Long Secondary Periods in Pulsating Red Supergiant Stars

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Abstract: About one-third of pulsating red-giant stars shows long secondary periods, an order of magnitude longer than their primary radial pulsation period. The cause of these long secondary periods is not known. Kiss *et al.* (2006) have recently used Fourier analysis to study the variability of 48 pulsating red supergiant stars, using several decades of visual measurements collected by the American Association of Variable Star Observers (AAVSO). They found long secondary periods in several stars. (We use the term "period" with some caution because, in many cases, the datasets are not sufficiently long to show whether there are true periods present, or just "characteristic time scales" of variability.) In this paper, we use self-correlation analysis to study the variability of the same stars. For other types of variable stars, we have found this method to be a useful adjunct to Fourier analysis, especially if the variability is only semi-regular. We find that about one-third of pulsating red supergiants show long secondary periods that are coherent over a few cycles or more. Although the cause of the long secondary periods is not definitely known, they may be linked to rotation of large, long-lived convection cells across the disc of the star.

Resumé: Environ un tier des étoiles géantes rouges à pulsations présente des longues périodes secondaires, à un niveau de magnitude plus grande que celle de leur période de pulsation radiale primaire. La cause de ces longues périodes secondaires est inconnue. Kiss *et al.* (2006) ont récemment utilisé une analyse Fourier pour examiner la variabilité de 48 étoiles supergéantes rouges à pulsations, en se servant de mesures visuelles accumulées durant plusieurs décénies par l'Association américane d'observateurs d'étoiles variable (AAVSO). Ils ont découvert de longues périodes secondaires dans plusieurs étoiles. [Nous employons le terme "période" avec prudence car dans plusieurs cas l'accumulation des données n'est pas suffisamment longue pour s'assurer qu'en effect il y a de vraies périodes, et non seulement une "variabilité caractéristique de la période échantillonnée."] Dans ce rapport, nous utilisons une analyse d'autocorrélation pour examiner la variabilité de ces mêmes étoiles. Pour d'autres types d'étoiles variables, nous avons trouvé cette méthode un ajout utile à l'analyse Fourier, particulièrement si la variabilité est semi-régulière. Nous trouvons qu'un tier des étoiles supergéantes à pulsations montre de longues périodes secondaires qui sont cohérentes durant quelques cycles ou plus. Quoique la cause des longues périodes secondaires n'est pas définitivement connue, elle pourrait être liée à la rotation des grandes cellules de convection de longue durée à travers le disque de l'étoile.

Introduction

This paper presents an example of the important contributions that amateur astronomers — in this case, those who measure the changing brightness of variable stars for the American Association of Variable Star Observers (AAVSO) — can make to astronomical research.

Red supergiants are the coolest, largest, most luminous stars, up to a thousand times larger in radius than the Sun. They are massive young stars in the final rapid stages of thermonuclear evolution. They undergo a complex variety of physical processes, including convection, pulsation, and extensive mass loss, which causes most of them to be shrouded in gas and dust.

They are also all variable, though not strictly periodic, being classified as SRc if they are semi-regular, or Lc if they are irregular. They vary typically on time scales of hundreds to thousands of days, and amplitudes up to a few magnitudes. Some bright examples are α Ori (Betelgeuse), α Sco (Antares), and μ Cep ("The Garnet Star"). These three stars have relatively low-amplitude variability. VX Sgr shows the most spectacular SRc variability; its amplitude is several magnitudes.

The terms "semi-regular" and "irregular" are rather vague. "Semi-regular" means that the variability is not strictly periodic. This could arise because of short- or long-term variability in the amplitude, period, phase, or shape of the light curve, or because of the presence of another form of variability, such as an additional period, or because the cause of the variability is intrinsically non-periodic. "Irregular" means that there is no evidence of any periodic behaviour in the star. In reality, there is a continuous spectrum of behaviour in red giants and supergiants, ranging from relatively periodic, like VX Sgr and S Per, to irregular.

David G. Turner (Saint Mary's University, Halifax) and his colleagues have been especially active in studying these stars in the last few years, both in terms of their fundamental properties (Turner 2006) and the long-term behaviour of a specific star, BC Cyg (Turner *et al.* 2006).

Kiss *et al.* (2006) studied 48 SRc and Lc stars using visual observations from the American Association of Variable Star Observers (AAVSO) International Database. That paper is a good source of information about SRc stars and shows long-term light curves for many of them. The mean time span of the data was 61 years. Most of the stars showed a period of several hundred days that could be ascribed to radial (in-and-out) pulsation. This is the most basic and common kind of pulsation; it's the kind that is observed in Cepheids, Mira stars, *etc.* Two or more periods were found in 18 stars. In some cases, the second period could be an additional radial mode. In other cases, the second period was an order of magnitude longer than the radial period, and could be classified as a long secondary period, similar to those that have been found in about a third of pulsating red giants, *e.g.* by Percy *et al.* (2001), and whose cause is unknown (Wood *et al.* 2004).

Wood and his colleagues considered (and rejected) a wide range of possible mechanisms for the long secondary periods in red giants; this same list might apply to red supergiants. The most promising mechanism is rotational variability. Rotational velocities of red supergiants are not well known, but their rotational periods would not be inconsistent with the long secondary periods; the equatorial rotational velocities are a few km/s and the radii are a few hundred solar radii, so the rotational periods would be a few thousand days. And red supergiants have giant bright convective cells that could provide a basis for rotational variability, though the characteristic lifetimes of giant convective cells may be shorter than the long secondary periods (Gray 2008). Incidentally, this remarkable study of the long-term spectroscopic variability of Betelgeuse, by David F. Gray (University of Western Ontario, London) was possible only because of the fine instrumentation that he has developed for a 1.2-m telescope, his regular access to this facility, his patience in accumulating data over many years, and his long experience in interpreting the spectra of cool stars.

The primary purpose of this paper was to study the long secondary periods independently, using the same datasets as Kiss *et al.* (2006), but using self-correlation analysis. This method has proven, for other types of stars, to be a useful adjunct to Fourier analysis, especially for stars that are not strictly periodic. This has been demonstrated through its use in many refereed papers, such as Percy *et al.* (2001: red giants), Percy *et al.* (2003: RV Tauri stars), Percy *et al.* (2004: Be stars), and Percy *et al.* (2006: T Tauri stars).

Data

Visual measurements of the brightness of 48 stars came from the AAVSO International Database, spanning up to a century, but averaging 61 years (Kiss *et al.* (2006)). The accuracy of the measurements is typically 0.15 to 0.3 magnitude. The intercept of the self-correlation diagram on the vertical axis is a measure of the mean error of observation (see below), and is consistent with this estimate of the accuracy.

Determination of Periods by Self-Correlation

Visual inspection of light curves and Fourier analysis (power spectrum analysis) is commonly used for period analysis of variable-star data. Self-correlation analysis — a form of variogram analysis (Eyer & Genton 1999) — has proven to be useful, in conjunction with the other two techniques, for some kinds of stars, especially if the stars are somewhat irregular, and if there are "aliases" in the power spectra due to regular gaps (*e.g.* seasonal gaps) in the data. It can detect characteristic time scales — here denoted τ — in the data. It determines the cycle-to-cycle behaviour of the star, averaged over all the data. The measurements do not have to be equally spaced.

The algorithm works as follows (Percy *et al.* 1993): for every pair of measurements, the absolute value of the difference in magnitude (Δmag) and the difference in time (Δt) is calculated. Then Δmag is plotted against Δt, from zero up to some appropriate upper limit Δt(max) which, if possible, should be a few times greater than the expected time scales, but a few times less than the total time span of the data. If the maximum Δt is as large as the total time span of the data, there will be few if any instances of Δt with this value.

The Δmag data are binned in Δt so that, if possible, there are at least a few points in each bin; the Δmag values in each bin are then averaged. The choice of the number of bins will depend on how many measurements are available. Increasing the number of bins increases the time resolution of the method, but it decreases the number of points in each bin, which decreases the accuracy of the average Δmag. In this study, we typically used 50 to 100 bins for each star. Figure 1 is a self-correlation diagram, a plot of average Δmag versus average Δt in each bin; the accuracy of the points can be judged from the point-to-point scatter. The method is so simple that there is no equation involved — just the procedure described above!

If the variability is regular, with a period or characteristic time scale τ, then the average Δmag will be a minimum at multiples of τ. If several minima are present, we can conclude that the variability is reasonably regular. Each minimum can be used to estimate τ. The Δt of the Nth minimum, divided by N, gives a measure of τ, so the values derived from the several minima can be averaged to give a better estimate of the time scale. The scatter in these individually determined values gives an estimate of the error in the average value of τ.

If the variability were strictly periodic and the magnitudes had no error, then the minima should fall to zero, because measurements that are an integral number of cycles apart would always be exactly the same. In reality, the measurements have observational error. The value of Δmag for very small values of

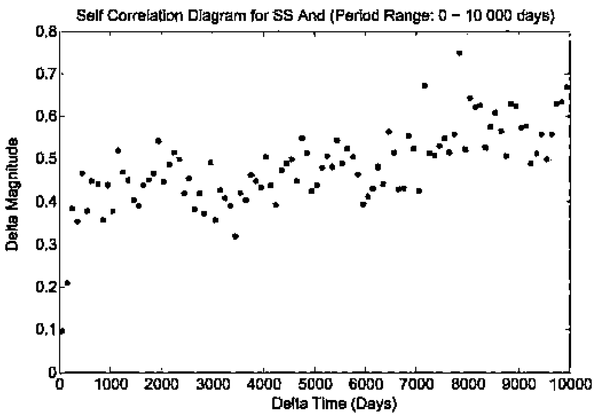

Figure 1 — An example of a self-correlation diagram — that of SS And. There are shallow minima at multiples of 3100 days, and maxima in between, indicating that there are variations on this timescale. This is an example in which the long secondary period self-correlation signal is weak. No long secondary period was reported by Kiss *et al.* (2006). The short period is 159±17 days (Kiss *et al.* (2006)). The intercept on the vertical axis is a measure of the average observational error. The depth of the minima is a measure of the amplitude of the variability.

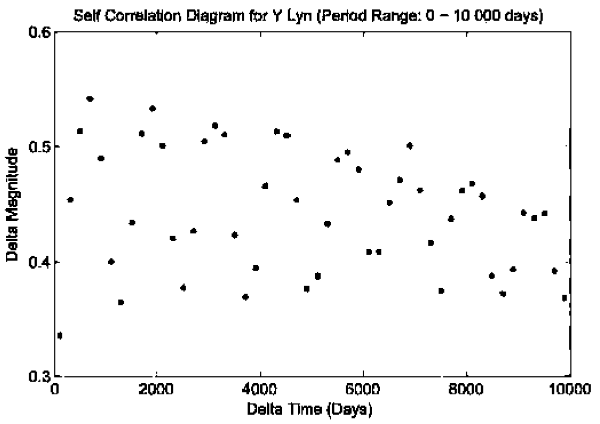

Figure 2 — The self-correlation diagram of Y Lyn. Here, the signal is very strong. The diagram is dominated by the 1240-day long secondary period. This is the same long secondary period reported by Kiss *et al.* (2006). The short period is 133±3 days, according to Kiss *et al.* (2006), Percy *et al.* (2001), and others.

Δt, *i.e.* the intercept on the vertical axis of the self-correlation diagram, will reflect observational error only, assuming that there is no variability on very rapid time scales. The height of the other minima above the zero line is also determined by the average error of the magnitudes, but also by the degree of irregularity, if any.

Measurements that are a half-integral number of cycles apart may have a Δmag ranging from zero to the full amplitude of the variations. As long as there is a sufficient number of Δmag values in each bin, the height of the maxima averages out to about half the peak-to-peak range of the light curve. The difference between the maxima and the minima is therefore a measure of the average amplitude of the variability. Specifically, it is about 0.45 times the average peak-to-peak range of the light curve (Percy *et al.* 2003).

The persistence of the minima to large Δt values is also determined by the degree of irregularity. The behaviour of the self-correlation diagram at a particular Δt depends on whether periodicity persists for that interval of time. For instance, if the periodicity remains coherent for only a few cycles, the minima will gradually disappear as Δt increases. Even if the self-correlation diagram does not show minima, it still provides a "profile" of the variability; the typical change in magnitude Δmag in a time of Δt.

Figure 1 shows the self-correlation diagram for SS And. In this case, the signal is weak, but there are minima of Δt at 3100, 6200, and 9300 days, and maxima half-way in between, suggesting a period of about 3100 days. The intercept on the vertical axis is about 0.1 magnitude; this is the expected observational error. The levels of minima 1, 2, and 3 are much higher than this: 0.4 to 0.5 magnitude. This result is an indication of the irregularity of the star. The difference between the maxima and minima is about 0.1 magnitude, so the corresponding full range of the light curve would be 0.1/0.45 or about 0.2 magnitude. The minima persist for at least three cycles; this is a measure of the coherence of the 3100-day time scale.

Figure 2 shows the self-correlation diagram for Y Lyn. In this case, the minima are very well-defined, and persist for at least eight cycles. The observational error, as measured by the intercept on the vertical axis, appears to be about 0.3 magnitude. The levels of the minima are not much higher than this error, so the contribution of the irregularity must be considerably less.

Note that the self-correlation diagram is constructed from measurements that are no more than Δt(max) apart, and is not the same as a light curve. Nor is it like a phase curve, which combines all measurements into a single cycle.

For reasons already mentioned, our method requires of the order of ten or more Δmag values in each bin, simply to produce a meaningful average Δmag. Although our method is not subject to "alias" periods due to the periodicity of the seasonal gaps in the data, there may be gaps in the self-correlation diagram if there are no pairs of measurements with certain values of Δt — due to long seasonal gaps in the data, for instance.

For a more detailed discussion of self-correlation, its nature, strengths, and weaknesses, see Percy & Mohammed (2004) and references therein; this reference is freely available on-line. One weakness of self-correlation analysis is that it is not very effective if the star has multiple periods with comparable periods and amplitudes. Our self-correlation software is publicly available at:

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www.astro.utoronto.ca/percy/index.html
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and a manual for its use is available at

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www.astro.utoronto.ca/percy/manual.pdf
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Unfortunately, the statistical properties of self-correlation are not known, especially since it is usually applied to stars that are intrinsically non-periodic. Our interpretation of diagrams such as Figures 1 and 2 is based on our 15 years of experience with the method, including comparison with results from Fourier analysis.

Results

The results are listed in Table 1. This table includes only stars that have or may have a long secondary period *i.e.* a "short" (radial) period has been identified, and there is an additional period that is significantly longer. In the table, the short period is taken from Kiss *et al.* (2006), who give values both from their own work, and from the literature. In a few cases, we have determined it from our self-correlation diagrams (recall, though, that the primary purpose of our project was to study the long secondary periods). The long periods in the last column are those determined by Kiss *et al.* (2006). Sample self-correlation diagrams are shown and described in the two figures. One figure shows a star in which the long secondary period is weak (SS And); the other shows a star in which it is strong (Y Lyn). A colon indicates that the value is uncertain.

Kiss *et al.* (2006) did not find periods in AO Cru, BI Cyg, IS Gem, KK Per, or PR Per. They found BO Car to be not variable.

Several stars have basic (short?) periods that are 1000 days or more, *i.e.* no shorter period has been reported; but one may still exist. They include: VY CMa (1450 days), TZ Cas (1400 days), BU Gem (2400 days), RS Per (4400 days), KK Per (2500 days), and PR Per (2600 days).

The following stars do not appear to have detectable long secondary periods, or have long secondary periods greater than 10,000 days, which is the upper limit of our calculations: NO Aur, UZ CMa, VY CMa, RT Car, CK Car, PZ Cas, W Cep, T Cet, WY Gem, RV Hya, W Ind, XY Lyr, S Per, T Per, XX Per, AD Per, BU Per, FZ Per, PP Per, VX Sgr, AH Sco, α Sco, W Tri.

Discussion and Conclusions

Of the 48 stars, 19 show long secondary periods, 23 do not, and the other 6 are marginal. The incidence of long secondary periods is thus about the same as in pulsating red giants.

In 14 stars, our long secondary periods agree, to within the uncertainties of each method, with those determined by Kiss *et al.* (2006) (though, in two stars, Kiss *et al.* (2006)'s error bars were especially large). In 11 stars, we found long secondary periods that were not reported by Kiss *et al.* (2006).

The long secondary periods are typically 5-10 times the short period *i.e.* the two are correlated, but there is considerable scatter in the relation between the two. The fact that there is a correlation, however, suggests that the long secondary periods are such that they are related to the size of the stars, because the short (radial) period is determined primarily by the size of the star.

The amplitudes range from 0.02 to 0.3 magnitude, for the confirmed long secondary periods. Only four stars have long secondary period amplitudes greater than 0.1 magnitude, so the long secondary periods could be produced by a relatively subtle process.

There is no obvious correlation between amplitude of the long secondary period and the spectral type, or the short (radial) period *i.e.* the largest amplitudes are not necessarily found in the coolest or largest stars.

The coherence of the long secondary periods is determined by noting the number of minima in the self-correlation diagram. It ranges from very low (a cycle or two) to very high (ten cycles or more). Those that are notably coherent are: EV Car, μ Cep, and Y Lyn. There are also stars with long periods, but without obvious short periods, that are coherent: VY CMa, PZ Cas, and BU Gem, but it is not clear whether these periods are true long secondary periods.

Once again, self-correlation has proven to be a useful adjunct to Fourier analysis, which was used by Kiss *et al.* (2006) in their study of these stars. In a few cases, we have been able to detect long secondary periods that were not reported by Kiss *et al.* (2006). This may be because Fourier analysis assumes the variability to be periodic, whereas self-correlation analysis can detect characteristic time scales that are less coherent. In other cases, we have been able to determine the long secondary period somewhat more precisely than Kiss *et al.* (2006), using several individual minima in the self-correlation diagram. Again, the differences may occur because the two methods make different assumptions, and the behaviour of some of the stars may vary from decade to decade. So, we do not claim that self-correlation analysis is a better tool, just an additional tool.

Both Kiss *et al.* (2006), on the basis of the structure of photometric power spectra, and Gray (2008), on the basis of long-term spectroscopic observations of Betelgeuse, propose that the short period is a radial pulsation period, driven stochastically (randomly) by convective motions whose time scale is similar to that of the pulsation period. The lifetimes of these modes are several periods. Neither Kiss *et al.* (2006) nor Gray (2008) comments specifically on the nature and origin of the long secondary periods. If there are giant bright convective cells present and their lifetimes are long, then the long secondary periods could be rotational; however, some of the long secondary periods in Table 1 are very long lasting, which would require that the lifetimes of the convective cells were much longer than the rotation periods.

It is also possible that rotational variability could be caused by some other temporary bright or dark features, but large convection cells are expected to be present in red supergiants, and are expected to be temporary.

Finally, it should be obvious that the study of stellar variability on time scales of decades is challenging. Fortunately, organizations such as the AAVSO have been facilitating systematic, long-term visual measurements of such stars for a century or more. May their work continue for another century!

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Table 1 — Long Secondary Period Determinations from Self-Correlation Analysis

Star	LSP (d)	Δm	Short Period (d)	Long Period (d)
SS And	3100	0.10	159±7	—
BO Car	2900	0.10	not variable	not variable
CL Car	3000:	0.10:	229±14, 490±100	—
EV Car	1150	0.15	276±26	820±230
IX Car	4500	0.30	408±50	4400±2000
TZ Cas	3500	0.10	—	3100
ST Cep	3200	0.05	—	3300±1000
μ Cep	4100	0.04	860±50	4400±1060
AO Cru	4000:	0.30	—	—
RW Cyg	3200	0.08	580±80	—
AZ Cyg	2000:	≤0.1	495±40	3350±1100
BC Cyg	3000:	0.05	720±40	—
BI Cyg	3000	0.10	—	—
TV Gem	2600	0.08	426±45	2550±680
BU Gem	2400	0.05	—	2450±750
IS Gem	5500:	0.01	—	—
αHer	1400	0.02	124±5	500±50, 480±200
Y Lyn	1300	0.20	133±3	1240±50
αOri	2100	0.02	388±30	2050±460
W Per	3000:	0.03	500±40	2900±300
RS Per	4400	0.06	—	4200±1500
SU Per	3500	0.03	430±70	3050±1200
KK Per	2500:	0.02	—	—
PR Per	2600	0.02	—	—
CE Tau	3600	0.03	140-165	1300±100