HISTORY AND DESCRIPTION

OF THE

WATER TELESCOPE

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ROYAL OBSERVATORY, GREENWICH.

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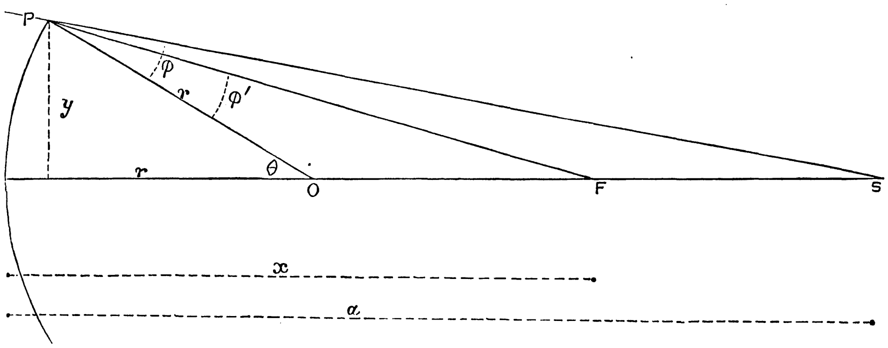
In the year 1867, attention was called by Professor Klinkerfues, of Gottingen, to an investigation of the passage of a ray of light, on the Undulatory Theory, through a definite thickness of refracting medium; and to the effect which would be produced on the apparent value of the coefficient of Sidereal Aberration. The result of the investigation was, that the coefficient would be increased by a calculable quantity, which would be sensible even in the effect of the thickness of the object-glass of an ordinary astronomical telescope. To test this theory, Professor Klinkerfues inserted, in the telescope of a transit-instrument, a tube 8 inches long filled with turpentine, and observed on the same day the transits of two groups of stars on whose R.A. the effect of aberration would be respectively maximum + and maximum —. The observation appeared to support the theory.

The physical importance of this theory appeared to me so great as to make a verification of the experiment very desirable. Viewing the position of γ Draconis (the birth-star of modern astronomy) with reference to the zenith of Greenwich, I decided that it would be best to test the theory by observation of the apparent zenith-distance of that star, by means of a vertical telescope filled with a refracting fluid. For observations which might occupy a considerable time, it was evidently most convenient that the fluid should be water. I therefore planned generally an instrument adapted to my purpose, and submitted the fundamental ideas to Mr. Simms, by whom the details were carried out with his well-known ability. The general understanding was, that the length of the telescope should be about 36 inches (the length of the column of water is in fact 35·3 inches); that it should be furnished with means of filling and emptying the tube, and with means of cleaning the glasses, under the peculiar conditions of this telescope; that it should carry two levels; that, in order to avoid the danger to which the flint glass would be liable if in contact with water, the flint lens of the object-glass should be the external lens; and that a micrometer should be used, adapted to rapid reversion of the instrument, similar to one which I planned many years ago for the Reflex Zenith Tube. Estimates of expense being prepared, and the sanction of the Lords Commissioners of the Admiralty being received, I proceeded to investigate the necessary curvatures of the lenses. I had the great advantage of examination of my work in all parts, and repetition of it to a great extent, by my friend E. J. Stone, Esq., then First Assistant of the Royal Observatory of Greenwich, now Astronomer of the Royal Observatory of the Cape of Good Hope. The following are the heads of the investigation:—



A ray of light, coming from the left, and directed towards the point S, is received at the point P upon the spherical surface of a refracting medium whose center of curvature is O, and is made by refraction to pass in the direction P F. The index of refraction from the external medium at the left side to the refracting medium within the spherical surface is μ. Other notations as in the diagram.

The equation $\mu = \frac{\sin \phi}{\sin \phi'}$ gives

$$\mu = \frac{a-r}{\mathrm{PS}} \cdot \frac{\mathrm{PF}}{x-r}$$

and expressing P F and P S in terms of y, to the second power,

$$\mu\left(\frac{1}{r}-\frac{1}{x}\right)\cdot\left\{1-\left(\frac{1}{r}-\frac{1}{a}\right)\frac{y^2}{2a}\right\} = \left(\frac{1}{r}-\frac{1}{a}\right)\left\{1-\left(\frac{1}{r}-\frac{1}{x}\right)\frac{y^2}{2x}\right\}.$$

For $\frac{1}{a}, \frac{1}{r}, \frac{1}{x}$, put A, R, X, respectively; then, on reduction,

$$\mathrm{X} = \mathrm{A}.\frac{1}{\mu} + \mathrm{R}.\frac{\mu-1}{\mu} + (\mathrm{R}-\mathrm{A})^2.\frac{\mu-1}{\mu^3}.\left(\mathrm{R}-\mathrm{A}\,(\mu+1)\right).\frac{y^2}{2}.$$

which formula, with proper substitutions, applies to every surface.

In the process following, the radii of the four surfaces of the two lenses (supposed positive when the convexities are turned to the outside, or to the left hand of the diagram), beginning at the external surface, will be represented by r_1, r_2, r_3, r_4, respectively. The thickness of the lenses at the middle (a constant in each, independent of y) will be neglected. The refractive index from air into the first lens is called m, that from air into the second lens is p, and that from air into water is s.

At the first surface, $\mu = m$; and, supposing the light to come from a star at an infinite distance, $\mathrm{A}_1 = 0$. The formula becomes

$$\mathrm{X}_1 = \mathrm{R}_1.\frac{m-1}{m} + \mathrm{R}_1^3.\frac{m-1}{m^3}.\frac{y^2}{2}.$$

At the second surface, $\mu = \frac{1}{m}$, $A_2 = X_1$, and the formula becomes

$$X_2 = m\,X_1 + R_2\,(1-m) + m^2\,(1-m).\,(R_2-X_1)^2.\left\{R_2 - X_1.\,\frac{m+1}{m}\right\}\frac{y^2}{2}.$$

At the third surface, or first surface of the second lens, $\mu = p$, $A_3 = X_2$, and the formula becomes

$$X_3 = \frac{1}{p}\,X_2 + R_3.\,\frac{p-1}{p} + \frac{p-1}{p^3}\,(R_3-X_2)^2.\left\{R_3 - X_2\,(p+1)\right\}.\frac{y^2}{2}.$$

At the fourth surface, considered as refracting into air, $\mu = \frac{1}{p}$, $A_4 = X_3$, and thus

$$X_4 = p\,X_3 + R_4\,(1-p) + p^2\,(1-p).\,(R_4-X_3)^2\left\{R_4 - X_3\,\frac{p+1}{p}\right\}\frac{y^2}{2}.$$

Finally, at the fourth surface, considered as refracting from air into water, $\mu = s$, $A_5 = X_4$, and the distance of ultimate convergence in water is $x_5 = \frac{1}{X_5}$, where X_5 is thus determined,

$$X_5 = \frac{1}{s}\,X_4 + R_4\,\frac{s-1}{s} + \frac{s-1}{s^3}\,(R_4-X_4)^2.\left\{R_4 - X_4\,(s+1)\right\}.\frac{y^2}{2}.$$

Making the substitutions in succession, we obtain for X_5,

$$(R_1-R_2)\,\frac{m-1}{s} + (R_3-R_4)\,\frac{p-1}{s} + R_4\,\frac{s-1}{s}$$

$$+\frac{y^2}{2}\left\{\begin{array}{l} R_1^3.\,\frac{m-1}{m^2 s} \\ \left\{R_2 - R_1\,\frac{m-1}{m}\right\}^2.\left\{R_2 - \frac{m^2-1}{m^2}\,R_1\right\}.\,\frac{m^2\,(1-m)}{s} + \\ +\left\{R_3 - (R_1-R_2)\,(m-1)\right\}^2.\left\{R_3 - (m-1)(p+1)(R_1-R_2)\right\}.\,\frac{p-1}{p^2 s} \\ +\left\{R_4 - (R_1-R_2)\,\frac{m-1}{p} - R_3\,\frac{p-1}{p}\right\}^2.\left\{R_4 - \frac{m-1}{p}.\frac{p+1}{p}\,(R_1-R_2) - R_3\,\frac{p^2-1}{p^2}\right\}.\,\frac{p^2\,(1-p)}{s} \\ +\left\{R_4 - (R_1-R_2)\,(m-1) - (R_3-R_4)\,(p-1)\right\}^2.\left\{R_4 - (s+1)\,(m-1)\,(R_1-R_2) - (p-1)\right. \\ \qquad \left.(s+1)\,(R_3-R_4)\right\}\frac{s-1}{s^3} \end{array}\right\}$$

The last collection of terms represents the spherical aberration.

Let the adopted focal length of the combination be expressed by u, and let $U = \frac{1}{u}$, where U consists only of the first line of the expression above. Also let us assume that the indices of refraction of two certain rays of the spectrum are m, p, s, and m', p', s', respectively; the corresponding values of U being U and U′. Then

$$U = (R_1-R_2)\,\frac{m-1}{s} + (R_3-R_4)\,\frac{p-1}{s} + R_4\,\frac{s-1}{s},$$

$$U' = (R_1-R_2)\,\frac{m'-1}{s'} + (R_3-R_4)\,\frac{p'-1}{s'} + R_4\,\frac{s'-1}{s'}.$$

The condition of achromatism requires that these two quantities be equal. Thus we find, as the equation of achromatism,

$$0 = (R_1-R_2)\left(\frac{m-1}{s} - \frac{m'-1}{s'}\right) + (R_3-R_4)\left(\frac{p-1}{s} - \frac{p'-1}{s'}\right) + R_4\left(\frac{s-1}{s} - \frac{s'-1}{s'}\right).$$

Mr. Simms, at my request, examined the indices of refraction of several lines of the spectrum in the two kinds of glass which he proposed to employ, preferring the determinations for C and F; and with these I combined Fraunhofer's indices for water (Munich Transactions, vol. 5, 1814). The following are the numbers:—

Flint Glass, m (for C) 1·6257, m' (for F) 1·6433.
Crown Glass, p (for C) 1·5126, p' (for F) 1·5211.
Water, s (for C) 1·3317, s' (for F) 1·3378.

Substituting these, the equation of achromatism becomes

$$0 = (R_1 - R_2) \times 0·01101 + (R_3 - R_4) \times 0·00460 + R_4 \times 0·00342.$$

From this we obtain

$$(R_1 - R_2) = -(R_3 - R_4) \times 0·4178 - R_4 \times 0·3106$$
$$= - R_3 \times 0·4178 + R_4 \times 0·1072$$
$$\text{and } U = + (R_3 - R_4) \times 0·18862 + R_4 \times 0·10314$$
$$= + R_3 \times 0·18862 - R_4 \times 0·08548.$$

We have now to find the equation for destroying spherical aberration.

It was thought probable that the air-bubbles, which would undoubtedly form in the water and would rise to the upper surface, would unite into a large bubble (more easy of extraction) under the center of the object-glass, if a slight concavity were given to it. (This expectation has not been supported by the event.) The radius of curvature of the lower surface of the second lens, or r_4, was therefore assumed to be + 50 inches, or $R_4 = + 0·02$. The original intention was that the focal length of the combination should be 30 inches, or $U = 0·03333$. The equation just found for achromatism, and that for the value of U, with the assumed numerical values of R_4 and U, enabled us to express $R_1 - R_2$ in terms of R_3. Assuming $R_1 = v.\ R_3$, all are expressed in terms of R_3, v, and constant numbers. The values assumed for m, p, and s, were mean values of the refractive indexes, namely, 1·6345, 1·5169, 1·3348. On substituting these in the expression which multiplies $\frac{y^2}{2}$, and making it = 0, the following equation is at length obtained:—

$$0·5735 \,.\, v^2 + 0·4474 \,.\, v = 0·4716.$$

Adopting the positive value of the root, $v = 0·597$; and then, with little trouble, the following radii of curvature of the lens-surfaces are found:—

r_1 = 9·03 inches.
r_2 = 5·38 ,,
r_3 = 5·39 ,,
r_4 = 50·00 ,,

all being convex towards the anterior face of the telescope.

Subsequently it was determined to increase the focal length from 30 to 36 inches, and the numbers above-given were all increased in that proportion.

As it was intended that the micrometer of the eyepiece should be placed as near as practicable to the plane glass which closes the lower end of the tube, no further investigation was made for the effect of the small thicknesses of glass and air through which the rays of light must pass before complete convergence.

The lenses of the object-glass were ground in conformity with the numbers above. The definition of the star was at first bad, in consequence of the lower lens having been so forced upon the cement in the object-glass-cell that it received a slight cylindrical bend. On replacing it with greater care, the definition became very good.

After thus giving the principal steps of the investigation relating to this important point, I will now proceed with the detailed account of the general construction of the instrument.

Explanation of the Plate illustrating the description of the Water-Telescope.

Figure 1 is a view from the east of the instrument as complete. The telescope is on the eastern side of the vertical spindle; this is one of the two positions proper for the measure of the star's zenith-distance. The eyepiece is in the position proper for observing the star, with the telescope on this side. If the instrument had been turned 180° in azimuth, it would have been in a fit state for the other measure of zenith-distance; the telescope, in the view, would then have been partly hidden by the spindle; and the eyepiece must have been turned 180° in azimuth upon the instrument, so that in the view it would still have pointed in the same direction as in figure 1.

Figure 2 is a view from the same side, the instrument having been rotated only 90°. The telescope is now in the position proper for observing the transit of the star across the wires. The eyepiece has not yet been turned in azimuth; it must be turned, so that in the view it will point as in figure 1.

Figure 3 is a view of the complete instrument from above; the positions of the telescope and eyepiece corresponding to those in figure 1.

Figure 4 is a vertical section of the telescope only, on a larger scale, transverse to the levels.

Figure 5 is a view of the lower end of the telescope from below. The sponge and sponge-rod, which are in the interior, are represented in shade.

Figure 6 is an elevation or lateral view of the micrometer and eyepiece. The longitudinal dimension of the micrometer-box is presented to the eye, and the eyepiece is supposed to be turned into the same position.

Figure 7 is the fixed plate of the micrometer-box, with its system of crossed wires.

Figure 8 is a plan of the micrometer, the covering of the box being removed. It shows the system of double-micrometer, the resting point of micrometer-screw (B) which carries the wires being in (A), which itself is carried by a micrometer-screw.

Figure 9 shows the mechanism of the counter of entire revolutions of the micrometer-screw, a similar mechanism being applied to each micrometer.

The following is a more detailed description of every part, referring to the letters upon the diagrams:—

AA, figures 1, 2, 3, is the stone pier on which the instrument stands. The form of the stones is accidental.

B, figures 1, 2, 3, is the iron-base-plate (attached to the stone by plugs let into the stone) to which the iron plate is fixed by screws.

C, figures 1, 2, 3, is a wooden plate attached by its edge to the edge of the iron plate, by means of an iron strap passing round the edge of the iron plate. It is intended to support the right arm of the observer, when his right hand is employed in turning the micrometer-head.

D, figures 1, 2, 3, is the upright part of the support of the instrument, which is cast in one piece with its base. The base, and the screw-bolts by which it is connected with the iron base-plate B, are seen in figures 1, 2, 3. The horizontal section of the upright part diminishes towards the top; its plan is a circular segment with an edge-bar at the back.

E, in figures 1, 2, 3, is the upper part of the instrument-support, not in the same cast with D, but fixed to it by three strong screws, shown in figure 3. In the other extremity of E, figure 3, will be seen the horizontal Y, in which the vertical pivot of the great spindle L turns.

F, in figures 1, 2, is a gas-pipe leading to the burner I.

G, in figures 1, 2, 3, is the tap of the principal gas-pipe which conveys gas to the two lamps I I. The pipe, being brought under ground, is led up to this height in order that the tap may be within reach of the observer's hand while he sits on the seat H; the pipe is then led down again to the ground and to the lamps I I. The observer can thus control the intensity of light of the two lamps.

H, in figures 1, 2, 3, is the observer's seat. The observer placed on it can command the eyepiece of the telescope, either when the telescope is on the east side, as shown in figure 3, or when it is on the west side.

I I, in figures 1, 2, 3, are the two gas-lamps, symmetrically placed on opposite sides of the axis of the great spindle L. Consequently, when the great spindle is turned through 180°, carrying with itself the telescope W and the illuminating reflector Y; if the reflector is properly adjusted to receive the light of one I in one position of the instrument, it will be properly adjusted to receive the light of the other I in the other position of the instrument.

KK, in figures 1, 2, the chimneys of the two lamps I I.

L, in figures 1, 2, 3, the great spindle, in the form of two brass cones united at their bases, with steel pivots at top and bottom.

MM, in figures 1, 2, 3, two arms fixed by a strong clamp to L; so adjusted that, when the great spindle is turned in azimuth in one direction, one of the arms M will strike one side of a stop-piece P which is fixed to the upright stand D; and when the spindle is turned the opposite way, the other arm M will strike the opposite side of P. Horizontal springs, bending downwards, are attached to the two sides of P; when an arm M is brought near to P, it rubs the lower surface of a spring, and the liability to shock is greatly diminished.

NN, in figures 1, 2, 3, screws (with locking nuts) in MM, whose points touch the opposite side of P in the rotation above mentioned. By means of these, the instrument can be so adjusted that with telescope East or telescope West the image of the star will run properly along the wires of the micrometer.

P, in figures 1, 2, 3, the stop-piece fixed to D.

Q, in figure 2, an arm attached to the great spindle L, in a position intermediate between the two arms MM. It is so short that it can pass the stop-piece P without touching it.

R, in figures 2, 3, a slider upon Q, which can be slipped as a clutch over P. The instrument is then held in such a position that the image of the star crosses the micrometer-wires at right angles.

SS, in figures 2, 3, two strong arms fixed to the spindle L, for carrying the telescope and its counterpoises.

TT, in figures 1, 2, 3, the clamps that hold the telescope.

VV, in figures 2, 3, the counterpoises.

W, in figures 1, 2, 4, the telescope tube.

XX, in figures 1, 2, 3, 4, the two levels. The frame which supports them is fixed by a ring to the telescope tube; the glass tubes rest near their extremities in brass Y's; they are covered by plates of glass.

Y, in figures 1, 2, 3, 4, the illuminating reflector, which receives the light of one of the lamps I I, and throws it down to the eyepiece.

Z, in figures 1, 2, 4, the object-glass-cell. It fits so nicely in the top of the tube of W that it can be adjusted by three pairs of screws (thrusting and supporting), but is almost perfectly water-tight.

a, figure 4, is the object-glass of the telescope, consisting of a concavo-convex of flint glass, whose radii of curvature are about 9·0 and 5·3 inches; and a meniscus of crown glass, whose radii of curvature are about 5·3 and 50·0 inches. The flint glass is placed at the top, to avoid contact with the water. The meniscus is fixed into the object-glass-cell with a watertight cement.

b, figure 1, is the lip or spout, opening at the top, by which water is poured into the telescope-tube W. The tube is so completely filled with water that there is a slight upwards-pressure against the object-glass. It is apparently impossible to prevent air bubbles from accumulating under *a*; but a bent tube is inserted through *b*, and the bubbles are sucked out by a slight action of the observer's mouth.

c, *c*, figures 1, 2, 4, are two thermometers for indicating the temperature of the water in the tube; their bulbs are within the tube and their scales without it. The thermometers have been carefully compared. The temperature of the upper part of the water is almost always higher by 2° Fahrenheit than that of the lower part.

d, figure 4, is the plane glass at the bottom of the tube.

e, figures 4 and 5, is a sponge for wiping the upper surface of the glass *d*. It is carried by a rod which slides through the ball of a ball-and-socket joint.

f, figure 5 (also seen in figure 1), is a tap for discharging the water from the tube.

g, figures 5, 6, 8, is the micrometer case.

(A) and (B), in figures 1, 2, 5, 6, 8, are the two micrometer-heads, distinguished by those letters, which are engraved upon them.

h, figure 7, is the system of crossed wires on the fixed plate of the micrometer. The actual number of crosses is 27; but only 25 are required. The breadth visible in the field of view of the eyepiece is so much limited that only a succession of simple crosses, × ×, &c., is visible.

i, figure 8, is the first moveable plate of the micrometer, moved by the screw (A). The *appui* or mechanical rest of (A) is against *g*, but its screw is "tapped," or acts by screw-power, in *i*, drawing the plate *i* in opposition to two spiral springs. The plate *i* does not carry any wires, but serves as *appui* for (B).

k, figure 8, is the second moveable plate, moved by the screw (B). The *appui* of (B) is against a projecting part of *i*; its screw is "tapped" in *k*, drawing it in opposition to two spiral springs. The plate *k* carries 26 wires.

l, figure 8, the 26 wires, at intervals of 250″ nearly. It was intended that the interval should be nearly double the mean zenith-distance of γ Draconis; but the artist, in calculating the linear interval of the wires, omitted to consider the peculiar action of the water in the tube. The intervals of the crosses in *h* are sensibly the same as those of the wires *l*.

The peculiar utility of this combination of micrometer movements and wire-mountings may now be explained.

First, to ascertain every interval of the wires *l*. Since either the micrometer screw (A) alone, or the micrometer screw (B) alone, will move the wire *l*, we may apply the following process to either micrometer. The micrometer is so placed that every wire of *l* nearly coincides with a cross of *h*. The reading of the micrometer at exact coincidence of each wire with its corresponding cross, wire 1 with cross 1, wire 2 with cross 2, &c., is obtained by very small motion of the micrometer, the eyepiece being slid along the whole length of the system of wires. Then the micrometer is turned through nearly one interval of wires, and, by a similar process, the readings at exact coincidence of wire 2 with cross 1, of wire 3 with cross 2, &c., are found. Thus it is plain that the intervals, wire 1 to wire 2, wire 2 to wire 3, &c., through the series of 26 wires or 25 intervals, are found in terms of the revolutions and parts of the micrometer employed, with a motion of the micrometer not exceeding one interval. Then the intervals of the wires in celestial arc are to be found by observing transits of the star across the wires, the instrument being placed in the position represented in figure 2. And by comparison of these two measures of intervals, the value of the scale of the micrometer will be found, in terms of celestial arc.

Second, to measure the double zenith-distance of the star. It matters not whether the observer begins with (A) or (B) nearest to him; we will suppose that (B) is nearest, as shown in figure 1. He reads micrometer (A), enters the reading in his book, places the instrument with N touching P, and reads the levels X X. He sits upon H, turns the eyepiece *q* to a convenient position, and, when the star has arrived near to the center of the field, he bisects it with the proper wire by the screw (B). Without leaving his place, and without reading the micrometer, he rapidly rotates the instrument till the other screw N touches the other side of P, and makes another bisection of the star (usually with another wire, which, if the wires are nearly at the star's double zenith-distance, will require very little motion of the micrometer) by the screw (A). Then he reads the two levels. Finally, he brings the instrument to a convenient position, and takes the new readings of (A) and (B). Thus every element for a complete double determination in reversed positions of the instrument is obtained, by observations of the star occupying only a few seconds of time. In the reduction of the observations, the equivalents of the readings of micrometer (A) and micrometer (B) must for each observation be combined; it is convenient that the figures on the micrometer-heads should be so arranged that these equivalents shall be added together.

Returning now to the diagrams:—

m, figures 5 and 6 (seen also in figure 1), is the rapid-motion screw, by which the eyepiece socket is moved from wire to wire. Above it is seen a small scale, upon which slides a pointer carried by *m*, which indicates the m^o of the wire seen in the center of the field of view.

n, figures 4, 5, 6, is the eyepiece socket.

p, figures 4, 5, 6, contains the diagonal or prismatic reflector.

q, figures 1, 2, 4, 5, 6 (seen also in figure 3), the eyepiece, containing three lenses in the horizontal part, and one in the part which enters the socket; with adjusting-power by movement of the short tube which carries the two lenses nearest to the eye. The eyepiece rotates freely in the socket, as is shown by its position in figure 5; and in the observation of the star, as described above, on reversing the position of the instrument, the position of the eyepiece with respect to the instrument must be reversed, so that the eyepiece will have the same position as before with respect to the observer.

r, figure 9 (partially seen in figure 6), a bevelled toothed wheel, carried by one of the micrometer heads; both (A) and (B) are furnished with this and the following apparatus.

s, figure 9 (partially seen in figure 6), a bevelled pinion, to which a rather rapid motion is given by the rotation of *r*.

t, figure 9 (partially seen in figure 6), a worm or screw on the axis of *s*, acting as an endless screw in the toothed wheel *v* (which is partially seen in figure 6), and causing *v* to rotate slowly.

w, figures 6 and 9, a graduated drum, connected with *v*, exhibiting the number of total revolutions of the micrometer (A) or (B) in convenient magnitude.

The following are the observations for determination of the elements employed in the reduction of the observations:—

OBSERVATIONS FOR MICROMETER-INTERVAL OF WIRES. 1870, SEPTEMBER 4.

First Series.				Second Series.				Interval Measured.	Measure of Interval.	Aggregate Interval Measured.	Measure of Aggregate.
Pointer.	No. of Cross.	No. of Wire.	Reading of B.	Pointer.	No. of Cross.	No. of Wire.	Reading of B.				
			r				r		r		r
										1—1	0·000
0·2	2	1	27·754	1·2	2	2	31·295	1—2	3·541	1—2	3·541
1·2	3	2	·805	2·2	3	3	·233	2—3	3·428	1—3	6·969
2·2	4	3	·754	3·2	4	4	·283	3—4	3·529	1—4	10·498
3·2	5	4	·777	4·2	5	5	·242	4—5	3·465	1—5	13·963
4·2	6	5	·706	5·2	6	6	·216	5—6	3·510	1—6	17·473
5·2	7	6	·757	6·2	7	7	·226	6—7	3·469	1—7	20·942
6·2	8	7	·747	7·2	8	8	·250	7—8	3·503	1—8	24·445
7·2	9	8	·766	8·2	9	9	·238	8—9	3·472	1—9	27·917
8·2	10	9	·725	9·2	10	10	·199	9—10	3·474	1—10	31·391
9·2	11	10	·793	10·2	11	11	·286	10—11	3·493	1—11	34·884
10·2	12	11	·745	11·2	12	12	·250	11—12	3·505	1—12	38·389
11·2	13	12	·764	12·2	13	13	·250	12—13	3·486	1—13	41·875
12·2	14	13	·757	13·2	14	14	·234	13—14	3·477	1—14	45·352
13·2	15	14	·746	14·2	15	15	·229	14—15	3·483	1—15	48·835
14·2	16	15	·730	15·2	16	16	·234	15—16	3·504	1—16	52·339
15·2	17	16	·746	16·2	17	17	·278	16—17	3·532	1—17	55·871
16·2	18	17	·809	17·2	18	18	·270	17—18	3·461	1—18	59·332
17·2	19	18	·784	18·2	19	19	·279	18—19	3·495	1—19	62·827
18·2	20	19	·804	19·2	20	20	·281	19—20	3·477	1—20	66·304
19·2	21	20	·770	20·2	21	21	·251	20—21	3·481	1—21	69·785
20·2	22	21	·742	21·2	22	22	·250	21—22	3·508	1—22	73·293
21·2	23	22	·788	22·2	23	23	·253	22—23	3·465	1—23	76·758
22·2	24	23	·705	23·2	24	24	·230	23—24	3·525	1—24	80·283
23·2	25	24	·749	24·2	25	25	·230	24—25	3·481	1—25	83·764
24·2	26	25	·744	25·2	26	26	·233	25—26	3·489	1—26	87·253

The mean of the first five is $6^{r}\cdot994$; the mean of the last five is $80^{r}\cdot270$: the interval of means is $73^{r}\cdot276$; its half is $36^{r}\cdot638$.

FURTHER OBSERVATIONS FOR MICROMETER-INTERVAL OF WIRES.

1870, SEPTEMBER 4.

First Series.				Second Series.				Interval Measured.	Measure of Interval.	Aggregate Interval Measured.	Measure of Aggregate.
Pointer.	No. of Cross.	No. of Wire.	Reading of A.	Pointer.	No. of Cross.	No. of Wire.	Reading of A.				
			r				r		r		r
										1— 1	0·000
0·1	2	1	28·266	1·1	2	2	31·804	1— 2	3·538	1— 2	3·538
1·1	3	2	·317	2·1	3	3	·766	2— 3	3·449	1— 3	6·987
2·1	4	3	·264	3·1	4	4	·802	3— 4	3·538	1— 4	10·525
3·1	5	4	·284	4·1	5	5	·759	4— 5	3·475	1— 5	14·000
4·1	6	5	·217	5·1	6	6	·725	5— 6	3·508	1— 6	17·508
5·1	7	6	·278	6·1	7	7	·750	6— 7	3·472	1— 7	20·980
6·1	8	7	·267	7·1	8	8	·770	7— 8	3·503	1— 8	24·483
7·1	9	8	·282	8·1	9	9	·756	8— 9	3·474	1— 9	27·957
8·1	10	9	·236	9·1	10	10	·733	9—10	3·497	1—10	31·454
9·1	11	10	·303	10·1	11	11	·802	10—11	3·499	1—11	34·953
10·1	12	11	·265	11·1	12	12	·780	11—12	3·515	1—12	38·468
11·1	13	12	·275	12·1	13	13	·767	12—13	3·492	1—13	41·960
12·1	14	13	·275	13·1	14	14	·753	13—14	3·478	1—14	45·438
13·1	15	14	·259	14·1	15	15	·752	14—15	3·493	1—15	48·931
14·1	16	15	·260	15·1	16	16	·752	15—16	3·492	1—16	52·423
15·1	17	16	·260	16·1	17	17	·799	16—17	3·539	1—17	55·962
16·1	18	17	·320	17·1	18	18	·787	17—18	3·467	1—18	59·429
17·1	19	18	·286	18·1	19	19	·792	18—19	3·506	1—19	62·935
18·1	20	19	·313	19·1	20	20	·792	19—20	3·479	1—20	66·414
19·1	21	20	·277	20·1	21	21	·766	20—21	3·489	1—21	69·903
20·1	22	21	·256	21·1	22	22	·766	21—22	3·510	1—22	73·413
21·1	23	22	·295	22·1	23	23	·775	22—23	3·480	1—23	76·893
22·1	24	23	·222	23·1	24	24	·748	23—24	3·526	1—24	80·419
23·1	25	24	·270	24·1	25	25	·748	24—25	3·478	1—25	83·897
24·1	26	25	·260	25·1	26	26	·757	25—26	3·497	1—26	87·394

The mean of the first five is $7^{r}\cdot010$; that of the last five is $80^{r}\cdot403$: the interval of means is $73^{r}\cdot393$; its half is $36^{r}\cdot6965$.

DETERMINATION OF INTERVAL OF WIRES OF THE WATER-TELESCOPE IN ARC.

Day of Observation, 1870.	Times of Transit of γ Draconis over the Wires. No. of Wire.																									
	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1
	m s	s	s	s	s	s	s	s	s	s	s	s	s	s	s	s	s	s	s	s	s	s	s	s	s	m s
July 23	45. 11·7	39·0	6·4	34·7	2·0	30·0	56·8	25·0	52·7	20·0	47·7	16·0	43·5	10·4	38·3	6·2	33·8	1·9	28·8	56·6	24·2	51·8	19·0	46·8	14·4	56. 42·7
July 24	46. 54·5	22·5	49·5	18·5	46·0	..	39·8	8·2	35·7	3·0	31·0	58·5	26·5	53·5	20·9	49·0	16·6	45·5	11·6	39·4	6·7	34·8	2·0	30·2	57·4	58. 25·4

	July 23.	July 24.
	m s	m s
Mean of First five wires......	46. 6·76	47. 50·20
,, Last ,, ...	55. 46·94	57. 29·96
Interval	9. 40· 8	9. 39·76

The mean of these intervals is $9^{m}.\ 39^{s}\cdot97$, and the half-mean is $4^{m}.\ 49^{s}\cdot985$, which corresponds to the polar angle $1^{\circ}.\ 12'.\ 29''\cdot775$.

The mean of the declinations of γ Draconis on July 23 and July 24 is $51^{\circ}.\ 30'.\ 31''\cdot64$.

Hence the half-arc of great circle, joining the mean of the first five wires with the mean of the last five wires, expressed in seconds of arc, is equal to

$$\frac{\text{sine } 1^{\circ}.\ 12'.\ 29''\cdot775 \times \text{cosine } 51^{\circ}.\ 30'.\ 31''\cdot64}{\text{sine } 1''},$$

of which the logarithm is 3·4325005.

The logarithms of the same half-arc in micrometer-revolutions are

By Micrometer A 1·5646246
By Micrometer B 1·5639318

Hence the values of one revolution, expressed in seconds of arc, are

For Micrometer A $73''\cdot7693$
For Micrometer B $73''\cdot8871$

OBSERVATIONS FOR MICROMETER-INTERVAL BETWEEN WIRES 13 & 14. 1870, SEPTEMBER 4.

Order of Readings.	Pointer.	Cross.	Wire.	Reading of A.	Order of Readings.	Pointer.	Cross.	Wire.	Reading of A.
				r					r
1	12	14	13	25·815	2	13	14	14	29·285
3				·815	4				·289
5				·815	6				·292
7				·814	8				·283
9				·822	10				·282
11				·811	12				·283
13				·816	14				·287
15				·817	16				·286
17				·812	18				·275
19				·820	20				·285
Mean				$25^{r}\cdot8157$	Mean				$29^{r}\cdot2847$

The interval 13–14 is $3^{r}\cdot4690$ of A = $255''\cdot91$

OBSERVATIONS FOR MICROMETER-INTERVAL BETWEEN WIRES 14 & 15. 1871, MARCH 8.

Order of Readings.	Pointer.	Cross.	Wire.	Reading of A.	Order of Readings.	Pointer.	Cross.	Wire.	Reading of A.
				r					r
1	12·6	15	14	27·749	2	13·6	15	15	31·244
3				·751	4				·238
5				·749	6				·242
7				·752	8				·242
9				·751	10				·242
11				·749	12				·243
13				·750	14				·240
15				·751	16				·243
17				·751	18				·246
19				·754	20				·246
Mean				$27^{r}\cdot7507$	Mean				$31^{r}\cdot2426$

The interval 14–15 is $3^{r}\cdot4919$ of A = $257''\cdot60$

OBSERVATIONS FOR MICROMETER-INTERVAL BETWEEN WIRES 12 & 13. 1871, APRIL 3.

Order of Readings.	Pointer.	Cross.	Wire.	Reading of A.	Order of Readings.	Pointer.	Cross.	Wire.	Reading of A.
				r					r
1	10·6	13	12	27·774	2	11·6	13	13	31·262
3				·774	4				·264
5				·773	6				·260
7				·772	8				·260
9				·774	10				·263
11				·775	12				·260
13				·777	14				·258
15				·772	16				·260
17				·768	18				·262
19				·776	20				·261
Mean				$27^{\rm r}{\cdot}7735$	Mean				$31^{\rm r}{\cdot}2610$

The interval 12–13 is $3^{\rm r}{\cdot}4875$ of A = $257''{\cdot}28$

OBSERVATIONS FOR MICROMETER-INTERVAL BETWEEN WIRES 13 & 14. 1871, AUGUST 17.

Order of Readings.	Pointer.	Cross.	Wire.	Reading of A.	Order of Readings.	Pointer.	Cross.	Wire.	Reading of A.
				r					r
1	11·6	14	13	27·170	2	12·6	14	14	30·641
3				·169	4				·641
5				·166	6				·641
7				·164	8				·642
9				·162	10				·635
11				·171	12				·639
13				·173	14				·638
15				·167	16				·640
17				·170	18				·644
19				·170	20				·641
Mean				$27^{\rm r}{\cdot}1682$	Mean				$30^{\rm r}{\cdot}6402$

The interval 13–14 is $3^{\rm r}{\cdot}4720$ of A = $256''{\cdot}13$

For valuation of the divisions of the two levels carried by the telescope, the following process was adopted:—

The object-glass having been removed, and the tube filled with water as usual, a delicate portable level $(x\,y)$ was placed on the top of the tube, and adjusted to parallelism with the two levels $(a\,b)$ $(c\,d)$, attached to the telescope. By moving the foot of the instrument, the following sets of large and small readings for the three levels were obtained, two readings of each level being made in each position of the instrument. The numbers given are the sums of the readings for the two ends of the bubble.

	Large Readings.			Small Readings.		
Levels	$(c\,d)$	$(a\,b)$	$(x\,y)$	$(c\,d)$	$(a\,b)$	$(x\,y)$
1872, May 9.	215·9	160·0	84·4	151·3	92·8	55·7
	216·2	160·0	84·0	151·6	92·9	55·5
	217·5	160·1	84·0	147·1	86·5	53·9
	217·5	160·6	84·2	147·3	86·6	53·2
	219·4	163·0	85·6	148·0	87·3	53·4
	219·5	163·2	85·0	148·0	88·2	53·5
May 14.	209·1	156·5	77·4	152·7	94·6	53·8
	209·3	156·7	77·3	152·8	95·0	54·0
	213·9	158·8	79·7	154·8	98·1	55·2
	213·9	158·9	80·5	154·8	98·4	55·7
	215·5	159·6	79·8	149·5	91·2	52·5
	215·2	159·4	80·1	149·9	91·4	52·7
	215·8	159·8	79·8	151·9	93·6	52·2
	215·9	159·6	79·4	151·7	94·1	52·3

Levels	Large Readings. ($c\,d$)	($a\,b$)	($x\,y$)	Small Readings. ($c\,d$)	($a\,b$)	($x\,y$)
	217·3	160·0	79·5	150·2	90·9	51·0
	217·1	159·9	79·2	150·2	91·1	50·4
	218·2	160·5	80·2	148·7	89·7	48·7
	218·7	160·1	80·1	149·2	89·8	49·4
	221·8	163·9	82·4	150·2	91·2	53·0
	221·7	163·8	83·0	150·4	91·6	52·6

$$\text{Whence } \frac{1 \text{ div. of } (c\,d)}{1 \text{ div. of } (x\,y)} = \frac{566{\cdot}9}{1319{\cdot}1}\,;\ \frac{1 \text{ div. of } (a\,b)}{1 \text{ div. of } (x\,y)} = \frac{566{\cdot}9}{1369{\cdot}4}$$

The portable level was then attached to the clamping circle of the Transit-Circle, and the following readings of the circle and level made for determination of the value of the level divisions :—

	Large Readings. Corrected Circle-reading.	Level-reading.	Small Readings. Corrected Circle-reading.	Level-reading.
	′ ″	d	′ ″	d
1872, May 16.	50. 51 ·13	103 ·8	49. 27 ·53	37 ·8
	50. 39 ·34	95 ·7	49. 33 ·92	41 ·4
	50. 47 ·23	100 ·5	49. 37 ·92	43 ·8
	50. 44 ·72	98 ·8	49. 34 ·70	42 ·5
	50. 44 ·40	98 ·5	49. 35 ·03	41 ·7
	50. 45 ·17	98 ·7	49. 28 ·12	37 ·3
	50. 46 ·40	100 ·0	49. 33 ·78	41 ·5
	50. 44 ·94	99 ·0	49. 31 ·23	39 ·7
	50. 49 ·73	102 ·1	49. 35 ·43	42 ·6
	50. 47 ·87	100 ·7	49. 35 ·37	42 ·6

Whence 1 div. of $(x\,y) = 2''{\cdot}480$
and therefore 1 div. of $(a\,b) = 1{\cdot}027$
1 div. of $(c\,d) = 1{\cdot}066$.

The value given by Mr. Simms for one division of each level was 1″·00. And, as that value was obtained by simple mechanical means, and is supported generally by this investigation, it has been thought most prudent to retain it.

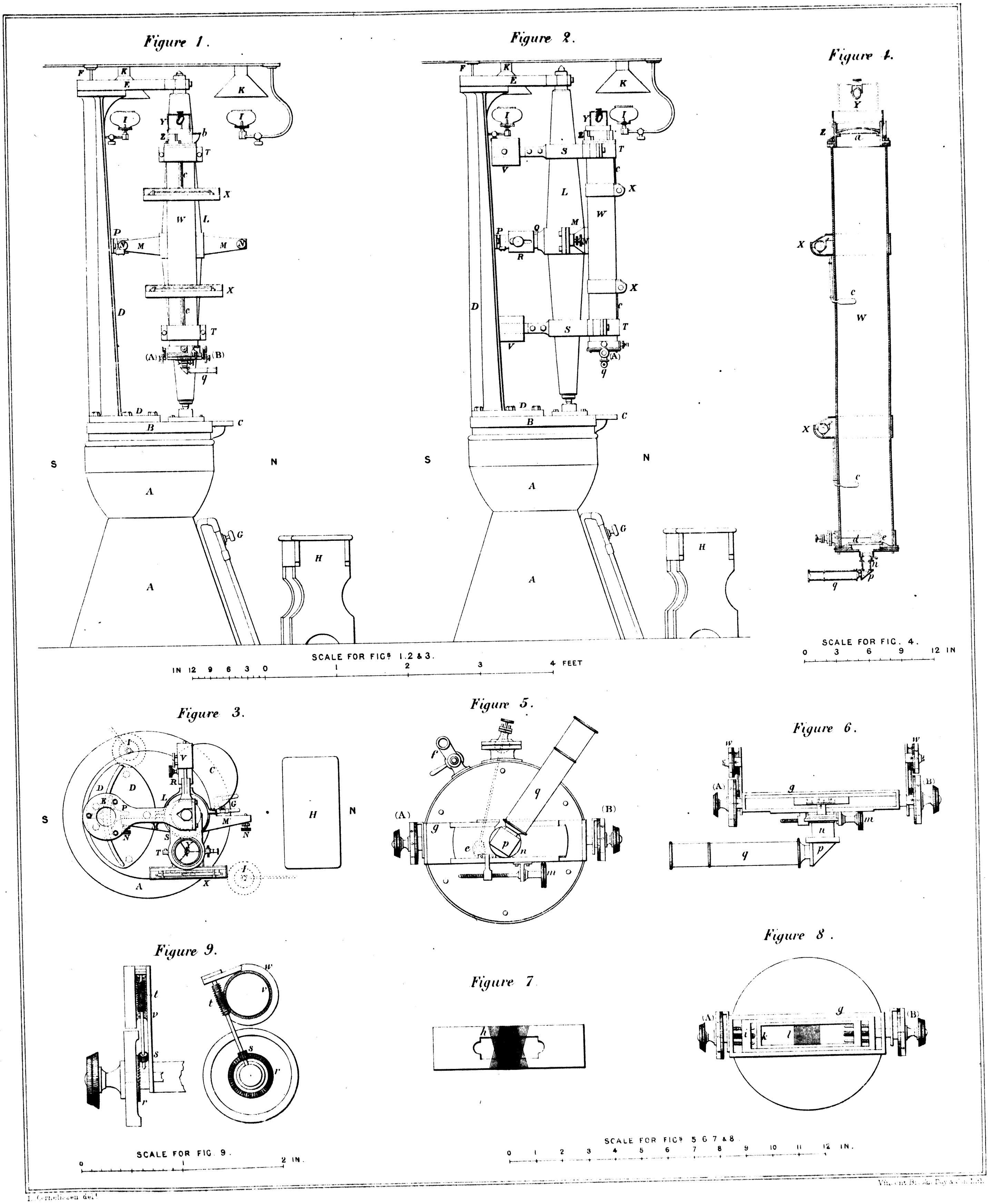


Figure 1.
Figure 2.
Figure 4.
S
N
S
N
SCALE FOR FIGS 1.2 & 3.
IN 12 9 6 3 0 1 2 3 4 FEET
SCALE FOR FIG. 4.
0 3 6 9 12 IN
Figure 3.
Figure 5.
Figure 6.
S
N
Figure 9.
Figure 7.
Figure 8.
SCALE FOR FIG 9.
0 1 2 IN.
SCALE FOR FIGS 5 6 7 & 8
0 1 2 3 4 5 6 7 8 9 10 11 12 IN.