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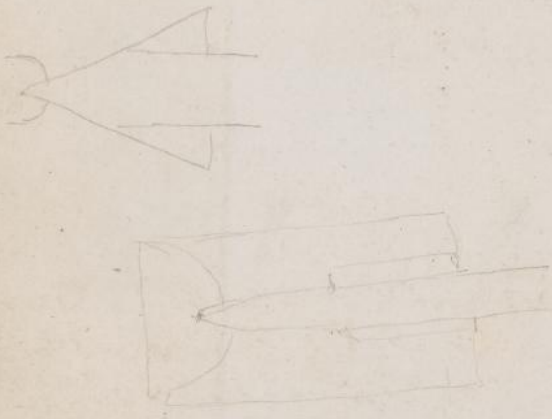
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Herschel's Pamphlets — Method of computing the orbit of a double stars





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Original

Method of determining the Period of a Variable Star

Let $t, t' t'' \ldots\ldots$ be the observed times of maxima of a variable star, an approximate period of which is known. Let P be an assumed period, and y a correction to this assumed period, so that the true period $= P + y$

Let T be the approximate assumed time for any maximum arbitrarily selected as an epoch; x, a correction to this assumed time, so that the true epoch $= T + x$.

Also let m be the number of periods elapsed between T and t', m' be the number between T and t' &c. &c. m being positive after the epochal maximum, and negative before.

We have for each observed maximum,

$$\begin{aligned} T + x + m(P+y) &= t \\ T + x + m'(P+y) &= t' \\ T + x + m''(P+y) &= t'' \\ &\vdots \end{aligned}$$

or

$$\begin{aligned} x + my + [T + mP - t] &= 0 \\ x + m'y + [T + m'P - t'] &= 0 \\ x + m''y + [T + m''P - t''] &= 0 \\ &\vdots \end{aligned}$$

And a solution of these equations by least squares will give the required corrections x and y, which applied to T and P, will give the true epoch and period, respectively.

For an example, see page 5-8

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Determination of the Period of δ Cephei from observations by J. F. J. Schmidt 5 given in Ast Nach. no—
(Solved, as an example of the method given on ~~preceding~~ first page)

Assumed Epoch T = 1863 Jan 21^d 9.0
" Period P = 5^d 9^h.00

Observed Maxima	N	bP	Computed Maxima	O − C	corr. (x + my)	v	
		d h	d h	h			
1862 Jan 5 9.5	−71	−381 15	Jan 4 18	−15.5	+16.0	+0.5	
31 20.5	66	354 18	31 15	−5.5	14.9	+9.4	
Feb. 6 16.5	65	349 9	Feb 6 0	−16.5	14.7	−1.9	
Mar 27 13.0	56	301 0	Mar 26 9	−28.0	12.6	−15.4	
Apr 22 12.0	51	274 3	Apr 22 6	−6.0	11.4	+5.4	
28 8.0	50	268 18	27 15	−17.0	11.2	−5.8	
May 24 18.5	45	241 21	May 24 12	−6.5	10.0	+3.5	49 −0.6
30 1.5	44	236 12	29 21	−4.5	9.8	+5.3	
June 5 1.0	43	231 3	June 4 6	−21.0	9.6	−11.4	
9 19.0	42	225 18	9 15	−4.0	9.4	+5.4	
16 1.5	41	220 9	15 0	−25.5	9.1	−16.4	
20 21.0	40	215 0	20 9	−12.0	8.9	−3.1	40 −2.4
25 17.0	39	209 15	25 18	+1.0	8.7	+7.7	
July 1 20.0	38	204 6	July 1 3	−17.0	8.4	−8.6	
6 23.0	37	198 21	6 12	−11.0	8.2	−2.8	
11 20.5	36	193 12	11 21	+0.5	8.0	+8.5	35 −1.0
17 19.5	35	188 3	17 6	−13.5	7.7	−5.8	
22 21.5	34	182 18	22 15	−6.5	7.5	+1.0	
28 13.0	33	177 9	28 0	−13.0	7.3	−5.7	
Aug 2 17.0	32	172 0	Aug 2 9	−8.0	7.1	−0.9	
7 18.5	31	166 15	7 18	−0.5	6.8	+6.3	30 +4.2
13 2.5	30	161 6	13 3	+0.5	6.6	+7.1	
18 16.5	29	155 21	18 12	−6.5	6.4	−0.1	
23 18.5	28	150 12	23 21	+2.5	6.1	+8.6	
29 16.5	27	145 3	29 6	−10.5	5.9	−4.6	
Sept 3 21.5	26	139 18	Sept. 3 15	−6.5	5.7	−0.8	
8 21.0	25	134 9	9 0	+3.0	5.4	+8.4	25 +1.3
14 15.5	24	129 0	14 9	−6.5	5.2	−1.3	
19 18.0	23	123 15	19 18	0.0	5.0	+5.0	
25 16.5	22	118 6	25 3	−13.5	4.7	−8.8	
30 13.0	21	112 21	30 12	−1.0	4.5	+3.5	
Oct 5 17.0	20	107 12	Oct. 5 21	+4.0	4.3	+8.3	20 +0.6
11 12.5	19	102 3	11 6	−6.5	4.1	−2.4	
16 16.5	18	96 18	16 15	−1.5	3.8	+2.3	
22 2.0	17	91 9	22 0	−2.0	3.6	+1.6	
27 22.0	16	86 0	27 9	−13.0	3.4	−9.6	
Dec 4 1.0	9	49 15	Dec. 4 0	−1.0	1.8	−2.8	−15 +5.2
19 6.0	6	32 6	20 3	+21.0	1.1	+22.1	
24 22.5	−5	−26 21	25 12	+14.0	+0.8	+14.8	
1863 Jan 21 9	0	0 0	Jan 21 9	0.0	−0.3	−0.3	
27 0	+1	+5 9	26 18	−6.0	0.5	−6.5	+2 −4.4
Feb. 1 12	2	10 18	Feb. 1 3	−9.0	0.8	−9.8	
6 18	3	16 3	6 12	−6.0	1.0	−7.0	
11 18	4	21 12	11 21	+3.0	1.2	+1.8	
17 21	5	26 21	17 6	−15.0	1.5	−13.5	
22 18	6	32 6	22 15	−3.0	1.7	−4.7	+5 −2.7
27 12	+7	+37 15	28 0	+12.0	−1.9	+10.1	

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Observed Maxima		b	ΔP	Computed Maxima		C − O = n	corr. (x + my)	v	
			d h			h			
Mar 23	18	+18	+43 0	1883 Mar 5	9	[+39.0] reject.	−2.2		
Apr 1	2	13	69 21	Apr 1	6	+4	3.2	+0.8	
6	15	14	75 6		6 15	0	3.4	−3.4	
11	18	15	80 15		12 0	+6	3.6	+2.4	15 +1.4
16	18	16	86 0		17 9	+15	3.9	+11.1	
22	18	17	91 9		22 18	0	4.1	−4.1	
May 8	18	20	107 12	May	8 21	+3	4.9	−1.9	
14	16	21	112 21		14 6	−10	5.1	−15.1	20 (−8.0)
24	20	23	128 15		25 0	+4	5.6	−1.6	
30	17	24	129 0		30 9	−8	5.8	−13.8	
June 4	17	25	134 9	June	4 18	+1	6.1	−5.1	
9	20	26	139 18		10 3	+7	6.3	−0.7	25 (−1.7)
14	15	27	145 3		15 12	+21	6.5	+14.5	
20	6	28	150 12		20 21	+15	6.7	+8.3	
25	14	29	155 21		26 6	+16	6.9	+9.1	30 (+5.0)
July 1	0	30	161 6	July	1 15	+15	7.2	+7.8	
6	15	31	166 15		7 0	+9	7.4	+1.6	
12	3	32	172 0		12 9	+6	7.6	−1.6	
17	18	33	177 9		17 18	0	7.9	−7.9	
22	19	34	182 18		23 3	+8	8.2	−0.2	35 +2.2
27	18	35	188 3		28 12	+18	8.4	+9.6	
Aug 2	9	36	193 12	Aug	2 21	+12	8.6	+3.4	
7	15	37	198 21		8 6	+15	8.9	+6.1	
13	5	38	204 6		13 15	+10	9.1	+0.9	
18	15	39	209 15		19 0	+9	9.3	−0.3	40 +2.7
23	18	40	215 0		24 9	+15	9.5	+5.5	
29	3	41	220 9		29 18	+15	9.7	+5.3	
Sept 3	15	42	225 18	Sept.	4 3	+12	10.0	+2.0	
8	20	43	231 3		9 12	+16	10.2	+5.8	
14	15	44	236 12		14 21	+6	10.4	−4.4	45 +0.5
19	22	45	241 21		20 6	+8	10.6	−2.6	
24	19	46	247 6		25 15	+20	10.9	+9.1	
30	18	47	252 15		31 0	+6	11.1	−5.1	
Oct 6	3	48	258 0	Oct.	6 9	+6	11.3	−5.3	
11	3	49	263 9		11 18	+15	11.6	+3.4	50 −11.3
16	21	50	268 18		17 3	+6	11.8	−5.8	
21	20	51	274 3		22 12	+16	12.0	+4.0	
27	13	52	279 12		27 21	+8	12.3	−4.3	
Nov 1	15	53	284 21	Nov.	2 6	+15	12.5	+2.5	
6	22	54	290 6		7 15	+17	12.7	+4.3	
12	15	55	295 15		13 0	+9	13.0	−4.0	55 −0.3
17	14	56	301 0		18 9	+19	13.2	+5.8	
23	15	57	306 9		23 18	+3	13.4	−10.4	
28	18	58	311 18		29 3	+9	13.6	−4.6	
Dec 3	15	59	317 3	Dec.	4 12	+21	13.9	+7.1	60
9	13	60	322 12		9 21	+8	14.1	−6.1	−3.9
14	18	61	327 21		15 6	+12	14.3	−2.3	
20	18	62	333 6		20 15	−3	14.6	−17.6	
30	14	+64	+344 0		31 9	+18	−15.0	+30	

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n	b	s	b²	bs	s²	bn	sn	nn	n	b	s	b²	bs	s²	bn	sn	nn
-15.5	-71	-70	+5041	+4970	+4900	+1100.5	+1085.0	240.25	+4	+13	+14	+169	+182	+196	+52.0	+56.0	16
-5.5	66	65	4356	4290	4225	363.0	357.5	30.25	0	14	15	196	210	225	0.0	0.0	0
-16.5	65	64	4225	4160	4096	1072.5	1056.0	272.25	6	15	16	225	240	256	90.0	96.0	36
-24.0	56	55	3136	3080	3025	1568.0	1540.0	784.00	15	16	17	256	272	289	240.0	255.0	225
-6.0	51	50	2601	2550	2500	306.0	300.0	36.00	0	17	18	289	306	324	0.0	0.0	0
-17.0	50	49	2500	2450	2401	850.0	833.0	289.00	+3	20	21	400	420	441	+60.0	+63.0	9
-6.5	45	44	2025	1980	1936	292.5	286.0	42.25	-10	21	22	441	462	484	-210.0	-220.0	100
-4.5	44	43	1936	1892	1849	198.0	193.5	20.25	+4	23	24	529	552	576	+92.0	+96.0	16
-21.0	43	42	1849	1806	1764	903.0	882.0	441.00	-8	24	25	576	600	625	-192.0	-200.0	64
-4.0	42	41	1764	1722	1681	168.0	164.0	16.00	+1	25	26	625	650	676	+25.0	+26.0	1
-25.5	41	40	1681	1640	1600	1045.5	1020.0	650.25	7	26	27	676	702	729	182.0	189.0	49
-12.0	40	39	1600	1560	1521	+480.0	+468.0	144.00	21	27	28	729	756	784	567.0	588.0	441
+1.0	39	38	1521	1482	1444	-39.0	-38.0	1.00	15	28	29	784	812	841	420.0	435.0	225
-17.0	38	37	1444	1406	1369	+646.0	+629.0	289.00	16	29	30	841	870	900	464.0	480.0	256
-11.0	37	36	1369	1332	1296	+407.0	+396.0	121.00	15	30	31	900	930	961	450.0	465.0	225
+0.5	36	35	1296	1260	1225	-18.0	-17.5	0.25	9	31	32	961	992	1024	279.0	288.0	81
-13.5	35	34	1225	1190	1156	+472.5	+459.0	182.25	6	32	33	1024	1056	1089	192.0	198.0	36
-6.5	34	33	1156	1122	1089	221.0	214.5	42.25	0	33	34	1089	1122	1156	0.0	0.0	0
-13.0	33	32	1089	1056	1024	429.0	416.0	169.00	8	34	35	1156	1190	1225	272.0	280.0	64
-8.0	32	31	1024	992	961	256.0	248.0	64.00	18	35	36	1225	1260	1296	630.0	648.0	324
-0.5	31	30	961	930	900	+15.5	+15.0	0.25	12	36	37	1296	1332	1369	432.0	444.0	144
+0.5	30	29	900	870	841	-15.0	-14.5	0.25	15	37	38	1369	1406	1444	555.0	570.0	225
-6.5	29	28	841	812	784	+188.5	+182.0	42.25	10	38	39	1444	1482	1521	380.0	390.0	100
-2.5	28	27	784	756	729	70.0	67.5	6.25	9	39	40	1521	1560	1600	351.0	360.0	81
-10.5	27	26	729	702	676	283.5	273.0	110.25	15	40	41	1600	1640	1681	600.0	615.0	225
-6.5	26	25	676	650	625	+169.0	+162.5	42.25	15	41	42	1681	1722	1764	615.0	630.0	225
+3.0	25	24	625	600	576	-75.0	-72.0	9.00	12	42	43	1764	1806	1849	504.0	516.0	144
-6.5	24	23	576	552	529	+156.0	+149.5	42.25	16	43	44	1849	1892	1936	688.0	704.0	256
0.0	23	22	529	506	484	0.0	0.0	0.00	6	44	45	1936	1980	2025	264.0	270.0	36
-13.5	22	21	484	462	441	297.0	283.5	182.25	8	45	46	2025	2070	2116	360.0	368.0	64
-1.0	21	20	441	420	400	+21.0	+20.0	1.00	20	46	47	2116	2162	2209	920.0	940.0	400
+4.0	20	19	400	380	361	-80.0	-76.0	16.00	6	47	48	2209	2256	2304	282.0	288.0	36
-6.5	19	18	361	342	324	+123.5	+117.0	42.25	6	48	49	2304	2352	2401	288.0	294.0	36
-1.5	18	17	324	306	289	27.0	25.5	2.25	15	49	50	2401	2450	2500	735.0	750.0	225
-2.0	17	16	289	272	256	34.0	32.0	4.00	6	50	51	2500	2550	2601	300.0	306.0	36
-13.0	16	15	256	240	225	208.0	195.0	169.00	16	51	52	2601	2652	2704	816.0	832.0	256
-1.0	9	8	81	72	64	+9.0	+8.0	1.00	8	52	53	2704	2756	2809	416.0	424.0	64
+21.0	6	5	36	30	25	-126.0	-105.0	441.00	15	53	54	2809	2862	2916	795.0	810.0	225
+14.0	-5	-4	25	20	16	-70.0	-56.0	196.00	17	54	55	2916	2970	3025	918.0	935.0	289
0.0	0	+1	0	0	1	0.0	0.0	0.00	9	55	56	3025	3080	3136	495.0	504.0	81
-6.0	+1	2	1	2	4	6.0	12.0	36.00	19	56	57	3136	3192	3249	1064.0	1083.0	361
-9.0	2	3	4	6	9	18.0	27.0	81.00	3	57	58	3249	3306	3364	171.0	174.0	9
-6.0	3	4	9	12	16	+18.0	-24.0	36.00	9	58	59	3364	3422	3481	522.0	531.0	81
+3.0	4	5	16	20	25	+12.0	+15.0	9.00	21	59	60	3481	3540	3600	1239.0	1260.0	441
-15.0	5	6	25	30	36	-75.0	-90.0	225.00	8	60	61	3600	3660	3721	480.0	488.0	64
-2.0	6	7	36	42	49	-18.0	-21.0	9.00	+12	61	62	3721	3782	3844	+732.0	+744.0	144
+12.0	+7	+8	+49	+56	+64	+84.0	+96.0	144.00	-3	62	63	3844	3906	3969	-186.0	-189.0	9
									+18	+64	65	+4096	+4160	+4225	+1152.0	+1170.0	324
-282.5	-1266	-1219	+52296	+51030	+49811	+11918.5	+11636.0	+5692.75	+453.0	+1880	+1928	+83652	+85532	+87460	+19501.0	+19954.0	+6749.0

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[aa] [ab] [as] [an] [bb] [bs] [bn] [sn] [nn]
95 +614 +709 +170.5 +135948 +136562 +31419.5 +31590 +12441.75

Direct Solution

95	614	709	170.5
1.977724	2.788168	2.850646	2.231724
+170.5	135948	136562	31419.5
−141.04	3968.37	4582.37	1102.0
+ 29.46	131979.63	131979.63	30317.5
1.469233	5.120506		4.481693
n 9.491509	n 9.361187		
−0.3101	−0.2297		
x	y	31590	12441.75
		12728	306.00
		30817.5	12135.75
		30317.5	6964.35
		[Mn.3]	5171.40
			3.713608
6.844351			1.968483
9.780268	6.624619		1.745125
±0.7765	±0.0205		±7.457
εx	εy		ε

Inverse Solution

135948	614	136562	31419.5
5.133373	2.788168	5.135330	4.497199
+31419.5	95	709	170.5
− 190.4	2.77308	616.772	141.904
+31229.1	92.22692	92.228	28.6
4.494559	1.964858		1.456366
n 9.361186	n 9.49 1508		
−0.2297	−0.3101		
y	x	31590	12441.75
		31561.4	7261.50
		28.6	5180.25
		28.6	8.87
		n.n.3	5171.38
			3.713608
			1.968483
3.155649			1.745125
6.624618	9.780267		± 7.457
±0.0205	±0.7765		
εy	εx		ε

The assumed value for Epoch is 1863 Jan 21^d 9^h.00
" " " " Period " 5 9.00
The corrections are { for x = − 0.3101 = 0^h 19^m.4
" y = − 0.2297 = 0 13^m 47^s

Hence our corrected values are:—

δ Cephei { Epoch 1863 Jan 21^d 8^h 40^m.6 with a probable error of 31^m.9
Period 5.36543 days or 5^d 8^h 46^m.13^s " " " " " 7^s }

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Original

Interpolation

Method of deducing constants varying with the powers of the time, from a system of differences of a correct ephemeris.

We have the ~~common~~ interpolation formula

$$F^{(n)} = F + na + \frac{n^2}{2} b + \frac{(n+1)\,n(n-1)}{1.2.3} c + \frac{(n+1)\,n^2(n-1)}{1.2.3.4} d + \&c.$$

where a, c, e, g, &c. are the means of the odd differences above and below the line of the function from which we set out; while b, d, f., h &c. are the even differences themselves on that line.

Performing the ~~multiplications~~ indicated in the formula given above, and arranging the terms, we have

I $$F^{(n)} = I + \alpha n + \frac{n^2}{1.2} \beta + \frac{n^3}{1.2.3} \gamma + \frac{n^4}{1.2.3.4} \delta + \&c.$$

where

$$\alpha = a - \tfrac{1}{6} c + \tfrac{1}{30} e - \tfrac{1}{140} g + \tfrac{1}{630} i + \&c.$$
$$\beta = b - \tfrac{1}{12} d + \tfrac{1}{90} f - \tfrac{1}{560} h + \tfrac{1}{3150} j + \&c.$$
$$\gamma = c - \tfrac{1}{4} e + \tfrac{7}{120} g - \tfrac{41}{3024} i + \ldots\ldots \&c.$$
$$\delta = d - \tfrac{1}{6} f + \tfrac{7}{240} h - \tfrac{41}{7560} j + \ldots\ldots \&c.$$
$$\varepsilon = e - \tfrac{1}{3} g + \tfrac{13}{144} i - \ldots\ldots \&c.$$
$$\zeta = f - \tfrac{10}{40} h + \tfrac{13}{240} j - \ldots\ldots \&c.$$
$$\eta = g - \tfrac{5}{12} i + \ldots\ldots \&c.$$
$$\theta = h - \tfrac{1}{3} j + \ldots\ldots \&c.$$
$$\iota = i \ldots\ldots \&c.$$
$$\kappa = j \ldots\ldots \&c.$$

$\frac{1}{4}$ or $\frac{1}{40}$?

If the given ephemeris is for intervals of ω units (years, days, hours, &c. as the case may be) we have by (I), $I^{(n)}$ equal the value of the function for the argument $I + n\omega$; and to obtain the value for the ~~time~~ $I + t$, we take the value of $F^{(n)}$, putting $n\omega = t$ or $n = \frac{t}{\omega}$.

Consequently I is the same as

II $$F^{(n)} = I + \frac{1}{\omega} \alpha . t + \frac{1}{\omega^2} \cdot \frac{1}{2} \beta . t^2 + \frac{1}{\omega^3} \frac{1}{6} \gamma . t^3 + \frac{1}{\omega^4} \cdot \frac{1}{24} \delta . t^4 + \&c.$$

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Putting in this equation

$\alpha \frac{1}{\omega} = A \qquad \beta \cdot \frac{1}{2} \cdot \frac{1}{\omega^2} = B \qquad \gamma \frac{1}{6} \frac{1}{\omega^3} = C$ &c. &c.

we have

III $\qquad F^{(n)} = F + At + Bt^2 + Ct^3 + Dt^4 +$ &c. &c.

Example

As an example, I will take the one given in Chauvenet's Ast. Vol I pages 84-85, there performed by the common formula. From there we obtain

$\omega = 12^h$

Moon's R 1856 Mar 5 $0^h = 21^h\ 58\ 28.39$

$a = +\ 29\ 6.71$

$b = -\ 39.34$

$c = +\ 0.79$

$d = +\ 3.16$

$e = -\ 0.54$

From these data α, β, γ &c. ε are computed as follows

a + 1746.710	b − 39.340	c + 0.790			
$-\frac{1}{6}c$ − 0.133	$-\frac{1}{12}d$ − 0.263	$-\frac{1}{4}e$ + 0.135			
$+\frac{1}{30}e$ − 0.018					
α + 1746.559	β − 39.603	γ + 0.925	δ + 3.16	ε − 0.54	
log α 3.2421832	log β n 1.5977281	log γ 9.9661417	log δ 0.4996871	log ε n 9.7323938	
ac. log $\frac{1}{\omega}$ 8.9208188	ac log $\frac{1}{2\omega^2}$ 7.5406075	ac log $\frac{1}{6\omega^3}$ 5.9843050	ac log $\frac{1}{24\omega^4}$ 4.3030638	ac log $\frac{1}{120\omega^5}$ 2.5249125	
log A 2.1630020	log B n 9.1383356	log C 5.9504467	log D 4.8027509	log E n 2.2573063	

A + 145

Hence $A = +145.547$

$B = -\ 0.13751$

$C = +\ 0.000\,089\,217$

$D = +\ 0.000\,006\,349\,7$

$E = -\ 0.000\,000\,018\,085$

Now Now with these coefficients compute the moons R for Mar 5 6^h by formula III.

$F = 21\ 58\ 28.39$

$At = +\ 14\ 33.28$

$Bt^2 = -\ 4.95$

$Ct^3 = +\ 0.02$

$Dt^4 = +\ 0.01$

$Et^5 = \underline{\qquad 0.00}$

$22\ 12\ 56.75$

Which is the same result exactly, as given in Chauvenet, where it is obtained by the common interpolation formula.

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Method of Reduction of Transit Observations for Instrumental and Clock corrections, as practised in the Coast Survey

Copied from Coast Survey Report for 1855

M Mean of threads

F " " thread intervals

$\log F$

$\log \rho$ ρ is the correction for rate, and its logarithm is 0.000005 for a gain of one second daily $= 0.00119$ for a clock giving mean time

$\log \sigma$ σ is the ~~sign~~ correction $\log \sigma$ being additive to $\log F$ When $F \sec \delta$ is less than 2 min. it may be neglected.

$\sec \delta$

$\log R$

R $R = F \sec \delta . \sigma . \rho$ and is the quantity to be added to M for obtaining the time of transit over the mean of all the threads.

~~$T = M + R$~~

Levelings

b_0 The level correction in time corrected for inequality of pivots &c. It is positive for West end high.

α

δ

$\varphi - \delta$

$A = \frac{\sin(\varphi - \delta)}{\cos \delta}$

$B = \frac{\cos(\varphi - \delta)}{\cos \delta}$

$C = \sec \delta$

} $180 - \delta$ being used instead of δ when the star is below the pole

K = The diurnal aberration $= 0^s.021 \cos \varphi \sec \delta$. It is negative in upper, positive in lower culmination.

$B b_0$

$T = M + R$

$t = T + B b_0 + K$

$w = \alpha - t = \Delta t + A a \pm C c$

Cc

$w_0 = w \mp Cc$ Upper sign for Clamp West

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The collimation constant c is determined from reversals on circumpolar stars, and is to be obtained from the equation

$$t_E - t_W = \omega_W - \omega_E = 2Cc$$

A is positive except for stars between the zenith and north pole.
B " " " " " at lower culmination
C " " " " " " " "

The local time and azimuth are to be obtained thus: Assume an approximate value of the clock correction = θ for an arbitrary time T_0 and call $\omega - \theta = \omega'$ If the collimation is known and the corresponding correction applied, we have only to reduce the values of ω_0 for the several stars to the time T_0 by applying the correction for daily rate. Thus:

$$(\omega_0) = \omega_0 + \frac{t - T_0}{24^h} \times \text{daily gain}$$

and we have, putting $\Delta\theta = \Delta t - \theta$

$$\Sigma \Delta\theta + \Sigma Aa = \Sigma \omega'_0$$
$$\Sigma A\Delta\theta + \Sigma A^2 a = \Sigma A\omega'_0$$

whence we determine a, $\Delta\theta$, and thence Δt for the time T_0

If the collimation correction is not known from reversals, it will be necessary to determine it approximately from the equations of condition, which then become:—

$$\Sigma \Delta\theta + \Sigma Aa + \Sigma Cc = \Sigma \omega'_0$$
$$\Sigma A\Delta\theta + \Sigma A^2 a + \Sigma ACc = \Sigma A\omega'_0$$
$$\Sigma C\Delta\theta + \Sigma ACa + \Sigma C^2 c = \Sigma C\omega'_0$$

Before forming these equations the several values must be carefully scrutinized to avoid grouping together observations between which the instrumental adjustments have been changed. It is not safe to make use of these equations unless, at least, one reversal has taken place. The more nearly equal the number of stars observed in each position of the telescope, the more reliable will be the result.

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Name of star	Mag.	α 1840.0	Ann. Dec.	Sec. Var.	Proper Motion	δ 1840.0	Ann. Dec.	Sec. Var.	Proper Motion
		° ′ ″							
16 g Celæno	5.6	53 49 33.64				+23 46 49.58			
17 b Electra	4.5	50 47.59				36 16.24			
18 m	7	54 25.52				24 19 52.36			
19 e Taygeta	5	55 26.47				23 57 34.12			
Anon. 1	8	53 59 14.52				31 42.30			
" 2	8.9	54 0 55.25				57 25.09			
" 3	9	1 30.81				34 36.65			
" 4	8	1 52.66				49 45.23			
" 5	9	2 11.22				24 7 16.14			
" 6	9	2 49.03				23 48 57.30			
20 c Maja	5	4 46.31				51 43.12			
Anon 7	8	5 35.15				32 0.41			
21 k Asterope	7.8	5 48.99				24 2 56.40			
22 l	7.8	7 56.33				1 21.97			
Anon 8	8.9	10 56.98				23 41 26.35			
" 9	8.9	11 31.24				41 7.91			
23 d Merope	~~5~~8	12 37.28				26 39.23			
Anon. 10	8	14 15.83				45 4.35			
" 11	8.9	17 24.71				36 0.60			
" 12	7.8	22 1.14				24 1 4.70			
" 13	8.9	23 41.65				23 29 37.12			
" 14	9	25 11.96				15 52.45			
" 15	8.9	26 39.14				37 37.49			
" 16	9.10	26 58.66				18 58.24			
" 17	8	27 18.67				13 29.57			
" 18	8	27 20.05				38 16.98			
24 j	7.8	27 46.26				36 55.12			
Anon 19	8	28 1.82				18 9.11			
" 20	8	28 5.46				24 5 15.46			
" 21	8.9	28 41.90				9 22.06			
" 22	8	28 47.57				23 24 50.13			
" 23	8.9	29 36.40				10 40.14			
" 24	8	54 29 42.52				47 16.49			

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Name of star	Mag.	α 1840.0	Ann. Dec.	Sec. var.	Proper Motion	δ 1840.0	Ann. Dec.	Sec. var.	Proper Motion
25 η Alcyone	3.4	54 29 46.72				23 36 16.91			
Anon 25	8.9	32 5.30				6 35.59			
" 26	9	33 36.76				2 35.89			
" 27	8.9	40 39.14				49 12.64			
" 28	7	45 17.17				22 55 25.93			
" 29	8	44 45.04				23 50 53.36			
26 S	7.8	51 48.27				21 43.53			
27 f Atlas	4.5	54 53.68				38 30.41			
28 h Pleione	5.6	55 10.82				38 30.60			
Anon 30	8.9	55 39.24				23 31.50			
" 31	8	56 20.32				54 4.70			
" 32	8	57 35.11				53 11.52			
" 33	8.9	54 58 45.54				45 12.36			
" 34	7.8	55 3 36.72				13 7.69			
" 35	9	3 47.46				45 2.15			
" 36	9	5 59.14				43 26.57			
" 37	8	6 17.63				51 22.51			
" 38	8	7 6.69				21 22.86			
" 39	8	13 58.00				24 0 13.63			
" 40	7.8	55 20 31.97				23 28 18.94			

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Original

Original

17

Table of Sine Corrections

h	corr.	log corr.	h	corr.	log. corr.	h	corr.	log corr
0m 0	0.000		12 0	0.329		36 0	8.871	
15	0		30	372		30	9.245	
30	0		13 0	418		37 0	9.630	
45	0		30	468		30	10.025	
1 0	0		14 0	522		38 0	10.432	
15	0		30	580		30	10.849	
30	1		15 0	642		39 0	11.276	
45	1		30	709		30	11.715	
2 0	2		16 0	780		40 0	12.165	
15	3		30	855		30	12.627	
30	4		17 0	0.934		41 0	13.100	
45	5		30	1.020		30	13.584	
3 0	5		18 0	1 110		42 0	14.081	
15	7		30	1.205		30	14.589	
30	8		19 0	1.305		43 0	15.109	
45	10		30	1.411		30	15.642	
4 0	12		20 0	1.522		44 0	16.187	
15	15		30	1.639		30	16.744	
30	17		21 0	1.762		45 0	17.314	
45	20		30	1.891		30	17.897	
5 0	24		22 0	2.026		46 0	18.493	
15	28		30	2.168		30	19.102	
30	32		23 0	2.315		47 0	19.724	
45	36		30	2.470		30	19.359	
6 0	41		24 0	2.630		48 0	21.008	
15	46		30	2.798		30	21.670	
30	52		25 0	2.973		49 0	22.346	
45	58		30	3.156		30	23.036	
7 0	65		26 0	3.344		50 0	23.740	
15	73		30	3.540		30	23.459	
30	80		27 0	3.745		61 0	25.191	
45	89		30	3.956		30	25.938	
8 0	98		28 0	4.176		52 0	26.699	
15	107		30	4.404		30	27.476	
30	117		29 0	4.639		53 0	28.267	
45	128		30	4.883		30	29.073	
9 0	139		30 0	5.136		54 0	29.894	
15	151		30	5.397		30	30.730	
30	163		31 0	5.666		55 0	31.582	
45	176		30	5.945		30	32.450	
10 0	190		32 0	6.232		66 0	33.333	
15	205		30	6.529		30	34.232	
30	220		33 0	6.834		57 0	35.147	
45	236		30	7.149		30	36.078	
11 0	253		34 0	7.474		58 0	37.025	
15	350		20	7.808		30	37.989	
30	372		35 0	8.153		59 0	38.969	
46	395		30	8.507		30	39.966	
12 0	329		36 0	8.871		60 0	40.980	

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Original

Precession

Below are some neat and rigorous formulæ for the computation of the effect of precession, that are not given in any of the works on Spherical Astronomy.

Putting in the formulæ (658) in Chauvenet's Astronomy Vol I,

$$A = \alpha + z + \vartheta$$
$$A' = \alpha' - z' + \vartheta'$$

and also putting

$$\sin \delta = m \sin M$$
$$\cos \delta \cos A = m \cos M$$

we have

$$\sin \delta' = m \sin (M + \theta)$$
$$\cos \delta' \cos A' = m \cos (M + \theta)$$
$$\cos \delta' \sin A' = \cos \delta \sin A$$

from whence we obtain

$$\operatorname{tg} M = \operatorname{tg} \delta \sec A$$

$$\operatorname{tg} A' = \operatorname{tg} A \cos M \sec (M + \theta)$$

$$\operatorname{tg} \delta' = \operatorname{tg} (M + \theta) \cos A'$$

Where A' is greater or less than 180° according as A is greater or less than 180°

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On the Probable error of observations from the differences of the observations from the Arithmetical mean.

Coast Survey Report 1856 - p 307 - Translated from Ast. Nach. 1034

The determination of the probable error of observation by means of the sum of the differences, these being considered as essentially positive, from their arithmetical mean, deserves attention on account of its simplicity. It has therefore been thought proper to develop the formulæ referring to this case.

Let p be the true value of a quantity repeated by observed

$a\ a'\ a''\ a'''\ldots$ be the individual values, and obtained under like circumstances

$\varepsilon\ \varepsilon'\ \varepsilon''\ \varepsilon'''\ldots$ be the true errors of observation

n be the number of observations

We then have

$$\begin{aligned} a &= p - \varepsilon \\ a' &= p - \varepsilon' \\ a'' &= p - \varepsilon'' \\ &\&c. \quad \&c. \\ a^{(n-1)} &= p - \varepsilon^{(n-1)} \end{aligned}$$

The arithmetical mean of the observed values is

$$\frac{a + a' + a'' + \ldots\ldots a^{(n-1)}}{n} = p - \frac{\varepsilon + \varepsilon' + \varepsilon'' + \ldots\ldots \varepsilon^{(n-1)}}{n}$$

and their deviations from the mean are respectively

$$\frac{(n-1)\varepsilon - \varepsilon' - \varepsilon'' - \ldots\ldots}{n}$$

$$\frac{-\varepsilon + (n-1)\varepsilon' - \varepsilon'' - \ldots\ldots}{n}$$

$$\frac{-\varepsilon - \varepsilon' + (n-1)\varepsilon'' - \ldots\ldots}{n}$$

We designate these quantities by $f, f', f'', \ldots$ and their sum, considering them as positive, by Σf. If, as is here supposed, the observations are of the same weight, the same probable error ρ corresponds to each of the true

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errors ε, ε', ε'' ... also the probable error ρ' refers to the differences f, f', f'' ..., and is equal to

$$\rho\sqrt{\frac{(n-1)^2}{n^2} + (n-1)\frac{1}{n^2}} = \rho\sqrt{\frac{n-1}{n}}$$

The quantities f, f', f'' ... are only subject to the condition that their sum = 0, hence the sum of the positive, must equal the sum of the negative deviations.

According to Gauss, the average value of a number of such differences, having the probable error ρ', equals

$$\frac{\rho}{0.476936\sqrt{\pi}} = \frac{\Sigma f}{n}$$

hence

$$\rho' = \frac{0.476936\sqrt{\pi}\ \Sigma f}{n}$$

but

$$\rho' = \rho\sqrt{\frac{n-1}{n}}$$

hence

$$\rho = \frac{0.476936\sqrt{\pi}\ \Sigma f}{\sqrt{n(n-1)}} = 0.845347\ \frac{\Sigma f}{\sqrt{n(n-1)}}$$

We meet generally with the formula

$$0.845347\ \frac{\Sigma f}{n-1}$$

which gives the value of ρ a little too large.

Example.

The azimuth results of Webb station have been selected on account of the occurrence of one predominant value, (thrown out by the criterion) and their small number, both circumstances tending to bring the formula to a severer test. We have

Mark E of N	
6° 7′ 44.″88	−0.76
45.65	+0.01
45.89	+0.25
44.80	−0.84
46.55	+0.91
44.78	−0.86
[47.97]	+2.33
44.03	−1.61
45.62	−0.02
46.25	+0.61
6° 7′ 45.64	+4.11
	−4.09
	8.20

log 0.845347 — 9.927 04
log 8.20 — 0.913 81
a.c. log $\sqrt{90}$ — 9.022 88
$\rho = \pm 0.73$ — 9.863 73

And probable error of mean = ± 0.23

by means of squaring we obtain for same ± 0.76 and ± 0.24 respectively

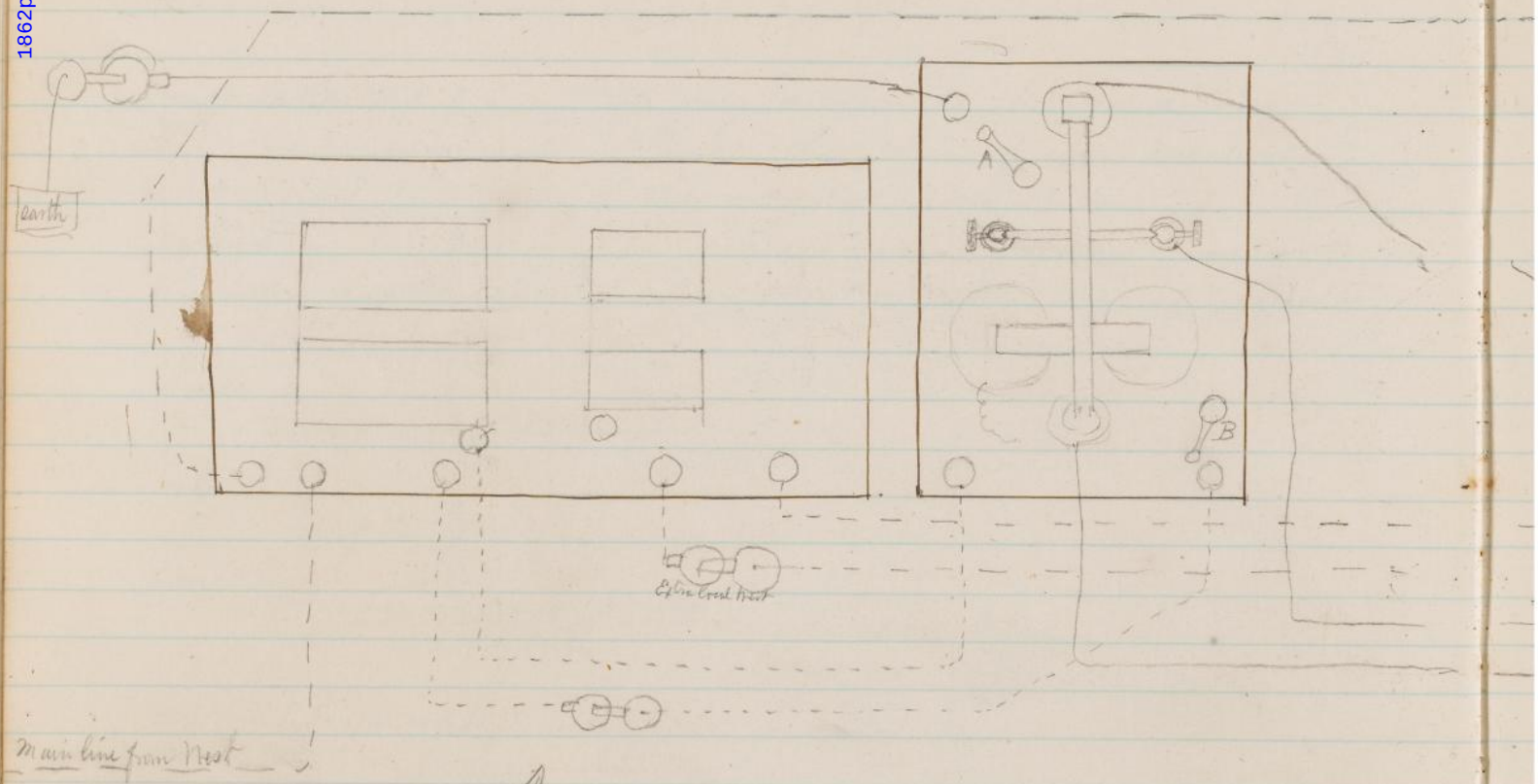
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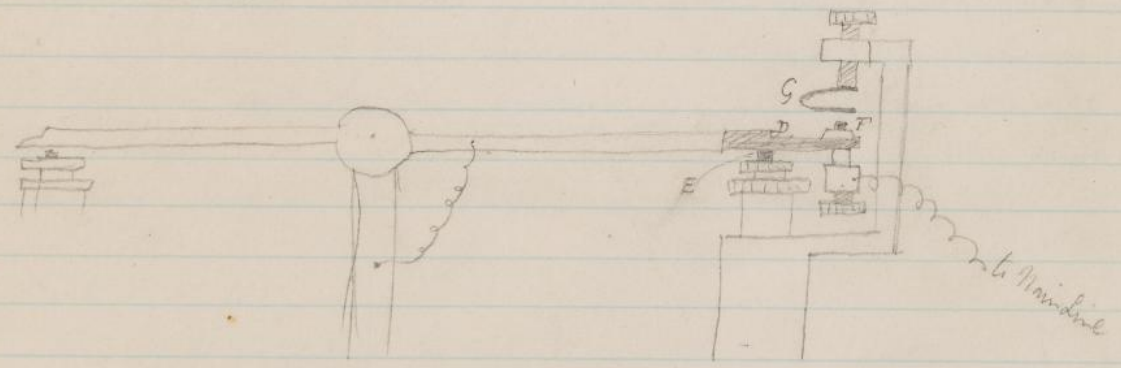
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Mikes Improved



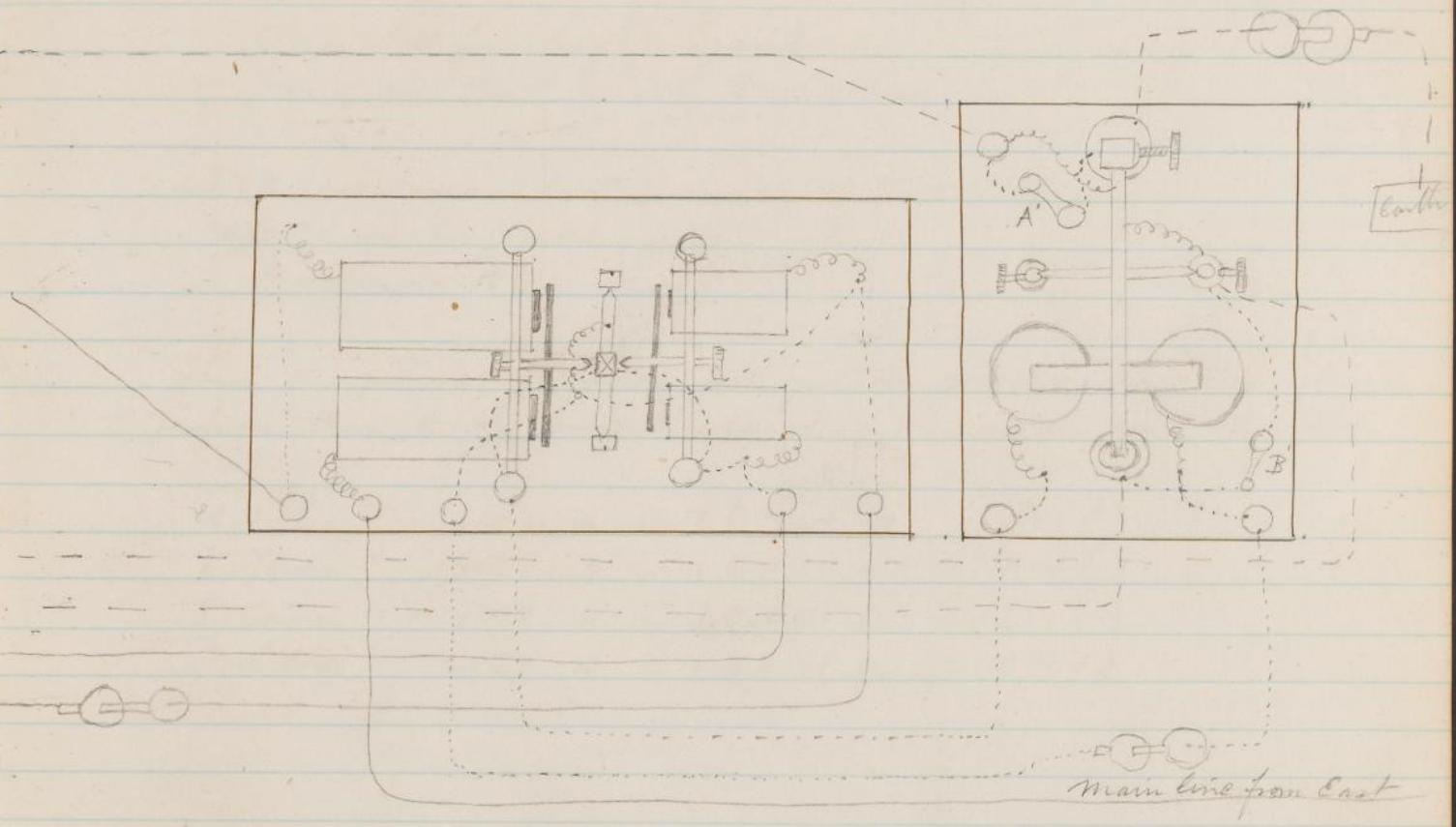
Fill out the connections identically like those of the left hand page



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Repeater



When A A' B B' are switched off the repeaters work through both ways.

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Formulas for Light-Equations of Four of the Short Period Variables

Let R = distance of Earth from Sun at time of observation
$\odot$ = Sun's longitude " " " "
t = year − 1855

Then the following formulas will give the corrections to (of minima observations) to reduce them to the mean distance of the earth from the star

For β Persei $\quad 460^{s}.2\ R \sin(\odot + 215^{\circ}\ 51'.1 - 0'.84\ t)$
λ Tauri $\quad 493.0\ R \sin(\odot + 211\ 23.4 - 0.84\ t)$
S Cancri $\quad 497.8\ R \sin(\odot + 143\ 42.2 - 0.84\ t)$
δ Librae $\quad 492.7\ R \sin(\odot + 46\ 44.5 - 0.84\ t)$

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Catalogue of

Including New Stars by

	No.	Star	1855 α	1855 δ	Ann. Prec. in α	Ann. Prec. in δ	Magnitude max.	Magnitude min.
			h m s	° ′	s	″	m m	m
1875 Nov 18	1	R Andromedæ	0 16 25	+37 46.4	+3.14	+19.″8	8.3-8.0	< 12.5
—	2	B Cassiopeiæ	16 47	+63 20.6	3.27	19.8		
Sept 4	3	T Piscium	24 29	+13 48.0	3.11	19.8	9.5-10.2	10.8-11.0
?	4	α Cassiopeiæ	32 18	+55 44.5	3.36	19.8	2.2	2.8
?	5	U Piscium	36 50	+ 6 30.7	3.09	19.8	9 ?	?
?	6	S Cassiopeiæ	1 9 4	+71 50.8	4.30	19.2	8	< 13
1876 Feb 6	7	S Piscium	10 0	+ 8 9.9	3.12	19.2	8.8	< 12
July 6	8	R "	23 10	+ 2 7.9	3.09	18.6	7.4-8.3	< 12
?	9	V "	46 43	+ 8 4.0	3.16	18.0	6,5 ?	8.5 ?
?	10	S Arietis	56 51	+11 49.7	3.21	17.4	10	< 13
Oct 4	11	R "	2 7 53	+24 22.9	3.39	16.8	7.7-8.5	12.5
1876 Feb 16	12	O Ceti	12 1	- 3 38.3	3.02	16.8	1.7-4	9 ?
?	13	ρ Persei	55 54	+38 16.5	3.81	14.4	3.4	4.0
1875 June 15d 12h 26m	14	β "	58 45	+40 23.6	3.87	14.4	2.3	4.0
✓	15	R "	3 20 50	+35 10.1	3.79	12.6	8.8	< 12
July 20d 9h 13m	16	λ Tauri	52 39	+12 4.6	3.31	10.8	3.4	4.3
✓	17	U "	4 13 22	+19 28.0	3.49	9.0	9.2	< 10.
✓	18	T "	13 33	+19 11.3	3.49	9.0	9.6	< 12
✓	19	R "	20 21	+ 9 50.1	3.28	8.4	8.2-9.2	< 13
✓	20	S "	21 16	+ 9 37.3	3.28	8.4	9.9	< 13
✓	21	R Orionis	51 6	+ 7 54.2	3.25	6.0	9	< 13
	22	ε Aurigæ	51 34	+43 36.2	4.79	6.0	3.5	4.5
	23	R Leporis	53 0	-15 1.7	2.73	6.0	6.0	9 ?
July 27	24	R Aurigæ	5 5 36	+53 25.0	4.82	4.8	6.6	12.7
	25	δ Orionis	24 36	- 0 24.6	3.06	3.0	2.2 ?	2.7
	26	α "	47 19	+7 22.6	3.25	+ 1.2	1	1.4
✓	27	R Monocerotis	6 31 15	+8 51.7	3.28	- 3.0	9.5	< 12
	28	ξ Geminorum	55 30	+20 46.7	3.56	4.8	3,7	4.5
75 March 26	29	R "	58 37	+22 55.4	3.62	4.8	6.8-7.2	< 10.5
75 Nov 29	30	R Can. Minoris	7 0 44	+10 15.0	+3.30	- 5.4	7	< 10.5

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Variable Stars

Dr. E. Schönfeld – 1866

No.	Elements – Paris Mean Time Epoch	Periods	Remarks
1	1862 Sept 3	402. days	
2			Nova 1572
3	1856 Nov 1	146.4 "	min. 1857 Jan 22
4	?	79.5 "	nur zeitweise var.
5	?	?	
6	1865 End of January		
7	1864 Nov 22	406.2 "	
8	1865 Jan 22	347 "	
9	?	?	
10	1861 End of January	?	
11	1861 Jan 18.9	185.2 "	
12	1865 Dec 29.13	331.3363	(See below *)
13	1854 Oct 30	?	irregular Period about 30 days
14	1865 Jan 2 $4^h\ 2.5^m$	$2^d\ 20^h\ 48^m\ 53^s = 2.8672801$	min. Period variable
15	1865 Mar. 11	210.	
16	1862 Feb. 16 $16^h\ 42.1^m$	$3^d\ 22^h\ 52^m\ 17^s = 3.95297455$	minimum
17	?	?	
18	?	?	
19	1859 Sept. 1	326.3	
20	1858 Jan 26	378.	
21	1857 Apr. 16	378.	
22	?	?	irregular
23	1859 May 21	439.	
24	1865 May 20	465	
25	?	?	irregular
26	?	196	irregular
27	?	?	
28	1857 Jan 7 $20^h\ 1^m$	$10^d\ 3^h\ 47^m\ 59^s + 1^s E^2$	minimum
29	1865 Feb. 6.5	370.	
30	1859 Mar. 16	339.	

*) $+10^d.48 \sin\left(\frac{360}{11}E + 250° i\right) + 18^d.16 \sin\left(\frac{45}{11}E + 27° 9'\right) + 33^d.90 \sin\left(\frac{45}{11} + 68° 3'\right) + 65^d.31 \sin\left(\frac{15°}{11}E + 178° 26'\right)$

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Catalogue of
including new Stars by

	No	Star	1855 α	1855 δ	Ann. Prec. in α	Ann. Prec. in δ	Magnitude Max.	Magnitude Min.
			h m s	° ′	s	″	m m	m
(May 1875 May 3	31	S Can. Minoris	7 24 51	+8 37.4	+3.26	−7.2	7.2 − 8.0	< 11
?	32	T " "	25 56	+12 3.0	3.34	7 2	9.5	< 13
" 1875 June 26	33	S Geminorum	34 20	+23 47.2	3.61	7.8	8.7	< 13
" " Sept 20	34	T "	40 36	+24 5.5	3.61	8.4	8.1 − 9.0	< 13
" " May 10	35	U "	46 30	+22 22.7	3.56	9.0	9.0 − 10 ?	13
" " July 14	36	R Cancri	8 8 4	+12 10.1	3.32	10.8	6.3 − 7.0	< 11
" " Nov 26	37	U "	27 28	+19 23.5	3.45	12.0	8.2 − 10.0	< 13
Paris Time May 18 d 13 h	38	S "	-35 39	+19 33.2	3.44	12.6	8.2	10.2
" May 25	39	S Hydrae	46 0	+ 3 36.8	3.13	13.2	7.5 − 8.5	< 12
(Miss) " June 1	40	T Cancri	48 23	+20 24.1	3.44	13.2	8.5	11.0
Mean " July 24	41	T Hydrae	48 37	− 8 35.4	2.92	13.2	7.0 − 8.0	< 12
2	42	α Hydrae	9 20 28	− 8 2.0	2.95	15.6	2.3	2.7
" May 14	43	R Leonis Min.	36 52	+35 10.6	3.62	16.2	7.0	< 11
" June 3	44	R Leonis	39 45	+12 5.9	3.23	16.2	5.3 − 6.3	9.8 − 10.5
		49 Leonis	10 27 26	+ 9 23.8	3.16	18.4		
" Aug 5	45	R Urs. Majoris	10 34 19	+69 32.1	4.38	18.6	6.8 − 7.6	12
	46	η Argus	39 27	− 58 55.4	2.31	18.6	1	6 − < 6
" July 2	47	S Leonis	11 3 21	+ 6 14.9	3.11	19.2	9	< 13
?	48	T Ε "	31 0	+ 4 10.5	3.08	19.8	10 ?	< 13
" July 3	49	R Comæ	56 49	+19 35.4	3.08	19.8	8	< 12
" Feb 24	50	T Virginis	12 7 10	− 5 13.8	3.08	19.8	7.9 − 8.4	< 12.5
" Aug 18	51	T Urs. Majoris	29 47	+60 17.2	2.77	19.8	6.5 − 7.1	12 − 13
	52	R Virginis	31 8	+ 7 47.2	3.05	19.8	6.5 − 7.1	10 − 10.5
" Mar 10	53	S Urs. Majoris	37 35	+61 53.3	2.66	19.8	7.6 − 8.2	10.5 − 11.0
" July 30	54	U Virginis	43 44	+ 6 20.6	3.04	19.8	7.5 − 8.0	10 − 12.5
" Apr 25	55	V "	13 20 19	− 2 25.2	3.09	18.6	7 − 9.5	< 13
" Apr 26	56	R Hydrae	21 48	− 22 31.8	3.27	18.6	4 − 5	10 ?
" Aug 19	57	S Virginis	25 26	− 6 26.8	3.13	18.6	5.7 − 7.5	< 11
?	58	T Bootis	14 7 18	+19 44.7	2.81	16.8	9.7 ?	< 13
" Mar 3	59	S "	18 1	+54 28.3	+2.01	16.8	8	< 12.5
" June 21	60	R Canelop.	14 28 54	+84 29.2	− 5.31	−16.2	7.2 − 8.2	12 ?

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Variable Stars

Dr. E. Schönfeld - 1866

No.	Elements - Paris Mean Time Epoch	Period	Remarkes
31	1859 Sept 12.	336^d	
32	?	?	
33	1857 Oct 10	294.	
34	1859 Mar. 21	287.	
35	1859 Feb. 20	97.1	irregular maximum
36	1857 Feb 20	353.6	Periode abnehmend ?
37	1858 Aper. 25	306.	
38	1865 Oct. 17 $16^h\,41^m$	$9^d\,11^h\,37.^m8 = 9.^d484583$	period not constant — Argelander's elements for minimum 1860 Jan 2^d 18^h 20.^{m}72 Paris mean time. Period = +9^d 11^h 38.^{m}1547 E + 0.^{m}003 E
39	1859 Aper. 2	256.4	
40	1859 Aper. 4	454.	minimum
41	1862 Feb. 7.4	289.1	
42	?	55 ?	nur zeitweis var. u. sehr irregular
43	1865 Feb. 25	373	next max 1867 Mar 13
44	1853 Mar. 16.13	312.086	minimum 1853 Aug 29 — Argelander 1867 — Schmidt 1867 — next minimum 1867 May 2
45	1859 Jan 15	302.3	next max 1867 Apr. 28
46	1843 Aperil		irregular
47	1864 Jan 21	190	1867 Mar 6
48	1865 Middle of April	?	
49	1857 Nov 10	358	
50	1862 Aper. 9	336	
51	1865 Jan 23	257.3	1867 Mar 6
52	1831 Sept. 12.92	145.7242	(see below *) — 1867 July 20
53	1860 June 1.8	224.8	
54	1861 Dec 8	207.6	
55	1860 Feb. 19	252.	
56	1859 May 18.1	447.85	Periode abnehmend ? — next min. 1867 May 7 — next max 1867 Dec. 17.
57	1860 Aper. 15.4	373.6	Next max 1867 June 14
58	?	?	
59	1865 July 10.	271.	
60	1864 July 23	265.7	1867 June 21

*) $+ 8.^d370 \sin(3° 41' 17'' E + 310° 15' 10'')$

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Catalogue of circumpolar New Stars by

	No.	Star	1855 α	1855 δ	Ann. Prec. in α	Ann. Prec. in δ	Magnitude Max	Magnitude Min.
Prof.			h m s		s	"		m
1875 June 29	61	R Bootis	14 30 48	+27° 22.1	+2.65	−15.6	7.2 - 8.0	12.5
〃	62	U 〃	34 8	+28 5.3	2.63	15.6	9.5 ?	12 ?
May 9d 12h	63	δ Libræ	53 14	−7 56.4	3.20	14.4	4.9	6.0
〃 (Oct 12)	64	S Serpentis	15 14 52	+14 50.3	2.81	13.2	7.6 - 8.3	13 ?
〃 Apr 16	65	S Coronæ	15 29	+31 53.5	2.44	13.2	6.5	12
〃	66	R 〃	42 36	+28 36.3	2.47	11.4	6.0	13 ?
Nov 24	68	R Serpentis	44 1	+15 34.6	2.76	11.4	5.7 - 7.1	< 11
(1876 Mar 24)	68	R Libræ	45 24	−15 48.1	3.39	10.8	9.3	< 13
		T Coronæ	15 54	+26 18	2.48	10.66	2.0	9.2
1875 Dec 5	69	R Herculis	59 43	+18 45.9	2.68	10.2	7.8 - 9.1	< 13
—	70	T Scorpii	16 8 25	−22 36.7	3.56	9.6	7	< 10
〃	71	R 〃	9 1	−22 35.0	3.56	9.6	9	< 13
〃	72	S 〃	9 2	−22 31.9	3.56	9.6	9.3 - 10.5	< 13
〃	73	U 〃	14 7	−17 32.3	3.44	9.0	9	< 13
〃 July 11	74	U Herculis	19 23	+19 13.6	2.65	8.4	7.0	11 ?
	75	ε 〃	23 53	+42 12.2	1.97	7.8	4.9	6.2
〃	76	T Ophiuchi	25 27	−15 49.2	3.42	7.8	10 ?	< 12
June 9	77	S 〃	25 55	−16 51.1	3.44	7.8	9.1	< 12.5
Dec 7	78	S Herculis	45 18	+15 11.4	2.73	6.6	6.3 7.5	12
—	79	— Ophiuchi	51 23	−12 40.0	3.36	6.0	5.5	< 10
Dec 2	80	R 〃	59 27	−15 53.7	3.44	5.4	8	< 12
	81	α Herculis	17 8 2	+14 33.5	2.73	4.2	3.1	3.9
—	82	— Serpentarii	21 57	−21 21.2	3.59	−3.6		
June 10	83	T Herculis	18 3 37	+30 59.9	2.27	+0.6	7.5 - 8.0	10.5 - 12.3
〃	84	T Serpentis	21 44	+6 12.5	2.93	1.8	10.5	< 13
〃	85	R Scuti	39 45	−5 51.4	3.21	3.6	4.7 - 5.7	6 - 9
June 12 13h 8m	86	β Lyræ	44 44	+33 11.8	2.21	3.6	3.5	4.5
June 7	87	R 〃	50 55	+43 45.5	1.83	4.8	4.3	4.6
Sept 25	88	R Aquilæ	59 23	+8 0.8	2.89	5.4	6.7 - 7.7	< 11
〃	89	T Sagittarii	19 7 52	−17 13.2	3.46	6.0	8	< 11
Mar 1	90	R 〃	19 8 11	−19 33.5	+3.52	+6.0	7 - 8	< 12

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Variable Stars

Dr. E. Schönfeld - 1866

No.	Elements Epoch	Paris mean Time Period	Remarks
61	1862 Feb. 2	222.5 d	minimum 1862 June 6 first max 1867 July 28.5
62	?	?	
63	1865 May 26.351	~~6.9834~~ 2,3274	minimum
64	1849 May 17.39	$362.303\ E - 0.188836\ E^2$	
65	1865 July 17	356.	Next. max 1867 June 29
66			irregular
67	1860 Aug 9.4	$351,74\ E - 0.167\ E^2$	Next max 1867 Apr. 30.6
68	1860 Apr. 30.	722.	
69	1860 Apr 19	317.1	Nova 1866
70			Nova ?
71	1860 April	?	
72	1865 June 26	?	
73	1863 May	?	
74	1865 April 30	413	Next. max Aug 3, 1867
75			irregular
76	1860 End of April	?	
77	1859 June 3	234.	
78	1865 Feb. 23	303.	" May 21, 1867
79	1848 May 3		Nova 1848
80	1857 July 27	304.6	
81			irregular
82			Nova 1604
83	1863 Mar. 25	165.2	min. 1863 June 18 " Oct 3 1867
84	?	310	
85	?	71.7	irregular
86	1855 Jan 6 $14^h\ 38^m\ 2^s$	$12^d\ 21^h\ 47^m\ 16^s.837 = 12.9078337$	(See below *) minimum
87	1862 Nov 2	46.	
88	1859 June 20	349,5	
89	?	?	
90	1859 Oct 27	467	

*) $+ 0^s.303977\ E^2 - 0^s.0000149454\ E^3$ - The phases of this star are: principle minimum $4^m.5$ - first max., $3^h.5$ in $3^d\ 2^h$, secondary minimum $3^m.9$ in $3^d\ 7^h.6$, second maximum almost $3^h.5$ in $3^d\ 3^h$, and then the principle minimum again in $2^d\ 9^h$.

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Paris – New York = $5^h\,5^m\,21^s$

Catalogue of
including New Stars by

	No	Star	1855 α	1855 δ	Ann. Prec. in α	Ann. Prec. in δ	Magnitude Max.	Magnitude Min.
			h m s	° ′	s	″	m	m
?	91	S Sagittarii	19 10 57	−19 17.1	+3.51	+6.0	10	< 12
1875 Sept 25	92	R Cygni	32 56	+49 52.5	1.61	7.8	6.2 – 8.0	13
—	93	11 Vulpeculæ	41 37	+26 57.7	2.46	8.4	3	?
June 6½	94	S Vulpeculæ	42 27	+26 55.7	2.46	9.0	8.5	9.5
Nov 17	95	X Cygni	45 0	+32 33.0	2.31	9.0	4.0 – 6.6	< 11
1875 May 17 (Paris m.t.)	96	η Aquilæ	45 5	+0 38.2	3.06	9.0	3.5	4.7
Mar 31	97	S Cygni	20 2 28	+57 34.2	1.26	10.2	8.8 – 9.2	< 13
apr 11	98	R Capricorni	3 10	−14 41.6	3.37	10.2	9	< 13
July 24	99	S Aquilæ	4 57	+15 11.5	2.76	10.2	8.9	11
June 25 (mins)	100	R Sagittæ	7 27	+16 17.4	2.74	10.8	8.3 – 8.5	9.8 – 10.0
?	101	R Delphini	7 58	+8 39.1	2.90	10.8	8.4	< 12.5
	102	P Cygni	12 27	+37 35.1	+2.21	10.8	3	< 6
?	103	R Cephei	34 37	+88 41.0	−42.0	12.6	5 ?	10 ?
June 21	104	S Delphini	36 24	+16 34.2	+2.76	12.6	8	11
Aug 5	105	T "	38 38	+15 52.4	2.78	12.6	8.4	< 13
(1876 Feb. 27)	106	U Capricorni	40 4	−15 18.8	3.35	13.2	10	< 12
1875 ?	107	T Cygni	41 24	+33 50.6	2.39	13.2	5	6.5 ?
May 6	108	T Aquarii	42 17	−5 40.9	3.17	13.2	7.5	13
June 25	109	R Vulpeculæ	57 56	+23 14.9	2.66	13.8	8	13
Oct 10	110	T Capricorni	21 14 0	−15 46.4	+3.32	15.0	9	< 13
1876 Apr 29	111	S Cephei	36 57	+77 58.2	−0.60	16.2	8.5	11.5
1876 Feb 23	112	μ "	39 4	+58 7.0	+1.83	16.2	4	5
1876 Mar 7	113	T Pegasi	22 1 49	+11 49.9	2.93	17.4	9.1	< 12
	114	δ Cephei	23 48	+57 40.4	2.21	18.6	3.7	4.9
1875 Mar 27	115	S Aquarii	49 20	−21 7.0	3.23	19.2	7.7	< 11
?	116	β Pegasi	56 45	+27 17.8	2.90	19.2	2.2	2.7
Oct 8	117	R Pegasi	59 22	+9 45.7	3.01	19.2	8	< 11
July 20	118	R Aquarii	23 36 19	−16 5.3	3.11	19.8	5.8 – 9.0	< 11
Sept 29	119	R Cassiopeiæ	23 51 4	+50 34.9	+3.01	+19.8	5.7 – 6.8	< 12.5

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Variable Stars

Dr. E. Schönfeld 1866

No.	Elements – Paris mean Time Epoch	Period	Remarks
91	1863 End of July ?	?	
92	1880 Aug 19	425 d	
93			Nova 1670
94	1864 Feb. 1.4	67.92	minimum Jan 1.6
95	1863 Aug 25	406.12	
96	1848 May 18 d 6 h 7 m	7 d 4 h 14 m 4 s = 7.d 17643 54	minimum
97	1865 July 8	323	
98	1865 Sept 20	349	
99	1864 June 28	122.5	
100	1861 July 26.4	70.58	minimum
101	1865 Aug 31	?	
102		(18 yrs)	Nova 1600
103	?	?	irregular
104	1865 May 25	283.	
105	1865 Aug 5	332	
106	1859 Dec 22	420	
107	?	?	
108	1865 Oct 20	205	
109	1861 Aug 10	138.6	minimum Oct 27
110	1855 Oct 25	270	
111	1862 Feb 1	470 ?	minimum
112	1854 Apr. 7	419	irregular. min. 1854 Sept 17
113	1864 Oct 31	374	
114	1840 Sept 24 d 20 h 23.9 m	5 d 8 h 47 m 39.97 s = 5.366435	minimum
115	1859 Dec. 9	279.35	
116		41. ?	nur zeitweise var.; irregular
117	1852 Dec. 31.	378	
118	1845 Oct 21	388.0	
119	1859 Apr. 6.5	430.0	

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Table of Annual Precessions

Giving the annual variations for the year 1880 in Right ascension and Declination for every 20ᵐ in right ascension and every 5° in declination.

[Copied from Proctor's Hand book of the Stars]

R.A.	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°	R.A.
ann. var. in decl.	±19.9	±19.9	±19.6	±19.3	±18.7	±18.1	±17.3	±16.3	±15.3	±14.1	±12.8	±11.4	±10.0	±8.4	±6.8	±5.2	±3.5	±1.7	±0.0	ann. var. in decln.
R.A.	12ʰ 0ᵐ 180°	11ʰ 40ᵐ 175°	11ʰ 20ᵐ 170°	11ʰ 0ᵐ 165°	10ʰ 40ᵐ 160°	10ʰ 20ᵐ 155°	10ʰ 0ᵐ 150°	9ʰ 40ᵐ 145°	9ʰ 20ᵐ 140°	9ʰ 0ᵐ 135°	8ʰ 40ᵐ 130°	8ʰ 20ᵐ 125°	8ʰ 0ᵐ 120°	7ʰ 40ᵐ 115°	7ʰ 20ᵐ 110°	7ʰ 0ᵐ 105°	6ʰ 40ᵐ 100°	6ʰ 20ᵐ 95°	6ʰ 0ᵐ 90°	R.A.
Decln. +85°	+3.06	+4.39	+5.70	+7.00	+8.26	+9.49	+10.66	+11.78	+12.83	+13.81	+14.70	+15.51	+16.22	+16.84	+17.34	+17.74	+18.03	+18.20	+18.26	−85°
+80°	3.06	3.72	4.37	5.01	5.64	6.25	6.83	7.39	7.91	8.40	8.84	9.24	9.59	9.90	10.15	10.35	10.49	10.57	10.60	−80°
+75°	3.06	3.50	3.93	4.35	4.76	5.16	5.54	5.91	6.25	6.57	6.86	7.13	7.36	7.56	7.73	7.86	7.95	8.01	8.03	−75°
+70°	3.06	3.38	3.70	4.01	4.31	4.61	4.89	5.16	5.41	5.65	5.88	6.06	6.23	6.38	6.50	6.59	6.66	6.70	6.72	−70°
+65°	3.06	3.31	3.56	3.80	4.04	4.27	4.49	4.70	4.90	5.08	5.25	5.40	5.53	5.65	5.74	5.82	5.87	5.90	5.92	−65°
+60°	3.06	3.26	3.46	3.66	3.85	4.04	4.22	4.38	4.54	4.69	4.83	4.95	5.06	5.15	5.23	5.29	5.33	5.36	5.37	−60°
+55°	3.06	3.23	3.39	3.56	3.71	3.87	4.01	4.15	4.28	4.41	4.52	4.62	4.71	4.78	4.85	4.90	4.93	4.96	4.96	−55°
+50°	3.06	3.20	3.34	3.47	3.61	3.73	3.86	3.97	4.08	4.18	4.28	4.36	4.44	4.50	4.55	4.59	4.62	4.64	4.65	−50°
+45°	3.06	3.18	3.29	3.41	3.52	3.63	3.73	3.83	3.92	4.00	4.08	4.15	4.22	4.27	4.31	4.35	4.37	4.39	4.39	−45°
+40°	3.06	3.16	3.26	3.35	3.45	3.53	3.62	3.70	3.78	3.85	3.92	3.98	4.03	4.08	4.11	4.14	4.16	4.18	4.18	−40°
+35°	3.06	3.15	3.23	3.30	3.38	3.46	3.53	3.60	3.66	3.72	3.78	3.82	3.87	3.91	3.94	3.96	3.98	3.99	3.99	−35°
+30°	3.06	3.13	3.20	3.26	3.33	3.39	3.45	3.50	3.56	3.61	3.65	3.69	3.73	3.76	3.79	3.81	3.82	3.83	3.83	−30°
+25°	3.06	3.12	3.17	3.22	3.28	3.33	3.37	3.42	3.46	3.50	3.54	3.57	3.60	3.63	3.65	3.66	3.67	3.68	3.68	−25°
+20°	3.06	3.11	3.15	3.19	3.23	3.27	3.31	3.34	3.38	3.41	3.43	3.46	3.48	3.50	3.52	3.53	3.54	3.55	3.55	−20°
+15°	3.06	3.10	3.13	3.16	3.19	3.21	3.24	3.27	3.29	3.32	3.34	3.36	3.37	3.39	3.40	3.41	3.41	3.42	3.42	−15°
10°	3.06	3.08	3.10	3.12	3.14	3.16	3.18	3.20	3.21	3.23	3.24	3.26	3.27	3.28	3.28	3.29	3.29	3.30	3.30	−10°
+ 5°	3.06	3.07	3.08	3.09	3.10	3.11	3.12	3.13	3.14	3.15	3.15	3.16	3.16	3.17	3.17	3.18	3.18	3.18	3.18	−5°
0°	3.06	3.06	3.06	3.06	3.06	3.06	3.06	3.06	3.06	3.06	3.06	3.06	3.06	3.06	3.06	3.06	3.06	3.06	3.06	0°
− 5°	3.06	3.05	3.04	3.03	3.02	3.01	3.01	3.00	2.99	2.98	2.97	2.97	2.96	2.96	2.95	2.95	2.95	2.95	2.95	+ 5°
−10°	3.06	3.04	3.02	3.00	2.98	2.96	2.95	2.93	2.91	2.90	2.88	2.87	2.86	2.85	2.84	2.84	2.83	2.83	2.83	+10°
−15°	3.06	3.03	3.00	2.97	2.94	2.91	2.89	2.86	2.84	2.81	2.79	2.77	2.76	2.74	2.73	2.72	2.71	2.71	2.71	+15°
−20°	3.06	3.02	2.98	2.94	2.90	2.86	2.82	2.79	2.75	2.72	2.69	2.67	2.64	2.62	2.61	2.60	2.59	2.58	2.58	+20°
−25°	3.06	3.01	2.96	2.90	2.85	2.80	2.75	2.71	2.67	2.63	2.59	2.56	2.53	2.50	2.48	2.47	2.45	2.45	2.44	+25°
−30°	3.06	3.00	2.93	2.87	2.80	2.74	2.68	2.62	2.57	2.52	2.48	2.44	2.40	2.37	2.34	2.32	2.31	2.30	2.30	+30°
−35°	3.06	2.98	2.90	2.82	2.75	2.67	2.60	2.53	2.47	2.41	2.35	2.30	2.26	2.22	2.19	2.16	2.15	2.14	2.13	+35°
−40°	3.06	2.97	2.87	2.78	2.68	2.59	2.51	2.42	2.35	2.28	2.21	2.15	2.10	2.05	2.02	2.00	1.97	1.95	1.95	+40°
−45°	3.06	2.95	2.83	2.72	2.61	2.50	2.40	2.30	2.21	2.12	2.05	1.97	1.91	1.86	1.81	1.78	1.75	1.74	1.73	+45°
−50°	3.06	2.93	2.79	2.65	2.52	2.39	2.27	2.16	2.05	1.94	1.85	1.77	1.69	1.63	1.58	1.53	1.50	1.49	1.48	+50°
−55°	3.06	2.90	2.73	2.57	2.41	2.26	2.11	1.97	1.84	1.72	1.61	1.51	1.42	1.34	1.28	1.23	1.19	1.17	1.17	+55°
−60°	3.06	2.86	2.66	2.47	2.28	2.09	1.91	1.74	1.58	1.44	1.30	1.18	1.07	0.98	0.90	0.84	0.80	0.77	0.76	+60°
−65°	3.06	2.82	2.57	2.33	2.09	1.86	1.64	1.43	1.23	1.05	0.88	0.73	+0.59	+0.48	+0.38	+0.31	+0.26	+0.22	+0.21	+65°
−70°	3.06	2.75	2.43	2.12	1.81	1.52	1.24	0.97	+0.72	+0.48	+0.27	+0.07	−0.10	−0.25	−0.37	−0.46	−0.53	−0.57	−0.59	+70°
−75°	3.06	2.63	2.20	1.78	1.37	+0.97	+0.58	+0.22	−0.13	−0.44	−0.74	−1.00	−1.23	−1.43	−1.60	−1.73	−1.82	−1.88	−1.90	+75°
−80°	3.06	2.41	1.75	+1.11	+0.49	−0.12	−0.71	−1.26	−1.78	−2.27	−2.71	−3.11	−3.47	−3.77	−4.02	−4.22	−4.36	−4.45	−4.48	+80°
−85°	+3.06	+1.74	+0.42	−0.87	−2.13	−3.36	−4.53	−5.65	−6.70	−7.68	−8.58	−9.38	−10.10	−10.71	−11.22	−11.61	−11.90	−12.07	−12.13	+85°
	180°	185°	190°	195°	200°	205°	210°	215°	220°	225°	230°	235°	240°	245°	250°	255°	260°	265°	270°	
	12ʰ 0ᵐ	12ʰ 20ᵐ	12ʰ 40ᵐ	13ʰ 0ᵐ	13ʰ 20ᵐ	13ʰ 40ᵐ	14ʰ 0ᵐ	14ʰ 20ᵐ	14ʰ 40ᵐ	15ʰ 0ᵐ	15ʰ 20ᵐ	15ʰ 40ᵐ	16ʰ 0ᵐ	16ʰ 20ᵐ	16ʰ 40ᵐ	17ʰ 0ᵐ	17ʰ 20ᵐ	17ʰ 40ᵐ	18ʰ 0ᵐ	R.A.
	−19.9	−19.9	−19.6	−19.3	−18.7	−18.1	−17.3	−16.3	−15.3	−14.1	−12.8	−11.4	−10.0	−8.4	−6.8	−5.2	−3.5	−1.7	−0.0	ann. var. in decln.
	24ʰ 0ᵐ 360°	23ʰ 40ᵐ 355°	23ʰ 20ᵐ 350°	23ʰ 0ᵐ 345°	22ʰ 40ᵐ 340°	22ʰ 20ᵐ 335°	22ʰ 0ᵐ 330°	21ʰ 40ᵐ 325°	21ʰ 20ᵐ 320°	21ʰ 0ᵐ 315°	20ʰ 40ᵐ 310°	20ʰ 20ᵐ 305°	20ʰ 0ᵐ 300°	19ʰ 40ᵐ 295°	19ʰ 20ᵐ 290°	19ʰ 0ᵐ 285°	18ʰ 40ᵐ 280°	18ʰ 20ᵐ 275°	18ʰ 0ᵐ 270°	R.A.

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Table of Precession Constants

for each decade from 1800 to 1900 inc.

Date	m	n	log n
1800	3.07082	20.0607	1.302346
1810	101	598	327
1820	120	590	309
1830	139	581	290
1840	158	573	272
1850	177	564	253
1860	196	555	234
1870	215	547	216
1880	234	538	197
1890	253	530	178
1900	3.07272	20.0521	1.302160

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Table Showing the Points in which The Ecliptic crosses the Meridians

Right ascension of Meridian		Declination of ecliptic + for right hand − " left hand Column of R.	Right Ascension of Meridian	
0 0	12 0	0° 0′	12 0	24 0
0 20	11 40	2 10	12 20	23 40
0 40	11 20	4 19	12 40	23 20
1 0	11 0	6 24	13 0	23 0
1 20	10 40	8 26	13 20	22 40
1 40	10 20	10 23	13 40	22 20
2 0	10 0	12 14	14 0	22 0
2 20	9 40	13 59	14 20	21 40
2 40	9 20	15 35	14 40	21 20
3 0	9 0	17 3	15 0	21 0
3 20	8 40	18 23	15 20	20 40
3 40	8 20	19 34	15 40	20 20
4 0	8 0	20 36	16 0	20 0
4 20	7 40	21 28	16 20	19 40
4 40	7 20	22 11	16 40	19 20
5 D	7 0	22 44	17 0	19 0
5 20	6 40	23 8	17 20	18 40
5 40	6 20	23 23	17 40	18 20
6 0	6 0	23 27	18 0	18 0

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Rule for determining the direction in which the needle of a galvanometer will turn.

Suppose my body is the conducting wire; the positive current flowing from head to foot; that I am lying parallel to the needle and facing it; Then the North end of the needle will always turn towards my right hand.

Rule for finding the pressure of a liquid on any surface

The pressure of a liquid on any surface is equal to the weight of a column of that liquid whose base is the surface in question and whose altitude is the depth of the centre of gravity of that surface below the upper surface of the liquid.

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Formulas for Falling Bodies

Let S be the space through which the body falls.
T " the time it takes to fall through S
V " " velocity at the end of the time
g " " space through which a body falls in one second of time; or $16\frac{1}{12}$ feet
$2g$ " " velocity at the end of one second

Then we shall have

$$S = gT^2 \qquad V = 2\sqrt{gS} \qquad T = \frac{V}{2g}$$
$$S = \frac{V^2}{4g} \qquad V = 2gT \qquad T = \sqrt{\frac{S}{g}}$$

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In this longitude determination
Mr. Dean & myself were stationed
at New Orleans and Messrs
Goodfellow and Blake at Galveston
S.C.C.J.

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Original

U. S. Coast Survey
Longitude Station New Orleans 1868.

(Written by me at Mr Dean's request)

I. Location of Station

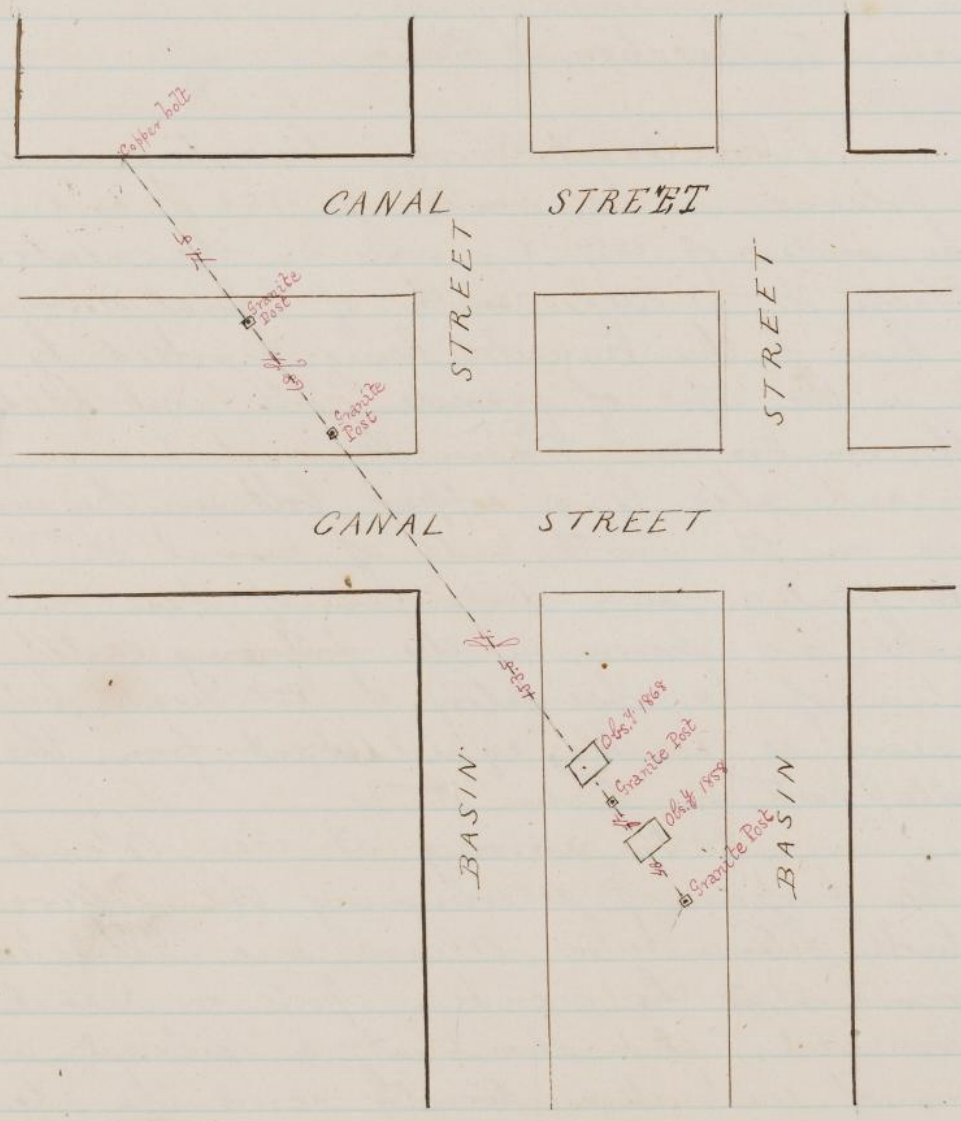
The Coast Survey observatory erected in New Orleans in the winter of 1856, and occupied in the winter of 1857-8, was on the neutral ground in Basin St., a little south of Canal St.:— the meridian line of the transit being marked by copper bolts in the tops of granite posts imbedded in the earth, ~~two~~ two on Basin St. and two on Canal St. —— and also by a copper bolt in the wall of a house on the north side of Canal St. The relative position and distances of these meridian marks are shown in the following sketch. The position of the observatory of 1858 was between, and as nearly as possible, equidistant from, the two granite posts on Basin St.

On arriving at New Orleans in 1868 for the purpose of determining the difference of longitude between New Orleans and Galveston, it was found that the meridian posts on Canal St. had been removed; and also that a railroad was in process of construction directly across the site of the station occupied in 1858. Accordingly the observatory was erected 36 ft. north of its former position; the centre of the transit being north of the Basin St. meridian posts 12.0 ft. & 60.0 ft. respectively. [See sketch]

On leaving the station the transit piers were left in position, and a bottle buried, with the top about a foot below the general surface of the ground, to mark the point immediately under the centre of the transit.

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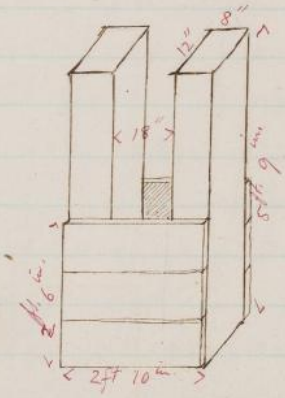


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II. Instruments.

The telescope used was C. S. Transit No 4, 2.75 in. aperture, and 46 in. focus. It was mounted upon two blocks of white oak, well seasoned, each 8 in. by 12 in., and 5 ft. 9 in. in length. These blocks, placed 18 in. apart, were sunk in the ground 2 ft. 6 in., and secured together by nailing pieces of plank firmly upon the north and south sides to the height of 2½ ft. from bottom, as in the figure: forming a sort of box, the top of which was level with the ground. It was hoped by this arrangement to attain a greater permanency of ~~instrument~~ instrumental adjustment than is secured by the ordinary plan of leaving the piers unconnected.



The Krille clock was mounted in the North-east corner of the observatory on a pier of seasoned yellow pine, 8 in. by 10 in., and 8 ft. 6 in. in length, sunk 3 ft. in the ground.

The chronograph used was Bond's Spring Governor C. S. No 1.

The interior arrangements of the observatory, & the telegraphic connections used for exchanging clock signals with Galveston, are exhibited in the accompanying sketch.

Both in sending and receiving signals the clock was made to break the local circuit.

In sending, the receiving magnet was cut out by closing a and b with a switch; and as this arrangement would leave local circuit open, the latter was closed by a small wedge inserted at the armature of the receiving magnet. The

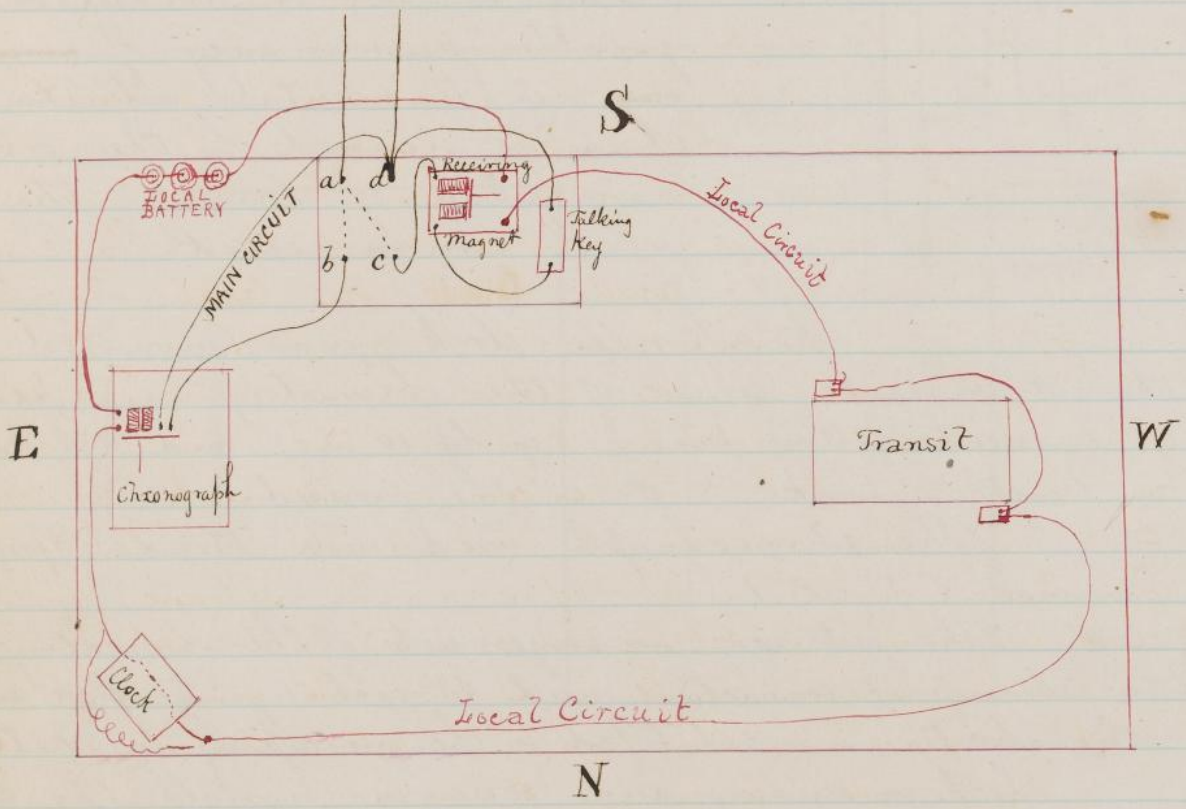
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clock breaks were then transmitted to the main line by the armature of the chronograph magnet.

In receiving, a and c were closed by switch and the wedge taken out, permitting the Galveston clock signals to come through the receiving magnet.

For the local battery, three Daniels cups of the form generally known as the Chester Battery, were employed.



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III. ~~The~~ Observations & Clock Signals

In determining the clock corrections, only the Standard stars of the American Ephemeris were employed. Time stars were, as a rule, observed only on the three middle tallies or fifteen threads. Where reversals on circumpolar stars were made, two tallies, or ten threads, were observed in each position of the instrument.

For determining the thread intervals eighteen complete transits of close polar stars were observed. And, on Mar. 14, a determination of the inequality of the transit pivots was made.

For determining the collimation correction, reversals on polar stars were made as frequently as possible. Care was also taken to observe about the same number of time stars in both positions of the instrument; and ~~thread~~ the level was read as often as circumstances permitted.

In exchanging clock signals each clock was made to send signals to the other station for five minutes, the eastern or N.O. clock sending first. Two such exchanges were made on each longitude night, and before each exchange the clock correction was determined by a set of observations.

Thus the programme for a night's work was as follows:

1. Set of obs^{ns}. for time and instrumental corr^{n}.
2. {N.O. sends clock sig^{s}. for 5 minutes.
   {Gal. " " " " " "
3. Set of obs^{ns}. for time and instrumental corr^{n}.
4. {N.O. sends clock sig^{s}. for 5 minutes
   {Gal. " " " " " "

In this way the difference longitude N.O – Gal. was determined on six nights; namely Feb. 7, 12, 13, 14, 15, & 17. On Feb. 7 however the N.O. clock sig^{s}. were not received in Galveston.

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After the operations for longitude were finished, Messrs. Dean & Goodfellow met at New Orleans, and, on Mar. 19, made a determination of their personal equation by observing the transits of 35 Stars. On each Star Tallies I and V were tapped by one observer, and II, III, & IV by the other; — the observers leading alternately.

IV. Telegraph Line.

The telegraph line between New Orleans & Galveston is constructed of N° 9 galvanized wire, with square glass insulators. The amount of battery power with which it was worked during the interval of the longitude experiments, was 86 cups, distributed as follows:—

Galveston	26	cups	Grove
Houston	25	"	"
New Orleans	35	"	Bi-chromate

At the time of the clock signal exchanges, all the relays on the line were generally cut out.

The Stations between New Orleans and Galveston, and their distances from the former, are as follow:—

New Orleans	La.	
Brashear City	"	80 miles
Franklin	"	100 "
New Iberia	"	125 "
Bayou Pierre Part	"	185 "
Lake Charles	"	230 "
Sassafras Island		260 "
Orange		265 "
Beaumont	Tex	289 "
Liberty	"	329 "
Houston	"	369 "
Virginia Point	"	412
Galveston	"	419 "

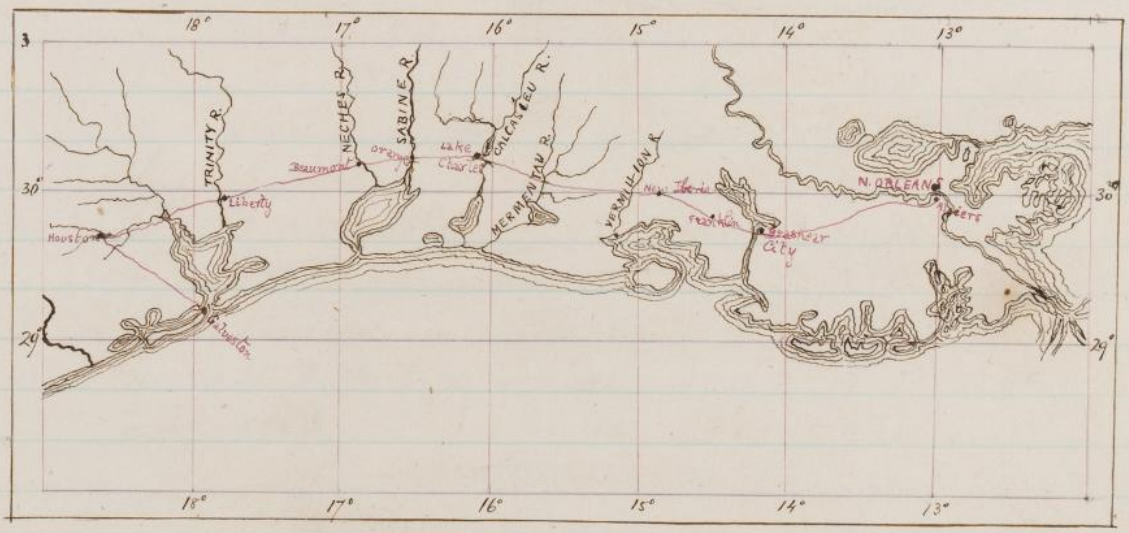
In addition to these, there are Railroad Offices at Algiers, La. & Houston, Tex., and Galveston, Tex.

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The following table gives the cables in the circuit, their lengths, and the number and kind of conductors.

Cable		Length in ft.	Conductors
Mississipi R.	N. O	2500	1 iron conductor
B. la Fourche		300	1 copper "
Berwicks Bay	Brash. City	2500	1 " "
Vermillion Bayou		200	1 iron "
Mermentau "		300	1 " "
Calcasieu R.	L. charles	400	1 " "
Sabine R.	Orange	1400	3 "
Neches R.	Beaumont	250	1 copper "
Trinity R		1000	1 " "
Galveston Bay	Galveston	10560	1 " "
		19410	



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Original

On the variable Star δ Librae

No 1700

[Original communication to the Astronomische Nachrichten]
Nov 1867.

I herewith send my observations, made during the past season, of the minima of δ Librae.

In the following list of comparison stars, a, f, g, h, and i, correspond to the b, c, d, e, and f, respectively employed by Prof. Schonfeld. [Ast. Nach. 1593] In forming my light scale, I have assumed, in accordance with his,

a = 0 i = 13

Letter	Name	Approx. Position 1870 α	δ	Mag.
a	B.A.C. 4945	14h 55.2m	−7° 3	0
b	" 5008	15 6.2	−10 31	0
c	Bessel Weisse 968	14 52.1	−4 28	1
d	15 Librae	14 49.7	−10 53	3
e	B.A.C. 5070	15 16.7	−11 54	4
f	178 Bode	15 31.7	−8 22	4½ c
g	15 Hev.	15 27.4	−8 44	8 d
h	31 ε Librae	15 17.2	−9 50	9
i	37 f "	15 27.1	−9 37	13

The first observation of a minimum of δ Librae was on Mar. 29, 1867. Comparisons on that date, though not sufficient for a determination of the time of minimum, showed that a fluctuation of light actually occurred, and thus confirmed Prof. Schonfeld's conjecture that the real period of this star is $2^d\ 7^h\ 51^m \pm$, and not $6^d\ 23^h\ 33^m \pm$, as had previously been supposed. The observations were as follow:

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$11^h\ 30^m$ Boston m. T. δ = h
15 16 " " = e
15 42 " " = e-1
16 8 " " = e-2
16 20 " " = e-2
16 40 " " = e-2

Daylight prevented further observation.

On subsequent dates determinations of the times of minima were made. Observations were generally begun an hour or more before minimum, and continued, at intervals of ten or fifteen minutes, until daylight, or the close approach of the star to the horizon. At each observation the variable was compared with at least two of the stars of the scale; generally with more.

Observed minima Boston m. T.						No. of observations Before min.	After min.	
1867	Apr.	26	2^d	14^h	51^m	5	3	good
	May	3		14	23	6	4	good
	"	10		14	13	2	3	uncertain
	"	24		13	12	6	4	good
	"	31		13	5	5	1	uncertain

These observations, after the application of the light equation, indicate the following corrections to the ephemeris in Ast. Nach. 1593, after adding $2^d\ 7^h\ 51^m$.

1867 Apr. 26 -28^m
May 3 -30
" 10 -13
" 24 -22
" 31 -4

I have never seen δ Librae brighter than h; ordinarily it is between that and g: At its faintest phase it is a trifle brighter than a, equal to c. That is, at maximum its light is $8\frac{1}{2}$, and at minimum, 1.

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Résumé of the Formulæ for computing parabolic elements of a comet's orbit

$t\ t'\ t''$	Times of observation
$\odot\ \odot'\ \odot''$	Longitudes of Sun
$A\ A'\ A''$	" " earth
$\alpha\ \alpha'\ \alpha''$	Geocentric Longitudes of comet
$\delta\ \delta'\ \delta''$	" Latitudes " "
$R\ R'\ R''$	Dist. of earth from sun
$\rho_1\ \rho_1'\ \rho_1''$	" " comet " earth
$\rho\ \rho'\ \rho''$	Cartate distances of comet from earth
$r\ r'\ r''$	Radii Vectores of orbit
$\beta\ \beta'\ \beta''$	Heliocentric Longitudes of comet
$\varpi\ \varpi'\ \varpi''$	" Latitudes " "
$\varepsilon\ \varepsilon'\ \varepsilon''$	Diff. of Hel. Long. of comet & earth
Ω	Longitude of ascending node
i	inclination
$w\ w'\ w''$	Arguments of Latitudes, or dist. from ascending node, on orbit
$\omega\ \omega'\ \omega''$	Arguments of lat reduced to ecliptic
$\chi = w'' - \alpha$	
$v\ v'\ v''$	True anomalies
D	Perihelion distance

$$\sin \varpi = \frac{\rho}{r} \operatorname{tg} \delta \qquad \sin \varpi'' = \frac{\rho''}{r''} \operatorname{tg} \delta''$$

$$\sin \varepsilon = \frac{\rho \sin(\odot - \alpha)}{r \cos \varpi} \qquad \sin \varepsilon'' = \frac{\rho'' \sin(\odot'' - \alpha'')}{r'' \cos \varpi''}$$

$$\begin{cases} \cot \omega = \operatorname{tg} \varpi'' \cot \varpi \operatorname{cosec}(\beta'' - \beta) - \cot(\beta'' - \beta) \\ \operatorname{tg}\left(\omega + \frac{\beta'' - \beta}{2}\right) = \frac{\sin(\omega'' + \varpi)}{\sin(\varpi'' - \varpi)} \operatorname{tg}\left(\frac{\beta'' - \beta}{2}\right) \end{cases}$$

Zenthum

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$\Omega = \beta - \omega$

$\operatorname{tg} \varphi = \operatorname{tg} \varpi \operatorname{cosec} \omega$

$\cos w = \cos \varpi \cos \omega$

$\cos w'' = \cos \varpi'' \cos (\beta'' - \Omega)$

$\operatorname{tg} \xi = \left(\frac{r}{r''}\right)^{\frac{1}{2}}$

$\operatorname{tg} \frac{1}{2} v = \cot \frac{1}{2} \chi - \left(\frac{r}{r''}\right)^{\frac{1}{2}} \operatorname{cosec} \frac{1}{2} \chi$

$\operatorname{tg} \frac{1}{2} v'' = -\cot \frac{1}{2} \chi + \left(\frac{r''}{r}\right)^{\frac{1}{2}} \operatorname{cosec} \frac{1}{2} \chi$

$\operatorname{tg} \left(\frac{1}{2} v + \frac{1}{4} \chi\right) = \operatorname{tg} (45^\circ - \xi) \cot \frac{1}{4} \chi$

$D = r \cos^2 \frac{1}{2} v =$ Perhelion Distance

$\beta = A + \varepsilon$

$\beta'' = A'' + \varepsilon''$

$w'' - v'' = \pi - \Omega$

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Science Observer.

BOSTON, APRIL, 1882.

ELEMENTS AND EPHEMERIS OF COMET WELLS, 1882.

The following observations of Comet Wells, 1882, were made by Prof. Lewis Boss, at Dudley Observatory, and are communicated with his permission.

		Albany M. T. d.	h.	m.	s.	α 1882.0 h.	m.	s.	δ 1882.0 °	′	″
1882.	March	19	15	18	12	17	54	51.68	33	29	49.7
		24	11	47	20	18	3	10.50	36	10	25.5
		30	11	39	8	18	14	26.85	39	55	12.4

These observations were corrected for aberration and parallax by means of Prof. Boss' Ephemeris, published in Science Observer Circular No. 23. From these observations I have computed the following

ELEMENTS.

$T =$ June 10.3076. Greenwich M.T.

	°	′	″	
$\pi =$	54	04	24	Mean Eq. 1882.0.
$\Omega =$	204	43	11	
$\omega =$	209	21	13	
$i =$	73	38	45	

$\log. q = 8.769098$

Middle Place $(c - o)$. $\quad \Delta \lambda \cos. \beta = -0''.9$ $\quad \Delta \beta = -1''.0$

From these elements I have derived the following

EPHEMERIS.

Greenwich midnight.			α 1882.0 h.	m.	s.	δ 1882.0 °	′	Log. △.	Light.
1882.	April	7.	18	31	48	45	37.9	0.1325	2.37
		11.	18	42	32	48	57.0	0.1082	2.90
		15.	18	55	14	52	32.7	0.0841	3.56
		19.	19	10	53	56	24.4	0.0605	4.40
		23.	19	31	03	60	29.7	0.0378	5.46
		27.	19	58	32	64	42.5	0.0165	6.79
	May	1.	20	38	13	68	49.4	9.9971	8.49
		5.	21	37	53	72	22.2	9.9802	10.63
		9.	23	04	03	74	26.9	9.9663	13.41
		13.	00	44	33	73	58.9	9.9559	16.94
		17.	2	09	15	70	47.2	9.9497	21.69
		21.	3	07	00	65	39.3	9.9480	28.37
		25.	3	44	34	59	20.0	9.9511	38.58
		29.	4	09	43	52	11.0	9.9596	56.51

The unit of light is that of the first observation — March 19.

For interpolation and extension of this ephemeris, the following equations of the coördinates are added:—

$$x' = r\,[9.961876] \sin (126^\circ\ 44'\ 13'' + v)$$
$$y' = r\,[9.861033] \sin (\ 61^\circ\ 14'\ 38'' + v)$$
$$z' = r\,[9.900941] \sin (197^\circ\ 17'\ \ 5'' + v)$$

H. V. Egbert.

Dudley Observatory, April 3, 1882.

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COMET WELLS, 1882.

To Editor of Science Observer:

From the elements communicated to you in this mail, by Mr. Egbert, it will be seen that the Wells Comet is likely to become an object of remarkable interest. These elements appear to be very exact, though too much should not be expected from comparisons based on star positions from Bessel and Lalande. I have not had leisure to examine the inferences which can be drawn from the elements in question, farther than a few leading points. In the first place, it is evident from a mathematical standpoint that this comet should have a brilliant career. Its light at perihelion should be more than three thousand times greater than at discovery. But it is a well known fact that great comets at perihelion always exceed in brightness the limits set for them by mathematical computation based on earlier observations. The display, whatever it is, will depend very largely on the physical constitution of the comet, rather than on the question of distance from the sun. I believe that this comet has a large and active nucleus; and if that proves really to be the case, in June we ought to see a tail of immense proportions. The brilliancy of the display will, however, suffer much from the presence of dawn and twilight; and merely as a spectacle this comet is not likely to take the highest rank. But to comet observers the behavior of the tail during the critical days, near June 10, will furnish phenomena of absorbing interest. Observations of the exact apparent location of the axis and boundaries of the tail among the stars will be throughout of the greatest importance. Star charts should be used and the position of the tail carefully laid down upon them as rapidly as practicable. The time of the observation should be noted to the nearest minute.

In the second place, though the comet will be visible for a long time after perihelion passage — until the middle or end of September, in all probability — it will not then be favorably located for observation in these latitudes. Observers near the Equator will have a better chance, and it is to be hoped that they will profit by it. It is hardly necessary to say that numerous observations, both before and after perihelion, of a comet which approaches the sun within five million miles, will be of inestimable service to astronomical science.

Any elements computed at present are liable to considerable apparent changes, but such changes will not materially affect the apparent path of the comet. I do not think the perihelion distance can be very much smaller than the figures given in Mr. Egbert's orbit.

Lewis Boss.

Dudley Observatory, April 3, 1882.

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Azimuth

[G.E.D. Davidson through Ed. Goodfellow]

1. By a close circumpolar star near its greatest elongation.

The operation for determining the azimuth of the star by this method will vary with the different kind of instruments used. With a non-repeating instrument, reading by micrometers, and the elongation mark not in the vertical plane passing through the star at or near Elongation, the scheme is, to observe several times upon the mark with the telescope direct and reversed: point the telescope to the star and make several level readings: observe upon the star about twice as often as upon the mark with tel. D & R, noting the time of each observation by a chronometer; reading the levels as at the beginning, and observing again upon the mark as before.

The mean reading of each of the micrometers is to be corrected for "run of micrometer" by the formula

$$\text{corr.} = \mp \frac{d}{2} + \frac{m.d}{D} \quad \text{when } f \gtrless b \qquad \text{I}$$

In this formula

D = the mean number of micrometer divisions between any two gradients.

f = the first or forward reading

b = the second or back "

$m = \frac{1}{2}(f+b)$

$d = (f-b)$

and a table of the corrections is made for each theodolite with the arguments f & $f-b$.

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To compute the azimuth of the star, and the hour angle at the pole for the greatest elongation, it is necessary first to correct the tabular apparent place of the star. For an hour angle of 6 hrs, the correction to the tabular apparent R of the star on account of diurnal aberration is a minimum, and the correction to the Δ may be applied to the final azimuth if desired.

The tabular apparent R of the star must be corrected for difference of longitude, for hour angle at the pole, and for terms of nutation involving 2 ☾; the latter found on pp. 422 & 423 of the British A.A. for every year

Corr.ⁿ R = Tabular app. R at U.C.
+ (diff. long. ± hour angle of star) × hourly diff.
II + corr. for terms of nutation for 2 ☾

Corrᵈ Δ = Tabular app. Δ at U.C.
+ (diff. of long. ± hour angle of star) × hourly diff.
III + 2 ☾ correction
+ Corrⁿ. for diurnal aberration.

The corrections for diurnal aberration are computed from the formula

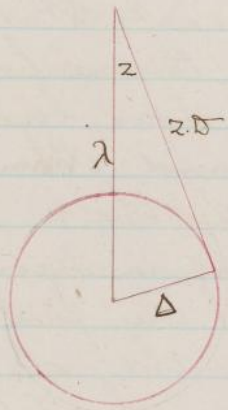
IV Diur. aberr.ⁿ in R = 0ˢ.0206 cos L. cos H cosec Δ
V " " " Δ = 0″.309 cos L sin H cos Δ

Or they may be taken from our table of corrections computed for 2 h & 8 hrs. min.ˢ and 51 inserted for every hour and for different latitudes.

With the corrected N.P.D, proceed to determine the Hour angle at the time of greatest elongation, and also the azimuth.

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$\cos He = \tan \Delta \tan L$ VII

$\sin He = \sin \Delta \sec L$ VIII

$\sin Z.D.e = \tan \Delta \cot Ze$ VIII

or $\cos ZDe = \cos \lambda \sec \Delta$

The sidereal time of $\frac{W}{E}$ elongation
= corr^d $R \pm He$
and the chronometer time of $\frac{W}{E}$ elong.
= corr^d $R \pm He$ + corr. for error of chron.
and this chron. error is $\pm$ when
the chron. is $\frac{\text{fast}}{\text{slow}}$ of sid. time.

Find the difference between each observed time and the chronometer time of elongation and call the separate intervals h, which become the arguments for obtaining from a table, the reductions of the corresponding ~~azimuth~~ obs^{ns}. to the azimuth at Elongation. The formula now used for the reduction is

$$\log. red.^{h.} = \log (\sin Ze . \cos Ze \operatorname{cosec}^2 He) + \log. \frac{2 \sin^2 \frac{1}{2} h}{\sin 1''} \pm \mu . \cot He \sin h \qquad IX$$

where μ = Modulus of common logs.

$\log \mu = 9.6377843$

He = Hour angle at the pole of the star at elongation

h = Intervals between obs$^{n.}$ and time of Elong$^{n.}$

Ze = Azimuth of star at Elong$^{n.}$

$\pm$ to be used after $\frac{E}{W}$ elong$^{n.}$ and before $\frac{W}{E}$ elong$^{n.}$

This formula is good for $1\frac{1}{2}$ hours on either side of elong$^{n.}$ for the Stars α & λ Urs. Min. and the first two terms only are needed when h is less than 15 minutes.

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The reduction is computed for each star observed for every minute of h, up to one hour from elongation, and for each half hour beyond. These results are then interpolated and tabulated for every ten seconds, with the differences for each second or tenth of a second.

The term $2 \sin^2 \frac{1}{2} p \operatorname{cosec} 1''$ is the first part of the usual red. to Merid. in the obs^ns. of Circum-Merid. Altitudes, but as the tables for it generally reach no farther than 35 min. it becomes necessary to compute it for our work. It answers for every station of course.

But if the table be not at hand, its construction may be avoided by writing

XI $\quad \text{Red}^{n} = [A] + [m] \pm n \cot He \sin h$

$[A]$ is computed once for all, for the particular star and station.

The values of $n \cot He \sin h$ are tabulated for every minute of time within the limits proper for the use of the formula, but it is necessary to compute them for every five or ten minutes only, and interpolate for the other minutes.

$[m]$, beyond the limit of the table of "Red. to Merid." may be replaced by $[\text{versin } h] - [\sin 1'']$ and in order to avoid the trouble of continually subtracting $[\sin 1'']$, $[A]$ may be replaced by $\frac{(\sin Ze \cos Ze \operatorname{cosec}^2 He)}{\sin 1''}$ which we will call A'

and the formula stands

XII $\quad \log. \text{Red}^{n} = [A'] + [\text{versin. } h] \pm n \cot He \sin h$

For Polaris this will give results within 0."1 at two and a half hours * from Elong^n

* See computation.

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For δ Ursæ Min. it is not advisable to use it beyond one hour, except in low latitudes.

But for rapid and preliminary field reductions, where the time from elong^{n}. is not more than 25 minutes, the following formula is close enough for Polaris

$$\text{Red}^{n} = \frac{2 \sin^2 \tfrac{1}{2} t}{\sin 1''} \tan Ze \qquad \text{XII}$$

Should a table of "Red^{n}. to Merid." not be at hand, the following is the same in another form, and is the original formula first investigated and proposed on the Coast Survey by Assist R. H. Tauntlevoy as hereafter noticed.

$$\text{Red.}^{n} = (112.5) \sin 1'' \times t^2 \times \tan Ze \qquad \text{XIII}$$

where t is the interval h in seconds of time from elongation.

{Diurnal Aberration}

If the tabular app. Δ have not been corr^{d} for diurn. Aberr. in the computation of H and Ze then the corr^{n} in Azimuth to the observed angle between the Star and the Elong^{n} Mark on account of Diurnal. Aberr. in Δ is $\pm$ if the Mark be $\frac{E}{W}$ of the true meridian; and is computed by the formula

$$\left.\begin{array}{l}\text{Corr}^{n} \text{ in Az. for} \\ \text{Diurn Aberr in } \Delta\end{array}\right\} = 0''.309 \cot ZD \cos L \qquad \text{XIV}$$

where ZD is the Zenith Dist. of the Star, and is computed by formula VIII or by the general formula

$$\sin ZD = \frac{\sin \Delta}{\sin Z} \cdot \sin H \qquad \text{XV}$$

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Or the following formula for the correction may be used

XVI $\left.\begin{matrix}\text{Corr}^{n}\text{ to Az. for} \\ \text{Diur. Ab. in } \Delta\end{matrix}\right\} = 0.309 \cos \Delta \sin^2 H$

Level Correction | The correction in azimuth to the observed angle between the star and elongation mark on account of inclination of the transit axis of the theodolite is ± if the star be to the $\frac{W}{E}$ of the mark; and is computed from the formula

XVII $$\text{Level Corr.} = \tan A \cdot d \cdot \frac{[W]-[E]}{n}$$

where A = Altitude of Star at time of obs^{n}; usually assumed equal to the Latitude of the Station *

d = the arc value of one division of the level scale

n = the number of level readings.

$[W]$ $[E]$ = the sums of the readings of the West and East ends of the level respectively

* [The change in alt^{n} of the Star near Elong. is
$\sin d A = \sin d H \cdot \sin \Delta$
or $d A'' = \sin d H \sin \Delta \operatorname{cosec} 1''$]

The stars to which this method of observing and reducing will apply are α, δ, γ λ Ursæ Min. γ δ Cephei; and the best of these is undoubtedly λ on account of its slow motion and small magnitude. It is readily recognized in the telescope by the grouping of the stars around it. α is too bright and large for delicate contact; and the others being further from the pole, the times of obs^{n}, and the error of chronometer have to be more accurately determined.

See example on next page for form of reduction of obsn^{s} for Azimuth made near elong. with the 30 inch Theod of the Coast Survey.

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Section I. Station Mount Clude Peninsula

Lat. = 43° 45' 32".18 Long = 7° 18' 53".04

Reduction of Obsns. for Azimuth

No of Obs.	Tel.	Time from Elongation	Reduct^{n} to Elong.	Corrected Mean Rdgs. of Theodolite	Theod. Rdgs. reduced to Elongation
1	D			45° 43' 1".6	
2	"			1.7	
3	"			1.5	
4	D	24m 50s.4	43".7	47 57 26.2	47 58 9.9
5	"	21 29.8	32.7	38.5	11.2
6	"	18 39.3	24.6	45.0	9.6
7	"	16 11.5	18.6	51.2	9.8
8	"	13 41.9	13.3	57.1	10.4
9	R	2 11.4	0.3	227 57 39.9	227 57 40.2
10	"	0 27.5	0.0	41.5	41.5
11	"	2 37.2	0.5	41.5	42.0
12	"	4 33.7	1.5	38.6	40.2
13	"	6 42.5	3.2	36.7	39.9
14	R				
15	"				
16	"				

September 19. 1849
2 Uss. Min. near Eastern Elong^{n}.
Chron. time Elong^{n} = 19h 9m 34.s5

Tel.	Mean Readg. of Elongation	Corr^{n} for want of Level	Corrected Means
D	47 58 10.18	−0.87	47 58 9.31
R	57 40.76	+1.97	57 42.73

Mean Rdg. of Star at Elong^{n}	47 57 56.02
Azimuth by Star	2 4 11.93
Rdg. of Meridian	45 53 44.09
" " Elongation Mark	45 42 54.00
Mark West of North	0 10 50.09
Corr^{n} for Diur. ab.	− 0.31
Mark West of North	0 10 49.78

In the usual repeating theodolites a different method of observing and recording is necessary; but all the star reductions and level corrections are essentially the same. The star is observed upon exactly as if it were a fixed object like the Elong^{n} mark, and the angle measured and repeated between the star and the mark, noting as usual the time of each obs^{n}. Our usual plan has been as follows:

Collimation of the cross lines reduced as much as possible; theodolite leveled and transit axis leveled; the illuminating lamps or lamps detached from the instrument and upon independent stands; vernier circle clamped;

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and the verniers read; level readings made with the telescope pointing to the star; 3 repetitions made on the star and the mark beginning on the left hand object; telescope reversed in the transit Y's, and three more repetitions made as before; level readings made as at beginning, verniers read for the six repetitions of the angle between the star and the mark, and the mean angle taken out. The time of each observation from the time of Elongation is then obtained, and the corresponding Reduction to Elongation taken from the previously computed table. The mean of these reductions is then applied to the mean angle observed, to which is also applied the level correction and the computed angle of the star at Elongation: the resulting angle between the Mark and the true meridian is thus obtained, to be finally corrected for the effect of diurnal aberration if the Δ have not been already corrected for it.

The following is an example of the data placed in the Record Book of Azimuth Obs.ns.

Section X Primary Triangulation

Ross mountain California

Lat. by Zen. Tel. Obs.ns = 38° 30′ 10.″03 used in determining following data

" from office 38 30 17.68 used in computing reduction table

Longitude " " 123° 6 10.99

" in time 8h 12m 24.s73

Table of R & Δ of λ Ursæ Min. at W. Long., corrected for diff. Long., hour angle, 2 Δ corr.n & for diur. aber.

"	1859 / 1860	R	Daily Diff	Δ	Daily diff	Sid. time W. Elong.	Daily Diff.	Azimuth W. Elong.	Daily diff
Dec	28	20h 2m 26.s28		1° 6′ 29.″2		1h 58m 54.s46		1° 24′ 57.″70	
Jan	2	23.75	−0.49	30.9	+0.34	52.05	−0.59	59.89	+0.44
	7	21.31	−0.49	31.9	+0.20	49.54	−0.50	61.16	+0.45
	12	18.85	−0.50	33.5	+0.32	46.98	−0.51	63.21	+0.41

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Method of Recording & Reduction

Station Ross. Mt. Sonoma Co. California. Dec. 27 P.M. 1859
Pleydolite U.S.C.S. 37 Observer G.D. Recorder E.H.F.

1 Elong. Mark d. i. | Verniers A & B | Means of A & B

λ Urs. Min. Sm. d. m. st. | 360° 0′ 15″ | 360° 0′ 15″
15

Level E 31.4 W 18.3
14.4 35.3
45.8 53.6

	Times of Obsn by Sid. chron 194	Times before W Elong.n	Reduction to Elongn.
D	1 25 56.5	34 47.6	58.77
	27 54.3	32 49.8	52.34
	29 3.5	31 40.6	48.72
			6)256.10
			42.68
R	1 32 48.0	27 56.1	37.90
	35 32.7	25 11.4	30.82
	36 55.0	23 49.1	27.55
			256.10

Levels E 19.2 W 29.2
35.1 13.1
54.3 42.3

2 Elong. Mark as bef. | 274° 53′ 24″ |
λ Urs. Min. " " | 24 | 274° 53′ 24″

Weather clear & cold. Wind. mod. N
Atmosphere clear, st.

	h	m	s
Sid. time W. Elong.	1	58	55.05
" chron fast at Elong.		1	49.15
Chron. time of W. Elong.	2	0	44.20

Rate gaining 0.s2 per hour

[W] = 95.9 Value of 1 div. of level scale
[E] = 100.1 = 0.″869
[W-E] − 4.2

−0.42 × 0.0865 = −0.″36 level corrn.

Mean obs^{d} angle	14°	11′	8.″50
Corr^{n} for want of level	−		0.36
Reduction to W Elong.	−		42.68
Azimuth of Star at Elong.	+ 1	24	57.26
Elong. Mark W. of N.	15	35	22.72

85 6 51.0
14 11 8.50

At Ross. Mt. Station seven nights obsns. for azimuth upon λ Urs. Min. with 34 sets of six observations each, gave the following results

Date 1859	No of Obs.	Resulting az. of Mark
Dec. 27	30	15° 35′ 19.″77
28	30	21.40
29	36	19.50
30	12	19.21
1860		
Jan. 9	30	23.24
10	36	20.43
11	30	20.72

Mean 15° 35 20.70 prob. error ±0.206

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2. When the theodolite is nearly in the vertical plane of the star (near elong^{n}) and the Elongation Mark, the aim being to place the mark so that it should be about the middle of the change of azimuth, during the proposed term of observation, the micrometer of the telescope may then be used to determine the angle between the mark and star.

This method becomes available with a Transit Inst^{t}. whilst the long transit axis and the more delicate level are advantages, and the facility with which the observations can be made, recommend it for those cases where an inferior theodolite must otherwise be used, and where a mark can be placed in a proper position.

It however requires the determination of the arc value of the micrometer, and the regularity of the screw. It also requires another transit, or a sextant to determine the error of the chronometer, so that the same transit be used upon another block and readily placed in the meridian by a proper mark.

Great care is necessary, in centreing the theodolite, used for referring the mark to the lines of the triangulation, in the line of collimation of the transit when pointed at the mark, especially if the latter be not far distant. It may be simply and accurately effected in the following manner: insert a pencil in the centre of a cork, and fit the latter into the eyetube of the transit ~~instrument~~ telescope, after removing the eyepiece. With the point of the pencil describe a mark on the stand under the vertical position of the telescope using the motion of the diaphragm tube to get the proper length. Reverse the telescope in its Y's and repeat the operation. Then the

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middle between the two lines described will be accurately in the vertical plane in which the theodolite is to be mounted.

Specimen of Work from C.S. Report 1856, p. 209

W. Elongation of Polaris

Time of elongn by chron. $7^h\ 6^m\ 0^s$

Arc value of 1 div. of Micrometer 2".18

" " " " Level scale 2".00

Increase of the Micr. readgs. corresponds to a movement of the telescope from North to West

Object	Lamp	Level Readings	Times of Observation	Micrm. Rdgs.	Time from Elongn ^{m} ^{s}	Redn to Elongn. "
Elong. Mark	E		$7^h\ 15^m\ 5^s$	186.0		
		D 30.5 30.5		186.1		
		R 30.5 30.5		185.8		
α UrsMin.			7 18 59.5	177.8	12 59.5	− 9.7
			19 21.5	177.2	13 21.5	10.2
			19 47.0	177.2	13 47.0	11.0
	W		21 55.5	175.3	15 55.5	14.5
			22 16.5	175.9	16 16.5	15.3
			22 45.5	175.0	16 45.5	16.2
		R 30.0 31.0				− 12.82
		D 31.5 29.5				
Elong. Mark		122.5 121.5	7 23	187.0		
		$-1^d.00$		186.0		
		$= -0''.15$		187.0		

Readings of Mark Lp. East = 186.0

" " " " West = 186.7 Mean 186.32

Readings of Star Lp. East = 177.4

" " " " West = 175.4 Mean 176.40

Mark W. of Star (micr. div.) + 9.92

" " " (arc) + 21".62

Reduction to Elongation − 12.82

Correction for level − 0.15

Mark W. of Elongation + 8".65

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Original

Computation of Precession.

I. Precession in Right Ascension & Declination

1. For stars not near the pole.

When the mean right ascension and declination of a star not very near the pole are given for any date, the corrections necessary to obtain the mean right-ascension and declination at another date may be found by multiplying the interval in years between the two dates by the annual precessions in right ascension & declination corresponding to the middle of the interval.

The formulæ for computing the annual precessions are

$$\begin{aligned} \text{Ann. Prec. in } \alpha &= m + n \sin\alpha \tan\delta \\ \text{" " " } \delta &= n \cos\alpha \end{aligned} \qquad \text{I}$$

in which the constants m & n are to be found by the following:

$$\begin{aligned} m &= 46''.0623 + 0''.0002849\,[y - 1800] \\ n &= 20''.0607 - 0.0000863\,[y - 1800] \end{aligned} \qquad \text{II}$$

where y is the date for which the annual precessions are to be computed.

The values of m in arc and time, and of n and its logarithm, are given in the following table for every decade from 1800 to 1900 inclusive. The values for any date may be had by simple interpolation.

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Date	m arc	m time	n	log n
1800	46.0623	3.07082	20.0607	1.302346
1810	46.0650	3.07101	20.0598	1.302327
1820	46.0680	3.07120	20.0590	1.302309
1830	46.0709	3.07139	20.0581	1.302290
1840	46.0737	3.07158	20.0573	1.302272
1850	46.0765	3.07177	20.0564	1.302253
1860	46.0794	3.07196	20.0555	1.302234
1870	46.0822	3.07215	20.0547	1.302216
1880	46.0851	3.07234	20.0538	1.302197
1890	46.0880	3.07253	20.0530	1.302178
1900	46.0908	3.07272	20.0521	1.302160

In accordance with the forgoing, the values of the quantities m, n, α & δ used in computing the annual precessions should be those corresponding to the middle of the interval between the two dates. But since the two last are not known, they must be found by computing the annual precessions using the α and δ given; and by means of these approximate precessions the α and δ at the middle date may be found, with which the annual precessions may be calculated with all desired accuracy.

Example: — The right ascension and declination of α Hydræ for 1868.0 are given in the American Ephemeris for 1868, as follow

$\alpha = 9^h\ 21^m\ 6^s.06 \qquad \delta = -8^\circ\ 5'\ 15''.9$

What are the right ascension and declination for 1855.0

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In this case the middle date is 1861.5, and for this time m and n may be taken from the table, or obtained from (II) as follows:—

$(y - 1800) = (1861.5 - 1800) = +61.5$

$m = 46''.0623 + 0''.0175 \cdot = +46''.0798$

$n = 20''.0607 - 0''.0053 = +20''.0554$

We now proceed to find by (I) approximate values of the annual precessions using the α and δ given. The following is the most convenient form of computation.

time α	$9^h\ 21^m\ 6^s$	9 21 25
arc "	$140°\ 16'\ 30''$	140 21 15
δ	$-8\ \ 5\ \ 16$	−8 3 36
tan δ	n 9.15260	n 9.15109
sin α	9.80557	9.80485
log n	1.30223	1.30223
cos α	n 9.88600	n 9.88649
log n sin α tan δ	n 0.26040	n 0.25817
$n°$ "	− 1.8214	− 1.812
Ann. prec. in α (arc)	+44''.2584	+44''.2677
" " " " (time)	+ 2^s.9506	+ 2^s.9512
n cos α	n 1.18823	n 1.18872
Ann. prec. in δ	− 15''.425	− 15''.443

Multiplying these approximate annual precessions by 6.5 and subtracting the resulting quantities from the α and δ for 1868, we obtain α and δ for the middle date 1861.5

$\alpha = 9^h\ 21^m\ 25^s$ $\qquad$ $\delta = -8°\ 3'\ 36''$

with which and the same m and n as before, we obtain by a similar computation (carried) to the one just made, the accurate values,

Ann. Prec in α = + 2^s.9512 } for 1861.5

" " " δ = − 15''.443 }

multiplying these by 13, we get,

Prec. for 13 years in α = + 0^m 38^s.37

" " " " " δ = − 3' 20''.76

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which being subtracted from the right ascension and declination for 1868.0, give,

$$\left.\begin{aligned}\alpha &= 9^h\ 20^m\ 27^s.69\\ \delta &= -8^\circ\ 1'\ 55''.14\end{aligned}\right\} \text{for } 1855.0$$

When the interval for which the precession is to be computed does not exceed 6 or 8 years, the differences between the given right ascension & declination and those for the middle date are so small that their effect on the annual precessions is insensible, and hence a second approximation is unnecessary.

If in the preceding example, we had employed the values obtained by the first approximation, we should have found

$$\left.\begin{aligned}\alpha &= 9^h\ 20^m\ 27^s.68\\ \delta &= -8^\circ\ 1'\ 55''.37\end{aligned}\right\} \text{for } 1855.0$$

which differ but little from those given above, though in this case the interval is thirteen years.

When, as is often the case, the catalogue from which we wish to reduce the place of a star, contains approximate values of the precession, these may be used with great advantage in obtaining the coordinates of the star at the middle date between the epoch of the catalogue and the date to which we wish the star's place reduced. The accurate annual precessions may then be calculated by (I) as heretofore shown.

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2. For stars near the Pole.

The changes of the mean places (more especially of the right ascensions) of stars near the pole are so rapid and variable that we cannot in their case make use of the differential formulæ just given, but must have recourse to more rigorous processes

Let α, δ be the given right ascension and declination of a star at the time $1800 + t$

α', δ' be the right ascension and declination required for the time $1800 + t'$

The first step is to find the values of ε_1, ψ, and λ, for both dates, as follows:

$$\left.\begin{aligned} \varepsilon_1 &= 23^\circ\ 27'\ 54''.22 + 0.00000735\ t^2 \\ \psi &= 50''.3798\ t - 0.0001084\ t^2 \\ \lambda &= 0.15119\ t - 0.00024186\ t^2 \end{aligned}\right\} \text{III}$$

where t = the number of years from 1800, minus before, and plus after, that date.

Then, denoting the values of ε_1, ψ, and λ corresponding to $1800 + t'$ by accents (i.e. ε_1', ψ', λ') the constants z', z, and θ must be computed by the following formulæ.

$$\left.\begin{aligned} \tan \tfrac{1}{2}(z' + z) &= \tan \tfrac{1}{2}(\psi' - \psi) \cos \tfrac{1}{2}(\varepsilon_1' + \varepsilon_1) \\ \tfrac{1}{2}(z' - z) &= \frac{\tfrac{1}{2}(\varepsilon_1' - \varepsilon_1)}{\tan \tfrac{1}{2}(\psi' - \psi) \sin \tfrac{1}{2}(\varepsilon_1' + \varepsilon_1)} \\ \sin \tfrac{1}{2}\theta &= \sin \tfrac{1}{2}(\psi' - \psi) \sin \tfrac{1}{2}(\varepsilon_1' + \varepsilon_1) \end{aligned}\right\} \text{IV}$$

Having found by means of the first two equations of IV half the sum, and half the difference of z' and z, we have, of course,

$$z = \tfrac{1}{2}(z' + z) - \tfrac{1}{2}(z' - z)$$
$$z' = \tfrac{1}{2}(z' + z) + \tfrac{1}{2}(z' - z)$$

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Before proceeding further with the problem, I shall give an example of the calculation of the constants by formulæ (III) and (IV).

Example: — The mean place of a star being given for 1840, and that for 1870 being required, what are the values of z, z' and θ?

First by III we compute ε, ψ, and λ for 1840 and 1870.

	1840	1870
t	40	70
t^2	1600	4900
$+0''.00000735\ t^2$	+0.011760	+0.036015
	23 27 54.22	23 27 54.22
ε_1	23 27 54.231760	23 27 54.256015
$50''.3798\ t$	+ 33′ 35.192	+ 58 46.586
$-0''.0001084\ t^2$	− 0.17344	− 0.53116
ψ	+ 33 35.01856	+58 46.05484
$0''.15119\ t$	+ 6.0476	+ 10.5833
$-0''.00024186\ t^2$	− 0.3870	− 1.1851
λ	+ 5.6606	+ 9.3982

We have then

$\varepsilon_1 = 23^\circ\ 27'\ 54''.231760$ $\quad \varepsilon_1' = 23^\circ\ 27'\ 54''.256015$

$\psi = +\ 33\ 35.01856$ $\quad \psi' = +\ 58\ 46.05484$

$\lambda = +\ 5.6606$ $\quad \lambda' = +\ 9.3982$

and with these we can find z z' and θ by (IV) arranging our computation thus: —

	$\frac{1}{2}(\psi' - \psi)$	$+0^\circ\ 12'\ 35''.51814$
	$\frac{1}{2}(\varepsilon_1' + \varepsilon_1)$	23 27 54.243887
	$\frac{1}{2}(\varepsilon_1' - \varepsilon_1)$	+0.012127
(1)	$\sin \frac{1}{2}(\psi' - \psi)$	7.5638186
(2)	$\tan \frac{1}{2}(\psi' - \psi)$	7.5638216
(3)	$\sin \frac{1}{2}(\varepsilon_1' + \varepsilon_1)$	9.6000902
(4)	$\cos \frac{1}{2}(\varepsilon_1' + \varepsilon_1)$	9.9625128
(2)+(4)	$\tan \frac{1}{2}(z' + z)$	7.5263344
	$\frac{1}{2}(z' + z)$	$0^\circ\ 11'\ 33''.0394$
(5)	$\log \frac{1}{2}(\varepsilon_1' - \varepsilon_1)$	8.0837534
(6)	(2)+(3)	7.1639118
(5)−(6)	$\log \frac{1}{2}(z' - z)$	0.9198416
	$\frac{1}{2}(z' - z)$	8″.3146
(1)+(3)	$\sin \frac{1}{2}\theta$	7.1639088
	$\frac{1}{2}\theta$	$0^\circ\ 5'\ 0''.8390$

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Hence we have

$$z = +11' 24.''7248$$
$$z' = +11\ 41.3540$$
$$\theta = +10\ 1.6780$$

to which we will add

$$\sin \theta = 7.4649384$$
$$\tan \tfrac{1}{2}\theta = 7.1639093$$

which will be useful.

With the constants thus determined we may now proceed to determine α' and δ' by one of four methods, which I shall give, and illustrate by examples.

First Method: — Putting

$$\left.\begin{aligned} A &= \alpha + \lambda + z \\ A' &= \alpha' + \lambda' - z' \end{aligned}\right\} \quad V$$

we have

$$\left.\begin{aligned} \tan M &= \frac{\tan \delta}{\cos A} \\ \tan A' &= \frac{\tan A \cos M}{\cos (M + \theta)} \\ \tan \delta' &= \tan (M + \theta) \cos A' \end{aligned}\right\} \quad VI$$

By the first equation of (V) and the first two of (VI,) A' may be found, and then by the second of (V)

$$\alpha' = A' - (\lambda' - z')$$

and by the third of (VI) we can then obtain δ'.

The ambiguity as to the quadrant in which A' is to be taken, is removed by the fact that A' is greater or less than 180°, according as A is greater or less than 180°.

Example: — The mean place of λ Ursæ Minoris for 1840.0 is

$$\alpha = 20^h\ 21^m\ 38^s.302$$
$$\delta = 88°\ 49'\ 44.''456$$

Required the mean place for 1870.0.

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In the solution of this example we can make use of the constants deduced in the preceding example. We there found

$$\begin{aligned} z &= +11'\ 24''.7248 \\ z' &= +11\ 41.3540 \\ \theta &= +10\ 1.6780 \\ \lambda &= +\ 5.6606 \\ \lambda' &= +\ 9.3982 \end{aligned}$$

The computation of α' and δ' by (V) and (VI) may be arranged according to the following convenient form.

(time) α	$20^h\ 21^m\ 38.302$
(arc) α	$305°\ 24'\ 34.53$
δ	$88°\ 49'\ 44''.456$
$\lambda + z$	$+11\ 30.3854$
$\lambda' - z'$	$-11\ 31.9558$
A	$305\ 36\ 4.9154$
$\tan \delta$	1.6895110
$\sec A$	0.2349709
$\tan M$	1.9244819
M	$89°\ 19'\ 5''.728$
$M + \theta$	$89\ 29\ 7.406$
$\tan A$	n 0.1451077
$\cos M$	8.0754873
$\sec (M+\theta)$	2.0466508
$\tan A'$	n 0.2672458
A'	$298°\ 23'\ 20''.16$
$\tan (M+\theta)$	2.0466332
$\cos A'$	9.6771088
$\tan \delta'$	1.7237420
(arc) α'	$298°\ 34'\ 52''.11$
(time) α'	$19^h\ 54^m\ 19.474$
δ'	$88°\ 55'\ 3''.89$

Second Method: — Still preserving the notation in (V), we can find α' and δ' by the equations

VII
$$\begin{cases} \cos \frac{1}{2}(90° + \delta') \sin \frac{1}{2}(A' + C) = \cos \frac{1}{2}(90° + \delta - \theta) \sin \frac{1}{2} A \\ \cos \frac{1}{2}(90° + \delta') \cos \frac{1}{2}(A' + C) = \cos \frac{1}{2}(90° + \delta + \theta) \cos \frac{1}{2} A \\ \sin \frac{1}{2}(90° + \delta') \sin \frac{1}{2}(A' - C) = \sin \frac{1}{2}(90° + \delta - \theta) \sin \frac{1}{2} A \\ \sin \frac{1}{2}(90° + \delta') \cos \frac{1}{2}(A' - C) = \sin \frac{1}{2}(90° + \delta + \theta) \cos \frac{1}{2} A \end{cases}$$

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where all the quantities in the second member are known. Subtracting the 2nd equation from the 1st, and the 4th from the 3d, we obtain the values of $\tan \frac{1}{2}(A'+C)$ and $\tan \frac{1}{2}(A'-C)$. $\frac{1}{2}(A'+C)$ and $\frac{1}{2}(A'-C)$ should always be taken out between $0°$ and $+180°$; and their sum will give A', whence we have

$$\alpha' = A' - (\lambda' - z')$$

δ' may be found by getting $\cos \frac{1}{2}(90° + \delta')$ from the first or second equation of (VII). If the sine of $\frac{1}{2}(A'+C)$ is greater than its cosine, it will be more accurate to use the first equation; if the reverse, the second equation is the one to be used. The third or fourth equations should never be used for the purpose of getting δ', since for stars near the pole, $\frac{1}{2}(90° + \delta')$ is nearly $90°$, in which neighborhood an arc cannot be determined by its sine with any precision.

Example: — Let us take the same example that we have already solved by the first method. We have there given

$$\begin{aligned}
\alpha \text{ (time)} &= 20^h\ 21^m\ 38^s.302 \\
\text{(arc)} &= 305°\ 24'\ 34''.53 \\
\delta &= 88°\ 49'\ 44''.456 \\
\lambda + z &= +\ 11\ 30.3854 \\
\lambda' - z' &= -\ 11\ 31.9558 \\
\theta &= +\ 10\ 1.6780
\end{aligned}$$

and the calculation by VII will be the following:—

(over)

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	$\frac{1}{2}A$	152° 48′ 2.″458
	$\frac{1}{2}(90°+\delta-\theta)$	89 19 51.389
	$\frac{1}{2}(90°+\delta+\theta)$	89 27 53.067
(1)	$\cos\frac{1}{2}(90°+\delta-\theta)$	8.0673317
(2)	$\sin\frac{1}{2}A$	9.6599992
(3)	$\sin\frac{1}{2}(90°+\delta-\theta)$	9.9999704
(4)	$\cos\frac{1}{2}(90°+\delta+\theta)$	7.9425114
(5)	$\cos\frac{1}{2}A$	n9.9491077
(6)	$\sin\frac{1}{2}(90°+\delta+\theta)$	9.9999834
(7)	(1)+(2)	7.7273309
(8)	(4)+(5)	n7.8916191
(7)−(8)	$\tan\frac{1}{2}(A'+C)$	n9.8357118
	$\frac{1}{2}(A'+C)$	145° 35′ 15.″177
(9)	(2)+(3)	9.6599696
(10)	(5)+(6)	n9.9490911
(9)−(10)	$\tan\frac{1}{2}(A'-C)$	n9.7108785
	$\frac{1}{2}(A'-C)$	152° 48′ 4.″942
(11)	$\sin\frac{1}{2}(A'+C)$	9.7521609
(12)	$\cos\frac{1}{2}(A'+C)$	9.9164490
(7)−(11)	$\cos\frac{1}{2}(90°+\delta')$	7.9751700
(8)−(12)	$\cos\frac{1}{2}(90°+\delta')$	7.9751701
	$\frac{1}{2}(90°+\delta')$	89° 27′ 31.″943
	A'	298 23 20.119
	(arc) α'	298 34 52.075
	time α'	$19^h\ 54^m\ 19^s.472$
	δ'	89° 55′ 3.″886

Third Method:— A method somewhat more accurate than either of the preceding is to form

$$p = \sin\theta\,(\tan\delta + \tan\tfrac{1}{2}\theta\cos A)$$

VIII. and then to compute the differences $A'-A$, and $\delta'-\delta$ as follows:—

$$\tan(A'-A) = \frac{p\sin A}{1-p\cos A}$$

$$\tan\tfrac{1}{2}(\delta'-\delta) = \tan\tfrac{1}{2}\theta\,\frac{\cos\frac{1}{2}(A'+A)}{\cos\frac{1}{2}(A'-A)}$$

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Example :— Taking the same example as before the computation by (VIII) is as follows

A	$305^\circ\ 36'\ 4.''9154$
δ	$88\ 49\ 44.456$
$\cos A$	9.7650291
$\tan \frac{1}{2}\theta$	7.1639093
$\log \tan \frac{1}{2}\theta \cos A$	6.9289384
no " " "	0.0008491
nat. tan δ	48.9227640
$\tan \delta + \tan \frac{1}{2}\theta \cos A$	48.9236131
$\log(\tan \delta + \tan \frac{1}{2}\theta \cos A)$	1.6895186
$\sin \theta$	7.4649384
$\log p$	9.1544570
$\sin A$	$n\,9.9101368$
$\log p \cos A$	8.9194861
$1 - p \cos A$	0.9169220
a.c. $\log(1 - p \cos A)$	0.0376676
$\log p \sin A$	$n\,9.0645938$
$\tan (A' - A)$	$n\,9.1022614$
$A' - A$	$-7^\circ 12'\ 44.''746$
A'	$298\ 23\ 20.17$
$A' - (z' - z) = \alpha'$ (arc)	$= 298\ 34\ 52.13$
α (time)	$= 19^h\ 54^m\ 19^s.475$

$\frac{1}{2}(A' + A)$	$301\ 59\ 42.543$
$\frac{1}{2}(A' - A)$	$-\ 3\ 36\ 22.373$
$\cos \frac{1}{2}(A' + A)$	9.7241509
$\sec \frac{1}{2}(A' - A)$	0.0008608
$\tan \frac{1}{2}\theta$	7.1639093
$\tan \frac{1}{2}(\delta' - \delta)$	6.8889210
$(\delta' - \delta)$	$+\ 5'\ 19.''430$
δ'	$88^\circ\ 55'\ 3.''886$

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Fourth Method:— When the interval of time is very great the computation of the precession by the preceding methods does not give results of the highest accuracy. When, therefore the greatest precision is required, the following approximative mode may be resorted to advantageously. It is based on the principle that logarithmic changes for very small variations of the arguments are nearly uniform.

We have first the following trigonometric relations

IX
$$\left\{\begin{aligned} \tan\tfrac{1}{2}(\delta'-\delta) &= \frac{\cos\frac{1}{2}(A'+A)}{\cos\frac{1}{2}(A'-A)}\tan\tfrac{1}{2}\theta \\ \cot\tfrac{1}{2}(\delta'+\delta) &= \frac{\sin\frac{1}{2}(A'+A)}{\sin\frac{1}{2}(A'-A)}\tan\tfrac{1}{2}\theta \end{aligned}\right.$$

where as heretofore

V
$$\left\{\begin{aligned} A &= \alpha + \lambda + z \\ A' &= \alpha' + \lambda' - z'. \end{aligned}\right.$$

Assume an approximate value of α' and call it (α'). Let x be a correction, in seconds of arc, to the assumed value so that

$$\alpha' = (\alpha') + x$$

Substituting this in the second equation of (V) and making $a = (\alpha') + \lambda' - z'$ we get

$$A' = a + x$$

whence we see that

$$\tfrac{1}{2}(A'+A) = \tfrac{1}{2}(a+A) + \tfrac{1}{2}x$$
$$\tfrac{1}{2}(A'-A) = \tfrac{1}{2}(a-A) + \tfrac{1}{2}x$$

Now let c, s, c', s', represent the changes of the logarithms of $\cos\frac{1}{2}(a+A)$, $\sin\frac{1}{2}(a+A)$, $\cos\frac{1}{2}(a-A)$, $\sin\frac{1}{2}(a-A)$, respectively, corresponding to a change of $+1''$ in (α'); it is evident that, since for small variations of a quantity the changes in

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its logarithm are nearly uniform, when x is small,

$$\log \tan \tfrac{1}{2}(\delta' - \delta) = \log\left[\tan \tfrac{1}{2}\theta \, \frac{\cos \frac{1}{2}(a+A)}{\cos \frac{1}{2}(a-A)}\right] + \tfrac{1}{2}(c-c')x$$

$$\log \cot \tfrac{1}{2}(\delta' + \delta) = \log\left[\tan \tfrac{1}{2}\theta \, \frac{\sin \frac{1}{2}(a+A)}{\sin \frac{1}{2}(a-A)}\right] + \tfrac{1}{2}(s-s')x$$

Let us now for convenience put

$$\left.\begin{aligned} \tan d &= \tan \tfrac{1}{2}\theta \, \frac{\cos \frac{1}{2}(a+A)}{\cos \frac{1}{2}(a-A)} \\ \cot d' &= \tan \tfrac{1}{2}\theta \, \frac{\sin \frac{1}{2}(a+A)}{\sin \frac{1}{2}(a-A)} \end{aligned}\right\} \text{X}$$

and denote by τ and τ' the changes of the logarithms of $\tan d$ and $\cot d'$ for a change of $+1''$ in d and d', respectively. Then since the correction to the logarithm of $\tan d$ for a change of x seconds in (α') is $\frac{1}{2}(c-c')x$, the correction to d in seconds will be $\frac{\frac{1}{2}(c-c')}{\tau} x$; similarly, the correction to d' will be $\frac{\frac{1}{2}(s-s')}{\tau'} x$; Making

$$\frac{\frac{1}{2}(c-c')}{\tau} = \beta \qquad \frac{\frac{1}{2}(s-s')}{\tau'} = \gamma \qquad \text{XI}$$

we shall have

$$\tfrac{1}{2}(\delta' - \delta) = d + \beta x$$
$$\tfrac{1}{2}(\delta' + \delta) = d' + \gamma x$$

and consequently

$$\delta' = d' + d + (\gamma + \beta)x \qquad \text{XII}$$
$$\delta = d' - d + (\gamma - \beta)x \qquad \text{XIII}$$

from the second of which we get the value of the correction

$$x = \frac{\delta - (d' - d)}{\gamma - \beta} \qquad \text{XIV}$$

to be applied to our assumed value of α'.

In making the computation we first assume (α'); then with

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$$a = (\alpha') + \lambda' - z'$$
$$A = \alpha + \lambda + z$$

we get d and d' from (X), and β and γ from (XI). The correction x is then had from (XIV); and this applied to (α') will give the value of α' very near the truth, unless the correction x is large;— in which case it will be necessary to make another approximation, and perhaps, indeed, a third. In each approximation the value of α' obtained from the last is to be used as the ~~assum~~ assumed (α').

The required declination δ' is found by (XII).

It need hardly be mentioned that $c, c', s, s', \tau, \tau'$, may be most conveniently taken out of the tables at the same time with $\sin \frac{1}{2}(a+A)$ $\cos \frac{1}{2}(a+A)$ &c, &c... It should be strictly borne in mind that these quantities are the logarithmic changes corresponding to a change of one second in a positive direction of the arguments, and great care accordingly exercised with the signs.

Instead of taking the two consecutive seconds as given in the table, it will in most cases be more accurate to employ the average difference for ten seconds. Thus, if we wish to find the change in plus one second of the logarithm of cosine $(3^\circ\ 21'\ 17'')$, by taking one tenth of the difference of $\cos(3^\circ\ 21'\ 12'')$ and $\cos(3^\circ\ 21'\ 22'')$ we should get a more precise result than by using the difference from 16" to 17", or from 17" to 18".

Example:— I shall take the example already solved by the other methods, and, assuming $(\alpha') = 298^\circ\ 34'\ 19''.2000$, shall make two approximations. The degrees ~~and minutes~~ of arcs, and the characteristics and ~~first three~~ two decimals of logarithms will be

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omitted in the second approximation as they are the same as for the first.

	1st approx.	2nd approx.
α	305° 24′ 34.″53	
δ	88 49 44.456	
$\lambda+z$	+ 11 30.3854	
$\lambda'-z'$	− 11 31.9558	
(α')	298. 34 19.2000	34′ 49.″697
a	298 22 47.2442	23′ 17.″7412
A	305 36 4.9154	36 4.9154
$\frac{1}{2}(a+A)$	301 59 26.0798	59 41.3283
$\frac{1}{2}(a-A)$	− 3 36 38.8356	36 23.5871
cos $\frac{1}{2}(a+A)$	9.7240954	41468
sec $\frac{1}{2}(a-A)$	0.0008630	08609
tan $\frac{1}{2}\theta$	7.1639093	39093
Sin $\frac{1}{2}(a+A)$	n 9.9284651	84450
cosec $\frac{1}{2}(a-A)$	n 1.2008083	13174
tan d	6.8888677	89170
d	+ 2 39.696	2′ 39.″714
cot d'	8.2931827	16717
d'	88 52 29.096	52′ 24.″533
c	+ 33.7	same
c'	+ 1.3	
s	− 13.2	
s'	− 333.6	
τ	+27228.	
τ'	− 1073.	1071.
$\frac{1}{2}(c-c')$	+ 16.2	
$\frac{1}{2}(s-s')$	+ 160.2	
β	0.00	
γ	− 0.149	− 0.″1496
$\alpha'-\alpha$	88 49 49.400	49 44.819
$\delta-(\alpha'-\alpha)$	− 4.544	− 0.363
$\gamma-\beta$	− 0.149	− 0.1496
χ	+ 30.497	+ 2.426
(arc) α'	298 34 49.697	34′ 52.″123
(time)	19h 54m 19.s320	54m 19.475
$\beta+\gamma$	− 0.″149	− 0.″1496
$(\beta+\gamma)\chi$	− 4.″544	− 0.″363
$d+d$	88° 55′ 8.792	55 4.247
δ'	88 55 4.248	55 3.884

$\alpha' = 19^h\ 54^m\ 19.^s475$

$\delta' = 88°\ 55'\ 3.''884$

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This method is particularly applicable when a star's place is to be reduced to several dates; for, after the places for a few dates are computed, the rest will be deduced with great facility, as we can obtain from the known places values of (α') for the unknown dates so close to the truth that only a single approximation will generally be necessary.

Remark:— If we have given the constants z, z' etc. etc. for reducing a star's place from $1800+t$ to $1800+t'$, and it be required to make the reverse computation, that is to reduce the place from $1800+t'$ to $1800+t$, we can do so by putting

$$z = -z'$$
$$z' = -z$$

and changing the sign of θ

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II. Precession in Latitude and Longitude.

In order to complete the subject we shall show how to correct the latitude and longitude of a star for precession. The practical astronomer does not however ~~have~~ so often have occasion to deal with these coördinates as with right-ascensions and declinations.

If we have given the mean longitude (λ) and latitude (β) for any date ($1800 + t$), the corrections necessary for finding the mean longitude (λ') and latitude (β') for any other date ($1800 + t'$) may be obtained by multiplying the interval in years between the two dates by the annual precessions corresponding to the middle of the interval.

The annual precessions may be found as follows:—

$$\left.\begin{aligned} \text{Ann. prec. in } \lambda &= m' + n' \cos(\lambda - M) \tan\beta \\ \text{" " " } \beta &= -n \sin(\lambda - M) \end{aligned}\right\} \text{XV}$$

in which m', n', and M are constants, to be found by

$$\left.\begin{aligned} m' &= 50''.2411 + 0.0002268\,[y - 1800] \\ n' &= 0.4776 - 0.0000070\,[y - 1800] \\ M &= 172^\circ\,45'\,31'' + 33''.23\,[y - 1800] \end{aligned}\right\} \text{XVI}$$

where y is the date for which the annual precessions are to be computed.

The following table gives the values of the constants for the present century.

(over)

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Date	m'	n'	log n'	M
1800	50.24110	0.47760	9.679064	172 45 31
1810	50.24337	0.47753	9.679001	172 51 3
1820	50.24564	0.47746	9.678937	172 56 36
1830	50.24790	0.47739	9.678873	173 2 8
1840	50.25017	0.47732	9.678810	173 7 40
1850	50.25244	0.47725	9.678746	173 13 12
1860	50.25471	0.47718	9.678682	173 18 45
1870	50.25698	0.47711	9.678619	173 24 17
1880	50.25924	0.47704	9.678555	173 29 49
1890	50.26151	0.47697	9.678491	173 35 22
1900	50.26378	0.47690	9.678427	173 40 54

In computing the precessions for the middle date, the longitude and latitude for this date should, in strictness, be employed: and we might find these by a preliminary computation, using λ and β for the date $(1800+0)$. But in most cases that occur in practice, it will be sufficient to take into account only the first term m' of the annual precession in longitude, and to neglect the annual precession in latitude, as is done in the following example.

Example: — The mean longitude and latitude of λ Ursæ Minoris for the year 1840.0 were

$$\lambda = 88^\circ\ 13'\ 40.''756$$

$$\beta = 67\ 29\ \ 5.817.$$

What are the mean longitude and latitude for the year 1870.0 ?

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The middle date between 1840.0 and 1870.0 is 1855.0, and by (XVI) we find the constants for this date to be

$$m' = 50''.2411 + 0''.01247 = 50''.25357$$
$$n' = 0.4776 - 0.00039 = 0.47721$$
$$M = 172^\circ 45' 31'' + 30' 28'' = 173^\circ 15' 59''$$

To find the approximate longitude for the 1855 we multiply the value of m' by 15 years and add the result to the given longitude which gives $88^\circ 26' 14''$. Then the computation by (XV) may be made thus:

$\lambda - M$ =	$275^\circ 10' 15''$
β =	67 29 6
$\tan \beta$	0.3824
$\cos(\lambda - M)$	8.954848
$\log n$	9.678714
$\sin(\lambda - M)$	n 9.998229
$\log[n \cos(\lambda - M) \tan \beta]$	9.016016
"	$0''.10376$
Ann. prec. in λ	$50''.35733$
$\log[-n \sin(\lambda - M)]$	9.676943
Ann. prec. in β	$0''.47527$

Multiplying these precessions by 30 we have

Prec. in thirty years in longitude + $25' 10''.720$
" " " " " latitude + $14''.258$

which added to λ and β give for 1870.0

$$\lambda' = 88^\circ 38' 51''.476$$
$$\beta' = 67\ 29\ 20.075$$

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When the interval between $1800 + t$ and $1800 + t'$ is large the use of the annual precessions does not give results so precise as the computation of the differences $\lambda' - \lambda$ and $\beta' - \beta$ directly by the formulæ

XVII
$$\begin{aligned} \lambda' - \lambda &= \mu + \nu \cos(\lambda - M) \tan\beta \\ \beta' - \beta &= -\nu \sin(\lambda - M) \end{aligned}$$

where the constants are to be found from

XVIII
$$\begin{aligned} \mu &= (t' - t)[50.2411 + (t' + t)\, 0.0001134] \\ \nu &= (t' - t)[0.4776 - (t' + t)\, 0.0000035] \\ M &= 172^\circ\, 45'\, 31'' + t.\, 50''.241 - (t' + t)\, 8''.505 \end{aligned}$$

Example:— Same as preceding.

We have $t = 40$, $t' = 70$ and hence

$$\begin{aligned} \mu &= 25'\, 7''.607 \\ \nu &= 14''.3164 \\ M &= 173^\circ\, 3'\, 25'' \end{aligned}$$

Using formulæ (XVII), we get, by a computation exactly similar in form to that given in the last example,

$$\begin{aligned} \lambda' - \lambda &= +25'\, 10''.720 \\ \beta' - \beta &= +\quad 14.258 \end{aligned}$$

which applied to λ and β give

$$\begin{aligned} \lambda' &= 88^\circ\, 38'\, 51.476 \\ \beta' &= 67\ 29\ 20.075 \end{aligned}$$

which agree rigorously with the values found by the employment of (XV) and (XVI). Indeed, (XV, XVI) may be depended upon in all cases where the interval between $1800 + t$ and $1800 + t'$ is less than a century.

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Original

Personal Equation

The following is the method which I adopted in determining the relative personal equations of Messrs Dean Goodfellow & Searle from observations made for the purpose at Seaton Station Washington in 1867.

Suppose three observers R, S, T, have determined by observation, according to any suitable method, the differences of their personal equation R-S, R-T, and T-S. Were the observations perfect it is evident that the sum of R-T and T-S must equal R-S. But as each of these quantities must be subject to error, this condition will not in general be exactly fulfilled. The observed values cannot, then, be directly used, since they are in a certain degree contradictory. In order to reconcile this discordance and to arrive at the most probable values we can have recourse to the method which I now proceed to demonstrate.

If we call the most probable values of R-S, R-T, and T-S, a, b, and c, respectively these values must evidently conform to the condition

$$a - b - c = 0. \qquad \text{I}$$

Let us further put

A = observed value of R-S having the weight p'
B = " " " R-T " " p''
C = " " " T-S " " p'''

and represent by x, y, and z the corrections which must be applied to the observed values A, B, C, in

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order to obtain the most probable values a, b, c. Then

$$\text{II}\left\{\begin{aligned} a &= A + x \\ b &= B + y \\ c &= C + z \end{aligned}\right.$$

If we transpose these equations (II) and apply the weights we have

$$\begin{aligned} \sqrt{p'}\,[a - A] &= \sqrt{p'}\,x \\ \sqrt{p''}\,[b - B] &= \sqrt{p''}\,y \\ \sqrt{p'''}\,[c - C] &= \sqrt{p'''}\,z . \end{aligned}$$

By the principle of least squares the differential of the sum of the squares of the second members may be put equal to zero; consequently

$$\text{III} \qquad p'x\,dx + p''y\,dy + p'''z\,dz = 0.$$

But, substituting (II) in (I) and making

$$\text{IV} \qquad n = A - B - C ,$$

we find

$$\text{V.} \qquad x - y - z + n = 0$$

the differentiation of which gives

$$dx - dy - dz = 0$$

If the value of dx obtained from this be substituted in III we get

$$dy\,(p'x + p''y) + dz\,(p'x + p'''z) = 0$$

in which the variables y and z may be regarded as independent, and consequently the coefficients of dy and dz must be zero: hence

$$\text{VI}\left\{\begin{aligned} p'x + p''y &= 0 \\ p'x + p'''z &= 0 \end{aligned}\right.$$

Eliminating x from these by the aid of (V) we obtain

$$y\left[\frac{p'+p''}{p'}\right] + z - n = 0$$

$$y + \left[\frac{p'+p'''}{p'}\right]z - n = 0$$

and then, by eliminating y,

$$z\left[\frac{p'}{p'+p''} - \frac{p'+p'''}{p'}\right] + n\left[1 - \frac{p'}{p'+p''}\right] = 0$$

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Reducing this, and putting

$$P = p'''(p'+p'') + p'p'' \qquad \text{VII}$$

we have,

$$z = p'p''\frac{n}{P}$$

and by combining this with (VI)

$$y = p'p'''\frac{n}{P}$$

$$x = -p''p'''\frac{n}{P}$$

} VIII

These values of the corrections x, y, z substituted in (II) will give the most probable values of R-S, R-T, and T-S.

Example:— In June 1867, Messrs. Dean, Goodfellow, and Searle, met at Coast Survey Station in Washington, and observed for personal equation with U.S. Transit No. 4, by the method of alternate tallies. Their observations furnished the following results.

From 52 stars Dean-Searle = $-0^s.045$ with a mean error of $\pm 0^s.004$
" 34 " Dean-Goodfellow = $-0^s.025$ " " " " " $\pm 0^s.010$
" 50 " Goodfellow-Searle = $+0^s.042$ " " " " $\pm 0^s.006$

It is required to find from these the most probable system of values.

The reciprocals of the squares of the mean errors are in the ratio of 136 : 52 : 116 or, very nearly, of 5 : 2 : 4. Putting, then,

$p' = 5$ 6 $A = -0^s.045$
$p'' = 2$ 1 $B = -0^s.025$
$p''' = 4$ 3 $C = +0^s.042$

we have from (VII), $P = 38$; and from (IV), $n = -0^s.062$ then, by applying (VIII),

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$$x = +0.013$$
$$y = -0.033$$
$$z = -0.016$$

whence, by (II) we obtain the required most probable values

$$a = \text{Dean} - \text{Searle} = -0.032$$
$$b = \text{Dean} - \text{Goodfellow} = -0.058$$
$$c = \text{Goodfellow} - \text{Searle} = +0.026$$

which rigorously fulfil the required condition

$$a - b - c = 0.$$

Remark :— If the three determinations be of equal weight, that is, if

$$p' = p'' = p''' = 1$$

(VII) and (VIII) reduce to

$$-x = +y = +z = \tfrac{1}{3} n$$

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