

V. *On the Dependence of a Clock's Rate on the Height of the Barometer.*

By the Rev. Dr. ROBINSON.

Read June 10th, 1831.

IN MR. BAILY's valuable memoir, "On the Mercurial Pendulum," published in the first volume of the Society's Transactions, he has noticed the effect of the air's variable density on rate, and recommended to astronomers the examination of its amount and nature. Since its appearance, the researches of BESSEL and SABINE have thrown much light on the subject: yet it is still to be regretted that Mr. BAILY's suggestion has not been attended to; for, had any observer compared his clock's rate with the barometer even for a single month, he would have seen the inaccuracy of the received theory of oscillations performed in a resisting medium. Nothing, however, of this kind has (I believe) been published; and therefore I may be excused for offering the result of such a comparison to the notice of the Society.

The excellent transit-clock of this observatory was furnished by its maker, the late Mr. EARNSHAW, with a gridiron-pendulum of brass and steel, which is a little over-compensated, changing its rate 0^s.05 for 1° of the thermometer. This defect was not of sufficient importance to authorise any alteration of it, and the consequent interruption of observations necessitated by its adjustment; but the perusal of the above-mentioned memoir satisfied me so fully respecting the superiority of the mercurial pendulum, and the improvements in its construction which are there indicated are so considerable, that I determined to try it: more especially as the effect of the air's variable density must be less as the specific gravity of the pendulum is greater. On stating this to the Primate, I was authorised by his Grace to make the change; and, accordingly, a pendulum was constructed for the observatory by Mr. SHARP, of Dublin, with his usual care. The rod and uprights of the stirrups are round, and

the parts of the latter put together with peculiar firmness: in other respects it is identical with Mr. BAILY'S. After some ineffectual attempts to prove the compensation by passing heated air through the clock-case, I commenced its astronomical examination in October 1830.

The rate of a clock of good workmanship may be assumed to depend on three things: first, the rate at a given temperature and barometric pressure; secondly, on the height of the thermometer, which affects the length of the pendulum and the density of the air; thirdly, on the height of the barometer.*

As the changes of rate are minute, the equations of condition including them are of the form

$$0 = -R + x + by + cz \pm \varepsilon(R)$$

where R is the rate given by observation for n days' interval, positive when gaining; x the rate at 49° of the thermometer, and 29.5 inches of the barometer; b and c the depression of these instruments below those points; y and z the retardations corresponding to a rise of one degree and one inch respectively. The symbol $\varepsilon(R)$ is the error of the observed rate; and as it cannot be known, we must substitute for it a probable error derived from those of observation. Let s be the number of stars observed at the beginning, s' at the end of an interval, and ε the probable error of the determination of time by a single star: then we have

$$\varepsilon \times \sqrt{\frac{s+s'}{ss'}} = n \times \text{Prob. Err. of } (R) = n \times \varepsilon(R)$$

if the number of rates used be sufficiently numerous. For *each rate* the sum of the squares of the errors of observation is

$$\varepsilon^2 \times (s + s') = n^2 ss' \times \varepsilon^2(R)$$

* There may be three other disturbing causes. Irregularity in the transmission of the weight's action through the wheel-work, which, in this clock, appears not to exist sensibly; irregularity of the arc of vibration, produced by cold thickening the oil on the pallets, which sometimes occurred; and perhaps, also, the hygrometric state of the air may modify its resistance. Some facts connected with a different inquiry indicate this; but it could easily be decided with Captain SABINE'S vacuum apparatus.

and therefore the sum of the squares will be a minimum, if x , y , and z satisfy the equations

$$\begin{aligned} 0 &= S \left\{ n^2 ss' (-R + x + by + cz) \right\} \\ 0 &= S \left\{ n^2 ss' (-bR + bx + b^2y + bcx) \right\} \\ 0 &= S \left\{ n^2 ss' (-cR + cx + bcy + c^2z) \right\} \end{aligned}$$

To obtain b and c , a thermometer placed in the clock-case, with its bulb three inches before the pendulum-rod, and on a level with the cover of the jar, and a barometer with its cistern at the same level, were noted morning and evening at $\frac{1}{2}$ past 9; a time chosen, not merely as likely to give mean results for the whole day, but also because it nearly bisects the average period of evening observations. The thermometer was read through a square of plate-glass cemented in the door of the case; and this being screwed close, (as described in the Armagh observations for 1828), all free communication with the air is precluded, and the changes of temperature within it are slower and more regular than would be possible in clocks of the ordinary construction. This made observations at other than the stated periods unnecessary. The barometer was observed whenever its movements seemed irregular, and the mean for the day corrected, if necessary, by interpolation.*

It seems useless to give the details, as the astronomical part will be published in the *Armagh Observations*, and the rest is not available for any other purpose (except, perhaps, the barometric.) Indeed, the following table gives the data for forming equations of condition of the results observed from October 14, 1830, till December 21. After that, severe frost occurred (the thermometer being as low as 14°), and the shortening of the arc of vibration shewed that the clock required cleaning, which interrupted the examination.

* No corrections were made for capillarity, or expansion of the mercury or scale, these refinements appearing needless.

Date.	Rate.	b	c	n	s, s'	Remarks.
1830. Oct. 14		⁰	ⁱ			
15	+0,43	-2,5	-0,46	1	2,2	
16	+0,40	-0,5	-0,51	1	3,3	
17	+0,27	-0,0	-0,55	1	2,2	
22	+0,45	-4,6	-0,28	5	1,1	
26	+0,48	-3,3	-0,47	4	1,3	
28	+0,35	-1,3	-0,22	2	3,2	
29	+0,35	-0,3	-0,03	1	2,2	
Nov. 4						
6	+0,24	+2,6	+0,37	2	3,3	
7	+0,19	+3,3	+0,66	1	3,3	
8	+0,24	+6,1	+0,14	1	4,4	Arc of vibration shortened by a perceptible quantity.
10	+0,14	+2,9	+0,36	2	4,4	
13	+0,09	+4,3	+0,29	3	1,1	
16	+0,05	+5,6	+0,56	3	2,2	
17						
18	+0,12	+9,2	-0,16	1	3,3	Arc shortened 6' for two days, after which it became as at first, 2° 3'. The correction for this being applied, the rate would be -0,54.
20	-0,13	+6,0	+0,01	2	3,3	
22	-0,31	+4,5	+0,13	2	1,1	
Dec. 4	-0,36	+5,9	-0,16	12	1,1	
10	-0,38	+6,2	+0,43	6	1,1	
12	-0,20	+9,7	+0,24	2	4,4	Arc not changed.
17	-0,45	+7,7	-0,52	5	1,1	
20	-0,26	+8,6	-0,32	3	2,2	
21	-0,37	+7,1	-0,14	1	3,2	

Omitting the 14th, and forming the other equations as already described, we have for final equations

$$\begin{aligned}
 0 &= 25,26 + 634x + 2801,3y + 9,59z \\
 0 &= 728,683 + 2801,3x + 22131,49y + 286,397z \\
 0 &= 0,1885 + 9,59x + 286,397y + 71,0929z
 \end{aligned}$$

which give

$$x = +0,263; \quad y = -0,069;^* \quad z = +0,241$$

If z be multiplied by 29,5, we find the reduction to a vacuum for this pendulum = 7^s,11. That obtained by Captain SABINE, from his experiments, is 10^s,18; numbers nearly inversely as the specific gravities of the pendulums, which are 12,0 and 8,6.† The mere effect of the air's buoyancy on the mercurial pendulum is 0,15 for an inch of the barometer: only 0,62 of the above value of z .

This result, however, is not comparable with that obtained by the vacuum apparatus, unless the arc of vibration be independent of the air's resistance. Mr. BAILY has noticed the *possibility* of its varying with the barometer, but seems to doubt whether the change can be detected by observation, if it really exist. That it does not, may easily be shewn.

It was long since proved, by the Chevalier de BUAT, that a pendulum moving in a fluid *drags with it a portion of that fluid adhering to it*, whose inertia increases the time of oscillation, and *whose bulk is independent of the velocity*.‡ This bulk we cannot, in the present state of hydrodynamics, determine *à priori* so as to express the time; but, knowing the latter, it is given. From what precedes, its effect on the time is shewn to be about *half the correction for buoyancy*; and we therefore allow for it by making an equivalent diminution in the accelerating force. Let g' be the force so diminished, g that of gravity, r the length of the simple pendulum, u the square of the pendulum's velocity at the angular distance x from the lowest point, and the air's resistance (which in this case may be supposed to act at the centre of oscillation) = $-\frac{1}{2} au$; then

$$du - arudx = -2g'r \cdot \sin x \cdot dx$$

* The pendulum was therefore overcompensated: removing 1481 grains of mercury, y became +0.056; or, in one of this construction, 118.5 grains of mercury are equivalent to a change of compensation amounting to 0^s,01 for one degree.

† By estimation, from the weight of the rod and mercury, it may be 12.5.

‡ His researches were of course long antecedent to those of BESSEL: they, however, detract nothing from the merit of this illustrious philosopher, who, in all likelihood, knew nothing of them. Indeed, NEWTON himself anticipated them both. See the *Principia*, lib. ii. prop. 27, cor. 2, where it is pointed out that the motion communicated to the surrounding fluid will lengthen the time of oscillation.

the integral of which

$$u = e^{ax} \times C + 2g'r \times \frac{ar \sin x + \cos x}{a^2 r^2 + 1}$$

becomes, between the limits 0 and $-X$,

$$V^2 = \frac{2g'r}{a^2 r^2 + 1} \left\{ 1 + e^{aX} (ar \sin X - \cos X) \right\}$$

the expression of the square of the velocity which the pendulum must have, when passing the vertical, to make it ascend through X ; and which *in vacuo* would carry it through the arc whose versed sine $= \frac{V^2}{2rg}$. To compute V^2 we have $g' = g \times 0.99985$, for this pendulum:* as to a , we deduce it from the experiments of BORDA, which give the resistance of air to a plane surface one foot square $= V^2 \times 0.002$, the units being feet and pounds; and which also shew that the resistance of a cylinder is half that of its diametral plane. In my pendulum, this plane (with a small allowance for the traverses of the stirrup) $= 24$ inches, and its weight is 11 pounds, from which I derive

$$a = \frac{g}{33000}$$

The value of X is 2° , and thus we find that

$$\frac{V^2}{2gr} = 0.0006090$$

The versed sine of $X = 0.0006092$; and therefore, *in vacuo*, the arc of vibration would *decrease* about $1''.2$, which can have no effect on the rate.†

With respect to the barometrical variation of rate, having once determined its co-efficient z , nothing is easier than to allow for it when any sudden change occurs during the period of observation: and this will, in most cases, be sufficient. At the same time it must be admitted, that the accurate performance of his clock is so important to the astronomer as to warrant the mechanical compensation of any error which can be detected; and this can, in the present instance, be done with extreme facility.

* Assuming, with Mr. BAILY, that the pendulum's specific gravity is 10000 times that of air.

† This seeming paradoxical result depends on the air's buoyancy exceeding its resistance: the arcs in air and *vacuo* would be equal, if a were 96 times its actual value.

The method of effecting it is shewn in the figure, which represents a mercurial pendulum, having attached to its rod *two* syphon barometers TS, by clips at C and E. The legs of these must be of equal bore, and the arrangement will obviously have the effect (when the barometer falls the quantity c) of transferring from T to S two cylinders of mercury of the altitude $\frac{1}{2}c$ and the diameter of the tubes. If the points T be properly placed at a given state of the barometer, this transfer will produce a retardation equal to the acceleration which we have seen is caused by a fall of the barometer $= c$.

To determine their position, let M be the mass of the pendulum in its initial state; MK^2 its moment of inertia; A the distance of its centre of gravity from the axis of oscillation: then, by the equation

$$r = \frac{MK^2}{MA} = \frac{K^2}{A}$$

we have for the pendulum's length, after the barometric column has been shortened by the quantity c ,

$$r' = \frac{MK^2 + m(k'^2 - k^2)}{MA + m(a' - a)}$$

m being the mass of the two transferred cylinders of mercury, k and a the distances of the centres of gyration and gravity *before*, k' and a' *after*, the transfer. For these cylinders, on account of their being similar and equal in both places,

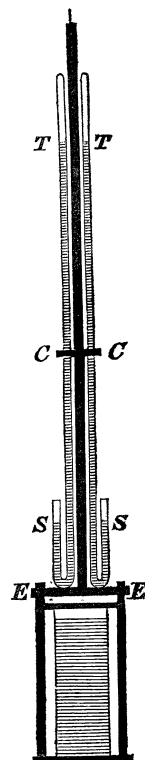
$$k'^2 - k^2 = a'^2 - a^2$$

and therefore, putting $\mu = \frac{m}{M}$ and ζ for the retardation in 24 hours,

$$\frac{r' - r}{r} = \frac{\zeta}{43200} = \frac{1 + \mu \left(\frac{a'^2 - a^2}{K^2} \right)}{1 + \mu \left(\frac{a' - a}{A} \right)} - 1 = \frac{\mu \left(\frac{a' + a}{K^2} - \frac{1}{A} \right) (a' - a)}{1 + \mu \left(\frac{a' - a}{A} \right)}$$

which, as μ is a small fraction, gives very nearly

$$\zeta = 43200 \frac{(a' - a)}{K^2} (a' + a - r) \mu$$



In this pendulum, K may be supposed equal to r ,* and putting δ for the distance of the points T from the axis, when the barometer stands at 29.5 inches = α , the expression becomes

$$\zeta = \frac{43200}{r^2} (2\delta + \alpha - r) (\alpha - \frac{1}{2}c) \mu$$

In this climate, c is scarcely ever one inch, and may therefore be neglected, and then ζ is proportional to μ , or to the barometric change. That it also may be equal to z , can be obtained by means of the arbitrary constants; but it must be observed, that δ cannot be less than $\frac{1}{2}(r - \alpha) = 4.81$, nor greater than 6 inches, unless the tubes of the syphons were to descend lower than the cover of the jar. Assuming the bore of the tubes such as to give μ a value consistent with any intermediate value, we may determine δ . Thus, the thickness of the tubes being = 0.1 (which makes $\mu = \frac{1}{2878}$, if $c = 1$) and ζ being = z , we have

$$0.24 = \frac{43200 \times 29.5}{(39.13)^2 \times 2878} \times (-9.62 + 2\delta)$$

whence $\delta = 5.08$.

Therefore, to make this compensation, when the tubes are attached to the pendulum, mercury must be poured into, or withdrawn from, the short legs, till the upper surface of the columns is distant from the point of suspension 5.08 *minus* half the height of the barometer above 29.5. If on trial the effect of atmospheric changes be still sensible, a few grains of mercury must be added to each tube; and the corresponding change of ζ being determined, will give the means of making a complete correction.

The oscillations of the mercury in the compensating tubes are not injurious, as they tend to isochronise the long and short vibrations, though

* Their difference, in my pendulum, is 0.28; but they must be yet more nearly equal, for the theorem $x = \frac{K^2}{A}$ is only true for a *solid mass*, in which the velocity of each particle of matter is determined by its distance from the axis of oscillation. The mobility of a fluid bob interferes with this; and its centres of oscillation and gravity would coincide, but for the impediments of viscosity, friction, &c. These make it true in small tubes; for I find that one of $\frac{1}{8}$ of an inch diameter, suspended by one extremity, oscillates in the same time, whether empty, or full of mercury, after allowing for the knob of glass at the lower end.

their extent, and the small mass disturbed by them, prevents this correction from being total.

I had previously recognised the influence of the barometer on the clock before the mercurial pendulum was applied; and, without employing the method of minimum squares, satisfied myself that the effect was even greater, being nearly 0·3 for an inch. In examining this point for any given clock, it is, however, to be remembered, that it is always necessary to make the probable errors of observation considerably less than the quantity sought; otherwise no conclusive result can be obtained. This may be done, either by taking the rate for intervals of several days, or by observing several stars on each night.

T. R. ROBINSON.

Armagh Observatory,
May 7, 1831.

Note by Mr. Baily.—Since the receipt of the above Memoir, as well as of the additional observations in the following page, and in fact whilst the present sheet was in the press, I have swung the *mercurial pendulum* in my vacuum apparatus. The mercurial pendulum actually employed in the experiment was exactly similar to the one above described by Dr. ROBINSON; except that the rod was flat, and $\frac{1}{2}$ an inch wide, as were also the sides of the stirrup. The height of the mercury was 6·8 inches: and the variation in the rate, for 1 inch of the barometer, (reduced to the thermometer at 32°), was, from a mean of 4 double experiments, agreeing very well with each other, equal to 0^s.42, which is greater than the value above deduced by Dr. ROBINSON, and about 2·33 times greater than the value deduced in the usual manner from the specific gravity of the pendulum.

F. B.

* * * Since the above was read, the following additional observations have been received from Dr. ROBINSON :—

Rates of the Transit Clock, Armagh Observatory.

Date.	Observed Rate.	b	c	s, s'	Obs. — Calc. Rate.
1831. June 8					
21	+0,135	+12,6	+0,02	4,4	—0,01
24	+0,10	10,9	+0,35	7,7	+0,04
28	+0,055	9,7	+0,20	6,6	—0,04
July 3	+0,064	9,5	+0,39	9,9	+0,02
5	+0,007	11,6	+0,36	12,12	—0,05
7	—0,02	13,0	+0,51	8,8	—0,04
9	+0,08	16,1	+0,46	4,4	+0,04
10}	+0,09	13,8	+0,16	5,5	—0,03
13}					
16	+0,04	14,5	+0,14	4,4	—0,08
15}	+0,07	14,2	+0,22	4,4	—0,03
17}					
20	+0,11	12,1	—0,02	4,4	—0,05
22	+0,14	10,8	—0,16	4,4	—0,05
23	+0,14	10,2	+0,08	4,4	+0,01
28	+0,01	12,5	+0,37	5,5	—0,05
31	+0,005	17,9	+0,39	4,4	—0,06
Aug. 1	+0,08	20,1	+0,40	6,6	+0,01
2}	+0,135	16,5	+0,13	6,6	+0,005
17}					
24}	+0,18	11,7	—0,03	4,4	+0,017
27}					
31	+0,08	11,2	+0,19	9,9	—0,02
Sept. 1	+0,28	8,4	+0,20	10,10	+0,19*
2	+0,11	8,0	+0,27	8,8	+0,04
4	+0,12	9,7	—0,01	4,4	—0,03

* The minute hand had been touched by the key in winding.

These give, by minimum squares, x , rate at $49^{\circ} = +0^{\circ},123$; y , change for 1° Fahrenheit = $-0^{\circ},003$; z , for 1 inch fall of barometer = $+0^{\circ},267$. The last column is the real error of the clock.