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The evolution of the theoretical bolometric albedo in close binary systems

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ABSTRACT

Until now the bolometric albedo in close binary systems was computed only for a few, and sometimes unrealistic cases. In this paper we present, for the first time, the evolution of the bolometric albedo as a function of the external flux, of the geometry and of the evolutionary status of an irradiated star. A new numerical method, based on the isoentropy at the bottom of the perturbed and the non-irradiated hemispheres, is introduced. A variant of this numerical method was already used by us to predict gravity darkening exponents and it is appropriate to radiative and convective envelopes. The procedure can also combine consistently the coupling interior–envelope–atmospheres models. The theoretical predictions are compared with the inferred values of the bolometric albedo available in the literature when possible. A theoretical transition zone is predicted for log $T_{\mathrm{eff}} \approx 3.8$ which coincides approximately with that for gravity darkening exponents. This value cannot be confirmed by the observations as a result of the high scattering of the empirical points. More extensive and careful analysis of light curves of eclipsing binaries is needed in order to clarify the scenario.

Key words: stars: atmospheres – binaries: close – binaries: general – stars: evolution – stars: fundamental parameters – stars: interiors.

1 INTRODUCTION

Unfortunately some aspects of the research of close binary systems did not advance with a similar rate to other fields of stellar physics. For example, up until very recently the light curves adopted only two values for the gravity darkening exponent β_1: 1.0 for radiative envelopes and 0.32 for late-type stars according to von Zeipel (1924) and Lucy (1967), respectively. This situation has changed somewhat after the papers by Claret (1998, 2000a) who was able to compute β_1 for any point of the Hertzsprung–Russell (HR) diagram. It was shown that this exponent is a function of the stellar mass, age, radius, metallicity, the adopted theory of convection, etc., that is, β_1 can be considered as a new evolutionary parameter. Indeed, the new tables of stellar evolutionary models incorporate for each track the values of the gravity darkening exponents (see tables 1–24 in Claret 1998). Recently Niarchos (2000) shown that these theoretical exponents are in satisfactory agreement with the observational data.

The situation is a little less problematic in the case of the limb-darkening coefficients since now that it is relatively easy to implement the new advances of stellar atmosphere modelling in the light-curve codes. Besides the intrinsic improvements incorporated in the stellar atmosphere models, new laws that describe better the specific intensity distributions were also introduced (Klinglesmith & Sobieski 1970; Manduca, Bell & Gustafsson 1977; Díaz-Cordovés 1990). As an additional step forward, actual intensities for several passbands are also now directly tabulated – instead of using a fitting – as a function of the emergence angle for each effective temperature, local gravity, metallicity, microturbulent velocity, etc. (Claret 2000b).

The so-called reflection effect presents some similarities with the case of the gravity darkening and perhaps the most important of them is that the users of light-curve codes are also restricted to only two values of the bolometric albedo A: 1.0 for atmospheres in radiative equilibrium and 0.5 for stars with convective envelopes. The first value was obtained by Eddington (1926) and Milne (1926). The paper by Milne was particularly important at that time because he derived a source function for an irradiated atmosphere. About 40 years later Sobieski (1965a,b) derived a more accurate source function for atmospheres perturbed by external fluxes. On the other hand, a fact of capital importance was pointed out by Hosokawa (1959) who established empirically that the bolometric albedo for late-type stars was different from 1.0, that is, when these stars are irradiated they do not re-emit simultaneously all the incident flux.

The first attempt to derive theoretically the bolometric albedo for convective envelopes came from a suggestion made by Lucy (1968). In words of Lucy ‘As in the gravity-darkening calculation (Lucy 1967), we must impose the condition that inward integrations of the equations governing the structure of the

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atmosphere all finish on the same adiabat.' Ruciński (1969) following such a suggestion determined that the mean value of the bolometric albedo is between 0.4 and 0.5; in any case different from that obtained by Eddington and Milne for radiative atmospheres. Since then it is usual to adopt 1.0 or 0.5 for the bolometric albedo, if the envelope of the star is in radiative or convective equilibrium with some few exceptions as in the case of the paper by Rafert & Twigg (1980). Of course, such a crude procedure may introduce many uncertainties in the analysis of light curves. This problem will surely be enlarged when more accurate light curves of eclipsing binaries are obtained systematically from space observations.

Given that no substantial improvements were introduced in this field in recent years, it is worthy to be studied by using updated physics as well as new numerical procedures. In order to try to improve this scenario we present in this paper a new method to compute the bolometric albedo based on the similar numerical procedure we have introduced to derive the gravity darkening exponent. As we shall see, the method is very versatile given that it makes possible to compute A whatever the mechanism, convective or radiative, that dominates in the stellar envelope. The resulting albedo not only will be a function of the geometry and of the incident flux but also of the evolutionary status of the stars. In the next sections we expound with more detail the numerical method and the influence of the input physics. A comparison with observational values, when possible, is also presented.

2 THE NUMERICAL METHOD

As previously mentioned, some attempts to describe the run of temperature in an irradiated atmosphere were performed by Milne (1926) and Sobieski (1965a) who found that the relationship between original and perturbed temperatures is given by

$$T^4(\nu,\tau) = T_{\rm o}^4(\tau) + \frac{\pi}{\sigma}F^*B(\nu,\tau), \tag{1}$$

where

$$B_{\rm Milne}(\nu,\tau) = \left[\left(\nu+\frac{1}{2}\right)\nu - \left(\nu^2-\frac{1}{4}\right){\rm e}^{-\frac{\tau}{\nu}}\right], \tag{2}$$

$$B_{\rm Sobieski}(\nu,\tau) = \left[\left(\frac{3}{4}\nu+\frac{1}{2}\right)\nu - \left(\frac{3}{4}\nu^2-\frac{1}{4}\right){\rm e}^{-\frac{\tau}{\nu}}\right], \tag{3}$$

where σ is the Stefan–Boltzmann constant, τ is the optical depth, ν is the cosine of the angle of incidence, $T(\nu,\tau)$ is the shell temperature considering the external flux, $T_{\rm o}(\tau)$ is the shell temperature in the absence of the external flux and F^* is the actual external flux. The exact solution for the function $B(\nu,\tau)$ was obtained by Chandrasekhar (1945) and more recently by Nariai & Murata (1987). We have used the exact solution for the grey case in the present approach. The functions $B(\nu,\tau)$ were computed for diverse values of the optical depth and angle of irradiation; the final results being stored in order to be used directly in the envelope–atmosphere calculations of the stellar evolution code. Fig. 1 shows a comparison between the solutions obtained by Milne (1926), Sobieski (1965a) and the exact solution. The three solutions are similar for grazing angles of incidence for any value of τ. As ν increases, the differences between Milne–Sobieski and the exact solution increase mainly in the interval $-2 \leq \log\tau \leq 1$. At the bottom, and for perpendicular irradiation, the Milne solution is around 20 per cent larger than the exact calculation. A more precise

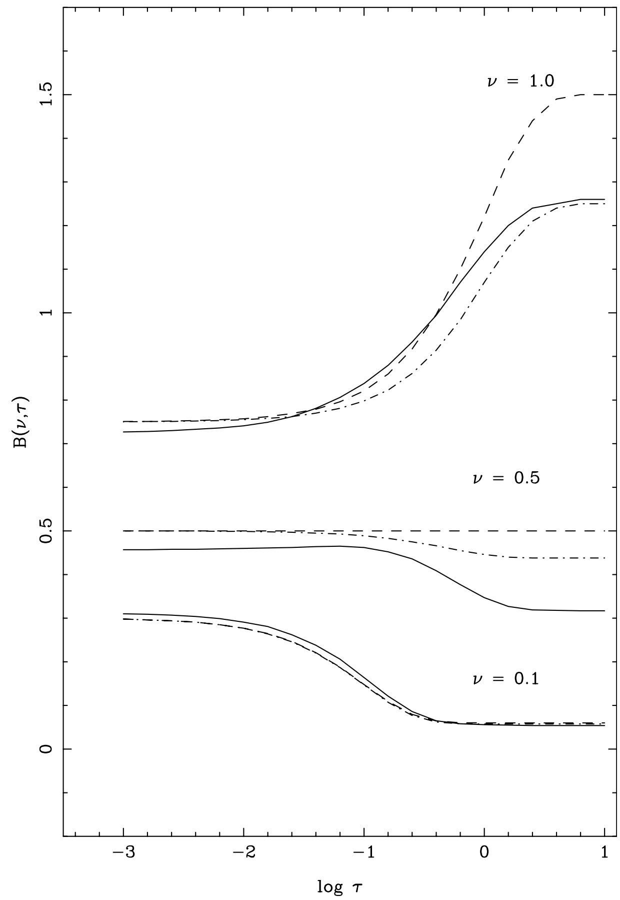

Figure 1. Solutions $B(\nu,\tau)$ for irradiated atmospheres: continuous line indicates the exact solution for the grey case, dashed represents the Milne solution while dotted–dashed one denotes the solution obtained by Sobieski.

source function can be derived from more realistic calculations like those derived from non-grey irradiated atmospheres. We are able not only to compute $B(\nu,\tau)$ for grey but also for the non-grey atmospheres using a modified version of the UMA code (Claret & Giménez 1992). We return to this point later.

The bolometric albedo is connected with the intrinsic and incident flux by

$$AF_r^* = \sigma(T_{\rm h}^4 - T_{\rm eff}^4), \tag{4}$$

$$(1-A)F_r^* = \sigma(T_{\rm eff}^4 - T_{\rm m}^4), \tag{5}$$

where $T_{\rm eff}$ is the effective temperature without external flux, $T_{\rm h}$ is the effective temperature of the star after irradiation, $T_{\rm m}$ is the effective temperature of a model which if irradiated by F_r^* would present the same entropy at the bottom of the envelope. The external radial flux is given by $F_r^* = \sigma T^{*4}r^2\nu$, where r is the apparent radius, T^* is the effective temperature of the irradiating star and the other symbols have their usual meanings.

As boundary condition we impose that both hemispheres of an irradiated star should present the same entropy deep inside the envelope. In order to simulate this configuration we shall use a numerical procedure similar to that developed to compute the gravity darkening exponent combining interior and envelope–atmosphere models (Claret 1998, 2000a). We first compute the envelope for a given point of the evolutionary track characterized by ($T_{\rm eff}$, $\log g$, X, Z) without perturbation. Two trial models (with arbitrary albedos) which take into account the contribution of the external flux through $B(\nu, \tau)$ are generated. Once these three models are computed an iterative procedure is activated in order to derive the irradiated model that presents the same entropy at the bottom of the envelope as the original model without irradiation. The process is repeated until the trial model and the definitive one differ only by a few degrees. These steps are repeated for each point of an evolutionary track.

A model with $2\,M_\odot$, solar chemical composition, $l/H_p = 1.52$ and $\alpha_{ov} = 0.2$ was selected as our reference. We select these physical ingredients because they reproduce, on average, the best observational data for double-lined eclipsing binaries. Moreover, such a model is very useful because during its evolution it crosses over the boundary between radiative/convective envelopes. In Fig. 2 we show the bolometric albedo as a function of the effective temperature of such a model during its evolution. We have adopted a relative flux $f^* = T^{*4} r^2 / T_{eff}^4 = 0.5$ and consider perpendicular direction of incidence. An HR diagram was also plotted for comparison. Instead of the two old values (1.0 or 0.5) now we have continuous values of A along the evolutionary track. For the higher effective temperatures we confirm the Eddington/Milne prediction, that is, $A \approx 1$. Note that we do not impose $l/H_p = 0$ to simulate radiative envelopes since this procedure is artificial although some authors use it to simulate radiative atmospheres even in models with very low effective temperatures.

The situation is a little more complicated for convective envelopes and fixed geometrical conditions. The albedo depends

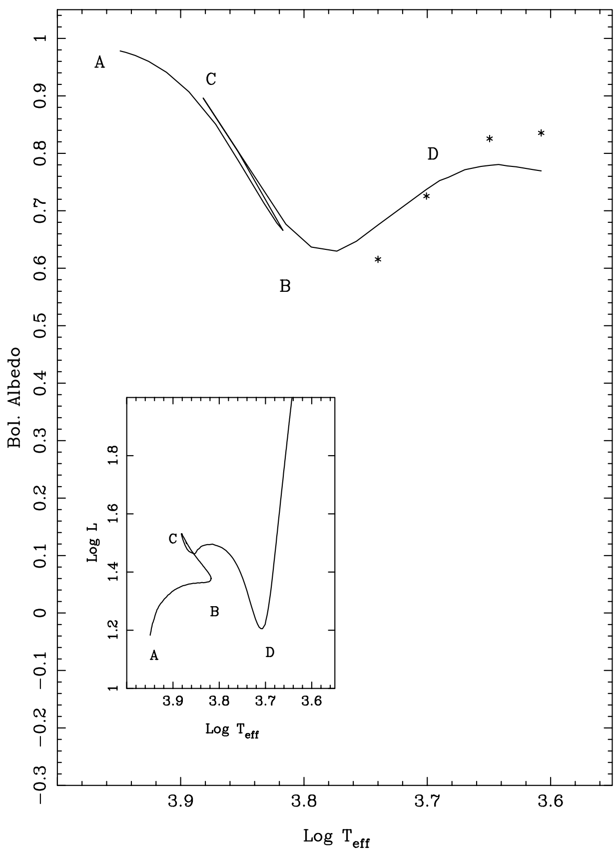

Figure 2. The evolution of the bolometric albedo for a $2\,M_\odot$ model. In the lower left corner the corresponding HR diagram is shown for orientation. Conditions of irradiation: $\nu = 1.0, f^* = 0.5$. See text for the meaning of the asterisks.

on the magnitude of the contribution of the convection, on the external flux, on the metal content and on the extension of the atmosphere. Note, mutatis mutandis, the similarities between the results shown in Fig. 2 with those of gravity darkening exponents. Progressively convection loses relevance, and A also falls progressively up to a transition zone. This transition zone coincides approximately with that of the gravity darkening calculations (see fig. 5 in Claret 1998). We have performed non-grey calculations combining our interior-envelope models with a stellar atmosphere code (Claret & Giménez 1992). The preliminary results, based on similar conditions of irradiation, are shown as asterisks in Fig. 2. The mean differences between the albedos obtained using grey and non-grey calculations are not large; we think that they are perfectly adequate for our present purposes. More elaborated non-grey calculations, combined with interior models, will be subject of a future work.

3 CHANGING THE INPUT PHYSICS

It should be recalled that the results presented in Fig. 2 are based on the hypothesis that the relative flux is constant during the evolution of the $2\,M_\odot$ model. In real binary stars the astrophysical parameters of both components as well as the orbital parameters change with time and that hypothesis may not be necessarily correct. Another limitation: if we assume synchronization then it is correct to talk about irradiated and non-irradiated hemispheres. However, this is not always true in eclipsing binaries. Even if the hypothesis of synchronism is correct there may appear currents which may penetrate into the non-directly irradiated hemisphere [Kirbiyik & Smith (1976) and Kirbiyik (1982)]. The eccentricity of the orbit or other sources of variability of the external flux, may also play an important role in the irradiation process. We expect to investigate the influence of these limitations on our global results in the near future.

The set of equations presented above gives rise to interesting questions. For example, as the combination of the configurations of irradiation is very large a point is: what set [T^*, r, ν] should one use to characterize a mean value of the albedo? Does such a definition make sense? In order to investigate how the conditions of irradiation affect the albedo we have generated irradiated envelopes for different incidence angles for $f^* = 0.5$. Fig. 3 shows the behaviour of the bolometric albedo as a function of ν. On the other hand, keeping the direction of irradiation constant, we have computed models for different values of f^* and the results can be seen in Fig. 4 for zero-age main-sequence (ZAMS) models. For fixed values of T_{eff} and log g, the bolometric albedo increases with the magnitude of the external flux: in some drastic cases the structure of the convective envelope changes in such a way that A tends asymptotically to 1.0, that is, it behaves as a radiative envelope. Fig. 5 illustrates, as an example of the change of conditions in irradiated envelopes, the run of the convective flux as a function of pressure: the continuous line indicates a standard model with $T_{eff} = 5530\,K$, $\log g = 4.54$ while the dashed ($F_r^* = 1.0$) and dotted ($F_r^* = 2.0$) lines indicate the effect of the irradiation on the convective flux in such an envelope. As the external flux increases and in order to keep the entropy constancy at the bottom, the envelope becomes more and more 'radiative,' i.e., the bolometric albedo tends to 1.0.

We have also checked the influence of the mixing-length, the hydrogen/metal content and the atmosphere extension on the bolometric albedo. Often, the importance of these parameters was evaluated by using isolated situations, many of them unrealistic. In

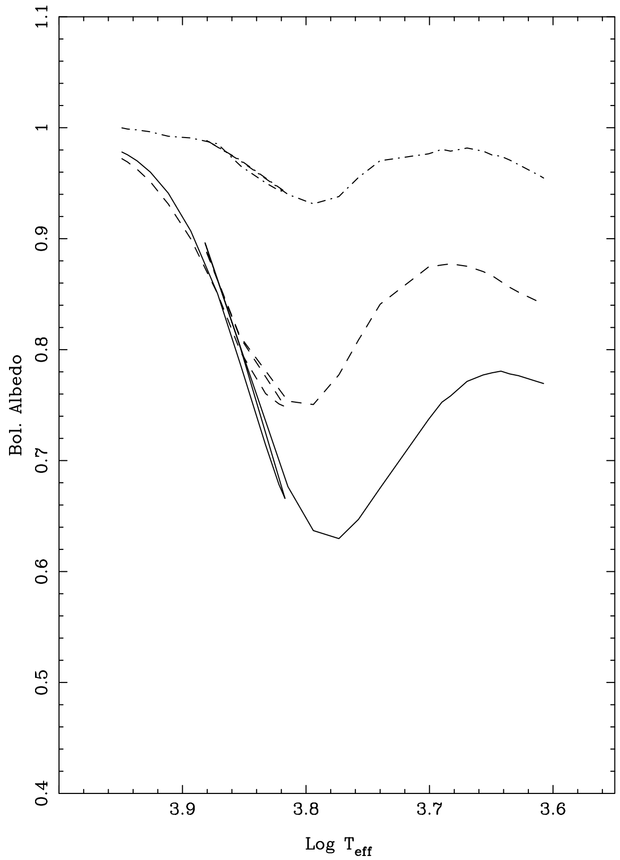

Figure 3. The effect of the angle of incidence on the evolution of the bolometric albedo for the 2 $M_\odot$ model. Conditions of irradiation: $\nu = 1.0$ (continuous line), $\nu = 0.50$ (dashed line), $\nu = 0.1$ (dotted–dashed line). All calculations were carried out for $f^* = 0.5$.

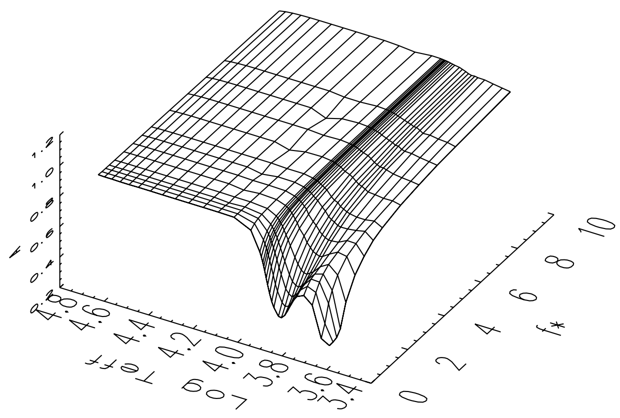

Figure 4. The influence of the external flux on the evolution of the bolometric albedo for ZAMS models. All calculations were carried out for $\nu = 1.0$.

order to try to make the direct comparison with observations easier, we have consistently adopted the properties of stellar models. In general, the bolometric albedo decreases as the mixing-length parameter increase due to the increase of the 'efficiency' of the convection. However, the differences in A due to the change of mixing-length are not very important, mainly if we consider the range of lH_p adopted in the current stellar models. On the other hand, the local gravity mimics the net effect of the increase of lH_p on the amount of convection: an increase of $\log g$ tends to increase

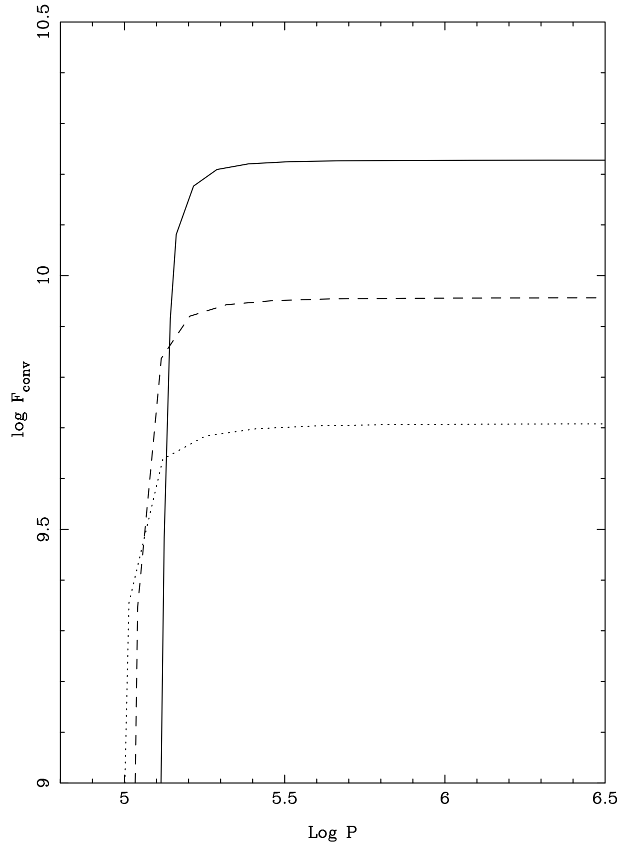

Figure 5. The run of the convective flux with the pressure for a standard (continuous line, $T_{\rm eff} = 5530\,\rm K$, $\log g = 4.54$) and irradiated models (dashed, $F_r^* = 1.0$ and dotted, $F_r^* = 2.0$)

the albedo (Claret & Giménez 1992). We show in Fig. 6 the behaviour of the bolometric albedo as a function of the effective temperature and of the hydrogen content for fixed values of the mixing-length parameter and metallicity. As known, the stellar radius and the effective temperatures increase with the mean molecular weight. Therefore an increase of the hydrogen content (lighter μ) reduces the effective temperature and luminosity. In this way (0.70, 0.2) models are hotter and brighter than the (0.80, 0.02) ones. The bolometric albedo follows a similar pattern, i.e., the hotter models present larger A than those with high hydrogen content. Note, in particular, that the bolometric albedo for the hottest model (0.80, 0.02) is smaller than 0.9. However, the mean differences among the three models are not so big.

4 CONFRONTATION WITH OBSERVATIONAL DATA AND FINAL REMARKS

We do not consider the check of the theoretical albedo against a *single* observational value as a reliable test, specially when A is taken as an input in the light-curve analysis. A more extensive comparison between observation and theoretical predictions concerning bolometric albedo is needed if one really wants to check the capability of the current theory of mutual irradiation. However, it is not a simple task to determine empirically the gravity darkening exponent and bolometric albedo from the analysis of light curves of close binaries systems. Often these are input parameters and they are keep fixed during the iterations. In spite of the intrinsic difficulties to treat such parameters, a

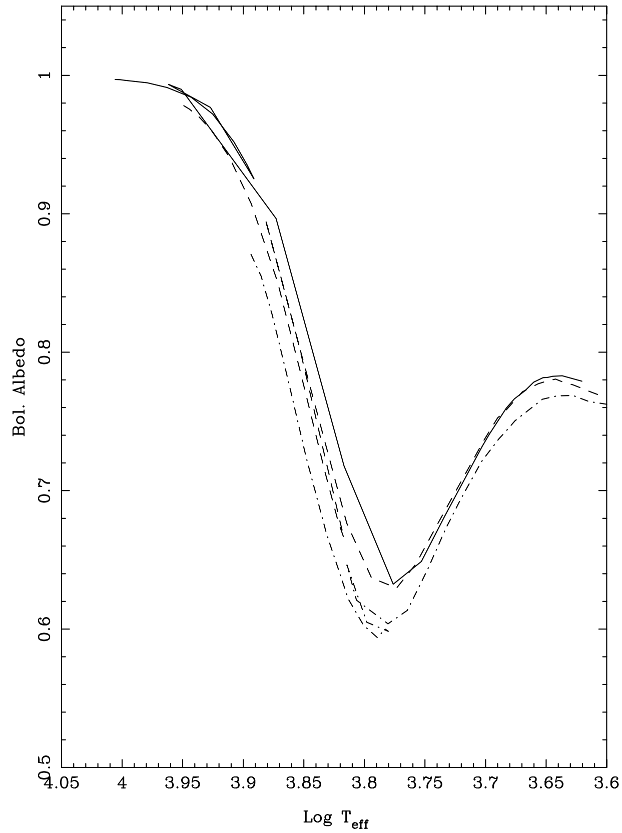

Figure 6. The hydrogen content and the bolometric albedo. The continuous line denotes $(X, Z) = (0.60, 0.02)$, the dashed line indicates $(X, Z) = (0.70, 0.02)$ and the dashed–dotted one represents $(X, Z) = (0.80, 0.02)$. All calculations were performed for $\nu = 1.0, f^* = 0.5$.

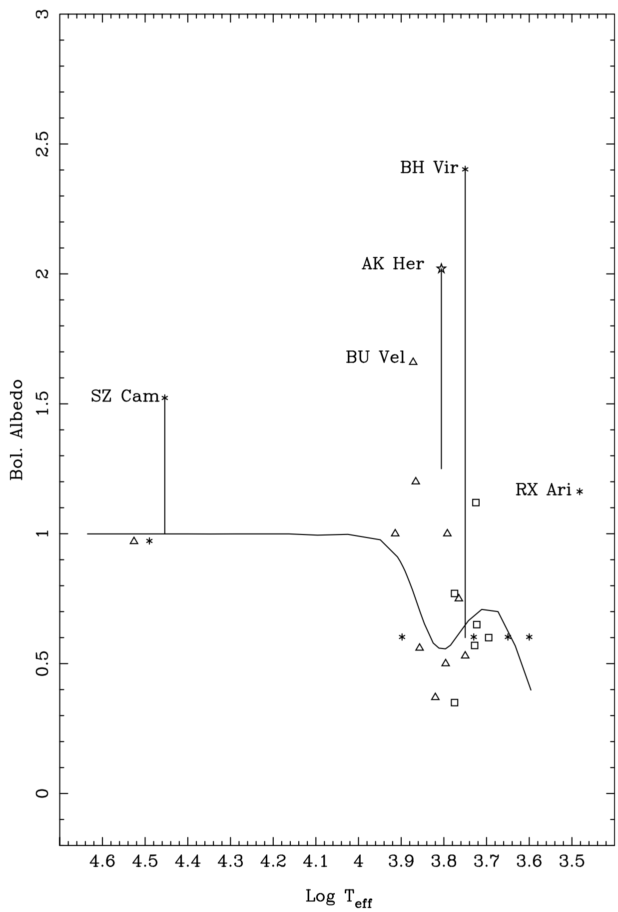

Figure 7. Comparison with the empirical bolometric albedos from Rafert & Twigg (1980). The asterisks represent detached systems, open squares the semi-detached, open triangles represent contact systems while stars indicate others. The theoretical calculations were performed for ZAMS models.

considerable effort was performed by Rafert & Twigg (1980) who used 31 systems to infer the bolometric albedo and gravity darkening exponent. They used the Wilson–Devinney code and they also introduced a scheme to get impersonal solutions.

The sample they considered is representative in the sense that the mentioned authors included detached, semi-detached and contact systems. Concerning effective temperatures there is a gap between 10 000 K and 28 000 K (Fig. 7). In order to compare such data with our theoretical predictions, we have performed calculations for ZAMS models ranging from 0.8 up to 40 $M_{\odot}$. It is noticeable the behaviour of the observational values of A for effective temperatures of the order or smaller than 6300 K. Though the values of A for some late-type components are found to be in good agreement with our theoretical predictions, near the theoretical transition zone we also found that observed albedos are clearly too high for their respective effective temperatures, some of them larger than 1. The scattering of the empirical points does not permit us even to distinguish clearly a transition zone.

The explanation of such behaviour is not simple. Several factors may influence the empirical determination of A. As commented, the most important is the intrinsic difficulty to extract information on A from a given light curve. Additional difficulties may come from the complicated physics of the systems themselves. For example, let us examine the case of AK Her. The primary has $\log T_{\rm eff} = 3.806$ (Woodward & Wilson 1977) and A is 2.58 or 2.02 depending on the adopted solution. The respective values of β_1 are 0.48 or 0.81. Several effects which would alter the energy balance of both envelopes may be present. Nagy (1985) modelled the light curves in the R and I bands. His table VI summarizes the discrepancies found for the bolometric albedo by different authors. Other complications were also detected in this system like period variation as quoted by Rovithis-Livaniou et al. (1999).

The case of BH Vir is similar to that of AK Her concerning the complications in their light curves. The system is intrinsecally variable (RSCVn type) and the depth of the primary eclipse changes by around a tenth of a magnitude (Koch 1967). Recent *uvby* H_{β} observations were reported by Clement et al. (1997a). They found a distortion in the light curve and an iterative procedure was introduced in the analysis of the light curve in order to get relevant information between eclipses after the rectification (Clement et al. 1997b). Combining their photometric ‘clean’ light curves with the spectroscopic data from Popper (1997) they finally obtain the absolute dimensions for this system with a good accuracy. Unfortunately, no details on the treatment of the mutual irradiation is given since it was calculated internally. A year ago Xiang, Deng & Liu (2000) reanalysed the light curves but they used the common procedure of keeping constant the bolometric albedo and gravity darkening exponent in such a way that no relevant information on A can be derived.

More recent observational data and light curves analysis than those used in the work by Rafert & Twigg are available for SZ Cam (Lorenz, Mayer & Dreschsel 1998). This system also shows additional complications since it probably presents third light (around 25 per cent of the total light). In addition there are not sufficient observational points in the light curve at the phases around 0.2. The parameters more interesting to us, the bolometric albedo and the gravity darkening exponents of both components, are fixed at their classical values. In spite of this, the synthetic light curve with $A = 1.0$ for both components shows acceptable deviations with respect to the observational points. Almost simultaneously Harries, Hilditch & Hill (1998) reported spectroscopic observations for SZ Cam. They concluded that the third body emits about 40 per cent of the total light of the system. These authors were able to fit the b and y light curves making use of normal albedo values.

Following Rafert & Twigg (1980) RX Ari shows unequal effective temperatures: 6800 K and 3000 K for the primary and secondary respectively. The same authors obtained a bolometric albedo too high for the spectral type of the secondary. The data for this system are conflictive. Two solutions were found by Wilson & Rafert (1980), one of them based on the classical values of the bolometric albedo. An additional difficulty comes from the fact that at that time the models of stellar atmosphere were not good enough to treat effective temperatures in the range where the secondary component of RX Ari seems to be.

The comments on the above stars serve as example of the problems inherent to the determination of the bolometric albedo from the light curves. Even new and more accurate observations do not improve significantly the scenario though modern determinations of A for those systems with extreme values of A (see vertical lines in Fig. 7) seem to be in better agreement with our predictions. The lack of an extensive and updated derivation of the bolometric albedo makes the confrontation between theory and observation incomplete and not definitive. Concerning the gravity darkening exponent Pantazis & Niarchos (1998) and Niarchos (2000) were able to derive them more accurately than before. It would be highly desirable that similar works as those by Rafert & Twigg (1980), Pantazis & Niarchos (1998) and Niarchos (2000) would be performed in order to test critically the theoretical predictions of the bolometric albedo. On the other hand, we plan in a near future to improve the level of the theoretical predictions introducing non-grey atmosphere models combined with interior models.

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