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A SPECTROSCOPIC PROOF OF THE METEORIC CONSTITUTION OF SATURN'S RINGS.

By JAMES E. KEELER.

THE hypothesis that the rings of Saturn are composed of an immense multitude of comparatively small bodies, revolving around Saturn in circular orbits, has been firmly established since the publication of Maxwell's classical paper in 1859. The grounds on which the hypothesis is based are too well known to require special mention. All the observed phenomena of the rings are naturally and completely explained by it, and mathematical investigation shows that a solid or fluid ring could not exist under the circumstances in which the actual ring is placed.

I have recently obtained a spectroscopic proof of the meteoric constitution of the ring, which is of interest because it is the first *direct* proof of the correctness of the accepted hypothesis, and because it illustrates in a very beautiful manner (as I think) the fruitfulness of Doppler's principle, and the value of the spectroscope as an instrument for the measurement of celestial motions.

Since the relative velocities of different parts of the ring would be essentially different under the two hypotheses of rigid structure and meteoric constitution, it is possible to distinguish between these hypotheses by measuring the motion of different parts of the ring in the line of sight. The only difficulty is to find a method so delicate that the very small differences of velocity in question may not be masked by instrumental errors. Success in visual observations of the spectrum is hardly to be expected.

Soon after the large spectroscope of the Allegheny Observatory was completed, in 1893, I attempted to determine the relative motions of different parts of the system of Saturn, by photographing the spectrum with the slit parallel to the major axis of the ring, but failed to obtain satisfactory results. The unfavorable atmospheric conditions at Allegheny, the strong yellow color of the objective of the thirteen-inch equatorial, and the yellow color of Saturn itself so reduced the intensity of the violet part of the spectrum that the negatives obtained with a sufficiently high dispersion were too weak and granular to admit of measurement. Another unfavorable circumstance was the fact that I had to guide the practically invisible image corresponding to the $H\gamma$ line by means of the visual image, which was greatly out of focus on account of the chromatic aberration of the visually corrected telescope. Having recently obtained excellent results in other directions with orthochromatic plates, by the use of which the difficulties mentioned above are to a great extent obviated, I was induced to repeat my earlier attempts, and obtained two fine photographs of the lower spectrum of Saturn on April 9 and 10 of the present year. The exposure in each case was two hours, and the image of the planet was kept very accurately central on the slit-plate. After the exposure the spectrum of the Moon was photographed on each side of the spectrum of Saturn, and nearly in contact with it. Each part of the lunar spectrum has a width of about one millimeter, which is also nearly the extreme width of the planetary spectrum. On both sides of the spectrum of the ball of the planet are the narrow spectra of the ansæ of the ring.

The length of the spectrum from *b* to D is 23 millimeters. The focus was adjusted on the line λ5352, a little above the position of maximum sensitiveness of an orthochromatic plate, in the yellow green. On both plates the densities of the different spectra are very nearly equal, and the definition is excellent. It is hardly necessary to say that all the lenses used in the apparatus are visually corrected objectives.

These photographs not only show very clearly the relative displacement of the lines in the spectrum of the ring, due to the opposite motions of the ansæ, but exhibit another peculiarity, which is of special importance in connection with the subject of the present paper. The planetary lines are strongly inclined, in consequence of the rotation of the ball, but the lines in the spectra of the ansæ do not follow the direction of the lines in the central spectrum; they are nearly parallel to the lines of the comparison spectrum, and, in fact, as compared with the lines of the ball, have a slight tendency to incline in the opposite direction. Hence the outer ends of these lines are less displaced than the inner ends. Now it is evident that if the ring rotated as a whole the velocity of the outer edge would exceed that of the inner edge, and the lines of the ansæ would be inclined in the same direction as those of the ball of the planet. If, on the other hand, the ring is an aggregation of satellites revolving around Saturn, the velocity would be greatest at the inner edge, and the inclination of lines in the spectra of the ansæ would be reversed. The photographs are therefore a direct proof of the approximate correctness of the latter supposition.

To apply more precise reasoning to the subject under consideration, let us determine the form of a line in the spectrum of Saturn when the slit is in the major axis of the ring, on the assumption that the planet rotates as a solid body and the ring is a swarm of particles revolving in circular orbits according to Kepler's third law. At present the motion of the system as a whole is neglected. The upper part of Fig. 1 represents the image of Saturn on the slit of the spectroscope (the scale above it applies to the instrument used at Allegheny), and the narrow horizontal line in the lower part of the figure represents an undisplaced line in the spectrum, or solar line.[1] Let this line be taken as the axis of x, and the perpendicular line through its center as the axis of y. The red end of the spectrum is supposed to be in the direction of the positive axis of y, and the camera and collimator of the spectroscope are assumed to have the same focal length, so that the breadth of the spectrum is equal to the length of the illuminated part of the slit. Corresponding points in the slit and spectral line will then have the same value of x.

Now let x, y, be the coördinates of a point on the displaced line,

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[1] The curvature of the line in a prismatic spectrum need not be considered.

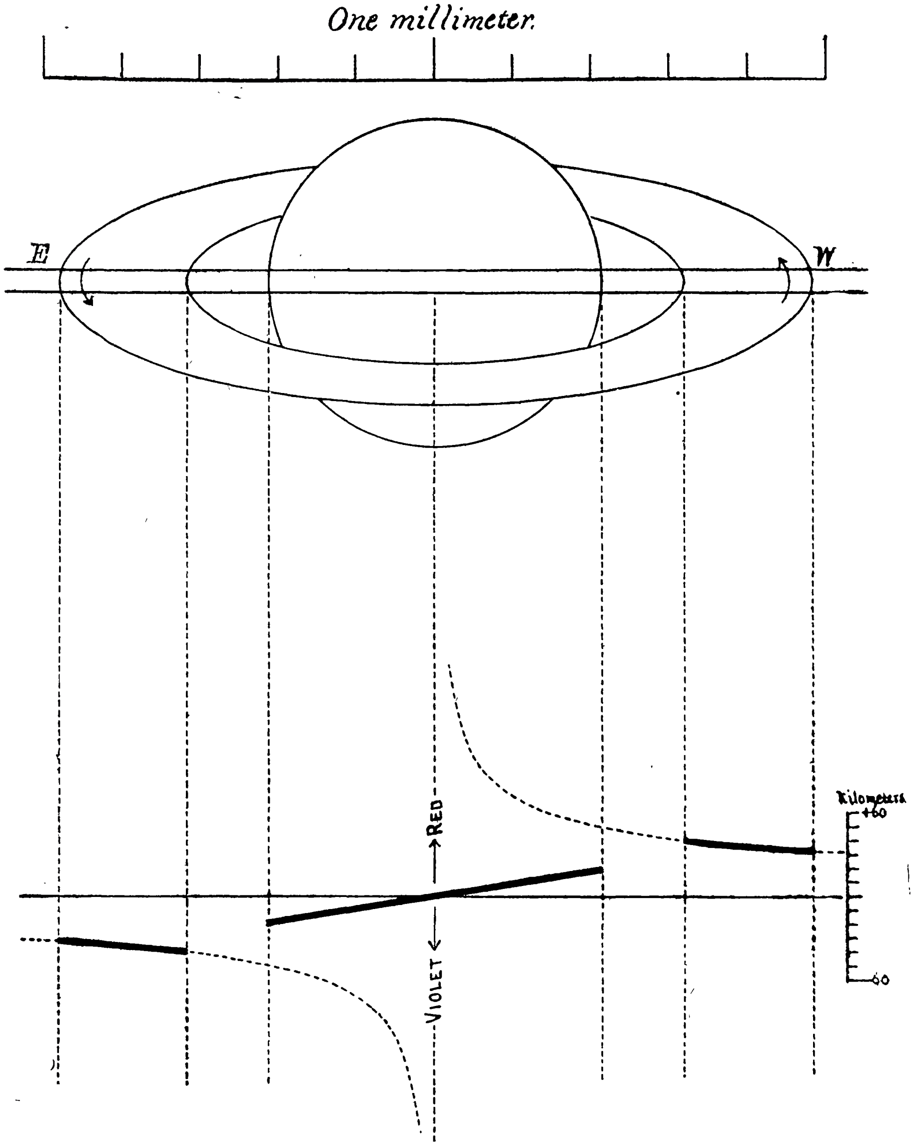

FIG. 1

v=velocity of point corresponding to x, y in the line of sight,

V'=velocity of a point on the equator of Saturn,

α=angle between the line of sight and the radius of Saturn which passes through the point corresponding to x, y,

2ρ=width of spectrum,

β=elevation of Earth (and Sun) above the plane of the ring.[1]

The displacement y is proportional to the velocity in the line of sight. Then we have

$$x = \rho \sin \alpha,$$
$$y = av = aV' \sin \alpha \cos \beta,$$
$$\frac{y}{x} = \frac{aV'}{\rho} \cos \beta = \text{constant}.$$

Hence the planetary line is straight, but inclined to the solar line at an angle

$$\phi = \tan^{-1} \frac{aV'}{\rho} \cos \beta.$$

To determine the form of a line in the spectrum of the ring, regarded as a collection of satellites, we have, by Kepler's third law,

$$T^2 = cR^3,$$

or, since $TV = 2\pi R$,

$$V^2 = \frac{4\pi^2}{cR}.$$

Since x is proportional to R and y to v (where v=velocity in the line of sight=$V \cos \beta$), we may write

$$xy^2 = b,$$

which is the equation to the curve of which the lines in the spectrum of the ring are a part. The curve is represented by the dotted line in the figure; it is symmetrical with respect to the axis of x, but only the upper branch has a physical meaning, and the curve corresponding to the other half of the image is obtained by taking both x and y with negative values.

In the equation $V = \frac{k}{\sqrt{R}}$, $\log k = 3.7992$ for the Saturnian system, R being expressed in kilometers and V in kilometers per

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[1] The slight error introduced by the assumption that the Earth and Sun are in the same direction from Saturn is inappreciable, when Saturn is anywhere near opposition.

second. The computed motions of different parts of the system are given in the following table. The gauze ring is not considered, as its spectrum does not appear on the photographs; the rings *A* and *B* are not separately distinguishable.

Object	*R*	Period of a Satellite at Distance *R*	Velocity	Velocity in Line of Sight April 10, 1895
	Kilometers	Hours	Kilometers	Kilometers
Outer edge of ring - -	135,100	13.77	17.14	16.35
Middle of ring - -	112,500	10.46	18.78	17.91
Inner edge of ring - -	89,870	7.47	21.01	20.04
Limb of planet - -	60,340	4.11	25.64	24.46
		Rotation		
Limb of planet - - -	60,340	$10^{h}.23$(A. Hall)	10.29	9.82

With the values given in the above table, and others which do not correspond to actual points in the system, the dotted curves were platted. For the ordinates, however, twice the values in the last column were taken, since the displacement of a line, due to motion in the line of sight, is doubled in a case of a body which shines by reflected and not by inherent light, provided (as in this case) the Sun and the Earth are in sensibly the same direction from the body. The planetary line is drawn to the same scale, and the heavy lines in the figure represent accurately the aspect of a line in the spectrum of Saturn, with the slit in the axis of the ring, as photographed with a spectroscope having about three times the dispersion of my own instrument.

The width of slit which I used ($0^{mm}.028$, or 7900^{km} on the surface of Saturn) is also represented in the figure.

If the whole system has a motion in the line of sight, the lines in the figure will be displaced toward the top or the bottom, as the case may be, but their relative positions will not be altered.

It is evident that in making a photograph of this kind the image must be kept very accurately in the same position on the slit plate, as otherwise the form of the lines shown in the figure would be lost by the superposition of points having different velocities. The second plate was made with special care, and as the air was steadier than on the first occasion, the definition is on the whole somewhat better than that of plate 1, although the difference is not great. On both plates the aspect of the spectrum is closely in accordance with that indicated by theory and represented in the figure. The planetary lines are inclined from 3° to 4°, and the lines in the spectra of the ansæ have the appearance already described. The slight curvature of the latter indicated by theory is, of course, unrecognizable. On account of the extreme narrowness of the spectra (barely more than a tenth of a millimeter) it was useless to attempt anything like a measurement of the inclination of the lines. The direction of such short lines is frequently masked by irregularities in the grain of the plate, and occasionally a line is considerably distorted. However, in fifty of the sharpest lines, in the region of best definition, only five were inclined in the same direction as the lines of the ball, while the rest were inclined as required by the theory, or elsewhere apparently parallel to the undisplaced lines of the lunar spectrum.

If the ring revolved as a whole, the displacement of lines in its spectrum would follow the same law as for a rotating sphere; that is, the lines would be straight and inclined, their direction passing through the origin. If the ring rotated in the period of its mean radius, a glance at the figure shows that the lines would practically be continuations of the planetary lines. Such an aspect of the lines as this would be recognizable on my photographs at a glance.

It will be seen from the foregoing considerations that the photographs prove not only that the velocity of the inner edge of Saturn's ring exceeds the velocity of the outer edge, but that, within the limits of error of the method, the relative velocities at different parts are such as to satisfy Kepler's third law.

Besides (1) the proof of the meteoric constitution of the rings, explained above, each line of the photographs gives (2) the period of rotation of the planet, (3) the mean period of the rings, (4) the motion of the whole system in the line of sight. I have measured a number of lines on each plate and compared the results with the computed values of the corresponding quantities.

The most accurate method[1] of measuring the relative displacement of the opposite ends of a line in the spectrum of the planet is to measure the angle ϕ.

The value of ϕ depends upon the dispersion and other constants of the spectroscope employed, as well as upon quantities which are independent of the instrument. If we let $L =$ the velocity of light in kilometers per second $= 299{,}860$; $\lambda =$ the wave-length of the measured line in tenth-meters; $D =$ the linear dispersion of the photographed spectrum at the position of the same line, expressed in tenth-meters per millimeter; $\rho =$ half the width of the spectrum in millimeters;—we have by Doppler's principle, allowing for the double effect already mentioned,

$$y = x \tan \phi = \frac{2v\lambda}{DL},$$

or,

$$v = x \tan \phi \, D \frac{L}{2\lambda},$$

from which we obtain the velocity in the line of sight at the limb ($V' \cos \beta$) by placing $x = \rho$. That is,

$$V' = \frac{\rho D L \tan \phi}{2\lambda \cos \beta}.$$

The value of ρ is computed from the angular semi-diameter of the planet and the focal length of the telescope. It cannot be obtained accurately by measurement of the photograph, because the borders of the spectrum are indistinct. For my instrument at the time of observation, $\rho = 0^{\mathrm{mm}}.2134$. D is obtained from a wave-length curve constructed from measurements of a standard plate of the solar spectrum made with the same apparatus, and ϕ is directly measured under a microscope provided with a position circle.

The relative displacement of a line in the spectra of the ansæ is measured directly, the micrometer wire having first been

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[1] This method is due to Deslandres (*C. R.* **120**, 417), and I have found it to be very satisfactory. The conclusions in Deslandres' article, with respect to the motion in the line of sight of bodies which are not self-luminous, are not new, although they are treated more fully than elsewhere.

placed parallel to the lines of the comparison spectrum. If δ is this measured interval, the mean velocity of the ring is

$$V'' = \frac{DL\delta}{4\lambda\cos\beta}.$$

The displacement could also be determined by measuring the angle which the line joining the centers of the two short lines of the ansæ makes with the comparison lines. I have found the direct method to be preferable.

There remains the motion of the whole system in the line of sight, which has hitherto not been considered. It is best determined by comparing the mean of the positions of the lines in the spectra of the ansæ with the corresponding line of the comparison spectrum. The results for this motion are unsatisfactory, as might be expected from the circumstances of observation. Owing to the fall of temperature during the rather long exposure of two hours, and the fact that the lunar spectrum was photographed at the end, and not in the middle of the exposure to the planet, the two spectra are relatively displaced by an amount which is about ten kilometers greater than that due to the motion of Saturn in the line of sight. I have therefore made no careful measurements of this displacement. For the reasons given above, the planetary lines are somewhat less sharp than the lines in the lunar spectrum, which was photographed with an exposure of only six minutes.

The results of all the measurements are given in the following tables:

PHOTOGRAPH NO. 1, APRIL 9, 1895.

λ	D	ϕ	Velocity of Limb	$C—O$	δ	Mean Velocity of Ring	$C—O$
Tenth-meters	Tenth-meters	° ′	Kilometers	Kilometers	Millimeters	Kilometers	Kilo-meters
5324.3	27.55	3 36	10.92	—0.63	0.0456	18.54	+0.24
5328.4	27.65	4 24	13.39	—3.10	0.0464	18.92	—0.14
5371.6	28.77	3 11	9.99	+0.30	0.0404	17.01	+1.77
5383.5	29.09	3 20	10.56	—0.27	0.0362	15.37	+3.41
5429.9	30.37	3 8	10.27	+0.02	0.0402	17.67	+1.11
			11.03	—0.74		17.50	+1.28

PHOTOGRAPH NO. 2, APRIL 10, 1895.

λ	*D*	φ	Velocity of Limb	*C—O*	δ	Mean Velocity of Ring	*C—O*
Tenth-meters	Tenth-meters	° ′	Kilometers	Kilometers	Millimeters	Kilometers	Kilo-meters
5324.3	27.55	2 11	6.62	+3.67	0.0468	19.03	—0.25
5328.4	27.65	3 19	10.09	+0.20	0.0412	16.81	+1.97
5371.6	28.77	2 42	8.47	+1.82	0.0436	18.35	+0.43
5383.5	29.09	3 13	10.19	+0.10	0.0420	17.84	+0.94
5429.9	30.37	3 49	12.51	—2.22	0.0468	20.56	—1.78
			9.58	+0.71		18.52	+0.26

The results from both photographs are

Velocity of limb = 10.3 ± 0.4 kilometers,

Mean velocity of ring = 18.0 ± 0.3 kilometers;

the computed values being 10.29 and 18.78 kilometers respectively.

Although there seems to be no systematic difference between the two plates, the results for each differ by more than the probable error. With photographs on so small a scale, distortions of the lines are produced by the irregular deposit of even a few particles of silver; hence it is advisable to measure a large number of lines instead of multiplying observations on a few of them.

The number of lines in the table is however sufficient for the present purpose.

As I have already pointed out, it is necessary to guide the telescope with extreme accuracy in making such photographs as those described in the present paper, and the method which I have used is so simple and effective that a short account of it may be of interest.

The spectroscope is fully described in *Astronomy and Astro-Physics*, **12**, 40, January, 1893, and the prism-train used in these observations is there shown in Plate VII. The slit is observed during an exposure by a small "broken" telescope, which receives the rays reflected from the first surface of the prism nearest to the collimator.

To prepare for an observation of Saturn, the slit is shortened until its length is equal to the computed length of the image (major axis of the ring). A small bar, which is wider at one end than at the other, is cut out of thin metal, and placed across the field of the diagonal telescope. If the bar is approximately of the right width, then, by throwing the image of the slit a little above or below the center, and by rotating the eyepiece, which carries the bar with it, the bar can be made to very nearly cover the image, leaving a very short length of slit uncovered at each end. When the telescope is directed to Saturn the extreme ends of the ring appear from behind the (invisible) bar as two minute points or stars, and the attention of the observer is concentrated on keeping these stars equally bright. Any displacement in declination is indicated by their disappearance or unusual faintness. The photographs show that the guiding by this method is quite accurate. The spectra of the ansæ do not show any traces of the Cassini division, but it would probably be requiring too much to expect that they should do so, considering the small size of the image and the length of the exposure.

It is a question whether these observations could be better made with a larger telescope. If the same spectroscope were mounted on a large telescope, the width of the photographed spectrum would be greater, the lines would be longer, and their direction could be more definitely measured. On the other hand, the inclination of the lines would be diminished, since tan ϕ varies inversely with ρ, and it could not be increased by employing a greater dispersion, as the brightness of the spectrum, which would be the same for both telescopes, would hardly bear any further reduction. A material advantage would be that with the same slit-width a smaller area of the image would be included between the jaws, and hence at any part of the slit there would be fewer points having different velocities in the line of sight. On the whole, it seems to me that the advantage would lie with the large telescope. With a reflector, or a photographically corrected refractor, the photographs could be taken at the $H\gamma$ line, where the dispersion is more than twice as great as in the region near λ 5350, and the only difficulty in that case would be found in the yellow color of Saturn.

I have given a somewhat full account of these observations, partly because of the interest inherent in everything that relates to the magnificent system of Saturn, and partly because the successful application of the spectroscope to the measurement of celestial motions depends largely upon details of appliances and methods.

PLATE VIII

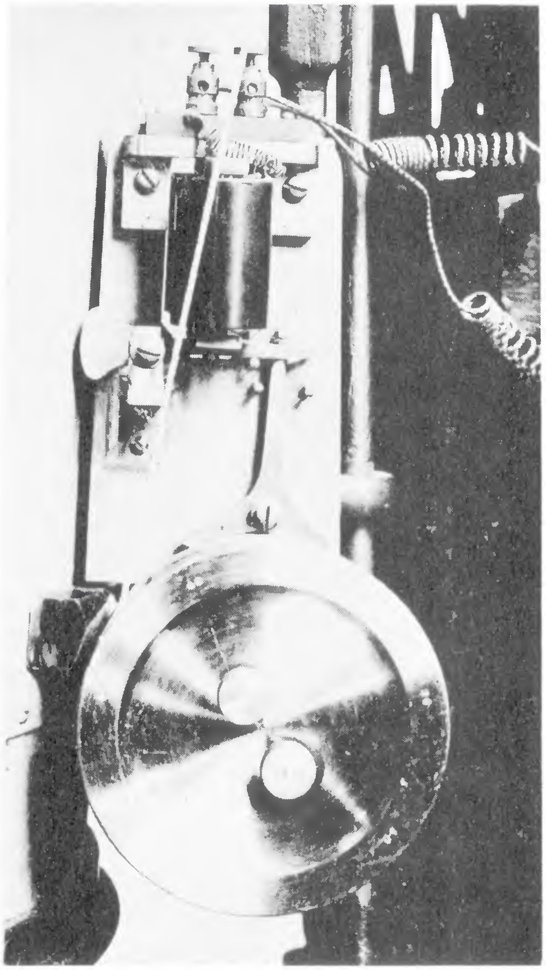

FIG. 1

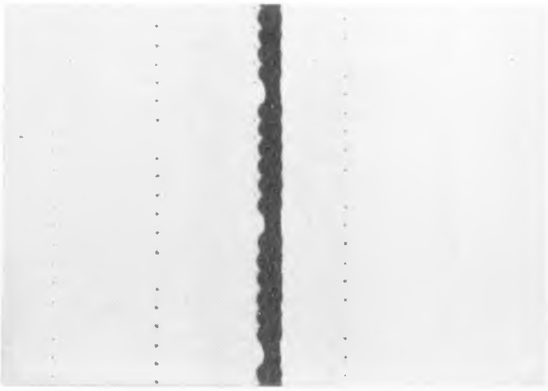

FIG. 2