ON AN APPARATUS FOR THE LABORATORY DEMONSTRATION OF THE DOPPLER-FIZEAU PRINCIPLE.

By A. BÉLOPOLSKY.

AN apparatus for this purpose was suggested by me in the year 1894,[1] and since then I have made numerous attempts to construct it. Thanks to a grant of three hundred dollars which I received early in 1898 from the "Elizabeth Thompson Science Fund," I have succeeded in my attempts this year. The other necessities, such as the spectroscopic apparatus, the electric current, etc., were supplied me by the Pulkowa Observatory. I express here my thanks to both of these scientific institutions.

The principle of the apparatus is as follows: If a source of light is reflected in two nearly parallel mirrors, the distance S of the nth reflection from the source itself may be expressed as

$$S = h + 2nx + l\,,$$

where h is the distance from the source to a plane midway between the two mirrors, l is the distance of the image from the same plane after n reflections, and x is the distance between the two mirrors. If we differentiate this expression according to t we obtain

$$\frac{dS}{dt} = 2n\frac{dx}{dt}\,.$$

$\frac{dx}{dt}$ is the velocity of the mirror. We see that although $\frac{dx}{dt}$ may itself attain no very large value, $\frac{dS}{dt}$ will be $2n$ times larger. If the mirror had, for instance, a velocity of 50 meters per second, its image after ten reflections would move with a velocity of $2 \times 10 \times 50 = 1000$ meters per second.

We can also show that the wave-length of a homogeneous beam of light after n reflections from plane, moving mirrors

[1] *Mem. Spettr. Ital.*, **23**; *A. N.*, No. 3267.

alters according to the direction of the motion.[1] We obtain the following expression for the wave-length λ after one reflection from a moving mirror:

$$\lambda_1 = \lambda_0 \left(1 \pm \frac{2v}{V} \cos \psi\right),$$

where λ_0 is the normal wave-length, v the velocity of the mirror, V the velocity of light, ψ the angle between the direction of motion of the mirror and the normal to its surface. If the beam is successively reflected from several mirrors, we shall obtain the following wave-lengths, provided that all the mirrors have the same velocity and that ψ is constant:

After the 1st reflection, $\lambda_1 = \lambda_0 \left(1 \pm \frac{2v}{V} \cos \psi\right)$.

After the 2d reflection, $\lambda_2 = \lambda_1 \left(1 \pm \frac{2v}{V} \cos \psi\right)$.

After the nth reflection, $\lambda_n = \lambda_{n-1} \left(1 \pm \frac{2v}{V} \cos \psi\right)$.

Hence we obtain, with sufficient approximation,

$$\lambda_n = \lambda_0 \left(1 \pm \frac{2nv}{V} \cos \psi\right).$$

The sign depends upon the direction of v. With a large enough value of n, $\lambda_n - \lambda_0$ will have an appreciable value, even if v remains comparatively small.

The apparatus must therefore move at least two mirrors as rapidly as possible in opposite directions. In its simplest construction it would consist of two wheels, like those of a water wheel, each rotating rapidly and carrying several mirrors. The axes of the wheels are so connected by gears that each pair of mirrors will come into a position near to parallelism at the same time. The wheels are of aluminum, of 250 mm diameter, and each carries eight silvered mirrors of size $20 \times 105 \times 3$ mm. The mirrors are so regulated by five adjusting screws that a beam falls upon the slit of a powerful spectrograph after n reflections by all of the eight pairs. The support of the wheels is of cast iron and weighs 175 pounds. Each wheel is placed on the

[1] KETTELER, *Astronomische Undulationstheorie.*

common shaft between two electric motors, of which there are four in all. With 50 volts and from 1.5 to 2 amperes per motor, they should make 6000 revolutions per minute. Two rheostats are used to introduce gradually the current from the storage batteries, and two switches, one for each pair of motors, permit a change in the direction of rotation as desired.

The shafts of the motors are nearly parallel, but displaced somewhat sidewise in order to make room for the beam before incidence and after repeated reflections. The apparatus is mounted upon a very solid wooden table A rather poor heliostat reflects the sunlight upon a slit 10 × 20 mm in front of the apparatus.

It soon appeared that the brightness of a beam undergoing repeated reflections falls off very much, and a spectrograph of large light-power is necessary for the production of spectrograms when moving mirrors are used. For this I employed three compound prisms, set at minimum deviation for λ 433, a collimator of 1.5 meters focus, and a camera of 1.75 meters, both in wooden mountings, the prism-box being of steel, however. The whole spectrograph rests upon four long wooden screws with lock-nuts. The stability of the apparatus is increased by weights, in all amounting to some 280 pounds, which are placed at different points upon the spectrograph.

The slit is at a distance of about a meter from the mirror which reflects the light last; and either half of the slit may be employed in turn by means of a device placed directly in front of the slit. A cylindrical condenser is introduced between this device and the apparatus. The slit is observed by the light reflected from the first surface of the first prism. A diaphragm is placed closely over the collimator objective in order to see that the rays pass through the objective centrally.

Directly in front of the plate are two slides, movable from outside, which permit the exposure of any desired portion of the plate, so that the central part of the plate can be covered up and the edges exposed to the light, or the reverse. The plate-holder can be pressed into position by a screw, and has a strong spring.

With this arrangement the experiment may be made as follows:

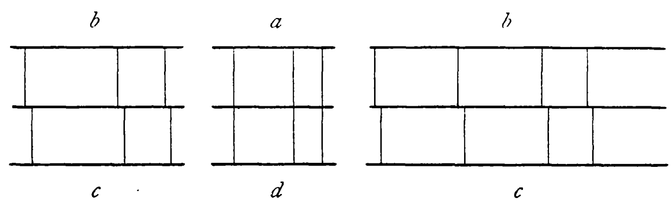
I. (*a*) One half of the slit open, say the upper half; mirror at rest; central part of plate exposed.

(*b*) Central part of plate covered, edges uncovered; mirror moving in one direction. Expose.

II. (*c*) The other half of the slit is now opened, while the first is covered; central part of plate covered, edges free; exposure made with mirrors rotating in the opposite direction.

(*d*) Mirrors at rest; central part of plate free; edges covered. Expose.

In this way we obtain upon a single plate the following spectra: Two spectra at the center of the plate, one above the



other, from mirrors at rest, to check the stability of all parts of the apparatus during the experiment; four spectra, two on each side of the central part, which should exhibit the displacements of the lines, two giving a displacement toward red, and two that toward violet.

The figure gives a schematic representation of the spectra obtained, the letters indicating the order of the spectra. *a* and *d* are the spectra for control of the stability; *b* and *c* the displaced spectra; hence the plate shows a double displacement of the lines.

The region of spectrum from $\lambda 438$ to $450 \mu\mu$ was employed, as the violet region was very faint in the reflected light (since the silvering of the mirrors transmits the violet rays). The lines found on the plate serve to compute the coefficient K for converting the measured displacements into kilometers per second.

To show that the dispersion alters very little on different plates, I give the measured distance in revolutions of the micrometer between the two lines λ 4456.24 and λ 4425.63, viz., January 27, 33.849 rev.; July 6, 33.877 rev.; August 9, 33.867 rev.

For these plates the following values of the coefficient K for λ 4444.18 were computed by the method of least squares:

1900	June 27,	$\log K =$	1.7878
	July 6,		1.7826
	Aug. 9,		1.7871

If the magnitude of the displacement of the lines was 0.010 rev., a difference of one unit in the second place of log K would make a difference of only 14 meters in the calculated velocity. It would therefore be permissible to use the mean value of log K $= 1.7858$ for all my plates.

The stellar spectrograph of this Observatory gives a precision seven times smaller in the determination of velocities in the line of sight; and since it has been shown that the velocity for stars of the second type can be determined with a probable error of ± 2 km per second for each plate, we might expect that this spectrograph could give a probable error of a few hundred meters.

The preliminary experiments, which have been in progress since the completion of the apparatus in April, have shown that a very long exposure is required to obtain a measurable spectrogram when the light is repeatedly reflected by moving mirrors. Thus sunlight reflected eight times required more than an hour, and more than two hours were necessary for two spectra. So long an exposure as this is hardly possible, for the summer sky is seldom free from clouds for so long a time, and variations in temperature and other causes may produce large changes in the parts of the spectrograph. I convinced myself that under favorable conditions no displacements of the spectral lines occurred, as was shown by two adjacent spectrograms taken two and one half hours apart.

To indicate the large loss by reflections from moving mirrors, I may say that with the mirrors at rest, quite strong spectrograms could be obtained with 2 seconds' exposure.

Thereafter I accordingly employed only the sixth reflection, which cut the exposure time down to 30 minutes for each spectrum.

I made use of a speed indicator for determining the velocity of rotation of the apparatus; and also employed the acoustical method of estimating the pitch when a piece of paper was held against the gears. These determinations yielded the following average results:

With a current of 4½ amperes (rheostat cut out) there were 2016 revolutions in 63 seconds, or 32 per second. With the same circuit, at another time, the pitch was estimated as *La* of the third octave, corresponding to 1740 vibrations per second. The wheel had 49 teeth, so that there were 35 revolutions per second. With 7¼ amperes there were 1512 revolutions in 34 seconds, or 44 per second. At another time 505 revolutions were counted in every 11½ seconds, giving the same number of revolutions per second as above. During the whole time of the experiment the ammeter showed no variations of over ¼ ampere, whence we may infer that the rotation of the dynamos was very constant.

The breadth of the mirrors being 20 mm, the largest diameter between the edges of two mirrors standing 180° from each other was 230 mm, and the least was 190 mm, whence it follows that with the sixth reflection and 32 revolutions per second the limits of the linear velocity were 276 and 230 meters per second. With 44 revolutions per second they were 389 and 318 meters per second. It is perhaps for this reason that the lines upon the spectrogams taken with rotating mirrors have a broader appearance than those taken with mirrors at rest.

The spectrograms were measured with a micrometer screw 65 mm long attached provisionally to the microscope, as the regular screw of 35 mm was too short for the purpose; 199.4 divisions of the head corresponded to one millimeter.

The plates were always so placed under the microscope that the readings of the head increased from red to violet. Only that half of the plate toward the violet was measured, as the length of the spectrograms was 100 mm. Settings (usually four or five on each line) were first made on the lines of the upper spectrum, then on those of the lower. The direction of the rotation of the mirrors, whether receding (−) or approaching (+) each other, is marked on the plate. I have obtained measurable plates only since June 27. The small number secured since that time is explained in part by the unfavorable weather and in part by the fact that I was also making spectrograms of the Sun with the 30-inch refractor. The measures of the plates and explanatory remarks follow.

JUNE 27, 1900.

Sixth reflection; exposure 30^{m}; exposures with mirrors at rest at beginning and end; the first motion was +; current of 4½ amperes; under microscope the upper spectrogram corresponded to a negative, the lower to a positive direction of rotation. The difference of the readings is always expressed as lines of upper − lines of lower spectrum.

Comparison spectrum (mirrors at rest)		Spectrum with rotating mirrors	
Line 1	−0.008 rev.	λ 4461.9	+0.011 rev.
2	−0.006	4462.1	+0.007
3	−0.022	4457.6	−0.008
4	−0.016	4456.0	−0.005
		4454.7	−0.010
Mean	−0.013	4451.7	+0.003
		4448.0	+0.003
		4444.0	−0.005
		4425.6	0.000

The absolute displacement (toward red in lower spectrum) equals 0.012 rev., corresponding to a velocity of 0.75 km per sec. The mirrors had a maximum velocity of 0.55 km per sec.

JULY 1.

The comparison spectra on this plate show an unaccountably large displacement, so that at first I rejected it altogether.

If these spectra are neglected, however, and those obtained from moving mirrors are treated independently, the micrometer thread being oriented by the dividing line between the spectra, a displacement is obtained which corresponds with sufficient accuracy to the velocity of the mirrors. The large displacement might be explained by a jar received by the spectrograph just after the first or before the last exposure.

The pitch indicated a velocity of rotation of the motor of 35 revolutions per second. Under the microscope the upper spectrum corresponded to a +, the lower to a − motion.

λ	Upper—lower spectrum
4459.4	−0.011 rev.
4457.9	−0.003
4456.0	−0.009
4451.8	−0.010
4448.0	−0.020
4444.0	+0.003
4442.5	−0.006
4425.6	−0.017
4415.7	−0.022
4408.0	−0.015
Mean	−0.011

The displacement corresponds to a velocity of 0.67, the maximum motion of the mirrors to one of 0.60 km per sec.

JULY 6.

Current = $7\frac{1}{4}$ amperes; motion at first in the negative direction; exposure 30^m. The temperature changed $0^\circ.4$ C. during the experiment. Under microscope the upper spectrum was due to −, the lower to + motion.

Comparison spectrum		Spectrum from rotating mirrors	
λ 4482.5	−0.074 rev.	λ 4462.0	−0.058 rev.
4482.5	−0.075	4456.1	−0.052
4476.2	−0.084	4451.8	−0.057
4468.7	−0.080	4436.0	−0.060
		4425.6	−0.053
Mean	−0.078 rev.	4418.0	−0.058
		4415.8	−0.064
		4407.9	−0.055
		Mean	−0.057

Absolute displacement (toward red in lower spectrum) $= 0.021$ rev.; velocity $= 1.28$; maximum velocity of mirrors $= 0.78$ km per sec.

JULY 9.

Current $= 7\frac{1}{4}$ amperes; exposure 30^m. Under microscope the upper spectrum was due to $+$, the lower to — motion.

	Comparison spectrum		Spectrum from rotating mirrors
1	+0.002 rev.	1	−0.007 rev.
2	+0.009	2	−0.028
3	+0.003	3	−0.009
4	+0.006	4	−0.011
5	−0.006	5	+0.009
Mean	+0.003 rev.	6	+0.002
		7	+0.006
		8	−0.0017
		9	−0.010
		10	−0.014
		Mean	−0.008 rev.

Absolute displacement (toward violet for lower spectrum) $= 0.011$ rev.; corresponding velocity $= 0.67$; maximum velocity of mirrors $= 0.78$ km per second.

AUGUST 7.

Current $7\frac{1}{4}$ amperes; first direction of motion —; under microscope upper spectrum corresponds to — motion.

	Comparison spectrum		Spectrum from rotating mirrors
1	+0.045 rev.	1	+0.080 rev.
2	+0.062	2	+0.080
3	+0.068	3	+0.060
4	+0.064	4	+0.062
5	+0.068	5	+0.074
Mean	+0.059 rev.	6	+0.068
		7	+0.063
		Mean	+0.070 rev.

Absolute displacement (toward red for lower spectrum) $= 0.011$ rev.; corresponding velocity $= 0.67$; maximum velocity of mirrors $= 0.78$ km per second.

AUGUST 9.

Current $7\frac{1}{4}$ amperes; exposure 30^m; first direction of motion —; under microscope the upper spectrum corresponds to — motion.

	Comparison spectrum	Spectrum from rotating mirrors	
1	+0.082 rev.	4461.8	+0.079 rev.
2	+0.071	4456.0	+0.100
3	+0.072	4451.8	+0.083
4	+0.077	4448.0	+0.086
5	+0.08	4444.0	+0.095
6	+0.075	4442.5	+0.079
7	+0.079	4437.1	+0.091
		4435.9	+0.088
Mean	+0.077 rev.	4425.6	+0.088
		4418.6	+0.087
		4417.9	+0.096
		Mean	+0.088 rev.

Absolute displacement (toward red for lower spectrum) = 0.011 rev.; velocity = 0.67; maximum velocity of mirrors = 0.78 km per second.

The velocities measured may be summarized as follows:

1900	From displacements	From rotation
June 27	0.73 km per sec.	0.46—0.55 km per sec.
July 1	0.67	0.50—0.60
July 6	1.28	0.64—0.78
July 9	0.67	0.64—0.78
Aug. 7	0.67	0.64—0.78
Aug. 9	0.67	0.64—0.78

Probable error of each velocity = ±0.17 km

These results are to be regarded as only the first experiments with the apparatus above described. Much remains in the way of its improvement, and it is especially desirable to put the wheels into a vacuum in order to avoid the resistance of the air. It is hoped that in time better results will be attained.

PULKOWA,
October 1900.