A discussion on the duration of central transits as seen from Earth

Darren Beard

The factors affecting transits of Venus and Mercury are discussed with particular attention to how these factors affect the maximum possible duration of a central transit. The secular variations of the orbits of the Earth, Venus and Mercury are considered so as to suggest secular variations in the maximum possible transit duration.

Introduction

Jean Meeus[1] gives the maximum time for a central transit of Venus as seen from the Earth as 7.9 hours and the corresponding time for a Mercury transit as 6.5 hours. These times are for a geocentric observer and assume that the planets are at their mean distances from the Sun and moving in circular orbits. This is likely to be a reasonably accurate time for actual Venus transits, since the orbit of Venus is the most circular of all the planets,[2] however, it was thought interesting to consider possible transit durations when allowing for the planets not to be at their mean distances during the transit. Actual transit durations for Mercury will differ markedly from the 6.5 hours approximation.

Theoretical transits

A central transit of a planet is one in which the centre of the disk of the transiting planet passes directly in front of the centre of the disk of the Sun as seen from the centre of the Earth.

To calculate the duration of a central transit, it is necessary to calculate the difference, v, between the angular speeds of the transiting planet and the Earth. If the distance of the transiting planet from the Sun is R_p, then the ratio of the distances planet–Sun and planet–Earth is given by

$$r = R_p / (1 - R_p) \qquad [1]$$

when the Earth is at its mean distance of 1 AU from the Sun.

During a transit, the angular speed of the planet as seen from the 'fixed' Earth is $r \times v$. If the angular diameter of the Sun at the time of the transit is known, the duration of the transit can be found.

For example, the mean motion of the Earth is 0.985609°/day and for Venus the mean motion is 1.60214°/day.[3] The difference is 0.616531°/day. Using mean distances for the planets from the Sun gives r = 2.61444. The apparent angular speed of Venus, $r \times v$, is therefore 1.611883°/day. At mean distance, the Sun has an angular diameter of 1919".26, so a central transit of Venus will last for

$$\begin{aligned} & 1919.26 / (3600 \times 1.611883) \text{ days} \\ = & \; 0.33075 \text{ days} \\ = & \; 7\text{h}\,56\text{m} \end{aligned}$$

Performing the calculation for Mercury results in a central transit duration of 6h 31m.

The above calculations have used mean distances of the planets from the Sun and mean orbital speeds. Actual transits do not occur with the planets at these distances. However, the method outlined above can be applied to actual transits. It is necessary to know the size of the Sun, and the distance of the planet from the Sun at the time of the transit, so that the angular diameter can be calculated. The orbital speed of the planet can be calculated from the basic physics of elliptical orbits.

The longest and shortest transits will occur when the planets are at perihelion or aphelion. For a transit to be observed, the inner planet has to be at, or very close to, one of the nodes of its orbit at the time of inferior conjunction. The longest and shortest situations considered here could not occur in practice since the coincidence of Mercury's or Venus's longitudes of perihelion with Earth's longitude of perihelion and with the lines of the nodes is not possible. However, these situations were calculated, since they are extremes and two of the three conditions can be met. That is, the circumstances required can almost occur.

Four situations were considered for calculation for transits of Mercury and Venus: Earth at perihelion with the transiting planet at perihelion and aphelion at the time of a transit, and Earth at aphelion with the transiting planet at perihelion and aphelion. Details concerning Mercury, Venus and Earth were calculated using the VSOP87 planetary theory which yields very accurate results for at least 4000 years before and after 2000 AD. The results of the calculations for Venus are shown in Table 1.

It may be seen that the shortest central transit times occur with the Earth at aphelion and Venus at perihelion during the transit. This should not be surprising, since the apparent diameter of the Sun is smallest when the Earth is at aphelion and Venus is moving fastest when it is at perihelion.

The longest arc lengths required for traversal by Venus to complete a transit occur when Venus is at perihelion and the Earth is at aphelion. However, this does not give the longest transits, because Venus is moving at its fastest

Table 1. Theoretical central transit times for Venus

Situation of planets	*Year*	*Central transit duration*
Earth perihelion, Venus perihelion	−2000	7h 54m
	+2000	7h 55m
	+6000	7h 56m
Earth perihelion, Venus aphelion	−2000	8h 09m
	+2000	8h 07m
	+6000	8h 05m
Earth aphelion, Venus perihelion	−2000	7h 45m
	+2000	7h 47m
	+6000	7h 48m
Earth aphelion, Venus aphelion	−2000	7h 57m
	+2000	7h 56m
	+6000	7h 56m

Table 2. Theoretical central transit times for Mercury

Situation of planets	*Year*	*Central transit duration*
Earth perihelion, Mercury perihelion	−2000	5h 22m
	+2000	5h 22m
	+6000	5h 21m
Earth perihelion, Mercury aphelion	−2000	8h 17m
	+2000	8h 17m
	+6000	8h 18m
Earth aphelion, Mercury perihelion	−2000	5h 23m
	+2000	5h 22m
	+6000	5h 22m
Earth aphelion, Mercury aphelion	−2000	8h 13m
	+2000	8h 13m
	+6000	8h 14m

and covers the distance more quickly than when it needs to traverse a shorter arc when it is near to aphelion.

The longest transits occur with the Earth at perihelion and Venus at aphelion during the transit. It can be seen from Table 1 that the durations of such transits appear to be decreasing by about 2 minutes every 4000 years.

The results for Mercury are shown in Table 2. As for Venus transits, the shortest central transit times occur when Mercury is at perihelion during the transit. The longest transits again occur with the Earth at perihelion and Mercury at aphelion. The reasoning is exactly the same as given for Venus. The differences are much larger for Mercury due to the higher eccentricity of the planet's orbit.

Orbital inclinations

The calculations above assume that the orbits of Mercury, Venus and Earth are coplanar,which is not in fact the case. The orbital inclination of Mercury is currently 7.0° and varies by less than 0.2° from this value during the period 4000 BC to 8000 AD. For Venus, the orbital inclination is currently about 3.4° and varies by less than 0.1° from this value during the period indicated.

The effect of the inclination is to cause the path of a transiting planet to be at an angle to the ecliptic. Since, at the time of a transit, both planets are moving roughly parallel to each other, the inclination of the transit path with respect to the ecliptic is larger than the orbital inclination of the planet concerned. For Venus, the inclination of a transit path to the ecliptic is about 8.8° while for Mercury it is about 8.2° for a perihelic transit and 10.8° for an aphelic transit.

The effect on the calculations of transit duration is that instead of using an arithmetical difference between the angular speeds of the planets, it is necessary to use the law of cosines so as to allow for the inclinations. When this is done, it is found that the error introduced in assuming coplanar orbits is less than 1%.

Actual transits

Let us now consider some actual transits of Mercury and Venus. Transits are rarely central. The Venus transit of 424 Nov 22 was nearly so, having a minimum separation of only 9.6".[1] However, it is possible to calculate what the central transit time would have been for any given transit if the minimum separation distance is known.

If the minimum separation is S, the angular radius of the Sun is R (both S and R must be measured in the same units) and the actual transit time is t_S, then by simple geometry, the central transit time is given by

$$t_{central} = t_S / \cos [\sin^{-1}(S/R)] \quad [2]$$

Since equation 2 and the other calculations refer to the centre of the disk of the transiting planet completing the transit, some calculations had to be made using the pub-

Table 3. Calculated central transit durations of actual Venus transits

Ascending node		*Descending node*	
Transit date	*Central transit duration*	*Transit date*	*Central transit duration*
−1998 Nov 18	7h 51m	−1641 May 20	7h 53m
−1755 Nov 18	7h 51m	−1398 May 20	7h 53m
−548 Nov 21	7h 52m	−1151 May 21	7h 53m
−305 Nov 22	7h 53m	−912 May 21	7h 53m
−62 Nov 22	7h 53m	−669 May 22	7h 52m
+181 Nov 22	7h 53m	−426 May 22	7h 53m
+424 Nov 22	7h 53m	−183 May 22	7h 52m
+667 Nov 23	7h 53m	+60 May 22	7h 52m
+910 Nov 23	7h 53m	+303 May 24	7h 52m
+1153 Nov 23	7h 53m	+546 May 24	7h 52m
+1396 Nov 23	7h 54m	+789 May 24	7h 52m
+2846 Dec 16	7h 54m	+1032 May 24	7h 52m
+3089 Dec 18	7h 53m	+1275 May 25	7h 52m
+3332 Dec 20	7h 54m	+2255 Jun 09	7h 51m
+3575 Dec 23	7h 54m	+2498 Jun 10	7h 51m
+3818 Dec 25	7h 53m	+2741 Jun 13	7h 51m
		+2984 Jun 14	7h 51m
		+3227 Jun 17	7h 51m
		+3470 Jun 19	7h 52m
		+3713 Jun 21	7h 52m
		+3956 Jun 23	7h 52m

lished values of transit times. The time for the transit of the centre of the disk was taken as the time from the midpoint of ingress to the time of the midpoint of egress. From published contact times,[1] the required times could be calculated.

To avoid scaling any errors in the tabulated values by large factors, only transits with a minimum separation of less than 500" were considered. The values obtained for Venus transits from 2000 BC to 4000 AD are shown in Table 3. The values for Mercury transits from 1600 AD to 2300 AD are shown in Table 4.

It can be seen that the times in Table 3 are in reasonable agreement with the times in Table 1. The Table 1 times are completely theoretical for situations which do not obtain for the actual orbits, although they are not far off. The times in Table 3 are accurate to about ±2 minutes.

Venus transits which occur at the ascending node are closest to the situation in Table 1 where both planets are at perihelion. The expected transit times are about 7h 55m. The actual calculated central transit times are indeed very close to this value. For transits at the descending node, the situation is closest to the Table 1 situation where both planets are at aphelion. The duration expected from Table 1 is about 7h 56m. The actual values are about four minutes shorter than this. This discrepancy is due to the fact that the planets are not actually both at aphelion during a transit in May or June. Currently, the longitude of perihelion of the Earth is about 25° from the line of the nodes whilst the longitude of perihelion of Venus is 54° from the line of the nodes. As a result, the planets are moving faster than they would be at their aphelion positions and so the actual transits are a few minutes shorter than the Table 1 times.

The times in Table 4 are in rough agreement with the theoretical times in Table 2. For Mercury transits at the ascending node, both Earth and Mercury are not very far from perihelion. The actual central transit times of about 5h 30m are only a few minutes longer than the shortest possible transits of 5h 22m for the current epoch. Conversely, the descending node transits are about 20 minutes shorter than the longest possible. The differences are due to the fact that the longitudes of perihelion for the planets and the line of the nodes do not all align.

All of the timings presented have been for a geocentric observer. It is interesting to note that for an observer on the Earth's surface, the duration is not the same as for a geocentric observer. The rotation of the Earth normally results in an observer seeing a shorter transit than would be observed from the centre of the Earth, but for some transits in polar regions the transit may actually last longer. For example, the Venus transit of 424 Nov 22, which was nearly central, had a geocentric duration of 7h 52m 36s, but for location 20° E, 70° S the duration was 7h 56m 11s (Meeus J., private communication).

Table 4. Calculated central transit durations of actual Mercury transits

Ascending node		Descending node	
Transit date	Central transit duration	Transit date	Central transit duration
1618 Nov 04	5h 29m	1615 May 03	7h 53m
1631 Nov 07	5h 27m	1661 May 03	7h 53m
1664 Nov 04	5h 28m	1707 May 05	7h 54m
1677 Nov 07	5h 27m	1753 May 06	7h 55m
1710 Nov 06	5h 28m	1799 May 07	7h 55m
1723 Nov 09	5h 28m	1832 May 05	7h 53m
1756 Nov 07	5h 27m	1878 May 06	7h 53m
1769 Nov 09	5h 29m	1924 May 07	7h 52m
1789 Nov 05	5h 31m	1970 May 09	7h 54m
1802 Nov 09	5h 27m	2016 May 09	7h 54m
1835 Nov 07	5h 29m	2095 May 08	7h 52m
1848 Nov 09	5h 27m	2141 May 09	7h 52m
1881 Nov 07	5h 29m	2187 May 11	7h 53m
1894 Nov 10	5h 27m	2233 May 12	7h 53m
1927 Nov 10	5h 28m	2279 May 13	7h 54m
1940 Nov 11	5h 28m		
1973 Nov 10	5h 28m		
1986 Nov 13	5h 29m		
2006 Nov 08	5h 31m		
2019 Nov 11	5h 28m		
2052 Nov 08	5h 30m		
2065 Nov 11	5h 28m		
2098 Nov 10	5h 29m		
2111 Nov 13	5h 28m		
2144 Nov 11	5h 28m		
2157 Nov 14	5h 29m		
2190 Nov 12	5h 28m		
2203 Nov 16	5h 30m		
2223 Nov 12	5h 31m		
2236 Nov 13	5h 28m		
2269 Nov 12	5h 30m		

Review of orbit details

Details concerning the actual orbits of Mercury, Venus and the Earth were calculated using the expressions given in Meeus.[4] These expressions are reasonably accurate for time periods from about 4000 BC to 8000 AD and certainly adequate for the discussion here where high accuracy is not required.

At around 1200 BC, the ascending node of the orbit of Venus coincided with the longitude of perihelion of the Earth. This meant that Venus transits occurring around this time had Earth at perihelion or aphelion. However, the longitude of the perihelion of Venus was about 38º away from this direction, so the comparison with Table 1 is not easy. The transits around this time happened not to be very central, so there are not many transits to match with in Table 3.

Not until about 8500 AD will the longitude of perihelion of Venus and the longitude of perihelion of Earth coincide. However, the line of the nodes will then be 80º from this direction.

The orbits of the Earth and of Venus are becoming more circular for the next several thousand years.[2] The Earth's orbital eccentricity will reach a minimum in about 29,500 AD whilst the orbit of Venus continues to become more circular until about 14,500 AD. As a result, it would be expected that the durations of central transits of Venus will tend towards the circular orbit approximation value of 7h 56m and be roughly independent of the precise timing of the transits. This trend is quite apparent from Table 1. Transits with Earth at perihelion and Venus at aphelion are decreasing in duration at a rate of about 2 minutes every 4000 years. Transits with Earth at aphelion and Venus at perihelion are increasing in duration at a rate of 1m 30s every 4000 years. The other situations listed in Table 1 have roughly constant times of 7h 56m. Central transits of Venus will be roughly 2 or 3 minutes longer in a few thousand years than they are today and the duration will be independent of the date of the transit.

For Mercury, the longitude of perihelion and the longitude of perihelion of Earth gradually separate from 4000 BC to 8000 AD. The longitude of the nodes also never coincides with either of the perihelion positions during this time interval. The coincidence of these directions was closest around 4000 BC and has diverged since then. It would therefore be expected that central transits of Mercury at the ascending node around 4000 BC would have been about 8 minutes shorter than those occurring currently. Central transits at the descending node would have been about 20 minutes longer than the current durations.

The orbit of Mercury, already the most eccentric of the planets apart from Pluto, will become gradually more eccentric. From Table 2 it can be seen that central transit durations with both planets at perihelion are slightly decreasing whilst the durations with both planets at aphelion are slightly increasing. The central transit durations will not converge on the circular orbit approximation value, at least for the next several thousand years.

Conclusions

Details concerning the theoretical aspects of transits of Mercury and Venus have been presented and these have been compared with values for actual transits. It seems that transits are slightly longer when the Earth is at perihelion and the transiting planets are at aphelion positions. This agrees with what would be expected by intuition. A review of the orbital details of the planets has indicated that there should be a slow trend towards slightly longer Venus central transit times during the next 10,000 years. Timings for Mercury transits will continue to diverge from the circular orbit value with large differences in duration between transits occurring at the ascending and descending nodes.

Address: 18 Cumberland Avenue, Chandlers Ford, Eastleigh, Hants. SO53 2JX. [darren_beard@uk.ibm.com]

References

1 Meeus J., *Transits*, Willmann–Bell Inc., 1989
2 Meeus J., *More Mathematical Astronomy Morsels*, Willmann–Bell Inc., 2002
3 Lang K. R., *Astrophysical Data: Planets and Stars*, Springer–Verlag, New York, 1992
4 Meeus J., *Astronomical Algorithms*, Willmann–Bell Inc., 1991

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