

Modeling the Diversity of Type Ia Supernova Explosions

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Abstract. Type Ia supernovae (SNe Ia) are a prime tool in observational cosmology. A relation between their peak luminosities and the shapes of their light curves allows to infer their intrinsic luminosities and to use them as distance indicators. This relation has been established empirically. However, a theoretical understanding is necessary in order to get a handle on the systematics in SN Ia cosmology. Here, a model reproducing the observed diversity of normal SNe Ia is presented. The challenge in the numerical implementation arises from the vast range of scales involved in the physical mechanism. Simulating the supernova on scales of the exploding white dwarf requires specific models of the microphysics involved in the thermonuclear combustion process. Such techniques are discussed and results of simulations are presented.

1. Introduction

SNe Ia are extremely bright cosmic explosions with properties that are more homogeneous than those of other astronomical transients. Furthermore, a correlation between the width and the decline rate of the *B*-band light curve (“width-luminosity relation”, WLR) allows to calibrate them (Phillips 1993; Phillips et al. 1999) and makes them the best distance indicators out to redshifts of about one. This correlation, however, is established only empirically on a set of nearby SNe Ia. Consequently, a quantification of systematic errors resulting from the calibration procedure is difficult to achieve. Moreover, aiming at distance determinations to far events, evolutionary effects could potentially obscure the measurements. It is clear that a sound understanding of the physics of SNe Ia is desirable in order to improve their precision as distance indicators in observational cosmology.

Besides adding to the ever growing cosmological SN Ia databases, observations in the past decade allowed to take a close look at a number of nearby events. It turned out that, although they form a remarkably homogeneous class, individual SNe Ia differ significantly in their properties (e.g. Benetti et al. 2005; Mazzali et al. 2007). Apart from variations within the “normal” (as defined by Branch et al. 1993) SNe Ia, there are distinct sub-classes which differ significantly from the bulk of events. A first step towards a physical understanding of the class of SNe Ia and to improve their quality as distance indicators is therefore to identify the origin of this diversity.

A model that reproduces a large range of observational characteristics will be discussed in the following. Recent numerical simulations suggest that it potentially can account for the “normal” SNe Ia. This, however, also implies that the distinct sub-classes have to be explained in different physical scenarios.

2. Astrophysical Model

SNe Ia are attributed to thermonuclear explosions of carbon-oxygen white dwarf (WD) stars. In order to evolve a WD to a state where a thermonuclear explosion can trigger, the supernova progenitor has to be a binary system. Different scenarios of the progenitor evolution have been suggested (see, e.g., Hillebrandt & Niemeyer 2000) and may account for different SN Ia subclasses. Here, we focus on the so-called Chandrasekhar-mass explosion scenario, where the WD has accreted matter from its companion so that its mass approaches the stability limit – the Chandrasekhar mass of $\sim 1.4 M_{\odot}$. Close to this mass limit, the density in the core of the WD increases dramatically so that carbon burning ignites. This, however, does not yet trigger the actual explosion process because convective cooling still moderates the burning. The resulting pre-explosion simmering phase lasts for about a century (Woosley et al. 2004) and is characterized by highly turbulent convective motions. Gradually, the background temperature increases so that finally one or many hotspots in the turbulent flow near the WD’s center undergo a thermonuclear runaway. Out of these “ignition sparks”, a flame starts to propagate. It incinerates the WD material and leads to an explosion of the star on time scales of 1–2 s. As discussed in Sect. 4., the flame propagation is highly sensitive to the initial conditions and the ignition geometry strongly affects the strength of the overall explosion process. Despite some recent progress (Höfllich & Stein 2002; Woosley et al. 2004; Wunsch & Woosley 2004; Kuhlen et al. 2006; Zingale et al. 2009), the physics of the ignition process and the turbulent simmering phase is still extremely challenging and realistic conditions are out of reach for numerical implementations. Therefore, the number and spatiotemporal distribution of ignition sparks remains uncertain.

After ignition near the center of the WD, the thermonuclear burning front propagates outwards. Two distinct modes of propagation, a subsonic deflagration and a supersonic detonation (see Landau & Lifshitz 1959) are consistent with the conservation laws of hydrodynamics. The first attempts to modeling SN Ia explosions (Arnett 1969; Arnett et al. 1971), however, showed that a prompt detonation converts the WD almost entirely into iron-group elements. This is in contradiction with the observed spectra of these events which show strong intermediate-mass element (IME) features. Such elements (like Si, Ca, and S) are synthesized in thermonuclear explosive burning at low densities and therefore the WD material must expand prior to incineration. This can only be achieved if the flame propagates subsonically, i.e. as a deflagration. Before reaching low densities, however, the flame burns the high-density core material to iron-group elements, predominantly ^{56}Ni . In its radioactive decay, this isotope releases Gamma-rays which are scattered down to optical wavelengths in the ejecta and make the supernova bright.

A laminar deflagration is a very slow process (see Timmes & Woosley 1992, for laminar flame speeds). In order to burn sufficient amounts of material to

explode the WD, the flame propagation must accelerate significantly. And indeed, such an acceleration is to be expected as the flame propagation from the WD's center outwards produces a buoyancy-unstable stratification of light and hot ashes under dense fuel in the gravitational field of the star. The ensuing Rayleigh-Taylor instability leads to shear flows between rising plumes of burning material and fuel downdrafts with Reynolds numbers of $\sim 10^{14}$. As a consequence of shear instabilities, a turbulent eddy cascade establishes. Down to the Gibson scale at which the laminar flame speed equals the eddy velocity, the flame interacts with turbulent eddies of this cascade. For most parts of the explosion, the Gibson scale lies orders of magnitude above the flame thickness which is only of the order of millimeters to centimeters. Therefore, turbulent eddies wrinkle and corrugate the flame on large scales without affecting the microphysics of the burning. In this flamelet regime of turbulent combustion, a sufficient flame acceleration is achieved to explode the WD (e.g. Reinecke et al. 2002; Gamezo et al. 2003; Röpke & Hillebrandt 2005; Röpke et al. 2007a). Only when the fuel density falls below $\sim 10^7 \text{ g cm}^{-2}$ (due to the WD expansion and in the outer layers of the star), the Gibson scale becomes smaller than the broadening flame structure. In this late stage of the explosion, turbulent eddies stir the internal flame structure mixing fuel and ash. Niemeyer & Hillebrandt (1997) suggested that this may cause the transition from an initial subsonic deflagration to a supersonic detonation in the delayed detonation SN Ia scenario (Khokhlov 1991). Microphysical studies (Lisewski et al. 2000; Woosley et al. 2009) indicate that entering the distributed burning regime alone may not be sufficient for causing a deflagration-to-detonation transition (DDT), but in addition turbulent velocities of $\sim 1000 \text{ km s}^{-1}$ at scales of about 10 km are required in this late burning regime. Analyzing three-dimensional simulations of deflagrations in WDs, Röpke (2007) found that such high turbulent fluctuations do occur in late stages of the explosion.

3. Numerical Techniques

The challenge in the numerical implementation of the above astrophysical scenario of thermonuclear supernova explosions arises from the multi-scale character of the problem (e.g. Röpke & Bruckschen 2008; Röpke 2008). The physically relevant range in spatial scales covers about 11 orders of magnitude. In particular, the representation of a thin thermonuclear flame on a grid comprising the entire WD and the flame/turbulence interaction require special numerical approaches (for a summary see Röpke & Schmidt 2009).

In the implementation used here, the flame propagation is modeled based on the level-set technique (Reinecke et al. 1999). It is associated with the zero-level set of a signed distance function G which is defined positive in the ashes and negative in the fuel. The advancement of the flame is described by a partial differential equation modifying this G -field in an appropriate way (see Reinecke et al. 1999). As an input, this requires the flame propagation speed, which, in the flamelet regime, is set by the turbulent velocity fluctuations on the scale of computational grid. These are derived from a subgrid-scale turbulence model (Niemeyer & Hillebrandt 1995; Schmidt et al. 2006).

Following Golombek & Niemeyer (2005) and Röpke & Niemeyer (2007), the detonation wave is propagated by a separate level-set function. This allows to prevent the detonation from unphysically crossing ashes left behind from the previous deflagration stage (Maier & Niemeyer 2006).

Another challenge arises from the expansion of the WD and the SN Ia ejecta. In order to reliably compute synthetic observables from the results of explosion simulations, these have to be followed to a state of hydrodynamically relaxed homologous expansion. This requires to simulate the explosion process for at least 10 s. An computational grid co-expanding with the ejecta allows to keep then on the domain (Röpke 2005) and with moving nested grids the flame front can be optimally resolved for a fixed number of grid cells (Röpke et al. 2006a).

Another scale problem arises from the discrepancy between hydrodynamic time scales and the time scales of some of the involved nuclear reactions. Currently, this problem is bypassed in our implementation by only following a very limited set of nuclear species in the explosion simulation and reconstructing the details of the nucleosynthesis in a post-processing step (e.g. Travaglio et al. 2004; Röpke et al. 2006a). This is achieved on the basis of passive tracer particles advected with the flow of the explosion which record trajectories of temperature, energy, and density. Both this tracer method and the expanding grid add Lagrangian components to our Eulerian formulation of the problem.

4. Results

Based on the numerical techniques described above, thermonuclear supernova explosion simulations in multiple spatial dimensions have been performed. While pure turbulent deflagrations are able to explode the WD, the resulting event is predicted to be on the faint end of the observed normal SNe Ia (Röpke et al. 2007a).

The delayed detonation scenario, in contrast, covers the range of normal to bright SNe Ia when the ignition of the deflagration is treated as a stochastic process (Röpke & Niemeyer 2007; Mazzali et al. 2007). In cases of few (García-Senz & Bravo 2005; Livne et al. 2005; Röpke et al. 2006b) or asymmetrically distributed ignition sparks (e.g. Calder et al. 2004; Röpke et al. 2007b; Townsley et al. 2007), the deflagration is weak. It burns comparatively little material and hence the production of ^{56}Ni and the energy release are low. Therefore, when the detonation triggers, it finds abundant unburnt material at high densities which it converts to additional ^{56}Ni . The resulting event is bright and energetic and the ejecta structure is set mainly by the detonation stage. Somewhat counter-intuitively, a vigorous ignition in many sparks distributed around the WD's center and a subsequent strong deflagration leads to an overall faint explosion. Here, the detonation finds an almost completely burnt core and outer layers that have been diluted by expansion. It therefore burns them predominantly to IMEs and does not significantly contribute to the ^{56}Ni production. Hence, the structure of the iron group material at the center of the ejecta is dominated by the large-scale buoyancy instabilities from the deflagration. The outer IME layers, however, are produced by the detonation and therefore smooth.

A thorough exploration of the deflagration ignition configuration as a parameter of the delayed detonation scenario was performed in a set of two-dimensional simulations (Kasen et al. 2009). Here, spherical ignition kernels of radius 6 km were placed close to the center of the star – in some models isotropically distributed around it and in others in a solid angle with an opening of less than 360° . In radius, a Gaussian distribution with a standard deviation of 150 km or 75 km was chosen. The number of ignition kernels ranged from 15 to 150 (see the Supplementary Online Material of Kasen et al. 2009). In addition to the ignition spark distribution, the DDT criterion was varied. The detonation was triggered in the distributed burning regime if the ratio of the turbulent velocity fluctuation to the flame speed exceeded a certain threshold. From the results of these simulations, synthetic observables were computed by means of radiative transfer calculations. These found good agreement of the models with observations both in color light curves and in spectra and their evolution. Furthermore, from these results the peak brightness and the decline rate of the B -band light curve could be determined for the individual models. These were found to follow the observational relation of Phillips et al. (1999).

5. Conclusions

Simulating the turbulent combustion in thermonuclear supernovae is a numerically challenging task. Specific techniques are required to correctly represent the thin flame and its interaction with turbulence on a wide range of spatial scales on a computational grid that comprises the full exploding WD. Here, a combination of a level-set based flame representation and subgrid-scale turbulence modeling were discussed as a solution to this problem. The implementation of this approach allows to simulate thermonuclear supernova explosions. A pure turbulent deflagration is found to be inconsistent with the properties of “normal” SNe Ia. These objects require a detonation stage following an initial deflagration. The detonation wave is again represented numerically with the level-set technique. Simulations of the delayed detonation scenario lead to synthetic observables in good agreement with the observations. If the ignition of the deflagration is modeled as a stochastic process, the diversity of luminosities found in normal SNe Ia is reproduced. Moreover, the correlation between peak luminosity and decline rate of the B -band light curve is reproduced. This is the first time that multidimensional hydrodynamical explosion models predict this relation. Since it forms the basis of the calibration of SNe Ia as cosmological distance indicators, a theoretical understanding of the correlation is required to improve the precision of SN Ia cosmology. Our model provides the first step in this direction.

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