Laying aside the question of the accuracy of the value of $\int n\delta e'^2$, the above theoretical values for the secular acceleration are correct to $\frac{1}{300}$ part of the whole in each case.

Haverford College, Pa.:
1897 *January* 11.

On a Photographic Transit Circle. By H. H. Turner, M.A., B.Sc., Savilian Professor.

1. The desirability of introducing photography into meridian observations, as a method of eliminating personal and other errors, has been recognised by many astronomers, and will probably be admitted by all, if this introduction can be accomplished without endangering the simplicity and stability of the present form of instrument. The following solution of this problem, which presents some advantages not realised in other forms, is offered for consideration.

2. It may be well in the first instance to enumerate the chief features of the present instrument—its advantages and disadvantages.

Features:

(*a*) Movable visual telescope with two micrometers at right angles.

(*b*) Two carefully made pivots, allowing one degree of freedom only.

(*c*) Graduated circle and microscopes.

(*d*) Chronograph.

All these, or their equivalents, may thus be allowed in any new arrangement.

Advantages:

(*e*) Stability, owing to there being only one degree of freedom, excepting in so far as the micrometers are used.

(*f*) Simplicity.

The advantages may generally be brought under one or other of these heads. It is unnecessary to specify them in detail unless we are going to propose any rearrangement of the instrument which will sacrifice any of them, and that is not my present purpose. It is more important to consider carefully how we may remove the

Disadvantages of the Present Instrument, viz.:

(*g*) Personal equation of all kinds.

(*h*) Flexure and irregular refraction.

(*i*) Limitation to comparatively bright stars.

3. Considering first (g), personal equation, photography is clearly useful here. In measuring a photograph different persons usually have a small 'error of bisection,' but this can be eliminated at leisure by turning the photograph round and remeasuring it, and a successful photographic method would thus be of the greatest advantage, not only in longitude work, where the absolute time is required, but in forming star catalogues, which are at present affected with the personality dependent on brightness of the star.

4. With regard to (h) I have already proposed (*Monthly Notices*, vol. liv. p. 412) an arrangement of a fixed telescope pointing to a plane mirror. The mirror can be made thick enough to be free from flexure without being nearly so heavy as a large telescope, and if carefully mounted in the meridian, in the same way as a transit is mounted, nothing would be lost of stability and very little of simplicity. The graduated circle (or equivalent) would now be attached to the mirror. It was further pointed out in the paper above cited that the effects of irregular refraction from the atmosphere of the observing room (small and much over-rated, as I believe these to be, see *Mem. R.A.S.* vol. li. pp. 110–116 and *Monthly Notices*, vol. liv. p. 488) are minimised by this arrangement, for the mirror may be placed practically in the open air at a distance from the observer, so that the light of a star travels as far as the mirror without much disturbance from irregular strata of air. On their way from the mirror to the observer the rays of light may suffer such disturbance, but this will be the same for all stars, whatever the zenith distance, and hence will go out in the zenith point.

5. With regard to (i), it is true that observations of stars and small planets to about the 10th magnitude are found possible at (*e.g.*) the Royal Observatory, Greenwich, with the transit circle of 8-inch aperture. But even then, for the fainter objects, illuminated wires must be used instead of the illuminated field, and the observations are found to lose in accuracy. With smaller instruments the range of available stars is more restricted, and it will not be possible for us at Oxford (with the Barclay transit circle of 4 in. aperture, and with no apparatus for illuminating the wires) to reobserve the great majority of the faint stars of the *Cambridge Catalogue* (1875·0), which we are using as reference stars for the work of the *Astrographic Catalogue*.

But one of the chief uses of photography is to bring faint objects within easier reach, and if we can use a fairly long exposure (of some minutes, say) there is no reason why an instrument of moderate power should not photograph stars to the 9th or 10th magnitude. We see at once, however, that to use photographic trails will not do; we are just as much restricted as before to bright stars. Something must move to follow the star, and that cannot be the telescope which we have already fixed, nor the mirror which is to have only one degree of freedom. Hence it must be the photographic plate. As the star travels across the field of the fixed telescope, which we may assume about 2° in breadth, the plate must be moved so that the star falls continuously on one spot, which thus receives an exposure of 8 minutes at the equator. This should be sufficient to photograph tolerably faint stars with a good photographic lens of modest dimensions. In any case, those observatories which possess instruments for the work of the Astrographic Chart would be able at some time to remove the object glass in its cell and mount it in a wooden tube, which would be all that is necessary in the case of a fixed telescope; and then an exposure of 8 minutes at the focus would give 11th or 12th magnitude stars.

6. Now it looks at first sight as though moving the plate were just as bad as moving the telescope or the mirror, were, in fact, giving up the idea of having only one degree of freedom. But by a simple device we can entirely obviate this objection. The plate need not be fixed to the telescope; all that is necessary is that *the position of the plate with reference to the fixed telescope at any instant should be recorded.* Suppose a spot of light from a fixed source to shine on the plate every second, the interruptions being made by the standard clock. As the plate travels across the field a series of dots will be formed which will show the irregularities of the motion. If there be two spots (one at the top and one at the bottom, say) we shall know the orientation of the plate as well as its position in the slide at any instant, and if the motion be tolerably uniform, such as can be secured by any one of a number of mechanical devices, the position of any star photographed can be referred to the mean of these seconds' spots on the plate. The process will be rather like referring a star to the several wires of the transit reticule, in fact the moving plate is now our chronograph, so that we are not really adding anything to the "features" specified in § 2. In the case of bright stars, the exposure would only be a few seconds, and the measures of the spots could be quickly made. In the case of longer exposures, of 8 minutes, say, it would probably not be necessary to measure the whole $8 \times 60 = 480$ spots; if every 10th or even 30th were measured, enough information would probably be obtained as to the regularity of the clock driving to show whether it was necessary to proceed further.

7. Before entering further into the details of the arrangement let us now sum up the features of the new instrument, and compare them with the ordinary transit circle. The following is a brief tabular comparison of this kind:—

Ordinary Transit Circle.	*Proposed Photographic Transit Circle.*
(*a*) Movable visual telescope with two micrometers at right angles.	(*a*) Fixed photographic telescope. The micrometers are not attached to the telescope, but are used to measure the photographic plate.

(*b*) Two carefully made pivots, allowing one degree of freedom only.

(*b*) The pivots attached to a plane mirror, which has thus one degree of freedom only.

(*c*) Graduated circle and microscopes.

(*c*) Retained, in connection with the mirror.

(*d*) Chronograph.

(*d*) A corresponding arrangement moves and records position of the photographic plate.

(*e*) Stability.

(*e*) Retained as in (*b*).

(*f*) Simplicity.

(*f*) The only additional apparatus is the plane mirror.

(*g*) Personal equation affects the observations.

(*g*) Personal equation minimised or eliminated.

(*h*) Flexure and irregular refraction.

(*h*) The flexure of a comparatively small mirror substituted for that of a long tube held by the middle. Effects of irregular refraction minimised, the mirror being in the open.

(*i*) Limitation to bright stars.

(*i*) Stars of 10th magnitude or fainter within reach.

8. This theoretical comparison seems to promise well, and I see no reason why it should break down in practice. There are, of course, numerous small points of detail to be worked out, but none of them present any great difficulties. Most of them, though perhaps not all, are considered in the following paragraphs. I am proceeding with the mounting of an instrument at the University Observatory, and a fuller account will be given after practical experience of the working.

9. *Movement of the Plate.*—So long as the driving is regular it is not necessary that it be *exactly* of the stellar rate unless we want a very faint star. If the stellar rate and the plate rate differ the star will trail slightly, but we shall make no sensible error in referring the middle of the trail to the mean of the seconds' marks.

10. Now stars of different declinations travel at different rates; hence we must have some contrivance for adjusting the rate at which the plate travels. Various devices might be used. There is an ingenious one already in use at Greenwich for the personal equation machine (*Monthly Notices*, vol. xlviii. p. 3); but perhaps the simplest would be to have a long graduated arm, pivoted at one end, the other moved by the clock; a string attached to the plate holder and passing over a fixed pulley (so that the pull on the plate is always in the same direction) could be attached to this arm at any distance from the pivot, and would thus pull the plate along at a suitable rate. But the details are still to be worked out.

11. *The Variable Chronographic Scale.*—For stars near the Pole the seconds' marks would be close together, while for

equatorial stars they would be better separated. The chronograph scale is thus wider for the latter than the former, and given errors of measurement affect the time of transit less in the ratio of the secant of declination. But it is easily seen that the error of transit in arc of a great circle is the same for all stars. There is, however, another inconvenience in a close scale, viz. that the spots would interfere with one another. This must be obviated by cutting out some of them. For instance, the observer might by hand hold down the electro-magnet or relay, &c. which admits the light periodically, only allowing it to rise every fifth or tenth second, as the case may be.

12. *Rotation of Plate.*—Another point to be considered is the rotation of the field of view, and the differential motion for stars in the same field, for one great advantage of this form of instrument is the possibility of photographing (and reducing together) a number of stars on the same plate. There are various mechanical contrivances by which a proper amount of rotation as well as of translation could be given to the plate; for the present I describe the simplest merely. Suppose for a moment the instrument is to be devoted to the reobservation of the (faint) stars in zone $+25°$. The plates taken with it will be of two kinds.

(A) Those for adjustments, for which bright stars may be used, though of all declinations.

(B) Those in zone $+25°$, which are faint.

Now construct a wooden arc of such a radius that the plate-holder will move along it so as just to suit zone $+25°$, and determine very accurately the point of the driving-arm which suits this exact declination, so that we get nice round discs for the stars in this zone readily. For other stars let the plate move along the same arc, but alter the rate to suit the star, though perhaps a rough approximation will suffice. The stars will trail slightly, but the exposures can be short, and no sensible error will be introduced. Indeed, we need not perhaps alter the rate of motion for some of the bright stars, but simply allow them to trail. When zone $+25°$ has been well observed we can make another arc for zone $+26°$ or any other zone. But if it be required to take faint stars in different zones on the same night we must make some linkage or epicycloidal arrangement for moving the plate.

13. *The Fixed Spots of Light.*—With regard to the spot or spots of light which mark the plate, there are advantages in the following arrangement: Let there be a point-source of light in the plane of the photographic film, but just beyond the field, above or below; which sends rays through the objective of the telescope. When the mirror is nearly normal to the telescope axis it will reflect these rays back through the objective to a point on the film (supposed at rest), which will become a line if the mirror is rotated gently. This line (l_1) obviously furnishes an important adjustment; if it is perfectly straight, the plane of the mirror is parallel to the horizontal axis, and if the point-source lies in its prolongation, the telescope is perpendicular to

the mirror. Further, if a mercury trough be placed below the mirror, and the mirror turned into a suitable position, another point or line (l_2) will be obtained showing the level adjustment of the horizontal axis. Now let there be a small, *fixed* mirror nearly normal to the telescope axis, which constantly reflects *a portion* of the light from the point-source on to the plate, at a point (l_3). The reflection from this mirror is to be the chronograph index, and is to be suitably interrupted by the clock. The distances $l_3 - l_1$ and $l_3 - l_2$, which can be recorded and measured whenever necessary, will furnish the necessary constants for reduction of the observations for collimation, level, and index error. The small fixed mirror will necessarily come between the objective and the large mirror, and will thus block out a portion of the objective; it should thus be as small as possible. In some cases a small circular patch of tin-foil, placed on the outer surface of the object glass, might serve the purpose. As already remarked, it will be better to have *two* point-sources of light, one above and one below, to check the orientation of the plate.

14. *Collimation of the Instrument.*—The method of determining this has been partly indicated in the last paragraph. There are three points to be considered :—

(α) The mirror must be placed parallel to its axis of rotation.

(β) We must determine where a plane through the centre of the objective perpendicular to the mirror axis cuts the plate.

(γ) We must determine the positions of the point-source and its reflected images with reference to the plane specified in (β).

As regards (α), we have always the expedient of reversing the mirror in its Ys, and comparing the images either of the spot of light or of a star.

As regards (β) and (γ), enough has been said in the last paragraph.

It may be remarked that the *correction* for collimation error with this instrument need not be made in the familiar way, but can be applied as an index error to the measures of position on the plate. Suppose the meridian on the celestial sphere were a continuous bright line, and that the axis of the mirror were truly horizontal E. and W. (*i.e.*, no level and azimuth errors), but the mirror inclined to it at any angle. Then, whatever be the position of the mirror, the image of the meridian on the photographic plate would be the same curved line, part of a small circle parallel to the meridian and at an angular distance from it equal to twice the tilt of the mirror. A star would be on the meridian when its image fell on this fixed line, the position of which can be determined with reference to the seconds' marks on the plate.

15. *Level Error.*—A striding level may be applied to the pivots of the mirror in the usual way, or the method of reflection from mercury indicated in § 13 may be used. Or stars may be observed by reflection from mercury.

16. *Azimuth Error and Clock Error* are found from stars in the usual way.

A Photographic Zenith Tube.

17. The general principle of a moving photographic plate, which is rendered equivalent to a fixed plate by the marking of a fixed index, would appear to be advantageously applicable to the zenith tube, especially the floating zenith tube. The actual tube is not of much consequence, since we are not using it to bind object-glass and plate, and may be dispensed with altogether, or detached from the objective and used as a shield from stray-light merely. The important point is to have the index which marks the plate rigidly attached to the floating objective. For the present it is not necessary to specify the way in which the marking of the plate shall be accomplished; but let us suppose some lamp, or lamp and prism, or other source of light, firmly attached to the floating objective, and so arranged that it makes a spot of light on a plate placed in the focus of the objective, and first suppose this plate fixed. Further assume that there are suitable arrangements for rotating the objective and its index appliance in any azimuth.

18. Now suppose a star at rest in the zenith. On exposing the plate it will make a spot on the film; and a second spot will be made by the index—let us say at distance d and in azimuth ϕ. Rotate the objective through 180°. If the centre of the objective remains accurately in the same place the star will focus on exactly the same spot as before; and the index-mark will be at the same distance, d, from it in azimuth $\phi + 180°$. If, however, the objective is shifted laterally in the process of rotating it (which would, indeed, generally be the case in such an arrangement), the star image would no longer fall on the same spot as before, but would deviate from it by the amount of the lateral shift. But the index-spot would be displaced in exactly the same manner, and hence it would still be in the same position relatively to the star image—viz., at distance d and in azimuth $180° + \phi$. (In fact a lateral shift of the objective and index together is equivalent to a movement of the photographic plate, which we are going to admit freely.) Thus the mean positions of the index and the zenith coincide; and, similarly, if the star is not actually in the zenith, but at some distance from it, we obtain its zenith distance from two photographs in reversed positions.

19. In symbols let x and y be the rectangular coordinates of the star referred to the index dot, measured in any fixed directions; and let their values for the first exposure be x_1, y_1, and for the second x_2, y_2. Then

$$x_1 = +d \cos \phi \qquad y_1 = +d \sin \phi$$
$$x_2 = -d \cos \phi \qquad y_2 = -d \sin \phi$$

Here we assume that the plate is rotated through exactly

180°. There will probably be a slight error in doing this—say $\Delta\phi$. Then for the second pair of equations we must write

$$x_2 = -d\,(\cos\phi - \sin\phi\Delta\phi) \qquad y_2 = -d\,(\sin\phi + \cos\phi\,.\,\Delta\phi).$$

Thus

$$(x_1 + x_2) = d\sin\phi\,.\,\Delta\phi \qquad y_1 + y_2 = -d\cos\phi\,.\,\Delta\phi.$$

We can thus reduce the effect of error of orientation $\Delta\phi$ on the x coordinate to an insensible quantity by making $d\sin\phi$ small, and on the y coordinate by making $d\cos\phi$ small. But $d\cos\phi$ and $d\sin\phi$ are the coordinates of the index spot with respect to the zenith. Hence we must adjust the spot to be as nearly as possible at the zenith. Or two spots at equal zenith distances would do equally well, and would check the orientation of the plate.

20. Now, as in the case of the transit-circle, let the plate move so as to follow the image of the star, and get the benefit of a prolonged exposure, so that we may photograph faint stars in the zenith; and let the index-spot be recorded every second only. We can determine their R.A.'s and zenith distances by referring the stars to the index-spots. Whenever a sufficiently bright star is passing near the zenith, we may reverse the instrument during the transit, obtaining two images of the same star for determination of index error; or we may reverse between the transits of two stars whose relative places are known; or again, we may reverse in opposite senses on two different nights between the transits of any two stars known or unknown.

21. Almost the only adjustment required is the centring of the index-spot. The only constant required is the scale value in arc on the plate, which is shewn by the scale of seconds marks when the star does not trail. In addition the plate should be horizontal; the bed on which it moves can be probably levelled with sufficient accuracy by mechanical means, or the plate might be floated on mercury.

22. If a series of plates, or perhaps a long film, be arranged on a slide, or on a rotating horizontal tray, so that each in turn is presented for exposure, the whole zone of stars which passes the zenith might be photographed every clear night with comparatively little labour. An occasional reversal of the object-glass would give the constant of reduction.

23. *Other Applications.*—The principle of a moving plate, marked by a fixed index-spot, is applicable generally to the case of any instrument for determining positions. As another example we may take the fixed camera proposed by Capt. E. H. Hills, R.E. (*Monthly Notices*, vol. lv. p. 89), for determining longitudes by photography; an arrangement which may also be used for determining the place of the Moon. So long as we fix the plate to the camera, we are limited to bright stars; but we can extend the observations to faint stars by moving the plate as above.

University Observatory, Oxford:
1897 *March* 5.

The Sidereal System Revised in 1896. By Maxwell Hall.

The original article (*Memoirs R.A.S.* vol. xliii. p. 157) was revised in 1887 (*Monthly Notices*, vol. xlvii. p. 521); the present revision is undertaken to meet the determination by photography at the Potsdam Physical Observatory of the radial velocities of fifty-one stars, the determination by photography at the Oxford University Observatory of the annual parallaxes of about twenty-six stars of the second magnitude conveniently situated in that latitude, and to meet the recognition of the greater accuracy of the stellar proper motions as deduced by Dr. Auwers.

In the present revision the latter proper motions are always used when possible.

Note I. contains a list of those stars whose proper motions have been taken from other authorities.

Note II. contains a list of observed parallaxes; the means have been adopted.

Note III. contains the radial velocities observed by Dr. Huggins, at Greenwich, and at Potsdam. It is disappointing to find so many contradictory results; in the case of many stars the adopted means seem very uncertain.

It will be noticed that the general effect of the Potsdam observations is to greatly reduce all radial velocities; and it will be found that the Oxford parallaxes are much larger than the computed parallaxes; consequently it appears that there are large errors among the constants of the system hitherto adopted; and these large errors must first be ascertained before we can possibly proceed.

From the revision of 1887 the value of ΠV was found to be $0''\cdot10$; and from the manner in which ΠV and V enter the computations it will be seen that if ΠV be still considered $0''\cdot10$, but that if V be reduced to 5 and Π increased to $0''\cdot02$, then all the radial velocities will be diminished about one-half, and all the parallaxes will be nearly doubled.

Let us consider the effect of these changes on the very few stars whose parallaxes and radial velocities have both been ascertained with considerable accuracy :—