the meridian circle or sidereal clock, and either the meridian circle or equatorial, into circuit with itself.

"A building is now in process of construction in which we hope to have mounted within a few months, a 9⅝ inch equatorial for photograhic purposes, and a concave grating by ROWLAND, 6 inch, with camera complete.

"In the field observatory we have two transits, a zenith telescope, an altazimuth, and several STACKPOLE sextants."

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## MEASUREMENT OF *JUPITER'S* SATELLITES BY INTERFERENCE.

BY A. A. MICHELSON.

It has long been known that even in a telescope which is theoretically perfect, the image of a luminous point is composed of a series of concentric circles with a bright patch of light at the common center. This system of circles can easily be observed by examining any bright star with a telescope provided with a circular diaphragm which diminishes the effective aperture. The appearance of the image is shown in Fig. 1. In the case of an object of finite angular magnitude the image could be constructed by drawing a system of such rings about every point in the geometrical image. The result for a small disc (corresponding to the appearance of one of the satellites of *Jupiter* as seen with a 12-inch telescope whose effective aperture has been reduced to six inches) is given in Fig. 2, the chief points of difference between this and Fig. 1 being the greater size of the bright central disc and the lesser clearness of the surrounding rings. The larger the disc the more nearly will the appearance of the image correspond to that of the object; and the smaller the object the more nearly does it correspond with Fig. 1, and the more difficult will be the measurement of its actual size. Thus, in the case just cited, the actual angular diameter is about one second of arc, and the uncertainty may amount to half this value or even more.

The relative uncertainty, other things being equal, will be less in proportion to the increase in the aperture, so that with the 36-inch telescope, the measurement of the diameters of *Jupiter's* satellites should be accurate to within ten per cent. under favorable conditions.

It is important to note that in all such measurements the image observed is a diffraction phenomenon—the rings being interference fringes and the settings being made on the position of that part of a fringe which is most easily identified. But such measurements must vary with the atmospheric conditions and especially with the observer—for no two observers will agree upon the exact part of the fringe to be measured, and the uncertainties are exaggerated when the fringes are disturbed by atmospheric tremors.

If, now, it be possible to find a relation between the size of the object and the *clearness* of the interference fringes, an independent method of measuring such minute objects will be furnished; and it is the purpose of this paper to show that such a method is not only feasible but in all probability gives results far more accurate than micrometric measurements of the image.

In a paper on the "Application of Interference Methods to Astronomical Measurements,"* an arrangement was described for producing such fringes, by providing the cap of the objective with two parallel slits, adjustable in width and distance apart. If such a combination be focused on a star, then, instead of the concentric rings before mentioned, there will be a series of straight equidistant bands whose length is parallel with the slits, the central one being brightest,† Fig. 3.

The general theory of these fringes may be found in the *Publications* A. S. P., vol. iii, no. 16, 1891. The general equation showing the relation between the *visibility* of the fringes and the distance between the slits is

$$V^2 = \frac{[\int \phi(x) \cos kx\, dx]^2 + [\int \phi(x) \sin kx\, dx]^2}{[\int \phi(x)\, dx]^2} \tag{1}$$

which reduces to the simpler form

$$V = \frac{\int \phi(x) \cos kx\, dx}{\int \phi(x)\, dx} \tag{2}$$

when the object viewed is symmetrical.

A number of applications of this formula are discussed in the former paper, but for the present purpose attention will be confined to the case in which the object viewed (or rather its projection) is a circular disc, uniformly illuminated.

* Philosophical Magazine, July, 1890.

† These will be superposed on another set of fringes due to diffraction from the edges of the slits; but the latter are too faint and broad to cause any confusion. Fig. 3 is in error, as the fringes should be in one group, and not in two.

Fig. 1.

Fig. 2.

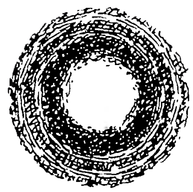
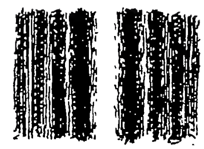


Fig. 3.



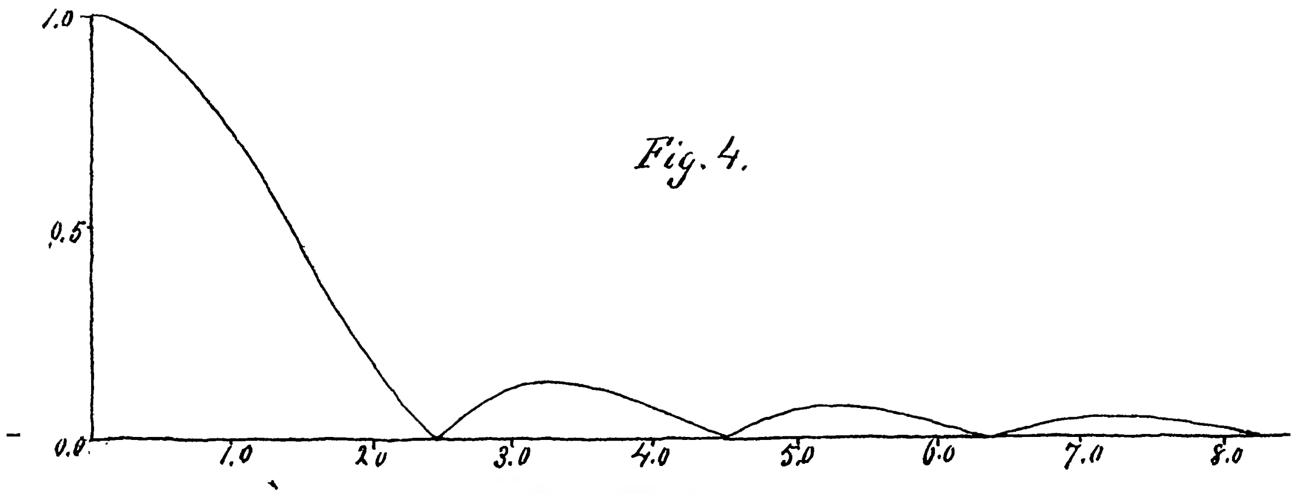



Fig. 4.

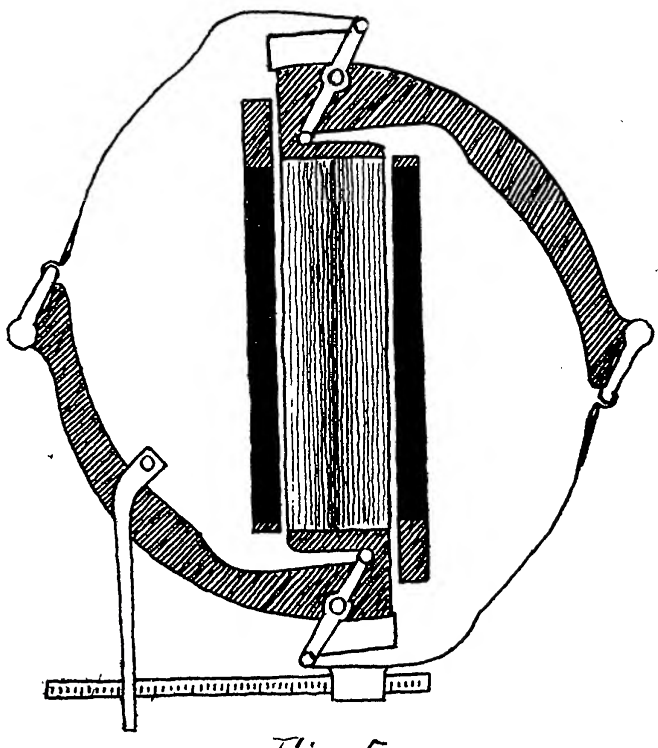


Fig. 5.

In this case equation (2) becomes

$$V = \int_0^1 \sqrt{1 - \omega^2} \,.\, \cos \pi \frac{\alpha}{\alpha_0} \omega \,.\, d\omega \qquad (3)$$

in which $\alpha$ is the angular diameter of the object, and $\alpha_0$ is the smallest angle resolvable by an equivalent aperture; that is, the ratio of a light-wave to the distance between the slits.

The curve expressing this relation is given in Fig. 4, in which the ordinates are values of the visibility of the fringes and the abscissæ are the corresponding values of the $\alpha/\alpha_0$.

From this it will appear that the fringes disappear at recurring intervals, and in a laboratory experiment as many as four such disappearances were noted, and the average error in the resulting value of $\alpha$, the angular magnitude of the disc, was found to be less than two per cent.

From the curve it is evident that the first disappearance is most readily and accurately observed, and for this we have

$$\text{whence putting } s \frac{\alpha}{\alpha_0} = 1.22;$$

for the distance between the centres of the slits, and taking for the wave-length of the brightest part of the spectrum 0.00055 mm.,* and dividing by the value of a second in radians we have

$$\alpha = \frac{1.38}{s} \qquad (4)$$

In consequence of the kind invitation extended by Professor HOLDEN it was decided to make a practical test of the usefulness of the proposed method at Mt. Hamilton.

For the preliminary experiments which are to be described it was thought desirable to use the 12-inch equatorial. Accordingly a cap, provided with two adjustable slits, was fitted over the objective and provided with a rod by means of which the distance between the slits could be altered gradually and at will by the observer, while the distance was measured on a millimeter scale attached to the sliding jaws. This arrangement which was constructed under the supervision of Mr. F. L. O. WADSWORTH, of Clark University, is shown in the accompanying diagram, Fig. 5.

With this apparatus the satellites of *Jupiter* were measured with results as given in the following table:

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* The wave-length will, of course, vary somewhat with the object observed but may be made constant by interposing a red glass.

TABLE I.

| No. of Satellites. | I. | II. | III. | IV. | Seeing. |
|---|---|---|---|---|---|
| | " | " | " | " | |
| August 2...... | 1.29 | 1.19 | 1.88 | 1.68 | Poor. |
| August 3...... | 1.29 | .... | 1.59 | 1.68 | Poor. |
| August 6...... | 1.30 | 1.21 | 1.69 | 1.56 | Poor. |
| August 7...... | 1.30 | 1.18 | 1.77 | 1.71 | Good. |
| Mean....... | 1.29 | 1.19 | 1.73 | 1.66 | |

These are the values of the angular diameters of the satellites of *Jupiter* as seen from the earth. To reduce these to *Jupiter's* mean distance these values are to be multiplied by 0.79 which give for the final values

| I. | II. | III. | IV. |
|---|---|---|---|
| 1".02 | 0".94 | 1".37 | 1".31 |

For the sake of comparison these values are recorded in the following table together with those given by ENGELMANN, STRUVE and HOUGH, and the last column contains some results kindly furnished by Professor BURNHAM with the 36-inch on the same date (August 7th) as the last of the series by A. A. M.

TABLE II.

| No. of Satellites. | A. A. M. | ENG. | ST. | HO. | BU. |
|---|---|---|---|---|---|
| I.............. | 1.02 | 1.08 | 1.02 | 1.11 | 1.11 |
| II............. | 0.94 | 0.91 | 0.91 | 0.98 | 1.00 |
| III............ | 1.37 | 1.54 | 1.49 | 1.78 | 1.78 |
| IV............. | 1.31 | 1.28 | 1.27 | 1.46 | 1.61 |

It was found impossible to see the reappearance of the fringes on increasing the distance, yet the results of Table I show that the disappearance could still be sharply marked. Indeed the concordance of the observations made under different circumstances on different nights was even closer than was expected. With a larger telescope both the brightness of the fringes and their distance apart will be increased, and it may be confidently predicted that the accuracy will then be even greater.

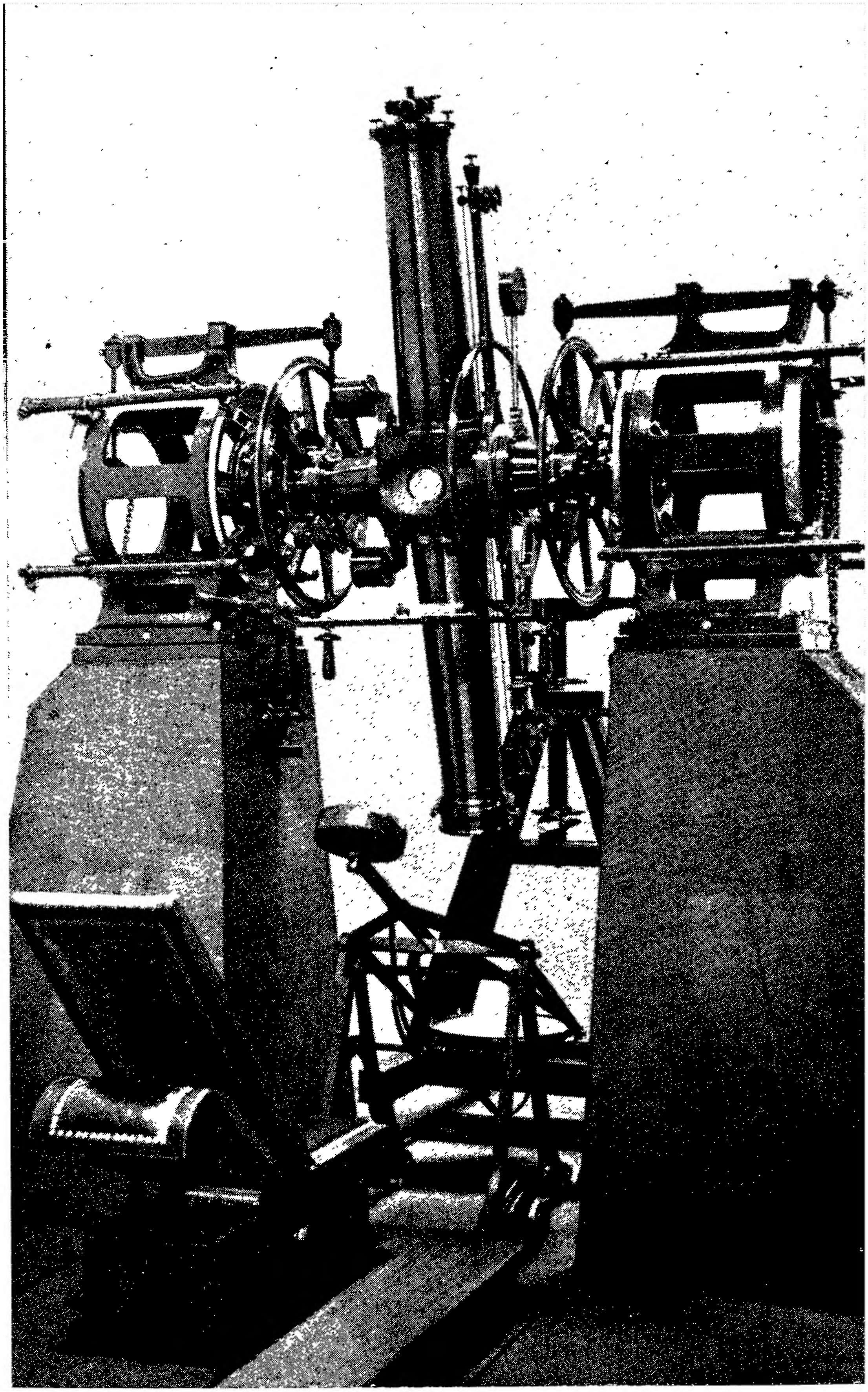
The values given in the second column "ENGELMANN" are probably more reliable than the succeeding ones, but it is well worth noting that the differences between the results obtained by interference agree with the others quite as well as these agree with each other.

It should also be noted that the distance between the slits was about four inches. It may therefore be stated that for such measurements as have just been described, a telescope sufficiently large to admit a separation of four inches—say a six-inch—suitably provided with adjustable slits is fully equal to the largest telescopes now used without them.

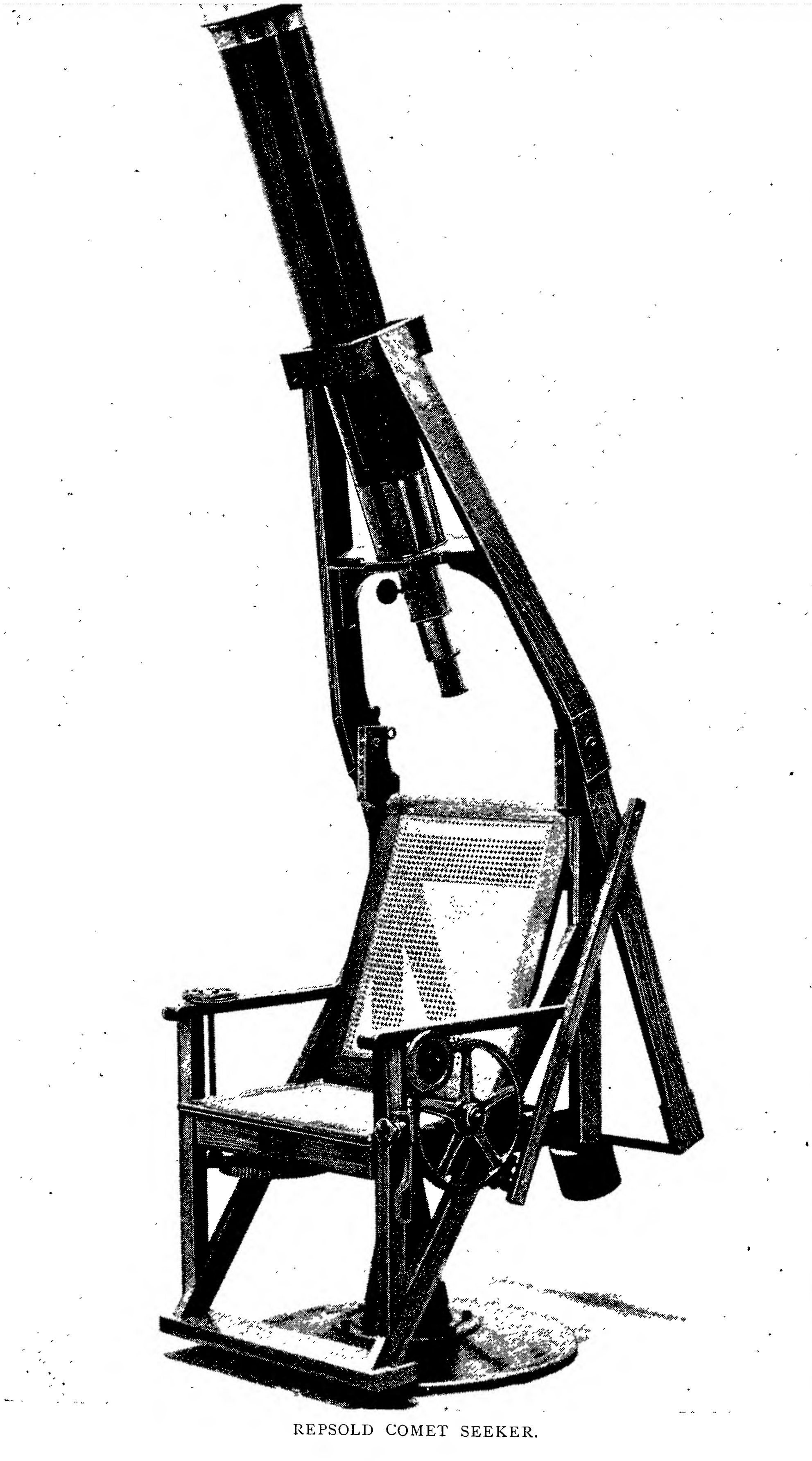
It is hoped that within a few months the 36-inch equatorial will be supplied with a similar apparatus and observations begun for the definitive measurement of the satellites of *Jupiter* and *Saturn* and such of the asteroids as may come within the range of the instrument.

In concluding, I wish to take this opportunity in expressing my appreciation of the courtesy of Director HOLDEN in placing all the facilities of the Observatory at my disposal, and of the hearty co-operation of all the astronomers of the Observatory, especially the valuable assistance of Professor W. W. CAMPBELL in making the observations.

MT. HAMILTON.



REPSOLD MERIDIAN CIRCLE.



REPSOLD COMET SEEKER.