NOTE ON THE DEFINITION, THE RESOLVING POWER AND THE ACCURACY OF TELESCOPES AND MICROSCOPES.

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Let us take the following nomenclature :

B = diameter of objective.
F = focal length of objective.
α = apparent semi-diameter of objective viewed from the object.
β = apparent semi-diameter of objective viewed from the image.
d = smallest distance between lines which can be clearly "resolved."
λ = wave-length of the light employed.
b = breadth of the diffraction fringes with this kind of light.
M = Magnification.
R = Resolution.
D = Definition.
A = Accuracy.

Then if M is the ratio of size of image to object,

$$M = \frac{\sin \alpha}{\sin \beta}$$

The *resolution* is measured by the closeness of two lines which can be clearly distinguished or "resolved." Let us therefore put

$$R = \frac{1}{d}$$

Now two lines are clearly distinguishable when the central fringes of their images are separated by the width of one fringe. The actual limit at which the resolution disappears may be anywhere between b and $\frac{b}{2}$. (See "Wave Theory," Lord RAYLEIGH, *Enc. Brit.*) But it can readily be shown that $b = \frac{\lambda}{2 \sin \beta}$ and $d = b/M$; hence,

$$R = \frac{2}{\lambda} \sin \alpha.$$

The *definition* of an objective is measured by the ease with which the forms of minute objects may be recognized. Thus, were it not for diffraction, D would be simply proportional to M. But for a given magnification the form of the image is clearer, or the definition greater as the fringes are narrower; hence, we may put

$$D = \frac{M}{b} = R$$

Definition is not capable of being so precisely formulated as Res olution, and would undoubtedly vary with the form of the object, its nearness to other objects, etc. In view of these uncertainties, it would scarcely be worth while to introduce a constant coefficient in the last equation.

The error of setting of the cross-hairs of an eye-piece on the middle of a diffraction band of sufficient width will be b/e where e is a constant not far from 100, and the corresponding error in distance would be $b/e \div M$. This smallest *measurable* distance is therefore e times as small as the smallest *resolvable* distance; hence,

$$A = eR.$$

These formulæ may be applied to the microscope (in which case the maximum values correspond to $\alpha = 90°$), or to the telescope (in which α is nearly zero, and angular measurements alone are of importance). Accordingly we obtain the following:

Microscope.	Telescope.
$M = 1/\sin \beta$	F
$R = 2/\lambda$	B/λ
$D = 2/\lambda$	B/λ
$A = e2/\lambda$	eB/λ

The formulæ for microscopes apply when the object is very near the lens. They show, first, that with a microscope of given length the magnifying power depends on the smallness of the objective, and on nothing else; second, that the resolution, definition and accuracy (upon which the usefulness of a microscope chiefly depends) are the same for all microscopes, no matter how large the objective may be, or how great the magnifying power, provided the latter be sufficient to show diffraction fringes; third, that these qualities vary inversely with the wave-length of the light employed.

There seems to be a prevailing impression that a microscope may have a high resolving power with but moderate definition, and *vice versa*. This may be due to the difficulty in giving an exact signification to the terms. If those here employed be admitted it is evident that the two qualities must go together.

In the telescope the size of the image and hence the magnification depends entirely on the focal length. The resolution is in this case the reciprocal of the smallest angular distance which can be clearly distinguished. It increases with the diameter of the objective and inversely with the wave-length. These formulæ may also be applied to the revolving mirror as used in galvanometers, etc., except that in this case the accuracy is doubled, so that

$$A = 2eB/\lambda.$$

The foregoing statements must be understood to refer to theoretically perfect lenses. If the lenses be imperfect the ratio of their performance to that of a perfect lens may be expressed by a constant depending on the accuracy of the surfaces and the nature of the glass.