II. *On a Determination of the Direction of the Meridian with a Russian Diagonal Transit Instrument.* By Captain Clarke, R.E., F.R.S.

Read 14 May, 1869.

The Diagonal form of Transit Instrument, namely, that in which the rays of light, instead of passing straight from the object-glass to the eye-piece, are bent at right angles by a prism in the central cube, and so pass out at one of the pivots—is not so well known in this country as in Germany and Russia. The advantages of the instrument are, that the observer without altering his position can observe stars of any declination, that the instrument requires but short pillars, and that the level can remain on the pivots whatever be the direction of the telescope, nor need it be taken off when the instrument is reversed.

The objections that may be urged *à priori* against this form of transit are, that the position of the observer is unfavourable, being seated at one side or the other of the instrument, and that there must be much difficulty in so fixing the central reflecting prism that it shall not vary within certain small limits, with changes of temperature for instance, and so cause the collimation centre to have a variable reading. The position of the observer at one side of the instrument is objectionable for two separate reasons, his weight tends to depress that side of the instrument at which he is sitting, while the warmth of his body tends to expand the upright near him. Thus he becomes a source of two level errors, of opposite signs however. But by taking pains to get a good foundation, it is always practicable to make the instrument quite insensible to the movements of the observer; and, indeed, this point has been always attended to with the utmost care in all the geodetical operations of the survey. And with respect to the effect of the warmth of the observer expanding the uprights, this is eliminated by the frequent reversal of the telescope, instruments of this class being always provided with a very easily manageable reversing apparatus. If the observer were to sit long at one side, certainly it would tell on the level, but this it is not advisable that he should do in any case, particularly as the telescope is so easily and rapidly reversed.

With respect to the fixity of the prism, the observations which have been made with two instruments of this class are very satisfactory.

Of these instruments, one was made in Russia, and the other—a copy of this—made by Messrs. TROUGHTON and SIMMS, of London. The former is the workmanship of M. BRAUER, of the Imperial Observatory at Pulkowa: it is the property of the Royal Observatory, and was kindly lent by the Astronomer Royal to the Director of the Ordnance Survey. The telescope of this instrument has a focal length of about 31 inches, and aperture of $2\frac{5}{8}$, the distance between the uprights is 19 inches. In the instrument made by SIMMS the telescope has 32 inches focal length and $2\frac{6}{8}$ aperture; and the micrometer box is so constructed that it can be turned at right angles so as to make meridian measures; the object being that the instrument may be used as a "Zenith Telescope."

In both of these instruments the reversing apparatus works very well. A very large number of observations have been made with the English instrument to test its stability under reversal, and it appears that the probable disturbance of the telescope in azimuth by one reversal is $\pm 0''\cdot193$, and the probable disturbance of level $\pm 0''\cdot133$. No special investigation has been made of these quantities for the Russian instrument, but there appears no reason to suppose that they are at all greater than in SIMMS'; they may, indeed, be smaller, as the telescope is much lighter. The transverse or pivot level of the Russian Transit is remarkable for its excellence, both in equality of the divisions, and in the quickness of its settling after any disturbance.

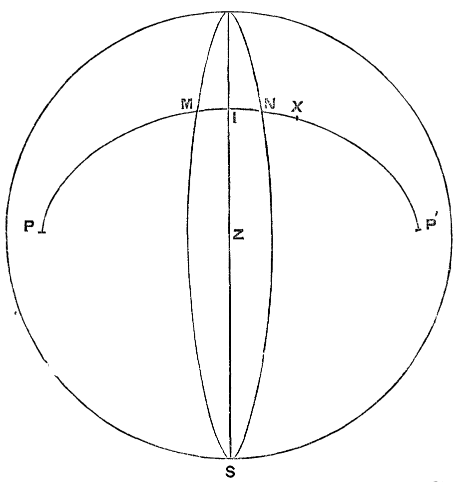
The Russian Transit was used at Findlay Seat, a mountain in Elginshire, 1104 feet high, from October 13th to 31st, 1868, for the purpose of determining the direction of the meridian. The reading of the collimation centre on the 14th was 1027·74 div. (one division $=0''\cdot835$) and on the 31st 1025·79 div. During this time the collimation was determined nearly every day with great care: the greatest and least values were 1027·78 and 1024·90 on the 16th and 28th respectively. On the 10th November the instrument was again set up at Southampton, and its collimation centre was found to be 1025·0, showing that the prism had remained undisturbed even by so long a journey.

But this class of instrument is subject to error through flexure of the transverse axis, and that to a very considerable amount. This first became apparent at Findlay Seat, the observations Circle East and Circle West could not be reconciled by the collimation centre derived from observations on a terrestrial mark. This led to determinations of the collimation centre by means of the Pole Star, and the collimation centre so found differed very sensibly from that resulting from the mark. A little consideration will explain this. The weight the transverse axis has to bear is greater than in the case of the ordinary transit instrument, because the weight of the counterpoise of the half telescope is of necessity considerably greater than the weight of the absent half of the telescope. If now we assume, and it can hardly be considered an assumption, that in any position of the telescope the central cube and the contained prism are simply depressed a certain constant amount, the effect will be that the image of a star in the field is always a certain distance vertically *below* its proper place, that is, the place it would have occupied if gravity did not depress the transverse axis. This being admitted, every micrometer reading of an object in the field requires a correction $-f \cos z$ where f is a constant which may be called the coefficient of flexure, and z is the zenith distance of the object observed. When the telescope is directed towards the zenith, the micrometer-screw head is downwards, and the large readings are in the lower part of the field, so that if the collimation centre were determined by a fixed point in the zenith the reading would be too great.

In order to determine the collimation centre at any zenith distance, an iron framework was constructed and attached to the central pillar in the dome of the observatory at Southampton, in such a manner that a collimating telescope could be mounted at any zenith distance from 0° to ±90°. The result of 172 determinations, extending over six days, is contained in the following table. At each position of the upper or moveable collimator, the collimation was determined alternately on this and on a second collimator fixed at 90° zenith distance, so that any change in the collimator might be thoroughly eliminated.

Zenith Distance of Upper Collimator.	Collimation Centre on Upper Collimator.	Collimation Centre on Collimator at $z = 90°$.	Difference.	Zenith Distance of Upper Collimator.	Collimation Centre on Upper Collimator.	Collimation Centre on Collimator at $z = 90°$.	Difference.
°	div.	div.	div.	°	div.	div.	div.
0	30·56	27·76	2·80	30	30·12	27·72	2·40
5	29·83	27·15	2·68	35	29·79	26·89	2·90
10	30·52	27·41	3·11	40	29·48	27·02	2·46
15	30·68	27·64	3·04	45	29·37	27·39	1·98
20	30·74	28·02	2·72	50	30·07	28·05	2·02
25	30·70	27·72	2·98	55	29·97	28·28	1·69

But the determination of the constant f is involved with the inequality of pivots. In order to trace this out clearly, suppose that the stand of the instrument is so levelled that the V's are at exactly the same height, then, if p be the inequality of the pivots, the circle-pivot being the larger, the inclination of the rotation axis Circle West will be $+p$, and Circle East $-p$.



Let P, P′ be the points in which the rotation axis meets the sphere Circle West and Circle East respectively. Z being the zenith, let S Z I be the vertical circle perpendicular to P Z P′; S N, S M, the great circles of of which P, P′ are the poles. Then $MI = NI = p \cos z$, where $ZI = z$. Let X be the point observed as collimating mark, $IX = x$: supposing then there were no flexure, we should have, if m, n were the micrometer readings of X, Circle West, and Circle East, μ_0 being the collimation centre, and D the value of one division of the micrometer screw,

$$\begin{aligned} XN &= x - p \cos z = (m - \mu_0)\, D, \\ XM &= x + p \cos z = (\mu_0 - n)\, D. \end{aligned} \qquad (1)$$

But the existence of flexure requires a correction to be made to each micrometer-reading, hence

$$\begin{aligned} x &= (m - f \cos z - \mu_0)\, D + p \cos z, \\ x &= (\mu_0 + f \cos z - n)\, D - p \cos z. \end{aligned} \qquad (2)$$

The difference of these gives

$$f \cos z = \frac{1}{2}(m+n) - \mu_0 + \frac{p}{D}\cos z. \tag{3}$$

If m_0, n_0 be the readings of a horizontal collimator, m_1, n_1 readings of a collimator fixed in the zenith,

$$\begin{aligned} \mu_0 &= \frac{1}{2}(m_0+n_0), \\ f &= \frac{1}{2}(m_1+n_1) - \frac{1}{2}(m_0+n_0) + \frac{p}{D}. \end{aligned} \tag{4}$$

On the other hand, when f has been determined by a large number of suitable observations at various zenith distances, if the instrument be collimated upon a point at a zenith distance z, the value of μ_0 may be inferred, for

$$\mu_0 = \frac{1}{2}(m+n) - \left(f - \frac{p}{D}\right)\cos z. \tag{5}$$

From equation (3) it appears that the difference between the collimation centres, as derived from a collimator at zenith distance z and a collimator at zenith distance $= 90°$, is

$$\left(f - \frac{p}{D}\right)\cos z = q \cos z.$$

The observations, of which results are given in the table above, give the values of $q \cos z$ for twelve values of z; and if we make the differences in the fourth and eighth columns successively equal to $q \cos z$, we have twelve equations, which, solved by the method of least squares, give

$$8{\cdot}850\, q = 26{\cdot}732. \tag{6}$$

This value of q, namely $q = 3{\cdot}021$, substituted in the equations, leads to the following system of residual errors:—

z.	Error.	z.	Error.	z.	Error.	z.	Error.
°	div.	°	div.	°	div.	°	div.
0	+0·22	15	−0·12	30	+0·22	45	+0·15
5	+0·33	20	+0·09	35	−0·43	50	−0·08
10	−0·13	25	−0·24	40	−0·15	55	+0·04

The sum of the squares of these errors is 0·5406, consequently making use of the weight in equation (6), we have

$$f = 3^{\text{div}}\cdot 021 + \frac{p}{D} \pm 0^{\text{d}}\cdot 050. \qquad (7)$$

There is a very considerable inequality in the pivots of this transit instrument. The result of 60 determinations is

$$p = 0''\cdot 650 \pm 0''\cdot 022. \qquad (8)$$

The value of one division of the level determined from numerous observations made on a level trier at the mean temperature during the period of the observations at Findlay Seat, is

$$l = 1''\cdot 810 \pm 0''\cdot 013. \qquad (9)$$

And the value of a division of the micrometer derived from transits of *Polaris* is

$$D = 0''\cdot 83450 \pm \cdot 00006. \qquad (10)$$

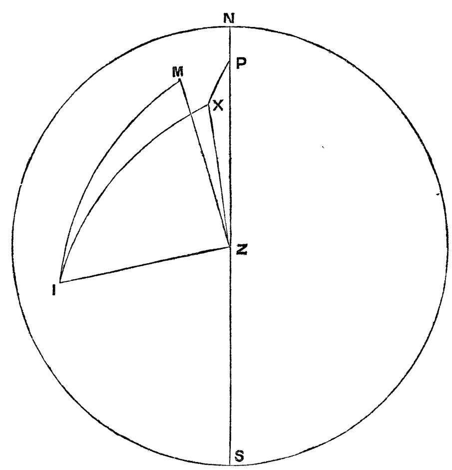
To determine the direction of the meridian at Findlay Seat with the Russian Transit Instrument the following method was adopted. Two marks (A) and (B) were erected, at nearly equal angular distances—a little more than two degrees—on either side of the north meridian line, so that the Pole Star crossed the vertical of each mark a short time before and after coming to its greatest eastern and western elongations. The marks were so placed that the observations could be made by daylight. Now, suppose the instrument directed to the mark, and adjusted by the azimuth screws until the centre wire is very nearly bisecting the mark, then if the telescope be directed to the altitude of the star, the star will at the proper time cross the different wires. If now, at known times, the position of the star in the field, or its distance from the central wire be measured by the micrometer, then since we know the azimuth of the star at any moment, it is clear that from every such measure we get a determination of the azimuth of the mark. Practically the observations were made thus: the instrument having been directed upon the mark and made as nearly level *as possible.*

1. The level is read.
2. The mark is bisected with the micrometer-wire.
3. The telescope being elevated to the star, the level is read, and reversed.
4. The star is bisected by the micrometer-wire three times in succession, the times being observed.
5. The level is read and telescope brought down to the mark.
6. The mark is bisected with micrometer-wire.
7. The level is read.

This completes one determination. The instrument is then reversed, and observations made in precisely the same manner, with the eye-piece in the reverse position. The observations commence when the star comes to about the first of the transit-wires, and are continued until it reaches the last wire. These outer wires are about $15\frac{1}{2}'$ distant from the centre wire. It will be seen by the arrangement of the observations as explained above that for each determination we have two level indications in each position of the level. The inclination of the rotation-axis is obtained by comparing the mean of the four west readings with the mean of the four east readings. The three observations of the star being made in close succession (a few seconds) the mean of the readings is assumed to correspond with the mean of the observed times.

The errors of the chronometers were determined by transits taken with this same instrument in the evenings.

With eye-piece east—the observer facing west—a star at its upper transit passes in the order of magnitude of the micrometer-readings—*or*, an object on the micrometer or eye-piece side of the vertical plane described by the telescope reads *less*, and an object on the opposite side reads greater than the collimation centre.



In the adjacent diagram, let P Z X M be the spherical positions of the Pole, the zenith, the star, and the mark; let also I be the point in which the rotation axis of the instrument meets the sphere, N S being the plane the meridian. At the known time of an observation of the star, let its azimuth from the south be α, and its zenith distance z. Let μ be the micrometer-reading of the star, μ' that of the mark, μ_0 the reading of the collimation centre. If D be the value of one division of the micrometer screw, then, supposing eye-piece West,

$$\mathrm{IX} = 90^\circ + (\mu - \mu_0)\,\mathrm{D}\,; \quad \mathrm{IM} = 90^\circ + (\mu' - \mu_0)\,\mathrm{D}.$$

For eye-piece East we have merely to alter the sign of D. Let, further, z' be the zenith distance of the mark, α' its azimuth, and b the inclination of the axis of rotation, then

$$\mathrm{ZI} = 90^\circ - b, \quad \mathrm{ZM} = z', \quad \mathrm{SZM} = \alpha',$$

and a being a small quantity let

$$\mathrm{SZI} = \alpha' - 90^\circ + a.$$

Finally, since $\mathrm{SZX} = \alpha$,

$$\mathrm{XZI} = 90^\circ - a + \alpha - \alpha'\,; \quad \mathrm{MZI} = 90^\circ - a.$$

Now

$$\mathrm{Cos\,IM} = \cos \mathrm{ZI} \cos \mathrm{ZM} + \sin \mathrm{ZI} \sin \mathrm{ZM} \cos \mathrm{IZM},$$
$$\mathrm{Cos\,IX} = \cos \mathrm{ZI} \cos \mathrm{ZX} + \sin \mathrm{ZI} \sin \mathrm{ZX} \cos \mathrm{IZX}.$$

Or

$$-\sin(\mu' - \mu_0)\,\mathrm{D} = \sin b \cos z' + \cos b \sin z' \sin a, \tag{11}$$
$$-\sin(\mu - \mu_0)\,\mathrm{D} = \sin b \cos z + \cos b \sin z \sin(\alpha' - \alpha + a). \tag{12}$$

In order to avoid the calculation of the precise azimuth and zenith distance of the star at each observation (mean of three star readings) we proceed as follows :—

Since the instrument is placed with its centre wire very nearly upon the mark, a is very small, only a few seconds, so also is b, consequently $\cos a = 1 = \cos b$. Also since $\alpha' - \alpha$ is only a few minutes, we may write for $\sin a \cos(\alpha' - \alpha)$ simply $\sin a$: thus equation (12) becomes

$$0 = \sin(\mu - \mu_0)\,\mathrm{D} + \sin b \cos z + \sin z \sin(\alpha' - \alpha) + \sin z \sin a$$

and substituting the value of $\sin a$ from (11)

$$0 = \sin b\,\frac{\sin(z' - z)}{\sin z'} + \sin(\mu - \mu_0)\,\mathrm{D} - \sin(\mu' - \mu_0)\,\mathrm{D}\,\frac{\sin z}{\sin z'} + \sin z \sin(\alpha' - \alpha) \tag{13}$$

In the triangle P Z X we have P Z X = 180 − α, and if φ be the latitude, h the hour-angle Z P X, and δ the north polar distance of the star, we have

$$-\sin z \cos \alpha = \cos \varphi \cos \delta - \sin \varphi \sin \delta \cos h,$$
$$\sin z \sin a = \sin \delta \sin h.$$

Multiply these by − sin α' and − cos α' and add, we get

$$\sin z \sin (\alpha' - \alpha) = - \cos \varphi \sin \alpha' \cos \delta + \sin \varphi \sin \delta \cos h \sin \alpha' - \cos \alpha' \sin \delta \sin h \quad (14)$$

Now let (α') be an approximate value of α', x the necessary correction to it, so that $(\alpha') + x = \alpha'$. Let (δ) be a mean value of the north polar distance of the star for the period over which the observations extended, $(\delta) + k$, the true N.P.D. for any particular observation. Let

$$\sin (\alpha') \sin (\delta) = \mathrm{H}, \quad (15)$$
$$\sin (\alpha') \cos (\delta) = \mathrm{I},$$
$$\cos (\alpha') \sin (\delta) = \mathrm{J},$$
$$\cos (\alpha') \cos (\delta) = \mathrm{K}.$$

Then

$$\sin \alpha' \cos \delta = \mathrm{I} - \mathrm{H}\,k + \mathrm{K}\,x, \quad (16)$$
$$\sin \alpha' \sin \delta = \mathrm{H} + \mathrm{I}\,k + \mathrm{J}\,x,$$
$$\cos \alpha' \sin \delta = \mathrm{J} + \mathrm{K}\,k - \mathrm{H}\,x.$$

If we substitute these in (14), and further put

$$\tan \mathrm{M} = - \frac{\mathrm{J}}{\mathrm{H} \sin \varphi} = - \frac{\mathrm{K}}{\mathrm{I} \sin \varphi} = \frac{-\cot (\alpha')}{\sin \varphi}, \quad (17)$$

$$\tan \mathrm{N} = - \frac{\mathrm{J} \sin \varphi}{\mathrm{H}} = - \cot (\alpha') \sin \varphi,$$

$$\mathrm{F} = \quad \mathrm{H} \cos \varphi \quad - \frac{\mathrm{K} \cos (h - \mathrm{M})}{\sin \mathrm{M}},$$

$$\mathrm{G} = - \mathrm{K} \cos \varphi \quad + \frac{\mathrm{H} \sin (h - \mathrm{M})}{\cos \mathrm{N}},$$

we get

$$\sin z \sin (\alpha' - \alpha) = - \mathrm{I} \cos \varphi \quad - \frac{\mathrm{J} \cos (h - \mathrm{M})}{\sin \mathrm{M}} + \mathrm{F}\,k + \mathrm{G}\,x,$$

or substituting this in (13) and dividing through by the sine of 1″, we have

$$o = b\frac{\sin(z'-z)}{\sin z'} \mp (\mu'-\mu_0)\,D\,\frac{\sin z}{\sin z'} \pm (\mu-\mu_0)\,D$$

$$+ F\,k + G\,x - \frac{I\cos\varphi}{\sin 1''} - \frac{J\cos(h-M)}{\sin M\sin 1''}, \tag{18}$$

where b, D, k, x are expressed in seconds. The upper signs apply to Circle West, the lower to Circle East.

Here b is to be derived from the known difference of pivots and the level indications. If w be the mean of the four readings of the west end of the bubble, e the mean of the four east readings, then from the manner in which the level has been used

$$b = \frac{1}{2}(w-e)\,l \mp p, \tag{19}$$

We have next to make the correction for flexure; that is, we have to add to μ and μ' the quantities $-f\cos z$ and $-f\cos z'$: thus (18) receives the increment

$$\pm f\cos z'\,.\,D\frac{\sin z}{\sin z'} \mp f\cos z\,.\,D = \mp f D\frac{\sin(z'-z)}{\sin z'}. \tag{20}$$

Equation (18) so modified becomes

$$o = \left\{\frac{1}{2}\,l\,(w-e) \mp (p + fD\right\}\frac{\sin(z'-z)}{\sin z'} \mp (\mu'-\mu_0)\,D\frac{\sin z}{\sin z'} \pm \ldots\ldots \tag{21}$$

From (7) we have $p + fD = 3{\cdot}021\,D + 2\,p = 3{\cdot}821$; the probable error of which is by (7) and (8)

$$\text{P. E. of } (fD + p) = \pm\sqrt{(0{\cdot}044)^2 + (0{\cdot}042)^2} = \pm\, 0{\cdot}061. \tag{22}$$

Now, finally, make

$$A = -\frac{I\cos\varphi}{\sin 1''} \mp (p + fD)\,\frac{\sin(z'-z)}{\sin z'}, \tag{23}$$

$$B = \frac{1}{2}\,l\,\frac{\sin(z'-z)}{\sin z'},$$

$$C = D\frac{\sin z}{\sin z'},$$

$$E = -\frac{J}{\sin M\sin 1''},$$

and (21) becomes

$$0 = A + B(w - e) \mp C(\mu' - \mu_0) \pm D(\mu_0 - \mu) + E\cos(h - M) + Fk + Gx. \quad (24)$$

The reduction of the observations by this formula becomes very simple. A, B, C, vary very slightly with the time, and are tabulated for every half hour; D, E, M, are constant; F and G are tabulated for every five minutes during the period occupied by the star in crossing the field.

With respect to μ_0, the values of this quantity, computed by equation (5) from daily observations on the marks, are shown in the following table for the days on which *Polaris* was observed for azimuth:—

	μ_0		μ_0		μ_0
	d		d		d
Oct. 14	1027·74	Oct. 20	1026·98	Oct. 26	1025·03
16	1027·78	21	1026·58	27	1025·05
17	1027·53	23	1026·49	31	1025·79
18	1027·28	25	1025·10		

For a mean value of the N.P.D. of *Polaris* during this same period we take $(\delta) = 1^\circ\ 23'\ 27''\cdot0$. The latitude of Findlay Seat is $\phi = 57^\circ\ 34'\ 50''\cdot0$, and the approximate azimuth and zenith distances of the marks are—

Mark (A) .. $(\alpha') = 177^\circ\ 45'\ 35''\cdot0$: $z' = 93^\circ\ 13'$,
Mark (B) .. $(\alpha') = 182^\circ\ 17'\ 15''\cdot0$: $z' = 93^\circ\ 2'$.

From these data, by equations (16) and (17) we get for mark (A),

$$\begin{aligned}
\log H &= 6\cdot9771784, \\
\log I &= 8\cdot5919406, \\
\log J &= 8\cdot3847777\ n, \\
\log K &= 9\cdot9995399\ n, \\
\log\tan M &= 1\cdot4811818\ ;\ M = 88^\circ\ 6'\ 31''\cdot0, \\
\log\tan N &= 1\cdot3340170\ ;\ N = 87^\circ\ 20'\ 47''\cdot9, \\
\log E &= 3\cdot6994395, \\
F &= 0\cdot0005 + [9\cdot99978]\cos(h - 88^\circ\ 6'), \\
G &= 0\cdot5355 + [8\cdot31167]\sin(h - 87^\circ\ 21'), \\
-\frac{I\cos\phi}{\sin 1''} &= -4321\cdot38,
\end{aligned}$$

whence

North-West Mark.

Time.	Z. D.	A.		B.	Log C.
		Circle W.	Circle E.		
h m	° ′				
8 30	32 54	−4324·70	−4318·06	0·783	9·6569
9 0	33 4	4324·69	4318·07	0·782	9·6589
9 30	33 13	4324·69	4318·07	0·781	9·6606
10 0	33 22	4324·68	4318·08	0·780	9·6624
10 30	33 29	−4324·68	−4318·08	0·779	9·6637

Time.	F.	G.	Time.	F.	G.
h m			h m		
8 25	·9388	·5428	9 30	·8050	·5479
8 30	·9311	·5432	9 35	·7919	·5483
8 35	·9229	·5437	9 40	·7784	·5486
8 40	·9143	·5441	9 45	·7645	·5489
8 45	·9052	·5445	9 50	·7502	·5492
8 50	·8958	·5449	9 55	·7356	·5496
8 55	·8859	·5453	10 0	·7207	·5499
9 0	·8755	·5456	10 5	·7054	·5502
9 5	·8648	·5460	10 10	·6898	·5506
9 10	·8536	·5464	10 15	·6738	·5509
9 15	·8421	·5468	10 20	·6575	·5512
9 20	·8301	·5471	10 25		·5516
9 25	·8177	·5475	10 30		·5519

For the North-East Mark (B) we have similarly,

$$\log H = 6{\cdot}9862329\ n,$$
$$\log I = 8{\cdot}6009951\ n,$$
$$\log J = 8{\cdot}3847636\ n,$$
$$\log K = 9{\cdot}9995258\ n,$$
$$\log \tan M = 1{\cdot}4721131\ n;\ M = -88^\circ\ 4'\ 7''{\cdot}4,$$
$$\log \tan N = 1{\cdot}3249483\ n;\ N = -87\ 17\ 26\ {\cdot}7,$$
$$\log E = 3{\cdot}6994355\ n,$$
$$F = -{\cdot}0005 - [9{\cdot}99977] \cos(h + 88^\circ\ 4'),$$
$$G = +{\cdot}5355 - [8{\cdot}31167] \sin(h + 87\ 17),$$
$$-\frac{I \cos \phi}{\sin 1''} = 4412{\cdot}42.$$

North-East Mark.

Time.	Z.D.	B. Circle W.	B. Circle E.	B.	Log. C.
h m 16 30	° ′ 33 20	4409·12	4415·72	0·778	9·6619
17 0	33 12	4409·11	4415·73	0·779	9·6603
17 30	33 2	4409·11	4415·73	0·780	9·6584
18 0	32 52	4409·10	4415·74	0·782	9·6564
18 30	32 42	4409·10	4415·74	0·783	9·6545

Time.	F.	G.	Time.	F.	G.
h m 16 35	−·7527	+·5492	h m 17 40	−·9068	+·5444
16 40	·7669	·5489	17 45	·9158	·5440
16 45	·7807	·5485	17 50	·9243	·5436
16 50	·7941	·5482	17 55	·9324	·5432
16 55	·8072	·5478	18 0	·9401	·5428
17 0	·8199	·5475	18 5	·9473	·5423
17 5	·8292	·5471	18 10	·9541	·5419
17 10	·8441	·5467	18 15	·9604	·5415
17 15	·8556	·5464	18 20	·9662	·5411
17 20	·8666	·5460	18 25	·9716	·5406
17 25	·8773	·5456	18 30	·9765	·5402
17 30	·8876	·5452	18 35	·9810	·5398
17 35	−·8974	+·5448	18 40	−·9850	+·5393

The following tables contain the whole of the observations on the two marks. After what has been said above, the nature of the quantities in the different columns requires no further explanation :—

North-West Mark.

Date.	Circle.	$w-e$	$\mu'-\mu_0$	$\mu-\mu_0$	Sidereal Time of Observation.	x	Azimuth of Mark.	Daily Mean.
1868. Oct. 14	E.	−2·40	−37·74	− 400·74	h m s 9 32 16·71	+3·70	° ′ ″ 177 45 38·70	
	W.	−0·05	+37·76	+ 611·06	9 44 41·70	+0·53	35·53	
	W.	+0·95	+38·06	+ 779·86	9 54 27·03	−1·00	34·00	
	E.	−1·68	−38·74	− 999·64	10 6 58·69	+4·00	39·00	″ 36·81
16	W.	−1·05	+29·42	− 111·78	8 55 24·98	+2·77	37·77	
	E.	−2·18	−33·48	− 41·48	9 7 48·64	+4·16	39·16	
	W.	+0·05	+32·62	+ 195·02	9 18 20·30	+1·96	36·96	
	E.	−1·58	−35·28	− 378·58	9 30 53·97	+2·46	37·46	
	W.	+2·23	+33·92	+ 535·62	9 40 27·97	+2·72	37·72	
	E.	−0·98	−35·58	− 861·58	9 59 29·63	+1·71	36·71	
	W.	+1·08	+34·12	+1068·82	10 10 20·96	+0·71	35·71	37·36
17	W.	−0·68	+45·07	+ 50·07	9 7 25·46	+5·22	40·22	
	E.	−1·20	−49·13	− 236·83	9 21 8·45	+4·55	39·55	
	W.	0·00	+48·87	+ 494·57	9 37 23·10	+4·74	39·74	
	E.	−1·38	−48·93	− 857·13	9 58 51·10	+3·06	38·06	39·39
18	W.	−1·95	+45·22	+ 438·22	9 33 44·91	+1·95	36·95	
	E.	+0·20	−48·78	− 638·08	9 46 31·91	+5·63	40·63	38·79
Oct. 20	W.	+0·43	+88·82	− 497·48	8 13 34·33	+1·59	36·59	
	E.	−1·35	−94·98	+ 297·02	8 36 9·33	+1·40	36·40	
	W.	−0·93	+86·22	+ 460·32	9 33 38·97	−0·15	34·85	
	E.	+0·28	−92·38	− 669·28	9 46 42·97	+0·42	35·42	
	W.	+2·15	+88·42	+ 899·42	9 59 41·31	+3·35	38·35	
	E.	+0·38	−93·38	−1137·28	10 12 38·30	+3·52	177 45 38·52	

North-West Mark (continued).

Date.	Circle.	$w-e$	$\mu'-\mu_0$	$\mu-\mu_0$	Sidereal Time of Observation.	x	Azimuth of Mark.	Daily Mean.
1868.					h m s		° ′ ″	
Oct. 25	W.	−0·95	+43·00	− 504·00	8 15 19·77	+0·07	177 45 35·07	
	E.	−5·58	−46·40	+ 345·00	8 33 35·43	+4·27	39·27	
	W.	+2·70	+44·20	− 181·60	8 48 40·43	+0·06	35·06	
	E.	+0·93	−45·30	+ 61·60	8 59 12·10	+1·65	36·65	
	W.	+3·25	+47·00	+ 181·30	9 16 47·77	+1·88	36·88	
	E.	−0·25	−45·80	− 303·50	9 25 34·43	+4·29	39·29	
	W.	+1·33	+48·30	+ 485·00	9 36 30·43	−0·09	34·91	
	E.	−0·53	−49·10	− 690·20	9 49 18·77	+4·06	39·06	
	E.	−0·28	−50·50	−1187·80	10 16 22·10	+3·76	38·76	″ 37·22
26	W.	−0·35	+42·57	− 390·53	8 28 28·83	+0·50	35·50	
	E.	−6·13	−46·33	+ 201·47	8 47 4·50	+5·47	40·47	
	W.	+0·03	+42·77	+ 21·67	9 5 3·50	+1·36	36·36	
	E.	−3·35	−42·03	− 193·23	9 18 8·50	+6·89	41·89	
	W.	−0·45	+41·57	+ 466·57	9 35 35·50	+2·99	37·99	
	E.	−0·70	−39·43	− 842·03	9 58 8·34	+2·31	37·31	
	W.	−0·68	+40·87	+1061·67	10 9 34·84	+2·60	37·60	38·16
31	W.	+1·60	−77·79	+1241·31	10 22 12·93	+2·01	177 45 37·01	37·01

North-East Mark.

Date.	Circle.	$w-e$	$\mu'-\mu_0$	$\mu-\mu_0$	Sidereal Time of Observation.	x	Azimuth of Mark.	Daily Mean.
1868.					h m s		° ′ ″	
Oct. 16	W.	−1·43	−49·28	+390·22	18 6 47·19	−1·46	182 17 13·54	
	E.	−2·58	+45·62	−469·68	18 18 11·86	+1·72	16·72	
	W.	+2·03	−45·88	+536·82	18 27 54·18	−0·59	14·41	
	E.	−1·88	+44·72	−595·88	18 40 20·85	+2·08	17·08	″ 15·44

North-East Mark (continued.)

Date.	Circle.	$w-e$	$\mu'-\mu_0$	$\mu'-\mu_0$	Sidereal Time of Observation.	x	Azimuth of Mark.	Daily Mean.
1868.					h m s		° ′ ″	
Oct. 17	W.	+1·53	−31·33	+243·47	17 49 59·27	−0·55	182 17 14·45	
	E.	−0·18	+30·27	−327·93	17 59 34·95	+0·63	15·63	
	W.	+0·40	−32·63	+410·47	18 8 33·62	+0·17	15·17	
	E.	−1·38	+28·27	−517·03	18 24 11·95	+2·09	17·09	″
	W.	+1·43	−29·73	+589·07	18 35 52·29	−0·78	14·22	15·31
20	W.	+1·60	−73·08	−880·38	16 30 58·87	−0·47	14·53	
	E.	+0·58	+73·42	+ 63·42	17 25 27·18	−1·50	13·50	
	W.	+2·60	−72·38	+107·02	17 39 32·01	+2·15	17·15	
	E.	+0·03	+71·32	−252·18	17 53 49·83	−1·42	13·58	
	W.	+0·35	−71·88	+408·12	18 11 6·82	+0·06	15·06	
	E.	−1·50	+70·82	−529·88	18 30 17·16	+0·74	15·74	14·93
21	W.	+0·13	−61·08	−558·98	16 49 42·10	−0·40	14·60	
	E.	−2·05	+60·82	+355·12	17 3 33·10	+3·02	18·02	
	W.	+1·33	−62·48	−186·68	17 15 9·93	+1·68	16·68	16·43
23	W.	−0·13	−66·69	−914·99	16 28 51·22	+1·31	16·31	
	E.	−0·85	+71·01	+681·21	16 43 3·55	+1·48	16·48	
	W.	+0·35	−74·49	−457·09	16 56 50·37	+1·44	16·44	
	E.	−0·05	+74·91	+287·81	17 8 48·03	−0·11	14·89	
	W.	0·00	−76·69	−133·89	17 19 39·02	+1·08	16·08	
	E.	−0·15	+76·51	− 11·69	17 31 50·67	−0·51	14·49	
	W.	+0·63	−77·59	+138·81	17 42 31·50	+0·09	15·09	
	E.	−0·35	+78·61	−240·79	17 53 14·65	−0·11	14·89	
	W.	+0·85	−79·99	+354·81	18 5 10·81	−0·83	14·17	
	E.	+0·08	+79·11	−431·99	18 16 3·47	+0·17	15·17	
	W.	+0·30	−79·89	+533·21	18 30 36·12	−1·71	13·29	
	E.	−1·30	+77·01	−580·19	18 41 30·11	+1·72	16·72	15·34
25	W.	+0·25	−66·30	−980·80	16 25 9·44	−0·29	14·71	
	E.	+0·90	+70·90	+752·20	16 38 53·11	+0·07	15·07	
	W.	+1·65	−72·60	−497·10	16 54 14·79	+0·35	15·35	
	E.	+0·65	+75·30	+332·00	17 5 47·45	−0·69	182 17 14·31	

North-East Mark (continued).

Date.	Circle.	$w-e$	$\mu'-\mu_0$	$\mu-\mu_0$	Sidereal Time of Observation.	x	Azimuth of Mark.	Daily Mean.
1868.					h m s		° ′ ″	
Oct. 25 (continued)	W.	+1·03	−72·90	−192·30	17 15 6·29	−1·52	182 17 13·48	
	E.	−0·43	+75·20	− 31·20	17 33 32·29	+0·53	15·53	
	W.	+0·38	−72·10	+160·90	17 44 19·29	−0·57	14·43	
	W.	+0·80	−73·40	+391·30	18 9 18·13	−1·03	13·97	
	E.	−1·80	+73·90	−468·60	18 21 0·80	+2·31	17·31	
	W.	+0·93	−71·90	+533·60	18 30 29·80	+1·24	16·24	
	E.	−1·05	+72·90	−565·60	18 38 10·46	+2·71	17·71	″ 15·28
27	W.	+0·10	+52·65	−323·05	17 1 6·61	−0·16	182 17 14·84	14·84

If we proceed to form the probable errors of a single determination from the numbers in the last column but one of these tables, we find for mark (A),

P. E. of a single determination	± 1″·24,
„ the general mean	± 0·196.

And for mark (B),

P. E. of a single determination	± 0″·82,
„ the general mean	± 0·126.

The reason why the observations upon mark (A) are not so good as those on (B) is, that the former was generally observed with difficulty in a bad light. When these marks were erected it was supposed that the observations would have been taken in the end of September, and the marks were so placed that (A) was to have been seen in the mornings by daylight. But the unavoidable delay of the observations to October, combined with the rapid shortening of the days and the earlier transit of the star, brought the observations of mark (A) into the morning twilight, so that the mark was almost always observed with difficulty, and a lamp had to be used at times. The observations on mark (B), taken in good daylight in the afternoon, are therefore to be considered as a fair test of the efficiency of the instrument; and when the inherent difficulty of determining the direction of the meridian in so high a latitude is duly considered, and the circumstances under which the observations were made, in stormy weather, on the top of a high mountain, until the party were fairly driven away by snow and the destruction of some of the tents, it will, doubtless, be admitted that the results here shown are strong evidence in favour of the stability and excellence of this transit instrument.

There remains one more correction to be applied to the azimuths deduced above. When the instrument was in the meridian, its centre was precisely over the centre-mark of the station at Findlay Seat. But when turned on to mark (A), the centre of the instrument was exactly 0·30 inch to the west of the centre of the station, and the same quantity to the east of the station when turned on to mark (B). This requires a correction of $-0''\cdot295$ to the observed azimuth of (A), and of $+0''\cdot273$ to the observed azimuth of (B). Hence

$$\text{Azimuth of Mark A} = 177^\circ\ 45'\ 37''\cdot29 \pm 0\cdot196,$$
$$\text{Azimuth of Mark B} = 182\ \ 17\ \ 15\cdot61 \pm 0\cdot126.$$

A further check upon these results is afforded by the direct measurement of the angle between the marks. The angle was measured most carefully with one of RAMSDEN's theodolites, and found to be

$$4^\circ\ 31'\ 37''\cdot43 \pm 0''\cdot232,$$

while the angle resulting from the absolute azimuths by the Russian Transit is

$$4^\circ\ 31'\ 38''\cdot32 \pm 0''\cdot233.$$

The preceding observations and calculations were made in connexion with the Ordnance Survey, but Colonel Sir HENRY JAMES has kindly given me his permission to present them to the Royal Astronomical Society.

The observations at Findlay Seat were made by Quarter-master STEEL and Serjeant BUCKLE, Royal Engineers.