

New Values of the Division Errors of the Greenwich Transit Circle, and their Effect upon the Observed North Polar Distances.

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It may be convenient to shortly enumerate the points considered in this paper as follows :—

The first part gives an account of the investigations of the division errors of the Greenwich Transit Circle, the probable errors of the results, the table of the newly determined values, and the corrections to the values adopted in previous years.

The second part traces the effect of the new values upon the Greenwich circumpolar observations, upon the discordances between the Cape and Greenwich Catalogues as given by Dr. GILL in the introduction to the Cape Catalogue for 1885, and upon the observations of the Sun at Greenwich for the twenty years 1877–1896, and also briefly considers the value of the refraction constant best suited to these three classes of observations.

The results derived in the second part of this paper may be summarised as follows :—

1. The new division errors do not appreciably affect the R—D discordance.
2. The new division errors fairly account for the observed discordance in the colatitude derived from stars at about 20° from the pole as compared with that from close circumpolars.
3. The new division errors largely reduce the discordances between the Cape and Greenwich observations.
4. Down to 75° Z.D. the Greenwich circumpolar observations are as well suited by Bessel's constant of refraction as by that adopted

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in the Pulkova Tables, and beyond that point neither system of refraction is satisfactory; and generally the data for deciding the question of the law of refraction are exceedingly scanty.

5. The outstanding differences between the Cape and Greenwich observations can be explained by a diminution of the constant of refraction of the *Tabulæ Regiomontanæ* by $\frac{1}{400}$ th part, or by the adoption of the Pulkova refractions.
6. While the Pulkova refractions make the Greenwich Sun observations agree better *inter se*, they introduce a marked discordance in the value of colatitude as derived from the Sun and from the circumpolar observations.

PART I.

1. In the Astronomer Royal's Report to the Board of Visitors for 1898 the following table exhibited the differences between the North Polar distances from observations above and below Pole, derived from the Ten-Year Catalogue for 1880 and the new Ten-Year Catalogue for 1890, arranged in zones of five degrees, and reduced with Bessel's and also with the Pulkova refractions :—

Excess of N.P.D. above Pole.

N.P.D.	Bessel's Refractions.		Pulkova Refractions.	
	1880 Catalogue.	1890 Catalogue.	1880 Catalogue.	1890 Catalogue.
0°-5°	-0'40	-0'32	-0'15	-0'07
5°-10°	-0'04	+0'07	+0'23	+0'34
10°-15°	+0'03	+0'07	+0'31	+0'35
15°-20°	+0'17	+0'22	+0'50	+0'53
20°-25°	+0'19	+0'11	+0'55	+0'47
25°-30°	-0'19	-0'19	+0'23	+0'23
30°-35°	-0'52	-0'33	-0'01	+0'18
35°-41°	-0'40	-0'38	+0'20	+0'22

The close agreement of the 1880 and 1890 Catalogues emphasised the discordances of the results from stars near the Pole and those between 15° and

25° N.P.D. in the Greenwich observations, and suggested a re-examination of the division errors.*

2. The Greenwich Transit Circle is ordinarily read by six microscopes placed at distances of 60° apart. The determination of the division errors consists essentially in subdividing arcs of 60°. By means of two supplementary microscopes placed at opposite ends of a diameter 20° away from two of the six microscopes used in reading the circle, the arc of 60° is subdivided at 20° and 40°. By means of two more supplementary microscopes placed at 25° from two of the regular microscopes, the arcs 0° to 20°, 20° to 40°, 40° to 60° are subdivided into arcs of 5°.

The corrections for division error now in use were determined from the mean of the observations made in 1851 and 1856, in both of which investigations the 0° division was taken as the initial division. The errors of the divisions at 20° and 40° are consequently best determined, and the divisions at 10°, 30°, and 50° are most likely to be affected by cumulative error. The 50° division is the one nearest the Pole, and the existence of a discordance here and at 20° from the Pole (near the divisions 10° and 30°) gives *prima facie* evidence that the adopted division error is erroneous at these points.

3. A re-determination of the errors of the 5° divisions was made in 1871, in which the 10°, 30°, 50° divisions were taken as the initial divisions. In the new determination made in 1898 June, the 15°, 35°, 55° divisions were taken as the initial divisions. As the same correction is applied for division error for all the divisions within 30' on both sides of the whole degrees, it is clearly advantageous to use such a mean of a number of divisions instead of simply the divisions belonging to the whole degrees. Instead of repeating measures of the same divisions of the circle the observations were extended to the three divisions on both sides of the principal division, and the mean for the seven divisions $-15'$, $-10'$, $-5'$, 0 , $+5'$, $+10'$, $+15'$ was considered as the division error of the principal division.

4. When the separate determinations, made in 1851, 1856, 1871, and 1898, were compared, it was seen that the 1851 determination was considerably in error, and therefore the Astronomer Royal considered that it would

* See Newcomb, "On the North Polar Distances of the Greenwich Transit Circle," in the *Astronomical Papers* published by the *American Nautical Almanac* Office, vol. ii. pp. 418-423.

be well to reject this and to make a fresh one, taking 5° as the initial division.

There are thus now four determinations, making a complete and symmetrical series, viz. :—

1856	with	$0^\circ, 20^\circ, 40^\circ$,	as primary divisions.
1871	„	$10^\circ, 30^\circ, 50^\circ$,	„ „
1898 (a)	„	$15^\circ, 35^\circ, 55^\circ$,	„ „
1898 (b)	„	$5^\circ, 25^\circ, 45^\circ$,	„ „

The mean of those four will give a result in which each of the five-degree divisions is equally well determined. The separate determinations are given below.

Reading of Pointer.	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°
1851	0 ^{''} 25	0 ^{''} 34	0 ^{''} 44	0 ^{''} 78	0 ^{''} 85	0 ^{''} 66	0 ^{''} 21	0 ^{''} 33	0 ^{''} 84	0 ^{''} 56	0 ^{''} 33	0 ^{''} 18
1856	0 ^{''} 20	0 ^{''} 33	0 ^{''} 43	0 ^{''} 67	1 ^{''} 14	0 ^{''} 67	0 ^{''} 21	0 ^{''} 32	0 ^{''} 62	0 ^{''} 79	0 ^{''} 45	0 ^{''} 29
1871	0 ^{''} 10	0 ^{''} 19	0 ^{''} 55	0 ^{''} 73	1 ^{''} 21	0 ^{''} 74	0 ^{''} 32	0 ^{''} 32	0 ^{''} 61	0 ^{''} 54	0 ^{''} 47	0 ^{''} 36
1898 (a)	0 ^{''} 18	0 ^{''} 21	0 ^{''} 55	0 ^{''} 66	1 ^{''} 14	0 ^{''} 75	0 ^{''} 22	0 ^{''} 16	0 ^{''} 47	0 ^{''} 65	0 ^{''} 43	0 ^{''} 34
1898 (b)	0 ^{''} 18	0 ^{''} 24	0 ^{''} 44	0 ^{''} 68	1 ^{''} 06	0 ^{''} 69	0 ^{''} 25	0 ^{''} 27	0 ^{''} 54	0 ^{''} 63	0 ^{''} 43	0 ^{''} 31
Mean of four	0 ^{''} 17	0 ^{''} 24	0 ^{''} 49	0 ^{''} 69	1 ^{''} 14	0 ^{''} 71	0 ^{''} 25	0 ^{''} 27	0 ^{''} 56	0 ^{''} 65	0 ^{''} 45	0 ^{''} 33

All the measures are originally referred to the readings of one of the microscopes used (*e.g.* A), and as reading of Microscope A = Pointer reading $-89^\circ 30'$, the divisions $0^\circ, 5^\circ, 10^\circ$, &c., under Microscope A, correspond to the Pointer readings $29\frac{1}{2}^\circ, 34\frac{1}{2}^\circ, 39\frac{1}{2}^\circ$, &c., but it is more convenient to refer to them as $30^\circ, 35^\circ, 40^\circ$, &c.

5. As these four determinations are quite independent and the different divisions are equally well determined, their accordancy gives a reliable indication of the accuracy of the result. From the above figures the mean discordance is found to be $\pm 0^{''}040$, and the probable errors of the resulting

division errors are $\pm 0^{''}040 \times \frac{1}{\sqrt{4-1}} \times 0.845 = \pm 0^{''}020$. We may conclude therefore that of the twelve five-degree divisions whose errors are determined, the error of the determination is less than $0^{''}02$ in six cases out of the twelve, less than $0^{''}04$ in ten cases out of the twelve, and less than $0^{''}07$ in all the twelve.*

* Chauvenet's *Astronomy*, vol. ii. p. 488, and Table IX.

6. A re-determination of the errors of the single degrees was next made by subdividing each arc of 5°. Five separate sets of measures were made, and instead of using only the principal division the observations were extended to the divisions at 10' and 20' on each side of it, and the mean for the five divisions -20', -10', 0, +10', +20', was taken as the division error of the principal division.

The determinations which were made in 1851 and 1856, and which consisted of four sets of measures of the whole-degree divisions, were re-reduced, with the new determination of the errors of the terminal five-degree divisions. The mean of the three determinations made in 1851, 1856, and 1898 was then taken, giving equal weights to the three determinations.

TABLE I.

Separate Determinations of Corrections for Division Error of Single Degrees made in 1851, 1856, and 1898, reduced with the values of the 5° divisions, arranged for readings of Microscope A.

(Microscope A + 89°·30' = Pointer reading.)

Reading of Mic. A.	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°	12°	13°	14°	15°	16°	17°	18°	19°	20°	21°	22°	23°	24°	25°	26°	27°	28°	29°
1851	0·22	0·38	0·16	0·02	"	0·37	0·41	0·43	0·46	0·54	0·80	0·79	0·73	0·56	0·49	0·47	0·57	0·34	0·40	0·25	"	0·30	0·32	0·17	0·32	0·17	0·27	0·28	0·27
1856	0·18	0·24	0·16	0·13	0·30	0·37	0·53	0·56	0·46	0·55	0·65	0·42	0·37	0·60	0·62	0·67	0·62	0·30	0·20	0·08	0·27	0·32	0·14	0·22	0·31	0·28	0·27	0·28	0·27
1898	0·38	0·48	0·29	0·24	0·17	0·32	0·46	0·38	0·58	0·36	0·34	0·48	0·74	0·75	0·52	0·54	0·50	0·70	0·55	0·46	0·35	0·20	0·42	0·31	0·28	0·27	0·28	0·27	0·27
Mean	0·26	0·37	0·20	0·04	0·28	0·37	0·47	0·47	0·53	0·57	0·59	0·54	0·56	0·61	0·54	0·59	0·49	0·47	0·20	0·28	0·31	0·23	0·28	0·27	0·28	0·27	0·28	0·27	0·27
Reading of Mic. A.	31°	32°	33°	34°	35°	36°	37°	38°	39°	40°	41°	42°	43°	44°	45°	46°	47°	48°	49°	50°	51°	52°	53°	54°	55°	56°	57°	58°	59°
1851	0·23	0·26	0·25	0·25	0·27	0·49	0·49	0·44	0·51	0·40	0·44	0·61	0·66	0·77	0·79	0·80	1·06	0·84	0·58	0·66	0·73	0·54	0·58	0·44	0·44	0·44	0·44	0·44	0·44
1856	0·23	0·22	0·26	0·08	0·31	0·51	0·48	0·43	0·65	0·41	0·55	0·64	0·66	0·84	0·73	0·58	1·04	0·97	0·79	0·82	0·72	0·56	0·49	0·45	0·45	0·45	0·45	0·45	0·45
1898	0·19	0·38	0·26	0·21	0·24	0·51	0·42	0·43	0·64	0·65	0·61	0·63	0·68	0·88	1·10	0·98	0·87	0·82	0·78	0·67	0·43	0·44	0·42	0·34	0·34	0·34	0·34	0·34	0·34
Mean	0·22	0·29	0·26	0·18	0·27	0·50	0·46	0·44	0·60	0·49	0·53	0·63	0·67	0·83	0·87	0·79	0·99	0·88	0·72	0·72	0·63	0·51	0·50	0·41	0·41	0·41	0·41	0·41	0·41

7. In the division of the known interval from the divisions 0° to 5° , the probable errors of the 1° and 4° divisions are equal, and those of the 2° and 3° are equal. The mean discordances of the three separate determinations from the mean of the three are found to be $\pm 0''.0683$ for the divisions next to the five-degree divisions, and $\pm 0''.0821$ for the divisions near the middle. The probable errors of the mean of the three determinations are $\pm 0''.041$ and $\pm 0''.049$ respectively. These values, however, do not include the errors of the terminal five-degree divisions.

It is of interest to compare these quantities with their theoretical values. Let x_0 and x_5 be the corrections to the terminal five-degree divisions, let l_1, l_2, l_3, l_4, l_5 be the measured lengths of the five nominal degrees. (A constant may be added to or subtracted from these.)

Let $l_1 + l_2 + l_3 + l_4 + l_5 + x_0 - x_5 = 5z$. Then $l_1 - z, l_2 - z, l_3 - z, l_4 - z, l_5 - z$ are the excesses of the five intervals 0° to $1^\circ, 1^\circ$ to $2^\circ, \&c.$, over actual degrees, and the corrections for division errors are

$$x_0, x_0 + l_1 - z, x_0 + l_1 + l_2 - 2z, x_0 + l_1 + l_2 + l_3 - 3z, x_0 + l_1 + l_2 + l_3 + l_4 - 4z,$$

and

$$x_0 + l_1 + l_2 + l_3 + l_4 + l_5 - 5z.$$

or

$$x_0, \frac{4x_0 + x_5 + 4l_1 - l_2 - l_3 - l_4 - l_5}{5}, \frac{3x_0 + 2x_5 + 3l_1 + 3l_2 - 2l_3 - 2l_4 - 2l_5}{5},$$

$$\frac{2x_0 + 3x_5 + 2l_1 + 2l_2 + 2l_3 - 3l_4 - 3l_5}{5}, \frac{x_0 + 4x_5 + l_1 + l_2 + l_3 + l_4 - 4l_5}{5}, \text{ and } x_5.$$

If ϵ_0 be the probable error of one of the terminal divisions, and ϵ of one of the measures $l_1, l_2, \&c.$, then the probable error for the two divisions next the terminals is

$$\frac{1}{5} \left\{ (4^2 + 1^2) \epsilon_0^2 + (4^2 + 1^2 + 1^2 + 1^2 + 1^2) \epsilon^2 \right\}^{\frac{1}{2}},$$

or

$$\frac{\sqrt{17\epsilon_0^2 + 20\epsilon^2}}{5},$$

and the probable errors of the two remaining divisions are

$$\frac{\sqrt{13\epsilon_0^2 + 30\epsilon^2}}{5}.$$

When ϵ_0 is taken = 0, the ratio of these quantities is $\sqrt{2} : \sqrt{3}$ or 1.22, and the ratio $0''.0683 : 0''.0821$ is 1.20, showing a satisfactory agreement.

Using the previously determined values of $\pm 0''.020$ for the probable error of the divisions $0^\circ, 5^\circ, 10^\circ$, &c.; and the value $\pm 0''.041$ for the internal error of the divisions $1^\circ, 4^\circ, 6^\circ, 9^\circ$, &c.; and $\pm 0''.049$ for the internal error of the divisions $2^\circ, 3^\circ, 7^\circ, 8^\circ$, &c., the above formulæ give

$\pm 0''.044$ for the probable error of the corrections for division error for $1^\circ, 4^\circ$, &c.,

and

$\pm 0''.051$ for the probable error of the corrections for division error for $2^\circ, 3^\circ, 7^\circ, 8^\circ$, &c.

The correction for division error thus determined is therefore probably not more than $\pm 0''.050$ in error in 24 of the 48 divisions considered, not more than $\pm 0''.10$ in 40 of them, and probably never more than $\pm 0''.17$ or $\pm 0''.20$.*

8. The divisions used for the close polar stars, namely, 45° to 55° , were determined with additional accuracy. For all the single degrees there were three determinations, consisting of 4, 4 and 5 sets of measures respectively, but for these divisions two additional determinations were made, consisting of 10 sets of measures each. The probable error of the division correction for $46^\circ, 49^\circ, 51^\circ$, and 54° is accordingly reduced to $\pm 0''.031$, and for $47^\circ, 48^\circ, 52^\circ$, and 53° to $\pm 0''.034$.

The finally adopted values of the corrections for division error, Table II., were obtained by taking the mean of all the sets of division errors for each microscope and combining the errors of the six microscopes afterwards; the means thus found were then smoothed by taking the sums of each pair, and then of these pairs again, and dividing the resulting values by four; but the values of the 5° divisions were not smoothed, being considerably more accurate than the others.

* Chauvenet's *Astronomy*, vol. ii. p. 488, and Table IX.

TABLE II.

Table of New Corrections for Division Error for Observations made with the Greenwich Transit Circle, and of Corrections to the Values previously adopted.

Pointe- Reading.	N.P.D.	N.P.D.	N.P.D.	New Cor- rection for Division Error.	Corrections to Green- wich Obs., 1851-56.	Corrections to Green- wich Obs., 1857-67.	Corrections to Green- wich Obs., 1868-79.	Corrections to Green- wich Obs., 1880-96.
0 to 1	39 to 40	99 to 100	-20 to -21	0.25	-0.06	+0.01	-0.03	-0.06
1 " 2	40 " 41	100 " 101	-19 " -20	0.28	-0.05	+0.01	-0.03	-0.06
2 " 3	41 " 42	101 " 102	-18 " -19	0.26	-0.10	-0.03	-0.08	-0.06
3 " 4	42 " 43	102 " 103	-17 " -18	0.24	-0.11	-0.03	-0.08	-0.07
4 " 5	43 " 44	103 " 104	-16 " -17	0.28	-0.08	+0.03	-0.03	-0.07
5 " 6	44 " 45	104 " 105	-15 " -16	0.35	-0.02	+0.03	0.00	-0.08
6 " 7	45 " 46	105 " 106	-14 " -15	0.46	+0.05	+0.03	+0.03	-0.06
7 " 8	46 " 47	106 " 107	-13 " -14	0.49	-0.02	-0.03	-0.03	-0.04
8 " 9	47 " 48	107 " 108	-12 " -13	0.47	-0.07	-0.05	-0.06	-0.02
9 " 10	48 " 49	108 " 109	-11 " -12	0.48	-0.02	+0.01	-0.01	-0.02
10 " 11	49 " 50	109 " 110	-10 " -11	0.56	+0.06	+0.06	+0.06	+0.03
11 " 12	50 " 51	110 " 111	-9 " -10	0.54	+0.03	+0.00	+0.01	+0.03
12 " 13	51 " 52	111 " 112	-8 " -9	0.57	+0.07	+0.07	+0.06	+0.02
13 " 14	52 " 53	112 " 113	-7 " -8	0.65	+0.10	+0.11	+0.10	-0.02
14 " 15	53 " 54	113 " 114	-6 " -7	0.72	+0.03	+0.09	+0.05	-0.01
15 " 16	54 " 55	114 " 115	-5 " -6	0.79	-0.04	+0.05	0.00	-0.02
16 " 17	55 " 56	115 " 116	-4 " -5	0.85	+0.03	+0.06	+0.05	+0.02
17 " 18	56 " 57	116 " 117	-3 " -4	0.86	+0.13	+0.07	+0.10	+0.13
18 " 19	57 " 58	117 " 118	-2 " -3	0.91	+0.22	+0.18	+0.20	+0.17
19 " 20	58 " 59	118 " 119	-1 " -2	1.10	+0.36	+0.29	+0.32	+0.16
20 " 21	59 " 60	119 " 120	0 " -1	1.01	+0.22	+0.04	+0.14	+0.07
21 " 22	60 " 61	120 " 121	0 " 1	0.88	+0.13	-0.10	0.00	+0.14
22 " 23	61 " 62	121 " 122	1 " 2	0.77	+0.20	-0.08	+0.06	+0.18
23 " 24	62 " 63	122 " 123	2 " 3	0.74	+0.24	-0.06	+0.09	+0.11
24 " 25	63 " 64	123 " 124	3 " 4	0.73	+0.13	-0.07	+0.02	+0.02
25 " 26	64 " 65	124 " 125	4 " 5	0.64	-0.04	-0.10	-0.07	-0.03
26 " 27	65 " 66	125 " 126	5 " 6	0.56	-0.11	-0.10	-0.11	-0.02
27 " 28	66 " 67	126 " 127	6 " 7	0.50	-0.09	-0.06	-0.08	0.00
28 " 29	67 " 68	127 " 128	7 " 8	0.42	-0.09	-0.05	-0.08	+0.01
29 " 30	68 " 69		8 " 9	0.29	-0.12	-0.10	-0.12	0.00
30 " 31	69 " 70		9 " 10	0.31	+0.03	+0.04	+0.02	+0.06
31 " 32	70 " 71		10 " 11	0.32	+0.05	+0.12	+0.07	+0.06
32 " 33	71 " 72		11 " 12	0.22	-0.08	+0.01	-0.04	+0.04
33 " 34	72 " 73	-47 " -48	12 " 13	0.16	-0.06	+0.03	-0.02	+0.02
34 " 35	73 " 74	-46 " -47	13 " 14	0.31	+0.13	+0.24	+0.17	+0.02
35 " 36	74 " 75	-45 " -46	14 " 15	0.32	-0.01	+0.03	+0.03	-0.12

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TABLE II.—*continued.*

Pointer Reading.	N.P.D.	N.P.D.	N.P.D.	New Correction for Division Error.	Corrections to Greenwich Obs., 1851-56.	Corrections to Greenwich Obs., 1857-67.	Corrections to Greenwich Obs., 1868-79.	Corrections to Greenwich Obs., 1880-96.
36° to 37°	75 to 76	-44° to -45°	15° to 16°	0" 39	-0" 10	+0" 01	-0" 05	-0" 14
37 " 38	76 " 77	-43 " -44	16 " 17	0" 47	- " 11	+ " 01	- " 06	- " 15
38 " 39	77 " 78	-42 " -43	17 " 18	0" 51	- " 14	- " 06	- " 11	- " 19
39 " 40	78 " 79	-41 " -42	18 " 19	0" 58	- " 17	- " 06	- " 12	- " 15
40 " 41	79 " 80	-40 " -41	19 " 20	0" 57	- " 25	- " 06	- " 16	- " 17
41 " 42	80 " 81	-39 " -40	20 " 21	0" 59	- " 27	- " 03	- " 15	- " 21
42 " 43	81 " 82	-38 " -39	21 " 22	0" 59	- " 32	- " 09	- " 21	- " 21
43 " 44	82 " 83	-37 " -38	22 " 23	0" 60	- " 30	- " 10	- " 20	- " 13
44 " 45	83 " 84	-36 " -37	23 " 24	0" 67	- " 09	- " 01	- " 06	+ " 02
45 " 46	84 " 85	-35 " -36	24 " 25	0" 61	- " 01	- " 07	- " 04	+ " 03
46 " 47	85 " 86	-34 " -35	25 " 26	0" 60	+ " 09	- " 04	+ " 02	+ " 05
47 " 48	86 " 87	-33 " -34	26 " 27	0" 59	+ " 16	- " 07	+ " 04	+ " 01
48 " 49	87 " 88	-32 " -33	27 " 28	0" 56	+ " 13	- " 15	- " 02	" 00
49 " 50	88 " 89	-31 " -32	28 " 29	0" 47	+ " 03	- " 19	- " 09	- " 02
50 " 51	89 " 90	-30 " -31	29 " 30	0" 50	+ " 13	- " 08	+ " 01	+ " 09
51 " 52	90 " 91	-29 " -30	30 " 31	0" 43	+ " 13	- " 09	+ " 02	+ " 16
52 " 53	91 " 92	-28 " -29	31 " 32	0" 31	+ " 05	+ " 04	+ " 04	+ " 20
53 " 54	92 " 93	-27 " -28	32 " 33	0" 29	+ " 09	+ " 29	+ " 18	+ " 15
54 " 55	93 " 94	-26 " -27	33 " 34	0" 35	+ " 16	+ " 28	+ " 21	+ " 11
55 " 56	94 " 95	-25 " -26	34 " 35	0" 32	+ " 10	+ " 07	+ " 08	+ " 06
56 " 57	95 " 96	-24 " -25	35 " 36	0" 29	+ " 08	- " 01	+ " 03	+ " 05
57 " 58	96 " 97	-23 " -24	36 " 37	0" 29	+ " 09	+ " 02	+ " 05	+ " 04
58 " 59	97 " 98	-22 " -23	37 " 38	0" 28	+ " 02	+ " 06	+ " 03	+ " 01
59 " 60	98 " 99	-21 " -22	38 " 39	0" 22	-0" 09	+0" 01	-0" 05	-0" 06

9. From the above table it will be inferred that the corrections for division errors with the transit circle have been changed four times since its erection : from 1851 to 1856 the values were derived from the observations made in 1851 ; from 1857 to 1867 the values were derived from the observations made in 1856 ; from 1868 to 1879 the values were derived from taking the mean of the observations made in 1851 and 1856 ; from 1880 to 1896 the last values were still used, but, in order to allow for the distance between microscope A and the pointer being $89^{\circ}30'$ instead of 90° , as it had till then been assumed to be, what had been taken as the correction for 1° —that was, for all pointer readings between $0^{\circ}30'$ and $1^{\circ}30'$ —was considered to be the correction for all pointer readings between 0° and 1° , and so on.

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10. Re-determinations were also made of the errors of some of the divisions used in the observations of the close Polar stars and of the Nadir, and previous determinations were collected together. These were re-reduced with the new values of the terminal divisions given in Table II. The resulting determinations were smoothed by taking means and means again just as in the case of the whole degrees. The errors of the separate 5' divisions thus smoothed and the corrections to the values previously adopted are given in the following tables :—

TABLE III.
Table of Corrections for 5' Divisions.
For Nadir.

Pointer Reading.	179° 30'	35'	40'	45'	50'	55'	180° 0'	5'	10'	15'	20'	25'	30'	No. of Sets.
Division Correction }	0"22	0"30	0"35	0"38	0"40	0"45	0"45	0"41	0"38	0"32	0"29	0"26	0"25	10

For Collimators.

Pointer Reading.	89° 30' or 269° 30'	35'	40'	45'	50'	55'	90° 0' or 270° 0'	5'	10'	15'	20'	25'	30'	No. of Sets.
Division Correction }	0"29	0"28	0"38	0"40	0"41	0"43	0"46	0"50	0"43	0"34	0"26	0"19	0"24	10

For Polar Stars.

Pointer Reading.	317° 30'	35'	40'	45'	50'	55'	318° 0'	5'	10'	15'	20'	25'	30'	No. of Sets.
Division Correction }	0"88	0"98	1"03	1"04	1"07	1"12	1"14	1"11	1"03	0"94	0"91	0"92	0"95	9

Pointer Reading.	319° 30'	35'	40'	45'	50'	55'	320° 0'	5'	10'	15'	20'	25'	30'	No. of Sets.
Division Correction }	1"10	1"09	1"14	1"13	1"09	1"05	1"03	1"06	1"09	1"11	1"15	1"15	1"06	14

Pointer Reading.	321° 30'	35'	40'	45'	50'	55'	322° 0'	5'	10'	15'	20'	25'	30'	No. of Sets.
Division Correction }	0"99	1"06	1"02	0"99	1"01	1"02	1"02	1"03	1"02	0"99	0"94	0"92	0"91	18

Pointer Reading.	322° 30'	35'	40'	45'	50'	55'	323° 0'	5'	10'	15'	20'	25'	30'	No. of Sets.
Division Correction }	0"91	0"98	0"97	0"90	0"87	0"92	0"98	1"00	1"02	0"99	0"92	0"86	0"82	4

Pointer Reading.	323° 30'	35'	40'	45'	50'	55'	324° 0'	5'	10'	15'	20'	25'	30'	No. of Sets.
Division Correction }	0"82	0"74	0"77	0"82	0"82	0"81	0"83	0"83	0"77	0"73	0"73	0"76	0"73	9

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Corrections to Values Previously Adopted for 5' Divisions.

Nadir.		Polaris.		Cephei 51.		δ Ursæ Min.		λ Ursæ Min.	
Pointer Reading.	Corr.	Pointer Reading.	Corr.	Pointer Reading.	Corr.	Pointer Reading.	Corr.	Pointer Reading.	Corr.
179° 35'	-0''07	322° 30'	+0''22	323° 55'	-0''02	324° 30'	-0''01	322° 10'	+0''28
40	+0''17	25	+0''25	50	-0''02	25	-0''03	5	+0''33
		20	+0''28						
		Polaris S.P.		Cephei 51 S.P.		δ Ursæ Min. S.P.		λ Ursæ Min. S.P.	
		319° 50'	+0''05	318° 25'	+0''25	317° 45'	+0''36	320° 5'	-0''02
		45	0''00	20	+0''31	40	+0''28	0	+0''04
				15	+0''31				

11. As in § 7 it is easily seen that, if ϵ_0 be the probable error of the division correction of the terminal divisions, and ϵ the probable error of one measure of a 5' space, and n the number of sets of observations, then the probable errors of the division corrections for the 7 five-minute divisions from the ends to the middle of each degree are

$$\begin{aligned} \frac{\epsilon_0}{12} \sqrt{\left(122 \epsilon_0^2 + 132 \frac{\epsilon^2}{n}\right)} \\ \frac{\epsilon}{12} \sqrt{\left(104 \epsilon_0^2 + 240 \frac{\epsilon^2}{n}\right)} \\ \frac{\epsilon}{12} \sqrt{\left(90 \epsilon_0^2 + 324 \frac{\epsilon^2}{n}\right)} \\ \frac{\epsilon}{12} \sqrt{\left(80 \epsilon_0^2 + 384 \frac{\epsilon^2}{n}\right)} \\ \frac{\epsilon}{12} \sqrt{\left(74 \epsilon_0^2 + 420 \frac{\epsilon^2}{n}\right)} \\ \frac{\epsilon}{12} \sqrt{\left(72 \epsilon_0^2 + 432 \frac{\epsilon^2}{n}\right)} \end{aligned}$$

When the errors of the terminal divisions are neglected, the probable errors of the division corrections of the divisions next, next but one, &c., to the ends determined from one set of observations are

$$\sqrt{\frac{11}{12}}\epsilon, \sqrt{\frac{20}{12}}\epsilon, \sqrt{\frac{27}{12}}\epsilon, \sqrt{\frac{32}{12}}\epsilon, \sqrt{\frac{35}{12}}\epsilon, \sqrt{\frac{36}{12}}\epsilon.$$

Now the mean discordances of ten sets of observations for these divisions are found to be

$$\pm 0''.157, \pm 0''.249, \pm 0''.357, \pm 0''.368, \pm 0''.405, \pm 0''.412.$$

Whence it follows that

$$\epsilon = \pm 0''.190.$$

As an illustration of the accuracy of the determinations of the different divisions in a degree we may take $\epsilon_0 = \pm 0''.05$ and $n = 12$; we then get

$$\begin{array}{cccccccccccccccc} 0^\circ & 5' & 10' & 15' & 20' & 25' & 30' & 35' & 40' & 45' & 50' & 55' & 1^\circ \\ \pm''050 & \pm''070 & \pm''083 & \pm''091 & \pm''097 & \pm''100 & \pm''101 & \pm''100 & \pm''097 & \pm''091 & \pm''083 & \pm''070 & \pm''050. \end{array}$$

Comparison of these probable errors with the observed quantities given on p. 150 at once shows the excellence of the division of the circle. The outstanding error introduced by taking the same division correction for $30'$ on each side of any principal division would seem to be very small indeed. Further, the size of these probable errors justifies the smoothing of the observations.

PART II.

We now propose to examine the effect of these new corrections :—

- (1.) On the R—D discordance.
- (2.) On the Greenwich circumpolar observations.
- (3.) On the accordance of the Greenwich and Cape observations.
- (4.) On the Greenwich Sun observations.

12. The effect of the new division errors on the R—D discordance for 1897 is shown in the following table :—

TABLE IV.

Limits of N.P.D.	Corr. to Circle Reading.		Sum of Corr.	Limits of N.P.D.	Corr. to Circle Reading.		Sum of Corr.
	R.	D.			R.	D.	
-28° to -27°	$-''05$	$+''18$	$+''13$	$+59^\circ$ to $+63^\circ$	$-''15$	$+''13$	$-''02$
-23 „ -22	$-''06$	$+''03$	$-''03$	$+64$ „ $+66$	$+''03$	$-''01$	$-''02$
-21 „ -18	$-''02$	$-''06$	$-''08$	$+67$ „ $+70$	$+''03$	$+''03$	$+''06$
-17 „ -15	$+''07$	$-''07$	$''00$	$+71$ „ $+73$	$-''02$	$+''03$	$+''01$
-14 „ -10	$+''14$	$-''03$	$+''11$	$+74$ „ $+77$	$+''12$	$-''15$	$-''03$
-9 „ -6	$+''02$	$''00$	$+''02$	$+79$ „ $+83$	$+''09$	$-''14$	$-''05$
-5 „ -1	$-''18$	$+''09$	$-''09$	$+84$ „ $+87$	$''00$	$+''02$	$+''02$
$+1$ „ $+4$	$-''10$	$+''08$	$-''02$	$+89$ „ $+92$	$-''03$	$+''15$	$+''12$
$+5$ „ $+10$	$+''03$	$+''02$	$+''05$	$+93$ „ $+96$	$-''07$	$+''05$	$-''02$
$+11$ „ $+14$	$''00$	$-''01$	$-''01$	$+98$ „ $+104$	$+''02$	$-''06$	$-''04$
$+15$ „ $+18$	$+''13$	$-''16$	$-''03$				

Inspection of this table shows that the effect on the R—D discordance is negligible. It was not considered necessary to examine any other year.

13. The large discordance in the value of the colatitude derived from close circumpolars and the stars about 20° from the Pole, a discordance which has been pointed out by the Astronomer Royal (*Mem. R.A.S.*, vol. xlv.), and by Professor NEWCOMB (Astronomical Papers issued by the *American Nautical Almanac* Office, vol. ii.), is seen in the short table at the beginning of this paper (p. 142). The effect of the new division errors on these discrepancies is shown in the following table:—

TABLE V.

Table of Mean Excess of N.P.D. above Pole of Circumpolar Stars, from the Introduction of the Greenwich Ten-Year Catalogue, 1880, pp. 48–54, arranged in order of N.P.D. for each 3° , corrected for the new division errors, and referred to—I. Bessel's Refractions and Colatitude $38^\circ 31' 21'' 90$ (Column 4); II. Pulkova Refractions and Colatitude $38^\circ 31' 21'' 75$ (Column 7).

(1) Mean N.P.D. of Group.	(2) Excess of N.P.D. above Pole.	(3) Correction for new Division Errors.	(4) Excess of N.P.D. above Pole with new Division Errors.	(5) Weight.	(6) Correction for Pulkova Refrac- tion and Colat. $21'' 75$.	(7) Excess of N.P.D. above Pole with new Division Errors.
$2^\circ 3'$	$-0^{\circ} 47'$	$+0^{\circ} 25'$	$-0^{\circ} 22'$	20	$-0^{\circ} 06'$	$-0^{\circ} 28'$
$5^\circ 4'$	$-0^{\circ} 16'$	$+0^{\circ} 01'$	$-0^{\circ} 15'$	21	$-0^{\circ} 05'$	$-0^{\circ} 20'$
$8^\circ 5'$	$-0^{\circ} 02'$	$0^{\circ} 00'$	$-0^{\circ} 02'$	36	$-0^{\circ} 03'$	$-0^{\circ} 05'$
$10^\circ 9'$	$-0^{\circ} 18'$	$+0^{\circ} 06'$	$-0^{\circ} 12'$	29	$-0^{\circ} 03'$	$-0^{\circ} 15'$
$13^\circ 8'$	$+0^{\circ} 25'$	$-0^{\circ} 09'$	$+0^{\circ} 16'$	47	$-0^{\circ} 02'$	$+0^{\circ} 14'$
$17^\circ 0'$	$+0^{\circ} 17'$	$-0^{\circ} 10'$	$-0^{\circ} 07'$	40	$0^{\circ} 00'$	$-0^{\circ} 07'$
$19^\circ 7'$	$+0^{\circ} 19'$	$-0^{\circ} 14'$	$-0^{\circ} 05'$	54	$+0^{\circ} 02'$	$-0^{\circ} 03'$
$22^\circ 6'$	$+0^{\circ} 19'$	$-0^{\circ} 09'$	$+0^{\circ} 10'$	64	$+0^{\circ} 03'$	$+0^{\circ} 13'$
$25^\circ 3'$	$-0^{\circ} 05'$	$+0^{\circ} 10'$	$+0^{\circ} 05'$	68	$+0^{\circ} 05'$	$+0^{\circ} 10'$
$28^\circ 4'$	$-0^{\circ} 22'$	$+0^{\circ} 20'$	$-0^{\circ} 02'$	78	$+0^{\circ} 08'$	$+0^{\circ} 06'$
$31^\circ 5'$	$-0^{\circ} 66'$	$+0^{\circ} 20'$	$-0^{\circ} 46'$	37	$+0^{\circ} 12'$	$-0^{\circ} 34'$
$34^\circ 5'$	$-0^{\circ} 25'$	$+0^{\circ} 10'$	$-0^{\circ} 15'$	25	$+0^{\circ} 18'$	$+0^{\circ} 03'$
$38^\circ 4'$	$-0^{\circ} 44'$	$-0^{\circ} 15'$	$-0^{\circ} 59'$	21	$+0^{\circ} 26'$	$-0^{\circ} 33'$
$41^\circ 7'$	$-0^{\circ} 64'$	$-0^{\circ} 22'$	$-0^{\circ} 86'$	7	$+0^{\circ} 35'$	$-0^{\circ} 51'$
$44^\circ 4'$	$-1^{\circ} 31'$	$-0^{\circ} 18'$	$-1^{\circ} 49'$	7	$+0^{\circ} 58'$	$-0^{\circ} 91'$
$47^\circ 6'$	$-4^{\circ} 94'$	$0^{\circ} 00'$	$-4^{\circ} 94'$	1	$+2^{\circ} 29'$	$-2^{\circ} 65'$

It may be remarked that the correction to the adopted colatitude is half the excess of N.P.D. above Pole with the sign changed.

Though the discordances shown in column (2) are not entirely removed in column (4), yet the systematic differences in the observations near the Pole and 20° from it are materially diminished, and it does not seem unlikely that what remains may be due to still outstanding division error and purely accidental errors.

14. Comparison of columns (4) and (7) shows that to about 70° Z.D. the refractions of the *Tabulæ Regiomontanæ* satisfy the observations better than those of the Pulkova Tables; from 70° to 75° the refractions of the Pulkova Tables are slightly better; and below 75° neither of the tables satisfies the observations.

15. In the Introduction to the Cape Catalogue for 1885 Dr. GILL discusses the Greenwich and Cape circumpolar observations and the differences between the N.P.D.'s at Greenwich and the Cape on the following suppositions:—

I.	Bessel's	refractions	and	Cape	observations,	<i>not</i>	corrected	for	R—D.
II.	„	„	„	„	„	corrected	for	R—D.	
III.	Pulkova	„	„	„	„	<i>not</i>	corrected	for	R—D.
IV.	„	„	„	„	„	corrected	for	R—D.	

The effect of the new division errors at Greenwich is to alter considerably the residuals in Dr. GILL's solutions, given on p. xli, and to modify the conclusions drawn from them; but as an error of sign seems to have been made in the comparison of theory and observation on p. xxxviii, we are repeating Dr. GILL's discussion, with slight modifications, but using his figures and tables.

Dr. GILL draws attention to a paper by Professor CLEVELAND ABBÉ in *Ast. Nach.*, 2761, in which it is pointed out that in tables of refraction, the height of the barometer being taken as an index of the density of the air, the readings of the mercurial barometer are not a true index at different points on the Earth's surface till they have been corrected for differences of gravity. Refraction tables applicable at Pulkova therefore need diminution at Greenwich and the Cape in the ratios $1-00072$ and $1-00225$.

Now, taking X and x to be corrections to the adopted colatitudes at Greenwich and the Cape, and Y and y the factors by which the refractions

should be diminished at the two places, Dr. GILL forms equations from the observations to give X, x, Y and y on the four suppositions mentioned above.

The values of $y - Y$ from observation are thus found by Dr. GILL to be

I.	II.	III.	IV.
+ '00001	+ '00321	- '00047	+ '00279

Now the theoretical value of this quantity should be $(1 - '00072) - (1 - '00225)$ or $+ '00153$, and therefore the discordances on the four suppositions are

- '00152	+ '00168	- '00200	+ '00126
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and not

+ '00154	+ '00474	+ '00106	+ '00432
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as given in the Introduction to the Cape Catalogue, 1885.

Thus solutions II. and IV. no longer showing abnormal corrections to the Pulkova refractions, the evidence, such as it is, is now slightly in favour of applying a correction for $R - D$ to the Cape results, instead of being strongly in favour of the opposite course.

16. As the discordances of $y - Y$ from its theoretical value are of different signs in the solutions I. and III. from those in II. and IV., we have re-solved the equations on the assumption that $y - Y = + '00153$, reducing the four equations to three.

We then find on the four suppositions

I.	II.	III.	IV.
$x = + '256$	$+ '042$	$+ '245$	$+ '040$
$X = - '056$	$- '192$	$- '053$	$- '191$
$y = + '00301$	$+ '00488$	$+ '00049$	$+ '00231$
$Y = + '00148$	$+ '00335$	$- '00104$	$+ '00078$

with the following residuals :—

TABLE VI.

Representation of the Various Solutions for determining the Errors depending on Latitude and Refraction in the Greenwich Catalogue for 1880 (corrected for the New Division Errors) and the Cape Catalogue for 1885 (Introduction to Cape Catalogue, 1885, p. xli).

Equation.	I. O-C.	II. O-C.	III. O-C.	IV. O-C.	Weight.	Adopted Corrections for New Greenwich Division Errors.
<i>Greenwich Circumpolars.</i>						
(1)	+ "02	+ "12	"00	+ "10	156	- "10
(2)	"00	+ '09	- '01	+ '08	132	- '08
(3)	- '14	- '07	- '16	- '08	144	+ '12
(4)	+ '03	+ '02	"00	+ '06	132	+ '20
(5)	- '06	- '02	- '09	- '03	110	+ '25
(6)	- '23	- '19	- '25	- '21	121	"00
(7)	- '27	- '25	- '30	- '27	110	- '09
(8)	- '01	"00	- '02	- '01	100	- '15
(9)	- '11	- '12	- '12	- '12	100	- '20
(10)	+ '33	+ '30	+ '33	+ '30	81	- '20
(11)	+ '09	+ '01	+ '08	+ '02	72	- '12
(12)	+ '25	+ '11	+ '29	+ '16	49	- '05
(13)	+ '18	- '08	+ '31	+ '06	30	+ '20
Sum of residuals }	$\pm 1'72$	$\pm 1'45$	$\pm 1'96$	$\pm 1'50$	...	...
<i>Cape Circumpolars.</i>						
(14)	- "09	- "22	- "03	- "18	137	...
(15)	+ '28	+ '18	+ '34	+ '21	174	...
(16)	- '13	- '18	- '09	- '17	110	...
(17)	- '34	- '30	- '32	- '32	7	...
Sum of residuals }	$\pm 0'84$	$\pm 0'88$	$\pm 0'78$	$\pm 0'88$	...	...

TABLE VI.—*continued.*

Equation.	I. O—C.	II. O—C.	III. O—C.	IV. O—C.	Weight.	Adopted Corrections for New Greenwich Division Errors.
<i>Greenwich and Cape.</i>						
(18)	+ "40	+ "42	+ "47	+ "49	52	— "01
(19)	— '14	— '03	— '14	— '02	117	'00
(20)	— '23	— '07	— '25	— '08	77	+ '12
(21)	— '17	+ '02	— '17	+ '01	156	+ '10
(22)	— '02	+ '17	— '05	+ '15	149	'00
(23)	— '18	+ '01	— '20	— '02	231	+ '03
(24)	— '19	— '02	— '21	— '05	243	— '14
(25)	+ '02	+ '16	— '01	+ '13	376	— '10
(26)	— '22	— '10	— '23	— '13	324	— '03
(27)	+ '04	+ '09	+ '02	+ '08	286	+ '11
(28)	+ '05	+ '05	+ '02	+ '03	372	+ '02
(29)	+ '18	+ '12	+ '17	+ '11	213	— '06
(30)	+ '06	— '10	+ '04	— '12	357	— '03
(31)	+ '14	— '13	+ '16	— '12	210	'00
(32)	+ '15	— '33	+ '32	— '16	144	+ '10
Sum of residuals }	±2'19	±1'81	±2'46	±1'70	...	...

From this solution and the residuals we conclude—

- (1.) Comparison of I. with III. and II. with IV. shows that the observations do not discriminate between Bessel's and Gylden's laws of refraction when the constants are suitably modified.
- (2.) The residuals are slightly smaller in II. and IV. than in I. and III., but the corrections to the colatitude at Greenwich seem too large, and the differences between the residuals near the Pole ((1) and (2)), from those at about 20° from the Pole ((6) and (7)), are increased.
- (3.) The smallest residuals would be obtained by taking a mean of I. and II. or of III. and IV. This solution would also make the difference of the refractions at Greenwich and the Cape agree with the theoretical value when they are determined as independent quantities (as in Dr. GILL's solution). It would make the Greenwich

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colatitude $21''\cdot78$, and the Cape colatitude $3''\cdot50$, agreeing with the results of the Cape vertical circle. It would imply the use of the Pulkova refractions practically unaltered at Greenwich, or a diminution of those of the *Tabulæ Regiomontanæ* by $\frac{1}{400}$ th part, and at the Cape the Pulkova refractions diminished in the ratio $1-00140$ or those of the *Tabulæ Regiomontanæ* in the ratio $1-00400$. This agrees almost exactly with the results deduced from the comparison of the Cape 1880 and Greenwich 1880 Catalogues by Professor TURNER and Mr. HOLLIS (*Mem. R.A.S.*, vol. li.).

17. Returning to the effect of the division errors on the Cape-Greenwich discordances, Dr. GILL finds the residuals of his equations would correspond (if there were no systematic errors) to a probable error of an observation of weight unity of

I.	II.	III.	IV.
$\pm 1''\cdot53$	$\pm 1''\cdot49$	$\pm 1''\cdot60$	$\pm 1''\cdot46$

in the four different solutions, whereas the amount of this error should be about $\pm 0''\cdot50$. It is, however, very satisfactory to find that when corrections for the new division errors are applied to the residuals given on p. xli of the Introduction to the Cape Catalogue, the residuals are so far reduced that the probable errors now come to be

* $\pm 1''\cdot18$	$\pm 1''\cdot10$	$\pm 1''\cdot34$	$\pm 1''\cdot09$
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These quantities, especially in the cases of II. and IV., where the Cape results are corrected for R-D, are by no means excessive, and it may be fairly concluded that the excesses of these quantities over $\pm 0''\cdot50$ are attributable to errors of division still outstanding.

The method employed by Dr. GILL has been strictly followed here for comparison, but it should be noted that of the 32 equations given by Dr. GILL 4 equations which show large residuals contribute about half the amount of the sum of the squares, so that little can be inferred from these figures.

18. We now come to the Sun observations, 1877-1896, which have been divided into two periods, namely 1877-1886 and 1887-1896, in order to test the consistency of the results and to detect any sign of personality which might appear to justify their rejection. In Appendix III. to the Greenwich Observations, 1887, the observations of the Sun for the years 1877-1886 were revised in order to reduce the N.P.D.'s to a uniform system

* The quantities given in an abstract in the *Monthly Notices*, lix. 2, p. 56, are erroneous.

of refraction, colatitude and R-D. Advantage was then taken of the revision to reduce the refractions to the readings of the Front Court thermometer in lieu of those of the exterior thermometer, as this latter thermometer, placed near the north wall of the transit circle room, was found to differ systematically from the meteorological standard (see *Report of the Astronomer Royal to Board of Visitors* for 1887, June 4, p. 7). The corrections given in column XI. of the following table are derived from a comparison of simultaneous readings made at the time of the transit of the Sun between 1887 January 1 and 1888 September 30—all the data then available—and column XII. gives the corresponding corrections to the tabular error for reduction to the Front Court thermometer, which were applied to the N.P.D.'s for 1877-86.

The Right Ascensions and North Polar distances for the second period 1887-1896 are taken direct from the different Greenwich volumes. The following table gives the mean monthly errors in R.A. and N.P.D. for each period of ten years; the errors in N.P.D. are corrected for the new division errors, and reduced to the two systems of—

- I. Bessel's refractions and colatitude, 21''·90.
- II. Pulkova " " 21''·75.

TABLE VII.

Mean Tabular Errors of the Sun in R.A. and N.P.D. for each month of the Ten-Year Periods 1877-1886 and 1887-1896.

Month	Approx. Mean N.P.D.	Tabular Error R.A.		Correction for Division Error.	Tabular Error N.P.D. I.		Corrections for Pulkova refractions and Colatitude 21''·75.	Tabular Error N.P.D. II.		Correction to Exterior Thermometer.	Reduction for Front Court Thermometer to Tabular Error N.P.D.
		1877-86.	1887-96.		1877-86.	1887-96.		1877-86.	1887-96.		
Jan.	111°	+ 0°064	+ 0°092	+ ''01	+ ''07	- ''02	- ''61	+ 0°68	+ 0°59	+ 0°5	+ 0°19
Feb.	103	+ ''018	+ ''071	- ''07	- ''02	- ''01	·48	+ ·46	+ ·47	+ 1°6	+ ·40
Mar.	92	+ ''011	+ ''051	+ ''11	- ''55	- ''24	·37	- ·18	+ ·13	+ 1°5	+ ·24
Apr.	80	+ ''017	+ ''058	- ''13	+ ''21	+ ''01	·31	+ ·52	+ ·32	+ 2°2	+ ·23
May	71	+ ''017	+ ''055	+ ''03	+ ''37	+ ''17	·25	+ ·62	+ ·42	+ 3°5	+ ·28
June	67	+ ''061	+ ''082	·00	+ ''42	+ ''04	·24	+ ·66	+ ·28	+ 4°3	+ ·28
July	68	+ ''045	+ ''080	·00	+ ''66	+ ''31	·24	+ ·90	+ ·55	+ 2°8	+ ·20
Aug.	76	+ ''050	+ ''082	- ''12	+ ''60	+ ''51	·30	+ ·90	+ ·81	+ 3°5	+ ·32
Sept.	87	+ ''055	+ ''092	·00	+ ''20	+ ''41	·33	+ ·53	+ ·74	+ 2°7	+ ·37
Oct.	100	+ ''075	+ ''079	- ''05	+ ''43	+ ''07	·44	+ ·87	+ ·51	+ 2°7	+ ·57
Nov.	109	+ ''097	+ ''132	+ ''01	+ ''28	+ ''10	·56	+ ·84	+ ·66	+ 1°7	+ ·55
Dec.	113	+ ''078	+ ''098	0°00	- ''07	- ''27	- ·66	+ 0°59	+ 0°39	+ 1°1	+ ·47

19. Converting these errors of R.A. and N.P.D. into errors of Ecliptic North Polar distance, and assuming that these can be represented by the formula

$$x \times \cos \odot + y \times \sin \odot + z,$$

and giving equal weights to each month's results, we obtain by the method of least squares

	1877-86.		1887-96.	
	I.	II.	I.	II.
x	0"00,	0"00,	+0"27,	+0"25
y	+0'29,	+0'06,	+0'18,	0'00
z	+0'12,	+0'51,	+0'03,	+0'42

The following table contains the corresponding errors in E.N.P.D. and the apparent errors of the formula :—

TABLE VIII.

Errors of the Sun in E.N.P.D. for each month of the Periods 1877-86 and 1887-96 on the two systems of

I. Bessel's refractions and colatitude, 21'90
 II. Pulkova „ „ 21'75

Month.	Error in Ecliptic N.P.D.				Apparent Error of Formula.			
	I.		II.		I.		II.	
	1877-86.	1887-96.	1877-86.	1887-96.	1877-86.	1887-96.	1877-86.	1887-96.
January .	+0"22	+0"20	+0"82	+0"80	-0"37	-0"36	-0"21	-0"27
February .	+0'07	+0'35	+0'52	+0'80	-0'12	-0'04	-0'19	-0'17
March .	-0'44	+0'08	-0'10	+0'42	+0'53	+0'60	+0'20	+0'25
April .	+0'28	+0'32	+0'58	+0'61	-0'01	-0'04	+0'03	+0'03
May .	+0'42	+0'35	+0'66	+0'60	-0'06	-0'10	-0'01	-0'03
June .	+0'46	+0'09	+0'70	+0'33	-0'05	-0'13	+0'14	+0'11
July .	+0'55	+0'12	+0'78	+0'36	-0'15	-0'21	-0'03	-0'04
August .	+0'33	+0'10	+0'61	+0'38	-0'04	-0'07	-0'19	-0'17
September .	-0'14	-0'14	+0'16	+0'16	+0'29	+0'36	-0'08	+0'01
October .	-0'04	-0'38	+0'39	+0'04	+0'05	+0'10	+0'09	+0'15
November .	-0'08	-0'37	+0'47	+0'17	-0'03	-0'01	+0'10	+0'10
December .	-0'12	-0'34	+0'54	+0'32	-0'05	-0'09	+0'14	+0'06

Comparing the correction to the colatitude given by the observations of the Sun with the results deduced from the stars for the period 1877-86, we find

		Bessel.	Pulkova.
☉	1877-86	22''02	22''26
Stars	„	21'87	21'75

from which it is clear that the effect of the Pulkova refractions is to make the Sun and Stars diverge considerably. From the corrections given in Table VII., above, we can easily find that the effect of using the Front Court thermometer instead of the Exterior thermometer in the Sun observations was

$$x = -0''09. \quad y = -0''10. \quad z = +0''33$$

and that therefore the above comparison between the Sun and Stars for the two different thermometers would be :—

		BESSEL.		PULKOVA.	
		Exterior Thermometer.	Front Court Thermometer.	Exterior Thermometer.	Front Court Thermometer.
☉	1877-86.	21'79	22'02	21'93	22'26
Stars.	„	21'87	21'87	21'75	21'75

It will be remarked that with Bessel's refractions the colatitude from the Stars agrees with the mean of the Sun observations from the two thermometers, but with the Pulkova refractions both Sun results are outside the value derived from the Stars, which would imply a thermometer reading even lower than the Exterior by an amount as large again as the adopted differences between the Exterior and Front Court thermometers. In other words, it would appear that no admissible thermometer readings would satisfy the discordances. At the same time, it must be noticed that the apparent error of formula indicates that the Pulkova refractions considerably reduce the differences of the monthly results; but perhaps the most remarkable feature in the table is the accordance between the results deduced from the two periods, an accordance which seems to preclude the idea of any large outstanding personalities, such as have been held to justify the rejection of the results of the Sun observations. It would, however, be very interesting to compare these results with Sun observations made at the Cape. Finally, to say that the Greenwich Sun observations support the refractions at present in use, and that comparison of the Greenwich and Cape N.P.D.'s supports a diminution in the constant of refraction equivalent to the adoption of the Pulkova Tables, as they stand, is only to repeat what has been frequently pointed out before.

