Popular Astronomy

Vol. LIII, No. 2 FEBRUARY, 1945 Whole No. 522

The Cause of Sun-Spots

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Introduction. Schwabe's solar observations from 1826 to 1859 established the sun-spot cycle. The researches of Wolf set its mean length at 11.1 years. This was increased to 11.2 years by the maximum of 1917.6. This last is not necessarily the final definitive value since from the maximum of 1836.5 to that of 1937.2 the lengths of the cycles are:

11.9 years	12.8 years	11.2 years
12.0 years	10.4 years	10.9 years
10.4 years	12.4 years	10.1 years

Here five cycles are less than Jupiter's period of 11.86 years and four are greater than it. In the light of these facts it would be very unscientific to assume that Jupiter has nothing to do with the sun-spots.

It is the aim of this paper to offer new and, what is believed to be, conclusive evidence that the sun-spots are caused mainly by the precessional action on the equatorial belt of the sun by the planets, Mercury, Venus, Earth, Jupiter, and Saturn.

In August, 1930, the writer published in the *Astronomical Journal* an article entitled "On the Cause of the Sun's Spots." The cause proposed was the precessional action mainly of Jupiter and Saturn on the sun. No attempt was made to show that Mercury, Venus, and the Earth were also effective in producing spots. In December, 1940, Popular Astronomy published an article of mine entitled "Planets and Sun Spots." Section 6 of that paper indicated briefly a method of proving that the precessional action of the Earth registered in the sun-spot record. That method is here detailed and extended to the five planets named in the preceding paragraph.

2. *Precessional Action.* The essential features of precessional action are easy to understand. They must register, however, if what follows is to be clear. Precessional action on any body is the result of two equal and opposite forces called a couple. It produces, or tends to produce, rotation. A couple has no single resultant and only an equal opposite couple can stop the tendency to rotation. The magnitude of a couple is measured by the product of one of the two forces and the length of the arm at which they act.

Tisserand ("Mecanique Celeste," Vol. II, pp. 92-94) and Moulton (*Astrophysical Journal,* Vol. 29, p. 260) both have proved that the equatorial diameter of the sun is greater than the polar diameter. All attempts to establish this observationally have failed (Abetti, "The Sun," pp. 82-84). According to Moulton the difference between the two diameters is 31.54 miles. That would make the *volume* of the solar bulge about 100 times the volume of the earth.

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The principal articles of this magazine, beginning with Volume **15** (1907), are listed in the INTERNATIONAL INDEX TO PERIODICALS.

The density of the bulge is not known but the surface gravity of the sun is 27.65 times that of the earth. Hence we will not be far wrong if we assume the density of the solar bulge to be .000001 of the sun's mean density. Calculation, using Formula (1) of Section 3 then gives for the couple of Jupiter on the sun 6.653×10^{11} kilometer metric tons. From Table I, Section 3, the combined couple of all planets is about three times that of Jupiter. That would be 2,000 million metric tons acting at the end of an arm 1000 kilometers long. This gives some idea of the magnitude of the forces at work.

Beginners in astronomy learn that the earth's equatorial diameter is over 26 miles greater than the polar. The moon's attraction on the near and far portions of the belt form a couple which tends to shift the plane of the earth's equator into the plane of the moon's orbit. But the earth is, so to speak, a rigid body and the equatorial bulge is firmly attached to it. Hence the net result is a continual shift in the direction of the plane of the equator accompanied by a conical motion of the earth's axis. Thus the earth spins *very slowly* like a top. Obviously the sun also exerts a couple and contributes to the resultant spin. It may be added that there would be no spin if the earth were a perfect sphere.

What Jupiter does may be described in simple terms as follows. Jupiter acts on the sun's equatorial bulge, which is gaseous and not rigidly attached to the body of the sun, and tends to shift the bulge into the plane of the planet's orbit. The sun's rotation forms the bulge and tends to maintain it in the same position with respect to the sun's axis. The interaction of these two sets of forces disturbs the solar equilibrium continuously. The result is not a minute spin of the sun like a top but a profound disturbance of the material in the thicker portion of the equatorial bulge. This is the cause of the sun's spots.

3. *Fundamental data.* The astronomical data of Table I are the basis of this whole paper. Formula (1) plays a vital role in the reasoning.

TABLE I

(1) Planet	(2) Inclination of orbit to sun's equator	(3) Longitude of ascending node of sun's equator on plane of orbit	(4) Longitude of perihelion 1930	(5) $(1+e)^3 / (1-e)^3$	(6) Mean Couple Earth = 1	(7) Extreme values of couple
Mercury	3°14′	327°34′	76°21′	3.480	.3250	.59, .17
Venus	3 47	251 50	130 35	1.041	1.138	
Earth	7 11	253 47	101 44	1.138	1.00	1.051, .951
Mars	5 34	261 50	334 46	1.753	.024	
Jupiter	6 2	248 20	13 11	1.301	1.89	2.18, 1.71
Saturn	5 29	237 5	91 40	1.398	.083	.0978, .0668
Uranus	6 25	253 48	169 31	1.329	.00186	
Neptune	6 23	240 14	44 1	1.056	.00054	

Webster's "Dynamics," page 301, gives the following formula for the magnitude of the precessional couple:

$$3\ km \times (C - A) \sin l \sin d \sqrt{(1 - \sin^2 l \sin^2 d)}/r^3 \qquad (1)$$

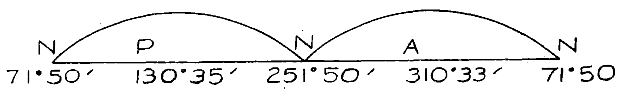
Here

k = the constant of gravitation.
m = the mass of the planet.
C — A = difference of the principal moments of inertia of the sun.
l = the longitude of the planet measured from the node of the sun's equator on the orbital plane.
d = the angle the sun's equator makes with the orbital plane.
r = the distance of the planet from the sun.

4. *Analyses of the Sun-Spot Data.* Numerous analyses of the sun-spot data have been made by various investigators. The purpose was to discover from them what component periods existed. The hope was that the periods found would lead to the discovery of the cause of the spots themselves. For reasons which will not be detailed none of the analyses were successful in this aim.

Preceding analyses of sun-spot data were carried out in absence of any known cause of the spots and in ignorance of all secondary periods. The latter were the result of the process used. In the present analysis dynamic reasoning indicated the secondary periods which should be present. It was necessary to devise a method which would show their presence. The procedure while laborious is simple and easily understood as it involves arithmetic only.

5. *Venus and the Sun-Spot Record.* Column 6 of Table I shows that the magnitude of the precessional action of Venus comes next to that of Jupiter, being equal to 60 per cent of it. Consequently, its action should be easily detected in the sun-spot record. To understand how this may be done the adjacent graph is helpful.



VENUS—PRECESSIONAL COUPLE

In one 225-day revolution of the planet its precessional action has two minima and two maxima. The minima occur at the nodes as the planet is then in the plane of the sun's equator and sin l, of formula (1), is zero. The maxima would come midway between the nodes were it not for the inversely varying cube of the distance, r, of the planet from the sun. This displaces the maximum slightly toward the perihelion point, P, and slightly away from the aphelion point, A.

To detect the period of Venus in the sun-spot record the 225-day period was divided into 15 parts of 15 days each. Then the sums of the sun-spot numbers for each successive 15-day interval were obtained from the *daily* sun-spot record. The range was from January 1, 1916, to April 29, 1936, comprising 33 revolutions of Venus. In all, 495 items were calculated from the sun-spot record. These items were

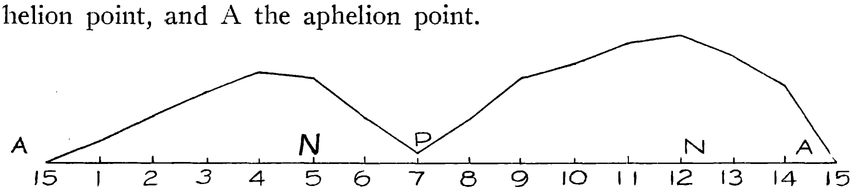
then ordered in fifteen columns with 33 numbers in each column. The fifteen sums are given in column A, Table II.

For convenience in making the 15-point graph, the least number 19434 was subtracted from each of the fifteen. The results are in column B. This practically eliminates the effect of all the planets but Venus. Each number in B is the cumulative effect of 33 equal impulses of Venus over its 225 day period. The calculations were so made that the first point of the graph comes at January 1. The curve is the graph of the numbers in B. The critical positions of Venus can be taken from the *Ephemeris* for any one of its revolutions. Those given are for the first revolution of Venus in 1916.

TABLE II

	A	B				A	B		
Jan. 1	19839	405	(1)		Apr. 30	21400	1966	(9)	
Jan. 16	20412	978	(2)		May 15	21722	2288	(10)	
Jan. 31	20959	1524	(3)		May 30	22241	2807	(11)	
Feb. 15	21486	2052	(4)	Node	June 14	22307	2873	(12)	Node
Mar. 1	21273	1839	(5)	Feb. 24	June 29	21843	2409	(13)	
Mar. 16	20414	980	(6)		July 14	21150	1716	(14)	Aph.
Mar. 31	19489	55	(7)	Per.	July 29	19434	0	(15)	July 22
Apr. 15	20481	947	(8)						

In all the graphs which follow, N denotes nodal points, P the perihelion point, and A the aphelion point.



VENUS—SUN-SPOT PRODUCTION
1916-1936, 33 REVOLUTIONS

The 15-point graph has two minima and two maxima. *The minima follow the nodes.* This shows a lag of the effect behind the cause. The curve shows unmistakable agreement with the precessional action of Venus. Inertia of the disturbed material would be expected to carry the effect beyond the point at which precessional action is zero.

To guard against errors and chance agreements a similar study was made over the interval January 1, 1870, to January 9, 1894, a total of 39 revolutions of Venus. The *daily* sun-spot numbers published in the *Journal of Terrestrial Magnetism and Atmospheric Electricity* begin in 1916. For this second interval the monthly means were used. These are the average number of spots per day for each month of the year. The orbital period was separated into 9 parts of 25 days each. The procedure used for the first three months of 1870 may be indicated as follows:

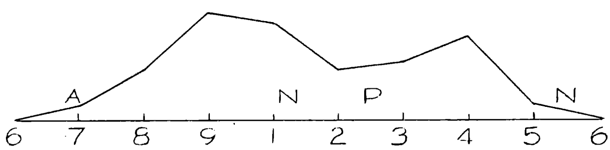
MONTLY MEAN

Jan.	77.3	$25 \times 77.3 = 1932.5 =$ Part 1
Feb.	114.9	$6 \times 77.3 + 19 \times 114.9 = 2646.9 =$ Part 2
Mar.	159.4	$9 \times 114.6 + 16 \times 159.4 = 3581.5 =$ Part 3

etc., for six more parts completing one revolution. This was carried out for 38 more revolutions. In this way a total of 9 × 39 or 351 items were taken from the sun-spot record. These were ordered in 9 groups of 39 numbers each. The sums of each 39 are in column A, Table III. The same procedure as before gives the 9-point graph.

TABLE III

	A	B		
Jan. 1	44952	2263	(1)	
Jan. 26	43702	1013	(2)	Node Jan. 3
Feb. 20	43950	1261	(3)	
Mar. 17	44628	1938	(4)	Per. Feb. 8
Apr. 11	42965	276	(5)	
May 6	42689	0	(6)	Node Apr. 24
May 31	42952	263	(7)	Aph.
June 25	43535	846	(8)	
July 20	45176	2487	(9)	

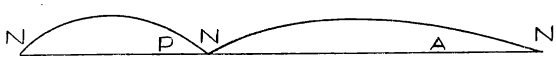


VENUS, 1870-1894, 39 REVOLUTIONS

This graph also has two maxima and two minima. The latter occur soon after Venus passes the preceding node. This graph shows the same significant agreement with the precessional action of Venus as the first one. The positions of aphelion and perihelion show why the minimum near aphelion is more marked than that near perihelion.

It is difficult to reduce the sun-spot numbers from the many observers so that they are homogenous. Moreover, there are marked disagreements between them and the area numbers. These are inevitable deficiencies in the methods of measuring the solar action. Hence exact parallelism can not be expected between the precessional curve of Venus and the two preceding sun-spot curves. The *two minima* coming just after the zeros of precessional activity are significant and conclusive agreements. So also are the *two maxima* which are always present.

6. *Mercury and the Sun-Spot Record*. The adjacent figure is a



MERCURY—PRECESSIONAL COUPLE

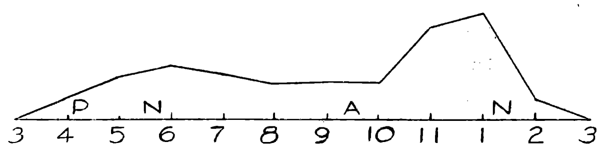
graph of the precessional action of Mercury showing the position of the four critical points: the two nodes, perihelion, and aphelion. It is very different from that of Venus. The eccentricity of the orbit is high 0.2056. Consequently, from node to node through perihelion is 33 days while the interval from node to node through aphelion is 55 days. The maximum couple in each arch tends to occur midway between the

nodes but is displaced slightly toward perihelion and away from aphelion.

The procedure used with Venus was followed with Mercury. The interval selected for study included the 50 revolutions of Mercury following January 1, 1916. The orbital period of 88 days was divided into 11 parts of 8 days each. From the daily sun-spot record the sums for each eight days of the entire interval were calculated. This gave a total of 550 items. The results were ordered in eleven columns and the sum of each 50 items was found. The same procedure as for Venus gave the following data (Table IV) and the eleven-point graph. Again the calculations were so made that the first point of the curve is January 1.

TABLE IV

	A	B		
Jan. 1	20417	2350	(1)	Node Jan. 5
Jan. 9	19363	296	(2)	
Jan. 17	18067	0	(3)	Per. Jan. 26
Jan. 25	18495	428	(4)	
Feb. 2	19026	959	(5)	Node Feb. 7
Feb. 10	19251	1184	(6)	
Feb. 18	19027	960	(7)	
Feb. 26	19201	834	(8)	
Mar. 5	18827	760	(9)	Aph. Mar. 10
Mar. 13	18896	829	(10)	
Mar. 21	19979	1912	(11)	



MERCURY, 1916-1929, 50 REVOLUTIONS

The curve shows two maxima and two minima. The effect lags behind the cause. Hence the two minima follow respectively the two nodes which are the two zeros of precessional action.

The curve is so different from the curves for Venus that there is a sharp feeling of surprise. According to Formula (1), Section 3, precessional action at perihelion is about 3.5 times as great as at aphelion. Why is this not reflected in the curve? It appears that the delayed minimum at point 3 of the curve is partly responsible. It seems also that the 33-day period is too short to permit Mercury's greater precessional period to register fully in the spot curve. In contrast the 55-day period allows the smaller couple to show a marked effect.

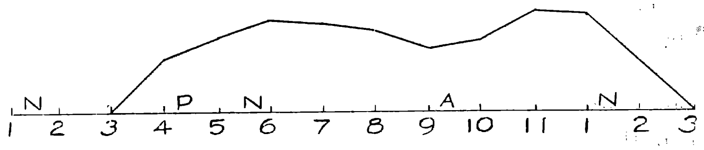
A further study of Mercury was made for an additional fourteen years previous to January 1, 1916; that is, back to January 1, 1902. The daily sun-spot numbers were not available so the eight-day sums were calculated from the monthly means. The whole interval from 1902 to 1928 covers 109 revolutions of Mercury; that necessitated finding 11 × 109 or a total of 1199 eight-day sums from the sun-spot record. Since the critical positions of Mercury can be taken from any

one of its revolutions the first one in 1916 was used. The sums for each interval and the numbers for the whole range 1902-1928 follow, Table V. The graph is for the 109 revolutions.

TABLE V

MERCURY 1902-1928 (109 REV.)

		1902-1916	1916-1928	1902-1928		
Jan.	1	13714	20417	34131	2264 (1)	Node Jan. 5
Jan.	9	13660	19363	33023	1156 (2)	
Jan.	17	13800	18067	31867	0 (3)	
Jan.	25	14201	18495	32696	829 (4)	Per. Jan. 26
Feb.	2	14430	19026	33456	1589 (5)	Node Feb. 7
Feb.	10	14659	19251	33910	2043 (6)	
Feb.	18	14782	19027	33809	1942 (7)	
Feb.	26	14598	19021	33619	1752 (8)	
Mar.	5	14499	18827	33326	1459 (9)	Aph. Mar. 10
Mar.	13	14586	18896	33482	1615 (10)	
Mar.	21	14226	19979	34205	2338 (11)	



MERCURY, 1902-1928, 109 REVOLUTIONS

It would seem that such marked results for Venus and Mercury would not have entirely escaped observation. As a matter of fact they did not, but the observations have been disregarded by the astronomical world or else looked on as doubtful.

Mrs. Maunder (*M.N.R.A.S.*, V. 67, pp. 451-479, 1906-1907) in her study of sun-spots found that as the sun's rotation brings new parts of the solar surface into view those parts bear more than the average number of spots. She concluded that the earth caused the number of spots to decrease. The correct reason, as the graphs for Venus and Mercury prove, is that the effect lags behind the cause.

Pocock studied spots in the interval 1902-1917 and confirmed Mrs. Maunder's results (*M.N.R.A.S.*, V. 79, November, 1918). De La Rue, Stewart, and Loewy found for Venus and Mercury the same result as Mrs. Maunder and Pocock did for the earth.

Schuster using only new spots reported on daily photographs, investigated the interval 1874 to 1909, and found that Mercury, Venus, and Jupiter all appear to have an influence on spot production. (*Proceedings of the Royal Society of London*, Vol. 85, pp. 309-323). Schuster concludes that the probability of the observed coincidences being accidental is one in 400,000.

Here is a considerable array of correct and significant observations. One reason that they have gone comparatively unnoticed is because the observers did not offer any explanation of the dynamical method by which the planets caused the spots.

There is a dynamical basis for precessional action as a cause of sun-

spots which may be outlined as follows:

First: There is the magnitude of the precessional couple of Jupiter on the sun which, by Formula 1 of Section 3, is of the order 1.47×10^{22} gram centimeters. This is small compared to 2.81×10^{29} gram centimeters, which is, according to Lord Kelvin, the moon's couple on the earth.

Second: Poincare has proved (Lamb, "Hydrodynamics," pages 677-680) that a fluid body will precess like a rigid one provided the period, P, of the disturbing body divided by the period, p, of the disturbed body is large compared to

$$(a^2 + c^2)/(a^2 - c^2)$$

of the disturbed body, that is:

$$P/p > (a^2 + c^2)/(a^2 - c^2) \text{ or } > [(r + x)^2 + r^2]/[(r + x)^2 - r^2] \text{ or } > r/x \text{ approximately.} \quad (1)$$

Here r is the sun's polar radius, r + x is its equatorial radius, and x according to Moulton is 15.77 miles. For the sun and Jupiter (1) becomes:

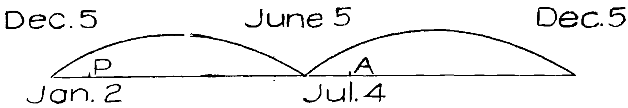
$$(11.86 \text{ years})/(24.65 \text{ days}) > (433200/15.77) \text{ or } 175.7 > 27470 \text{ or } 1 > 156 \quad (2)$$

Instead of the left member of (2) being greater than the right, it is far smaller and Poincare's condition is *not* fulfilled; that is, there is no possibility of the slightest axial spin of the sun. Hence the precessional action of Jupiter on the sun, which is considerable, must be expended in causing action of some type in and below the equatorial bulge. This is the dynamical foundation for sun-spots and also for the belts of Jupiter.

Third: There is also observational evidence that precessional action of the moon on the waters of the Atlantic Ocean has been observed in the motion of the Gulf Stream. This was established through a four year study by Lieutenant (later Rear Admiral) J. E. Pillsbury. He spent about a year on each of four sections of the Stream, anchored his boat and measured the flow at many points in each section. His calculations showed that 21 cubic miles of water flow through the Straights of Florida every hour. Moreover, and this is a significant discovery bearing on solar study, he found that twice in every lunar month, at high declination of the moon and at low declination, the character of the flow changes. (See Report of U.S. Coast and Geodetic Survey for 1893.)

High tide occurs at new and full moon and these do not occur regularly at high declination of the moon and at low declination. The greatest precessional action of the moon does occur at these times. Hence the result Pillsbury observed is due to precessional action of the moon on the waters of the Atlantic. The causes of the flow of the Gulf Stream are well known and no one lists precessional action of the moon (or the sun) as one of them. The slight variation of the flow noted is merely an index of precessional action.

7. *The Earth and the Sun-Spot Record.* Table I, column 6, shows that the precessional couple for the earth is less than that of Venus. The variations in precessional action are not so marked as for Venus for two reasons. First, because the magnitude of its couple is less than that of Venus and, second, as the figure below shows, perihelion and



EARTH—PRECESSIONAL COUPLE

aphelion are both near a node. Were each midway between the nodes the difference in the two maxima of precessional action would be large. As it is the difference is small and one is displaced slightly toward perihelion and the other slightly away from aphelion.

Calculation shows that the most active month is February and the next most active is September, but the difference in activity for August and September is small. The minimum of June 5 should be marked because the node and the aphelion are very near together. The node of December 5 is only 28 days from perihelion. Consequently, precessional action during November is nearly equal to that during December.

8. *The Area Numbers.* The sun-spot numbers and the Greenwich area measures are in good agreement in fixing the epochs of maximum and minimum solar activity. In an investigation such as this the area numbers are very inconvenient to use because the means are calculated for synodic revolutions of the sun instead of for calendar months. The length of a synodic revolution of the sun varies slightly throughout the years. For the interval 1874 to 1913, inclusive, however, Arctowski (*Memoria della Societa Spettroscopisti Italiani,* Vol. V, pp. 98-99) made the necessary reductions of the area numbers to means for calendar months. The sums of the 40 means for each of the twelve months of the year are given in Table VI.

TABLE VI

AREA NUMBERS 1874-1913

Jan.	501	May	452	Sept.	553
Feb.	535	June	413	Oct.	519
Mar.	484	July	526	Nov.	481
Apr.	451	Aug.	495	Dec.	497

The agreement with precessional action is good. The minima come in June and November and the maxima in February and September.

The sun-spot numbers over the same 40-year interval give about the same results as the area numbers, as shown in Table VII.

TABLE VII

Jan.	1261	May	1257	Sept.	1352
Feb.	1389	June	1303	Oct.	1311
Mar.	1295	July	1417	Nov.	1132
Apr.	1320	Aug.	1342	Dec.	1191

The sums of the monthly sun-spot means for the 100-year interval 1839 to 1938 are:

TABLE VIII

Jan.	4310	May	4490	Sept.	4580
Feb.	4690	June	4540	Oct.	4500
Mar.	4500	July	4540	Nov.	4400
Apr.	4350	Aug.	4670	Dec.	4410

Here the agreement with precessional action would be good were it not for the four summer months in which the numbers are too great. This is probably a seasonal effect due to the better "seeing" of the summer days and the greater number of hours of observation per day.

Mars, because of its small couple, will not be considered. The results for Mercury, Venus, Earth, Jupiter, and Saturn will be sufficient to complete the purpose of this investigation.

9. *Jupiter and the Sun-Spot Record.* Jupiter's orbital period of 11.862 years was divided into twelve equal parts of 0.988 year each. The sun-spot numbers for each 0.988 year were calculated and ordered into groups of twelve. Then the corresponding numbers for each of the twelve parts were added. The results follow.

Table IX shows Jupiter's sun-spot activity in each of 12 parts per orbital period. Two different intervals from 1834 to 1941 are given. The first is 1858 to 1942; the second is 1834 to 1942, inclusive.

TABLE IX

Orbital Parts	1	2	3	4
84 yr. 7 rev.	120.9	154.9	283.6	400.3
108 yr. 9 rev.	267.6	288.8	375.11	461.27
Orbital Parts	5	6	7	8
84 yr. 7 rev.	505.4	574.7	444.4	320.6
108 yr. 9 rev.	577.9	664.0	621.0	581.1
Orbital Parts	9	10	11	12
84 yr. 7 rev.	226.1	139.9	128.2	192.4
108 yr. 9 rev.	446.2	303.3	271.0	302.8

Subtracting 120.9 from each of the twelve values in line one gives the residuals which form a measure of Jupiter's sun-spot activity:

0 34 162.7 279.4 384.5 454 324 200 105 19 7.3 71.5

Reducing these for plotting:

0 .68 3.3 5.6 7.7 9.1 6.5 4.0 2.1 1.6 .14 1.43

These last give the first of the adjacent graphs. The numbers in line two, Table IX, give the residuals:

0 21.2 107.5 193.7 309.7 396.4 353.4 313.5 178.6 35.7 3.4 35.4

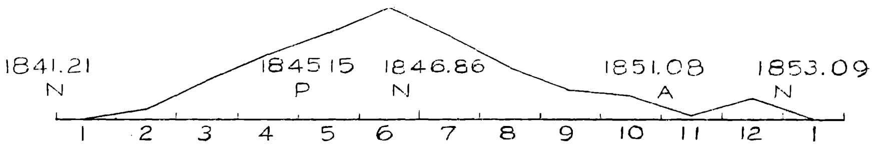
Reducing these for plotting:

0 .53 2.74 4.84 7.74 9.16 8.84 7.84 4.46 .9 .09 .88

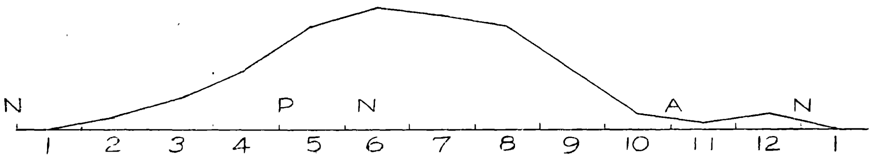
These numbers give the second graph.

In the first graph of Jupiter's action two maxima and two minima do *not* appear. For this the positions of the nodes, the perihelion and aphelion points are partly responsible. In addition the enormous couple of Jupiter and the lag of the effect after the cause contribute. The first

precessional impulse is 5.63 years long and is far greater than the second. From the node the action increases until about one and one-half years after perihelion passage. The magnitude of the forces present and the enormous area over which the activity is spread have a certain inertia which produces a considerable lag. The effect of both nodes is to turn the curve downward.



JUPITER, 1858-1942



JUPITER, 1834-1942

The second precessional impulse is 6.23 years long and far less than the first. It supports the activity, however, until it diminishes almost to zero at aphelion. After aphelion there is a slight rise; then the passage of the node brings zero again. The behavior of the curve *here indicates small but significant and conclusive agreements.*

The second graph of Jupiter's action may be interpreted in like manner. In this way the two precessional impulses of Jupiter are merged into a single one. This brings us to Newcomb's considered conclusion:

"Underlying the periodic variations of spot activity, there is a uniform cycle unchanging from time to time and determining the general mean of the activity" (*Astrophysical Journal*, Vol. 13, 1901, pp. 1-14).

10. *Saturn and the Sun-Spot Record.* Four times Saturn's period of 29.46 years is 117.84. The interval of reliable sun-spot observations from 1826 to 1943, inclusive, is 118 years. In that time, Saturn has made *four* complete revolutions around the sun. This is not great enough for the best results; nevertheless, definite confirmation of precessional action by Saturn can be obtained from these four revolutions.

The Wolf sun-spot activity of Saturn as measured by the sums of corresponding four-year means for each of the 29.5 years of the period is indicated by numbers as follows:

TABLE X

30.6	40.8	54.6	25.8	57.2
35.3	38.3	60.3	30.1	50.3
48.7	38.4	63.3	40.4	41.6
60.1	46.3	55.5	55.5	33.3
61.1	59.9	40.5	72.8	25.7
48.0	61.5	29.0	65.3	18.6 (29.5)

These results are rather irregular but if we smooth them by taking overlapping nine-year means their definite trend appears.

The nine-year means are in the first three columns which follow. Subtracting 42.2 from each of these gives the residuals in the last three columns, Table XI. These form a measure of Saturn's spot-producing action.

TABLE XI

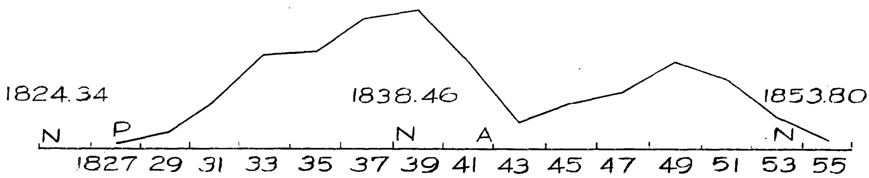
(1826)	42.2	51.6	46.3	0.0	9.4	4.1
	42.5	53.3	47.3	0.3	11.1	5.1
	42.8	53.4	48.8	0.6	11.2	6.6
	43.8	52.3	49.6	1.6	10.1	7.4
	44.6	50.0	49.2	2.4	7.8	7.0
	46.4	46.7	48.4	4.2	4.5	6.2
	49.1	44.4	44.9	6.9	2.2	2.7
	50.5	44.5	44.4	8.3	2.3	2.2
	49.9	45.9	44.1	7.7	3.7	1.9
	49.8	46.1	42.2	7.6	3.9	0.

Taking the means of each pair of adjacent numbers in the last three columns gives:

TABLE XII

1827	.15	1837	10.2	1847	4.6
1829	1.1	1939	10.7	1949	7.0
1831	3.3	1841	6.2	1851	6.6
1833	7.6	1843	2.3	1853	2.5
1835	7.65	1845	3.8	1854.5	.95

The fifteen-point graph of these follows.



SATURN, 1826-1943

The sun-spot curve of Saturn agrees with its precessional curve in having two minima and two unequal maxima. The effect of the nodes and aphelion is clearly reflected in the curve. The two precessional periods of Saturn are, respectively, 14.12 years and 15.34 years long. Calculation shows that the ratio of the activity of the shorter period to the longer is about 11 to 8. The magnitude of the forces involved and the enormous area over which spot activity is spread produce a considerable lag in the maxima and the minima. The period of Saturn is about 50 times that of Venus. Hence a much greater lag for Saturn than for Venus would be expected.

Periodogram Analyses. The existence of the periods found in this study have all been confirmed by the periodogram analyses of three independent investigators. In 1891 Schuster published in a Vienna journal the result of an examination of the interval 1880 to 1887 for *short periods,* and found one of 69.4 days. This corresponds to the interval from conjunction to opposition of Mercury and Venus which is 72 days.

Elsa Frankel sought *short periods* in the sun-spot data over the intervals 1880-1887, 1890-1899, and 1902-1911. She found a period of 68.5 days and one of 200 days. (*Pub. der Sternwarte des Eidg. Polytechnikunst zu Zurich,* Bd. V, S. 47.) Her first result confirms that of Schuster, to whose work she refers in the introduction to her paper. The second, 200 days, corresponds to the mean interval from conjunction of the earth and Jupiter to their opposition, which is 199.5 days.

The precessional action of Mercury, Venus, Earth, and Jupiter is confirmed by these investigations. Neither Schuster nor Elsa Frankel had any idea of what the periods they found meant. Elsa Frankel said they reminded her of the 88-day sidereal period of Mercury and the 225-day sidereal period of Venus.

Other short periods exist which have not been discovered by any periodogram analysis are:

Mercury-Earth	58 days	Venus-Earth	292 days
Venus-Jupiter	119 days	Mars-Jupiter	1.19 years

The Venus-Jupiter period is the strongest of all the short periods. Elsa Frankel said her work indicated another short period which she would have to defer to second research. These facts are of great significance for Dr. Nicholson points out:

"Averaged by months, they show that the cycle does not progress in a regular fashion but by a series of fluctuations which vary in lengths from five or six to as much as fifteen months. These fluctuations are smoothed out when the yearly means are taken." (The Solar Cycle, Leaflet No. 50, *P.A.S.P.*)

For long-period effects Clayton has the most striking results by what he calls harmonic analysis. Over the interval 1793 to 1936 he obtained among others the following periods in years: 9.93, 19.86, 11.17, 11.90, and 14.89 (*Smithsonian Miscellaneous Collections,* Vol. 98, Number 2). The synodic period of Jupiter and Saturn is 19.86 years and the interval from conjunction to opposition is half of this or 9.93 years, which is exact agreement. It may be noted in support of the 9.93 year period that Alter and Stumpf each obtained a 10-year period by periodogram analysis. The writer also obtained a period of 9.75 years from the area measures of the interval 1832 to 1935. The 11.17-year result is the standard mean length of the sun-spot cycle. Corresponding to 11.90 is Jupiter's sidereal period of 11.86 years. Half Saturn's sidereal period is 14.73 years which corresponds to the 14.89-year period found.

Thus three independent periodogram analyses confirm the precessional sun-spot activity of Mercury, Venus, Earth, Jupiter, and Saturn.

11. *Conclusion.* A simple procedure for five planets shows that each is effective in producing sun-spots. The curves obtained prove for each planet that its orbital period and its precessional period register in the sun-spot data. Each of the five curves shows what the underlying

dynamics is. The cause of the sun-spots is the precessional action of all the planets on a slightly ellipsoidal sun. Moreover, the dominant actor is Jupiter. Thus two road blocks to solar knowledge, a 7-year spot-cycle and a 17-year one, are now removed.

It will be illuminating to consider how the dynamics of precession works on the sun. An easily understood phenomenon of sun-spots is the solar equatorial belt which is nearly free of spots. The plane of the sun's equator makes an angle of 6° 2′ with the plane of Jupiter's orbit. For half a solar rotation with this 12 degree belt whatever motion is imparted to the material at and near the solar surface by Jupiter's precessional action is reversed during the next half rotation. Outside this belt the action is cumulative. Here is the reason why spots are "infrequent" from solar latitude 5° north to 5° south.

Three other crucial solar phenomena, the presence of *two* sun-spot belts, Spoerer's law, and the equatorial acceleration of the sun have been explained on the basis of precessional action in sections 11 and 12 of my 1930 article in the *Astronomical Journal.* Those explanations will not be repeated here. Section 10 of that paper also gives a much needed account of the sun-spot data before 1826.

If precessional action of the planets can produce sun-spots on a slightly ellipsoidal sun, conversely precessional action of the sun on certain planets should be discernible. Jupiter's equatorial diameter is nearly 6,000 miles greater than the polar. Jeffries places its gaseous envelope at about 4,000 miles deep. Jupiter's period of rotation is 9 hours and 50 minutes. It is not surprising, therefore, that the sun's precessional action on this planet produces *not spots* but ever changing continuous *belts*. Moreover, Jupiter's surface has a marked equatorial acceleration.

Confirmative evidence comes from long continued observations of Jupiter which have discovered two color changes in the belts. For the "tawny color," the period is 11.86 years, the planet's orbital period and it runs from equinox to the same equinox again. For the oppositely changing redness of the two belts, it reaches equality at the solstices and the two periods would be the precessional periods. (See article by the writer "On the Cause of Jupiter's Belts," POPULAR ASTRONOMY, November, 1929).

Similarly, Saturn has belts and its surface shows equatorial acceleration. Belts have been observed on Uranus and Neptune also but these planets are too far away to learn whether equatorial acceleration of their surfaces is present or not. Here are four additional instances of precessional action in the solar system which involve motions other than a conical spin of the axis of the rotating body.

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