by *P*, then by the known planets. One orbit, number 107, has such a high hyperbolic excess that it could only be due to the attraction of planet *R* in case it passed very near that body. There remain then but two orbits, numbers 239 and 446, whose hyperbolic character might be due to their aphelia lying on the opposite side of the Sun to the present position of the planet. These form such a small proportion of the total, that it seems proper to reject them, and to admit that there is no planet save *Q* capable of producing a hyperbolic velocity in a comet coming from the opposite quarter of the sky.

We are now in a position to be able to make an estimate of the total number of comets belonging to the solar system that under favorable circumstances might be expected to be visible to the human race. We need not now consider the inner groups. The periods of the comets of type *Q* are so much affected by the positions of the outer planets *P*, *Q*, and *R* that these periods cannot be used to advantage in the present computation. The period of planet *Q*, however, is 25900 years, and we may assume that most of the comets of type *Q* will appear at least once during that interval. Allowing a value of one-half to the comets of type *QR*, we find that in the last half of the last century 46.5 comets appeared belonging to type *Q*. In 25,900 years, therefore, we should expect to find 24,000 comets of this type. Allowing one-third of these to be duplicate appearances, we may estimate the total number of visible comets belonging to type *Q* as about 16000. Doubtless there are many more whose perihelion distances are so great that they would never become visible to us. The period of planet *R* is 500,000 years. During the last half of the last century 31.5 comets of this type have appeared. During a period of 500,000 years, 300,000 comets of this type should be visible. We may estimate, therefore, that there are about 200,000 comets belonging to type *R* that may come within the range of our telescopes. It is more difficult to estimate the total number of comets in the subtypes, and in classes *H* and *I*, but that the last two are very numerous can be learned from an inspection of Table LXI, p. 256. If we assume them to be about half as numerous as type *R* it is probable that we shall not be far out of the way. In that case, the total number of individual comets in the solar system, visible in our telescopes, and included in all the various types and subtypes, will be about 300,000.

Although the origin of the solar system is still shrouded in mystery, we can feel considerable assurance that for many millions of years the general conditions have been much as we find them to-day. At present we know that the planes of rotation of Jupiter and Saturn coincide closely with the planes of revolution of the orbits of their inner satellites, owing to the ellipsoidal form of the planets, and we can feel reasonably assured that the Sun's equator should in the same way coincide with the invariable plane of the solar system, did not some force either in the past or present interfere with it.

If we plot the positions in latitude and longitude of the north poles of the Sun, the orbits of the planets, and the invariable plane, we shall find that as measured from the last, the pole of the Sun lies in argument 330° and at a distance of 5°.7. The orbit of Mercury is the least inclined to the equator of the Sun, the angle being only 3°.3. Venus is next at 4°.0. The others range from 5°.5 to 7°.2, the last being the Earth, which is nearly a degree more remote than any other planet. The pole of the invariable plane lies nearly midway between those of Jupiter and Saturn.

Certainly this deviation of the Sun's equator from the invariable plan cannot be due to the Sun itself, nor to the action of any known planet. The very existence of such a deviation is strong evidence of the existence of some massive unknown body lying far outside of the plane of the ecliptic. The writer has shown, Astr. Nach. 1904, 164, 201, that the effect of the annual tide, produced by the Sun upon a planet will tend to cause the plane of rotation of the latter body to shift, so as to coincide with the plane of its revolution in its orbit. A detailed analytical discussion of this problem by F. J. M. Stratton, will be found in the M.N. 1906, 66, 374. In the present case the Sun takes the place of the planet, for we must remember that admitting the existence of planet *Q*, the Sun with its known planets is revolving about a point lying at present thirty-four units away from us, and in a period of 26,000 years.

Owing to the action of Q an annual tide of this period exists upon the Sun, and must affect its rotation in the same way that the rotation of the planets has been affected. Although Q is such a massive body, yet the tidal effect which it produces upon the Sun is less than we might expect, — far less for instance than that of Jupiter, or even than that of the Earth. The reasons for this are that the tidal force varies inversely as the cube of the distance, and that the revolution of Q is extremely slow. On the other hand Q is efficient because the plane if its orbit lies nearly at right angles to the plane of the Sun's equator. On account of its great distance and small inclination the effect of R is entirely negligible, but P is more or less effective, since it is so much nearer than Q. Its mass however is unknown.

The known planets being by far the most efficient in the production of tides, the plane of the Sun's equator must coincide most nearly with the planes of their orbits. Since it is inclined to them at an appreciable angle however, some other force must affect it, and P and Q are the only bodies capable of acting in this manner. Their ascending nodes lie in longitudes 350° and 93°, and the ascending node of the Sun's equator should therefore lie between them. Such we find to be the case, its longitude being 74°. Planet Q therefore seems to be more effective than P, and doubtless by means of this relation the ratio of the masses of P and Q might be computed. The longitude of the ascending node of the Sun's equator indicates that the direction of revolution of both P and Q is direct.

In looking over the ten chapters of Part III of this volume, one cannot but be struck by the number and the variety of ways in which planet Q makes its presence felt.

(1) In Chapter XII it is by means of the curve of aphelion distances, Figure 32, p. 193, where owing to the large number of associated comets, the maximum point at the distance of Q exceeds any other maximum save that due to Jupiter. Since new comets are constantly coming from Q, while the supply from Jupiter is limited, it will not be many years before the maximum at the distance of Jupiter is also surpassed. The distance of Q is studied in more detail in Chapters XVIII and XIX.

(2) Again on p. 198, it is shown how the presence of such a body could account for the numerous hyperbolic cometary orbits, and a little later its mass is computed by this means.

(3) In Chapter XIII we recognize its influence by the deviation to the south of the aphelia of classes A and B, described on p. 219.

(4) In Chapter XV its presence is shown by the concentration of the aphelia about the reference circle drawn parallel to its orbit, represented in Figure 40 p. 241. This concentration is shown still more markedly in Figure 47 p. 265.

(5) In Chapter XIX, p. 330, the distribution of the elliptic and hyperbolic aphelia along the reference circle of type Q, not only indicates the presence of a massive planet, but also determines the eccentricity, mean distance, and argument of perihelion of its orbit, its location, and the direction of its motion.

(6) In Chapter XX, on p. 363, the deviation of the Sun's equator, and also of the orbits of Mercury and Venus from the plane of revolution of the other planets is explained as due chiefly to the action of planet Q.

The distance of Q is determined by the method described in reference numbers 1 and 5, the mass by reference number 2, the node and inclination of the orbit by 4, the eccentricity, longitude of perihelion, direction of motion, and location by 5, and the direction of motion again by 6.

Summary

In Chapter XI the cometary orbits are grouped in classes and types, and tabulated chronologically. This classification is then expressed in percentage. In Chapter XII the orbits are classified according to their aphelion distances, and an explanation is suggested of the existence of so many hyperbolic orbits of small eccentricity. Incidentally the mass of the assumed planet Q is computed.

Chapter XIII deals with the periodic comets. It shows that their aphelia are located on the average to the south of the ecliptic, and that the angle increases rapidly with the increase of aphelion distance. Moreover, in the case of class B the angle is still increasing. For both classes the maximum deviation is found near longitude 170°. A study is made of the cause of the frequent appearance of new comets in class A, and also of their frequent disappearance, and of the relation of the appearances to solar activity. Finally an elementary study is made of the relative perturbing forces of four bodies in space, with a resulting explanation of the southerly deviation of the cometary aphelia.

In Chapter XIV the orbits of moderate eccentricity are discussed, and it is found that their aphelia tend to lie near a great circle of the sphere inclined at a considerable angle to the ecliptic. This is referred to as their

Reference Circle. By dividing the surface of the sphere into three equal areas it is shown that the likelihood that this concentration near the reference circle should be due merely to chance is less than one in one thousand. The total number of comets forming this group must be considerably in excess of all the periodic comets taken together. Finally, the earlier investigations of this portion of the subject are described.

In Chapter XV the orbits of high and of unknown eccentricity are discussed. It is found that the former are associated with two distinct reference circles, one of them passing nearly through the poles of the ecliptic. Many of the brightest comets, and most of those having hyperbolic orbits are associated with this reference circle. The brightest comets of this type move in elliptical orbits. Their aphelia are about equally distributed north and south of the ecliptic, while the hyperbolic comets usually come from the south. The so-called parabolic comets are divided nearly equally in their allegiance between the two reference circles. A summary is given showing the dominating type of orbit for each aphelion distance, that is to say the reference circle near which the aphelion is likely to be found. It is then shown that for a given aphelion distance, more than five times as many aphelia per square degree lie near the appropriate reference circle than is the case for any region remote from it. As we approach the reference circle itself, the density further rapidly increases.

In Chapter XVI it is shown that about one-half of the comets which are not classed as perodic can be divided into groups, the elements of whose orbits resemble one another. While the majority of the groups suggested by the earlier astronomers are found to be unsatisfactory, when brought to the test of an actual comparison of their elements, a large number of new groups has been found of which the individual members do resemble one another closely. Tables are furnished to facilitate the classification of any newly discovered comet with those that have appeared previous to 1910. An estimate, based on the returns of Halley's comet is made of the probable inaccuracies of the orbits deduced from the earlier observations. Evidence is produced to show that a number of the periodic comets appeared at dates earlier than those heretofore suggested, also that the periodic comet 1858 III, supposed to have disappeared, may have been recently observed as 1907 III. Finally by a process of elimination the individuals in all the groups have been examined to determine whether it is likely that any of these comets having a period exceeding one hundred years, has returned and been observed a second time. Four probable instances are indicated.

In Chapter XVII the elements of some of the more unusual orbits are tabulated, a possible origin of the asteroids and zodiacal light is suggested, and the retardation of Encke's comet and that of 1882 II discussed. Finally all the more remarkable individual peculiarities that have been exhibited by various comets are tabulated in a form for ready reference.

Chapter XVIII contains a more detailed discussion of the aphelion distances of the different types than had previously been possible. It is shown how we may compute the distance, eccentricity, and longitude of perihelion of a planet that has never been seen. An application of this plan is made as an illustration to the planet Jupiter. These elements for the assumed planet P are then computed.

In Chapter XIX the distribution of the aphelia along their reference circles is first described, and it is shown that this distribution is quite unsymmetical in certain cases. The well known lack of symmetry in the case of the aphelia of the comets dependent upon Jupiter is shown to be still more marked in the case of those dependent upon the outer known planets. This lack in the case of planets Q and R furthermore enables us to determine the eccentricity and argument of perihelion of their orbits. We are next led to discuss the hyperbolic comets and also those with long elliptical and parabolic orbits in connection with their possible origin outside of the solar system. Finally the direction of revolution and present location of planet Q are determined.

In Chapter XX the location and motion of the aphelia of the more eccentric asteroids is discussed. They are found to be more concentrated than the aphelia of the comets of type N, and to be moving faster. The motions of all these aphelia are explained as being due to the presence of planet R, and its direction of motion is believed to be like theirs direct. The mass of R is next determined. This is followed by an investigation of the present longitude of planet P. An estimate is next made of the total number of visible comets in the solar system. The inclination of the Sun's equator and that of the orbits of the planets Mercury and Venus to the invariable plane of the solar system is explained as due to the presence of planet Q. The various ways in which the existence of this planet has impressed itself upon our attention during the present investigation are next summarized. Finally the present summary of the contents of the various chapters, followed by a table of the elements of the orbits of the three assumed planets, and an appendix describing a search for planet *O* closes the volume.

The elements that we have adopted for the orbits of these planets, together with their masses and magnitudes, are given in Table LXXXVIII. Each original result is followed by the number of the page on which it is derived. The parallax and light days for *P* and *Q* are given at their present distances, those for *R* at its mean distance from the Sun.

TABLE LXXXVIII.

ELEMENTS OF THE ORBITS OF THE THREE ASSUMED PLANETS.

Quantity.	*P*	Page.	*Q*	Page.	*R*	Page.
Epoch	1900.0	..	1900.0	..	1900.0	..
Longitude	212°	361	95°	353	..	..
Latitude	−26°	"	+23°	"	..	..
Present distance	95	..	575	"	..	..
Mean distance	123	327	875	340	6250	341
Period	1,400	"	26,000	"	500,000	"
Annual motion	1400″	..	106″	353	2″.6	..
Argument of Perihelion	160°	327	105°	340	80°	341
Longitude of Node	351°	229	93°	245	234°	252
Inclination	37°	"	86°	"	26°	"
Eccentricity	0.35	328	0.54	340	0.20	341
Motion	Direct	364	Direct	345, 353, 364	Direct	360
Right ascension	$13^h\ 17^m$	..	$6^h\ 27^m$	353	..	..
Declination	−36°	..	+46°	"	..	..
Mass in terms of the Sun	..	..	0.06	"	0.03	360
Mass in terms of the Earth	..	..	20,000	..	10,000	..
Angular diameter	..	..	1″.6	353	0″.12	360
Magnitude	..	..	15.4	"	26	"
Annual parallax	36′ 11″	..	5′ 59″	..	33″	..
Distance in light days	$0^d.55$	..	$3^d.32$	..	$36^d.1$	..

APPENDIX.

A Photographic Search for Planet *O*.

The first published reference to planet *O* is in the Harvard Circular 144, dated November 30, 1908. The approximate position had however been computed the previous April, and instructions sent to Arequipa to have duplicate photographs of the region, with exposures of an hour, taken at intervals of about a week, with the 24-inch Bruce telescope. Owing to the northern declination of the region required, to the long duration of the cloudy season at Arequipa, and the consequent low attitude of the region when the photographs were taken, it was found when the plates were received, that the results were but little better than could have been obtained under favorable conditions at Cambridge, with the 8-inch Draper telescope and an exposure of 30 minutes. It was not considered that plates taken under these circumstances showed sufficiently faint stars to be serviceable for the investigation proposed.

Arrangements were accordingly made with the Rev. Joel H. Metcalf to take the requisite negatives with his 12-inch doublet. This instrument has a focal length of 87.5 inches, which corresponds to a scale of 39 millimeters to a degree. The field which it covers, giving practically equally good images throughout, measures 175 millimetres, or 4°.5, in diameter. It was proposed to have fourteen 8 × 10 plates taken, each with an exposure of one hour, and covering seven regions of the sky between right ascensions $7^h\,20^m$ and $8^h\,20^m$, and declinations +13° and +28°. Other regions, however, were also taken, so that a rather more extensive area of the sky was finally covered.

The method of search selected was that which has been previously employed at this observatory in the detection of variable stars. A light positive was printed by contact from one of the negatives of each region. These plates are in the D series and are numbered as follows: — 13793, 13801, 13804, 13922, 13923, 13925, 13946, 13947, 13954, 13955, 13957, 13958, 13974, 13975, and 13976. Where the sky of one negative of a pair was the darker, that one was selected as the plate from which to print the positive, otherwise there was no choice.

The positive was now placed on a retouching frame arranged for the purpose, with the film side up, and the other negative of the pair placed upon it film side down, and adjusted so that the stars coincided. A 2-inch eye piece was arranged to slide horizontally on a scale which could be shifted vertically 0.62 of an inch at a time. By this means all parts of the two plates could be examined systematically.

Two different methods of examination were employed. In the first the stars were made to exactly coincide. In this case the brighter stars appeared as black dots surrounded by clear white halos, while all the fainter stars vanished. Had the planet been detected, it would have appeared as a white spot with a black dot by the side of it, an appearance which would have been rather conspicuous. When one negative showed appreciably fainter stars than the other, it was found better to shift the upper plate very slightly to one side. All the stars were then visible, and appeared like small projections accompanied by shadows on one side of them. Knowing the direction in which the planet should move, the plate was shifted in such a direction, that if the planet were found, its apparent shadow would lie in the opposite direction to that of the stars, it would thereby become very conspicuous appearing like a hollow in place of an elevation.

A number of asteroids were detected by this plan, and a few variable stars, but in general the interval that was allowed to elapse between the dates of the two negatives was to small to permit much change in the brightness of the variables to have occurred. A few nebulae were also noted, and many small defects resembling stars. Since the two plates were taken but a few days apart, there was not time enough for a distant planet to have moved far, accordingly there was little trouble in recognizing the defects as such.

Table LXXXIX gives a list of the negatives employed. The first column gives the number of the negative in the MA series, and the second and third the right ascension and declination of its centre. The positions are also indicated on the chart Figure 67. The curved line represents the ecliptic and the blackened disk upon it the position of the planet for 1909 reduced to the epoch of 1855. It was convenient to adopt this epoch in this investigation since the negatives were identified on the DM. charts.

TABLE LXXXIX.

PLATES USED IN SEARCH FOR PLANET *O*.

Plate.	α 1855.		δ 1855.	Plate.	α 1855.		δ 1855.	Plate.	α 1855.		δ 1855.	Plate.	α 1855.		δ 1855.
	h.	*m.*	°		*h.*	*m.*	°		*h.*	*m.*	°		*h.*	*m.*	°
748	7	40	+28.6	777	8	0	+26.0	765	8	10	+21.2	770	8	00	+16.0
752	"	39	+28.0	746	7	10	+20.3	780	"	"	"	791	8	19	+15.7
753	8	00	+28.0	747	"	"	"	792	8	30	+20.8	795	"	"	"
754	"	"	"	736	7	31	+20.8	796	"	"	"	759	7	36	+13.0
771	7	38	+26.2	746*a*	"	"	"	767	7	38	+15.8	762	"	38	+13.3
781	"	"	"	764	7	50	+20.3	772	"	"	"	756	8	00	+13.5
773	8	00	+26.0	769	"	"	+20.2	766	7	59	+16.0	758	"	03	+13.8

In Part 2 of this volume, three causes were pointed out, either of which might render it difficult to find the planet. They were, first, the planet might be dark or reddish in color, so as to be extremely faint on the photographic plates, it might move in an extremely eccentric orbit, so as to deviate largely from its computed longitude, and third, its orbit might have a high inclination to the ecliptic, with the planet at present far from the node.

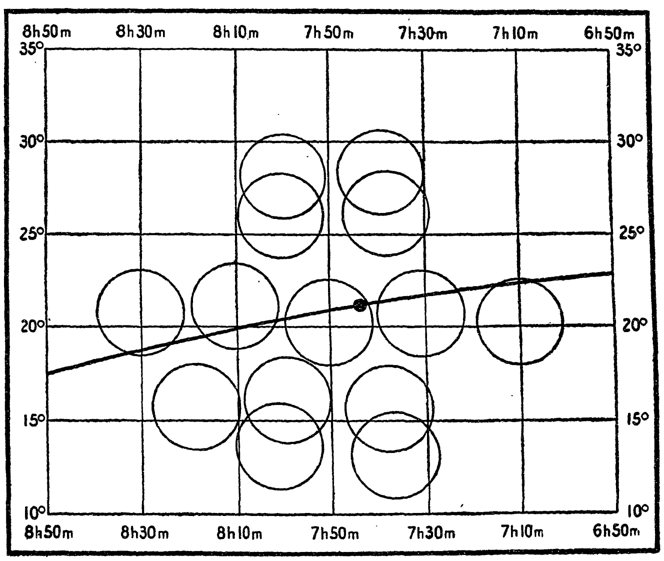

FIG. 67.

It is difficult to state how bright a planet might have been overlooked in the examination, as it was actually conducted. The computed magnitude of the object was 13.5, and it is thought that no planet as bright as photographic magnitude 15 would have been likely to have been missed, had it occurred within the region of good definition 4°.5 in diameter about the center of each plate, and indicated on the chart by the circles. Had the object appeared at the very corner of a plate, however, it might have been missed had it been fainter than magnitude 14.

It has been suggested in Chapter XIX that the observed lack of motion in a north and south direction of the aphelion of the orbit of Halley's comet between its last two appearances might have been due to the attraction of planet *O*. In a private letter Mr. Cowell states that unless the comet approached the planet within ten units, a delay of one day in the perihelion passage would involve a planet having a mass exceeding that of Neptune. Such a mass would increase the computed brightness by at least two magnitudes, but the planet would in that case be farther south than we have looked.

The computed right ascension of the planet upon January 0.0, 1909, was $7^h\,44^m$ (1855). The search along the ecliptic extended from $7^h\,00^m$ to $8^h\,40^m$ or a total distance of 25°. It is not thought likely that the longitude of the planet lay outside these bounds, but if it was a degree or so north of the ecliptic in the smaller longitudes, or the same amount south of it in the higher ones, it might not have been found.

The most plausible cause of the failure to find it hitherto, seems to the writer to be that it may have been more than 10° north or south of the ecliptic. When we consider the inclination of the orbits of Jupiter's sixth and seventh satellites, 28° and 26° respectively, to which it may perhaps, on account of its size and position, best be compared, we see that a maximum distance of search of 10° north and south from the ecliptic does not give us a very wide range of latitude for our examination. Le Verrier was perhaps fortunate in his investigation that the inclination of the orbit of Neptune is but 1°.8.

It has been found that the average time required to carefully examine a pair of plates by the method above described was four hours. In the central region of the central plate there were about 65 stars to the square centimeter, or 1000 stars per square degree. They were obviously more numerous in the regions preceding, towards the galaxy, and clearly less so in the regions following, but this figure is perhaps a fair average of the whole. This would give 30,000 stars on a plate, examined at the rate of two stars per second on the average. Since at the corners of thê plate fewer stars are shown, this will reduce the estimate to 25,000 stars on each plate. The total area covered is about twelve times that shown on a single plate, therefore we may conclude that the total number of stars examined in this investigation has been about 300,000.

END OF VOLUME LXI.