Journal of Astronomical History and heritage 4(1):15-28 2001

# Six stages in the history of the astronomical unit

**David W Hughes**
*Department of Physics and Astronomy,*
*The University of Sheffield*
*Sheffield S3 7RH, UK*
E-mail: d.hughes@sheffield.ac.uk

**Abstract**
Giovanni Antonio Rocca wrote "The problem of solar distance and parallax was one of the most important in astronomy, well worth a lifetimes work by any astronomer." (see Ricciolo, 1651:732). This paper briefly reviews the values obtained for the Earth-Sun distance throughout the history of astronomy, and divides the investigation of the astronomical unit into six stages. It is suggested that a similar six stages can be recognized in the history of many other fundamental parameters in our subject.

**Key words:** *astronomical unit*

## 1 INTRODUCTION

In a historical context, the fundamental parameters of astronomy fall into two categories: there are those quantities that our predecessors measured nearly correctly first time, and those that they did not. Staying firmly in the solar system, two examples of the first type of parameter are the radius of Earth and the distance between Earth and the Moon. Ever since Eratosthenes of Alexandria, around 235 BC, used his famous shadow-stick approach, at noon, near the summer solstice, and his near contemporaries timed the duration of the phases of a lunar eclipse, we (i.e. the enlightened non-flat-earthers) have known that Earth has a radius, $r_e$, of about 6400 km, and that the Moon is about 60 $r_e$ from Earth. As far as these parameters are concerned, all that has happened since the time of the Ancient Greeks is the dotting of a few i's and the crossing of a few t's. I do not belittle this 'dotting and crossing' because, for example, it led French astronomers in the late seventeenth century to the realization that Earth was not strictly spherical, and Edmond Halley, a few years later, to the discovery that the Earth-Moon distance was slowly changing with time, but these topics are not germane to the main thrust of the present paper.

A typical parameter of the second type is the mean distance between Earth and the Sun, the so called astronomical unit. The Greeks got this wrong, and not just a little wrong: their value was too small by a factor of twenty. So the Greeks, knowing the angular diameter of the Sun, thought that the Sun was a mere 5.4 times bigger than Earth, whereas it is actually about 109 times bigger. As there is an inverse cube relationship between the astronomical unit and the mass of the Sun (as calculated by its gravitational influence on Earth), and an inverse relationship between the astronomical unit and the values of stellar parallaxes (these being expected in post-Copernican days), the influence and importance of the astronomical unit in the history of astronomy is clear. This fundamental datum was referred to by Sir George Airy as "... the noblest problem in astronomy." (see Agnes Clerk, 1885:269), and this was earlier underlined by Robert Grant (1852:211) when he wrote "The determination of the distance from the sun to any of the planets revolving around him, is one of the most important problems of astronomical science."

The history of a parameter of the second type can be divided into six stages, these being:

(1) The Age of Ignorance
(2) The First Measurements and their Acceptance
(3) The Realization of Error
(4) The Period of Confusion
(5) The Diminution of the Standard Deviation about a Commonly-accepted Value
(6) The Era of Growing Disinterest.

According to Charles A. Young (1895:375), "The problem of finding the true value of the astronomical unit is difficult, because of the great disproportion between the size of the earth and the distance of the sun. The relative smallness of the earth limits the length of our available "base line," which is less than $^{1}/_{12000}$ part of the distance that is to be determined by it."

Table 1 lists a selection of the measurements made of the astronomical unit throughout history, together with certain values used in key-note texts, the latter being produced by authors reviewing the data that were available at the time. Notice that the solar parallax, $\pi$, is defined as being the angle, in seconds of arc, subtended by the equatorial radius of Earth at the Sun's mean distance. Figure 1 shows the AD 900 to 1980 subset of astronomical unit values, whilst Figure 2 shows the last two centuries in more detail, taken from Table 1.

Let us start by trying to define the onset times and durations of each of the six stages mentioned above. These have clearly been affected by improvements of specific astronomical instruments, the introduction of new techniques, and the vagaries of the rate of advance of astronomical theory. The temporal changes of these factors will be stressed in what follows.

Table 1. Measurements and accepted values of the Earth-Sun distance throughout astronomical history. The first column of values is in units of the Earth's equatorial radius ($r_e$ = 6378.140 km). The second column expresses the distance as a fraction of the value of the astronomical unit accepted at the present time (i.e. 1 au = 149,597,870.61 km = 23454.78 $r_e$). Note that the mean Earth-Sun distance = $r_e \times$ [tan (solar parallax)]$^{-1}$.

| Source | Value $r_e$ (km) | Value au |
|---|---|---|
| Aristarchus of Samos (280 BC) | 1150 | 0.04903 |
| Ptolemy/Hipparchus (135 BC) | 1210 | 0.05159 |
| Al-Battani (AD 900) | 1108 | 0.04724 |
| Tycho Brahe (1570) | 1150 | 0.04903 |
| Kepler (1604) ($\pi < 1'$) | > 3438 | > 0.14657 |
| Wendelin (1626) lunar dichotomy ($\pi < 1'$) | > 3438 | > 0.14657 |
| Wendelin (1630) | 14000 | 0.59689 |
| Wendelin (1635) planetary diameters ($\pi \approx 14''$) | ≈ 14733 | 0.62815 |
| Horrocks (1640) planetary diameters ($\pi = 14''$) | 14733 | 0.62815 |
| Langrenus (1650) | 3420 | 0.14591 |
| Riccioli (1651) lunar dichotomy | 7300 | 0.31124 |
| Huygens (1659) ($\pi = 8''.2$) | 25154 | 1.07246 |
| Hevelius (1662) ($\pi = 40''$) | 5156 | 0.21985 |
| Picard/Cassini (1669) ($\pi < 12''$) | > 17188 | > 0.73285 |
| Richer/Cassini,C.D. (1672) Mars parallax ($\pi = 9''.5$) | 21712 | 0.92570 ± 7000 |
| Flamsteed (1672) Mars parallax ($\pi = 10''$) | 20626 | 0.87941 |
| Picard (1672) Mars parallax ($\pi = 20''$) | 10313 | 0.43971 |
| Lahire (1672) Mars parallax ($\pi < 6''$) | 34377 | 1.46569 |
| Newton (1673) ($\pi > 20''$) | < 10300 | 0.43943 |
| Flamsteed (1673) ($\pi < 10''$) | > 20626 | 0.87941 |
| Halley (1678) Mercury transit ($\pi \approx 45''$) | ≈ 4583 | 0.19543 |
| Flamsteed (1681) ($\pi < 15''$) | > 13751 | 0.58628 |
| Newton's *Principia* 1st edition (1687) | 10000 | 0.42635 |
| Whiston (1696) ($\pi \approx 15''$) | ≈ 13751 | 0.58628 |
| Whiston (1701) ($\pi \approx 10''$) | ≈ 20626 | 0.87941 |
| Maraldi (1704) Mars parallax ($\pi = 10''$) | 20626 | 0.87941 |
| Newton's *Principia* 2nd edition (1713) | 20626 | 0.87941 |
| Newton's *Optics* (1717) ($\pi = 12''$) | 17188 | 0.73285 |
| Pond & Bradley (1717) Mars parallax ($9'' < \pi < 12''$) | 19600 | 0.83565 |
| Newton's *Principia* 3rd edition (1726) | 19600 | 0.83565 |
| Maraldi (1719), Cassini (1736) Mars parallax ($11'' < \pi < 15''$) | 16000 | 0.68216 |
| de Lacaille (1751) | 19800 | 0.84418 |
| Delises (1760) Venus transit ($10'' < \pi < 14''$) | 17000 | 0.72480 |
| de Lacaille (1761) | 21850 | 0.93158 |
| de Lacaille (1769) | 23940 | 1.02069 |
| Delambre (1770) Venus transit review ($\pi = 8''.6$) | 23984 | 1.02258 |
| Henderson (1832) Mars parallax ($\pi = 9''.125$) | 22604 | 0.96374 |
| Hansen (1854) ($\pi = 8''.97$) | 22995 | 0.98040 |

Table 1 (concluded).

| Source | Value $r_e$ (km) | au |
|---|---|---|
| Foucault (1862 ) c + aberration ($\pi$ = 8″.86) | 23280 | 0.99257 |
| Hansen (1863) parallactic inequality of the Moon ($\pi$ = 8″.92) | 23124 | 0.98589 |
| *Nautical Almanac* (1866) adopts $\pi$ = 8″.90 | 23176 | 0.98811 |
| Newcomb (1867) Mars parallax ($\pi$ = 8″.848) | 23312 | 0.99391 |
| Berlin Ephemeris adopts $\pi$ = 8″.85 | 23307 | 0.99369 |
| Leverrier (1872) parallactic inequity of the Moon ($\pi$ = 8″.95) | 23046 | 0.98259 |
| Stone (1872) ($\pi$ = 8″.91) | 23150 | 0.98700 |
| 1873 observations of Flora ($\pi$ = 8″.875) | 23241 | 0.99089 |
| Cornu (1874) c + aberration ($\pi$ = 8″.794) | 23455 | 1.00002 |
| Stone, 1874 Venus transit ($\pi$ = 8″.88 ± 0.04) | 23228 | 0.99033 |
| Airy, 1874 Venus transit ($\pi$ = 8″.754) | 23562 | 1.00459 |
| Puiseux (1875) Venus transit ($\pi$ = 8″.879) | 23231 | 0.99044 |
| French micrometer (1875) Venus transit ($\pi$ = 9″.05) | 22791 | 0.97173 |
| Todd (1875) Venus transit ($\pi$ = 8″.883 ± 0.34) | 23220 | 0.99000 |
| Lindsat & Gill (1877) Juno parallax ($\pi$ = 8″.815) | 23399 | 0.99763 |
| Tupman, Venus transit ($\pi$ = 8″.813 ± 0.033) | 23405 | 0.99763 |
| Hall (1879) ($\pi$ = 8″.879 ± 0.060) | 23231 | 0.99044 |
| Downing (1880) ($\pi$ = 8″.788 ± 0.018) | 23471 | 1.00070 |
| Gill (1881) Mars parallax ($\pi$ = 8″.78 ± 0.12) | 23493 | 1.00161 |
| Todd (1880) c + aberration ($\pi$ = 8″.803) | 23431 | 0.99899 |
| Todd (1881) Venus transit ($\pi$ = 8″.883 ± 0.034) | 23220 | 0.99000 |
| The consensus was that the 1874 Venus transits gave the au as (148.9 ± 2.6) × $10^6$ km, i.e. | | 0.9953 ± 0.017 |
| Values used by the USA's Astronomical Almanac: | | |
| 1834-1869 $\pi$ = 8″.5776 (Encke, 1824) | 24047 | 1.02525 |
| 1870-1889 $\pi$ = 8″.848 (Newcomb, 1867) | 23312 | 0.99391 |
| after 1890 $\pi$ = 8″.80 (Paris, 1896) | 23439 | 0.99933 |
| Faye (1881) c + aberration ($\pi$ = 8″.813) | 23405 | 0.99786 |
| 1882 transit of Venus ($\pi$ = 8″.911 ± 0.084) | 23147 | 0.98689 |
| Newcomb (1885) c + aberration ($\pi$ = 8″.805) | 23426 | 0.99877 |
| Gill (1890) asteroid parallax ($\pi$ = 8″.802 ± 0.005) | 23434 | 0.99911 |
| Proctor (1892) *Astronomy Old and New* ($\pi$ = 8″.811) | 23410 | 0.99809 |
| Hinks, Eros parallax visual (1901) ($\pi$ = 8″.806 ± 0.004) | 23423 | 0.99865 |
| Hinks, Eros parallax photographic (1901) ($\pi$ = 8″.807 ± 0.0027) | 23421 | 0.99854 |
| Hough, radial velocities of stars (1912) ($\pi$ = 8″.802 ± 0.004) | 23434 | 0.99911 |
| Noteboom, perturbations of Eros (1921) ($\pi$ = 8″.799 ± 0.001) | 23442 | 0.99945 |
| Jones & Halm, Mars parallax (1924) ($\pi$ = 8″.809 ± 0.005) | 23415 | 0.99831 |
| Jones (1924) paral. inequality of the Moon ($\pi$ = 8″.805 ± 0.005) | 23426 | 0.99877 |
| Russell Dugan Stewart (1926) ($\pi$ = 8″.803 ± 0.001) | 23431 | 0.99899 |
| Rabe (1950) ($\pi$ = 8″.79835 ± 0.00039) | 23443.58 | 0.99952 |
| IAU (1964) ($\pi$ = 8″.794) | 23455.17 | 1.00002 |
| IAU (1976) ($\pi$ = 8″.794148 ± 0.000007) | 23454.78 | 1.00000 |

## 2 THE FIRST TWO STAGES IN THE HISTORY OF THE ASTRONOMICAL UNIT

Stage 1, *The Age of Ignorance*, lasted from prehistory until about 280 BC. Before that time the Sun, Moon, planets and stars were 'a long way off', further away than the clouds, but there was little concept as to how far away they actually were. Pythagoras (circa 530 BC) and his school had the celestial bodies attached to equi-spaced crystalline spheres, and the ordering of these Earth-centred spheres in increasing radius (i.e. the Moon, Mercury, Venus, Sun, Mars, Jupiter, Saturn, and the stars) was simply a function of their mean angular velocity with respect to the background sky. By about 235 BC Eratosthenes had found that the Earth was about 6000 km in radius, and the Moon was about 88 $r_e$ away. Aristarchus of Samos reduced the later value to about 60 $r_e$ this being within about 1% of what is presently accepted.

Aristarchus used the lunar dichotomy method to estimate the Earth-Sun distance (for a more detailed explanation see, for example, Rogers, 1960). Here the quarter Moon is watched carefully until the terminator exactly bisects the disc. At this 'half moon' instance, the astronomer measures the Moon-Earth-Sun angle ($\alpha$). The Sun-Earth distance is then equal to the reciprocal of cosine $\alpha$ times the Earth-Moon distance. Aristarchus found that $\alpha$ = 87°, and thus the Sun-Earth distance = 1150 $r_e$. This figure gained great authority, but unfortunately the

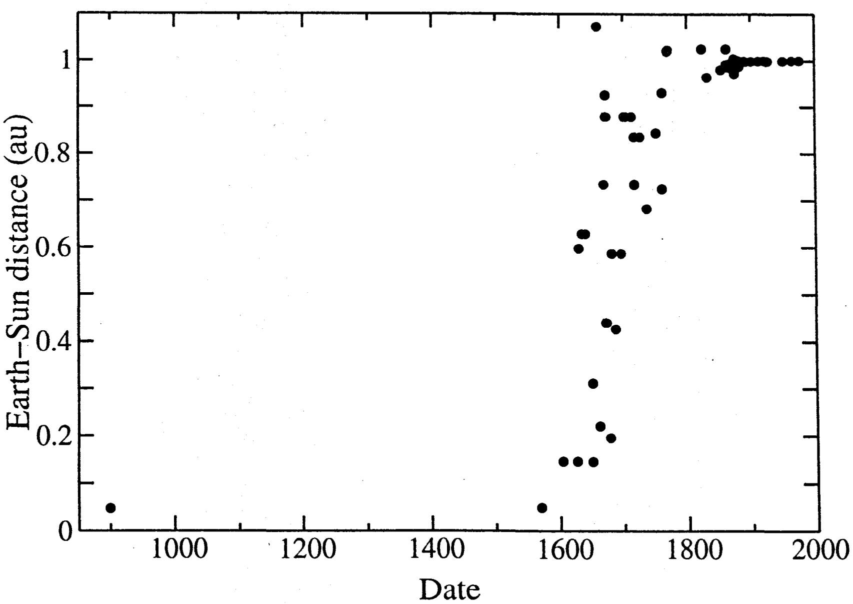


Figure 1. The measured, and accepted, values of the Earth-Sun distance for the period AD 900 to AD 2000. The ordinate is astronomical units, which are obtained by dividing the value (in km) given in each reference by 149,597,870.61 km. Notice how the investigations of the transits of Venus at the end of the eighteenth century led to a considerable decrease in the standard deviations.

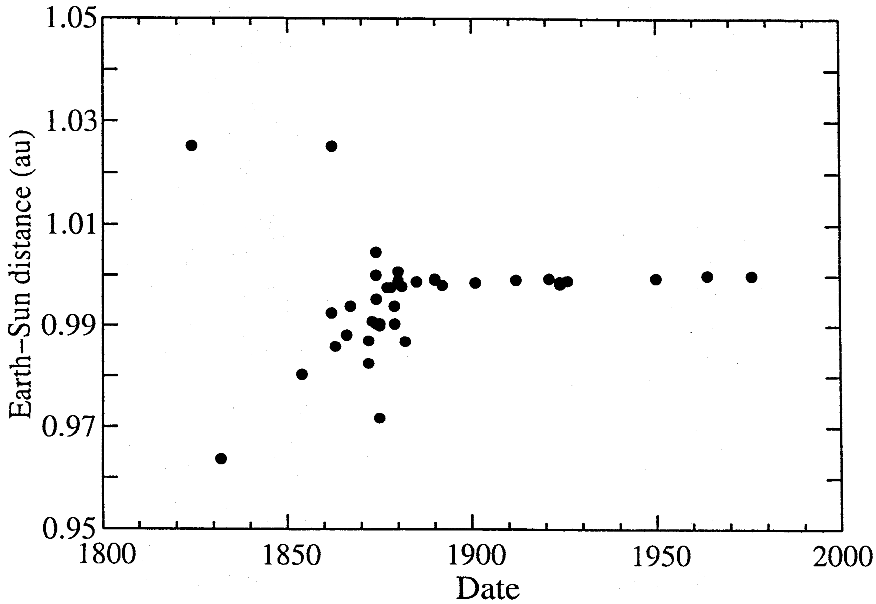


Figure 2. The measured, and accepted, values of the Earth-Sun distance between AD 1800 and 2000. Since 1885 the suggested value has never been more than 0.2% away from today's value. Notice that most results were under-estimates, and that it took about fifty years (between about 1885 and 1935) before the accumulation of many similar results seriously reduced interest in the astronomical unit.

method by which it was obtained is fatally flawed. First it is extremely difficult to estimate the time of dichotomy (a problem not helped by the fact that the mountainous nature of the Moon makes the terminator far from linear). The required angle, α, is also changing quickly as a function of the time. And further more, the fact that one is using a $\cos^{-1}$ function in the calculation means that the difference between $\alpha = 87°$ and $\alpha = 89°.333$ (the latter being the correct value) introduces an error of a *factor* of eighteen.

Hipparchus used an approach that depended upon the timing of the phases of lunar eclipses, and these timings led to a value for the lunar horizontal parallax (i.e Earth-Moon distance in terms of the Earth's radius). Combining this with the acceptance (from the work of Aristarchus) that the Sun was 19 times further away from Earth than the Moon (and thus that the solar parallax was $^1/_{19}$ the lunar parallax), led to the solar parallax being a little less than 3′ (Dreyer, 1906:183, quotes 2′ 54″), a figure that was accepted and publicized by Ptolemy. Hipparchus, however, realized that 2′ 54″ was probably an upper limit. He also noted that the correct value was not observationally obtainable at the time and might even be as low as zero.

Stage 2, *The First Measurements and their Acceptance*, lasted an amazing nineteen centuries, between the work of Aristarchus (and Hipparchus and Ptolemy) and Kepler's analysis of Tycho Brahe's observations of Mars. There was hardly any questioning in this interval; the distance of 1150 $r_e$ was simply accepted.

## 3 STAGE THREE: *THE REALIZATION OF ERROR*, and STAGE FOUR: *THE PERIOD OF CONFUSION*

As in many planetary endeavours, Johannes Kepler was a key player in the early seventeenth century. He can be regarded as the initiator of Stage 3, *The Realization of Error*, not because he had been tackling the problem of the Earth-Sun distance directly, but because his Martian observations indicated that the solar parallax could not exceed 1 minute of arc. His estimated Earth-Sun distance of < 3450 $r_e$ was regarded as very much a lower limit, and Kepler encouraged his contemporaries to return to the lunar dichotomy method and try and do it more accurately. Stage 3 only lasted about twenty years. Other astronomers quickly followed Kepler in doubting the 'Greek' distance, and these astronomers started to re-examine the problem. Attempts were made to improve the lunar dichotomy approach.

To this end Gottfried Wendelin (1626:11-12), noted that the Moon was at most 1° from quadrature at the time of dichotomy, so the solar parallax must be less than 1 minute of arc. Giovanni Battista Ricciolo (1651) was one of the last astronomers to apply the lunar dichotomy method carefully, getting $\pi$ = 28″. He advised observers to concentrate on the centre of the lunar disc and to do the measurements only when the Moon was close to the ecliptic. By the mid-seventeenth century most astronomers were convinced that this dichotomy method just would not work and at best would provide only a lower limit.

Kepler instigated a new, typically seventeenth century, approach to the problem, and one that relied upon the measurement and (erroneous!) interpretation of planetary diameters. He was not only convinced that successive nestings of a cube, a tetrahedron, a dodecahedron, an icosohedron, and an octohedron, inside spheres of ever decreasing diameter, might give an important clue as to the scale and form of the solar system, but was also convinced that "... nothing is more in concord with nature than that the order of the sizes should be the same as the order of the spheres." (Kepler, 1617:878) By 'size' it was left open to the reader as to whether diameter, surface area, or volume should be used. In 1635 Wendelin argued that the planetary diameters were directly proportional to their distances from the Sun, and further more, that all planets, seen from the Sun, had an angular diameter of about 28″, or at most 30″. This gave a solar parallax of $\pi$ < 15″. Kepler (Caspar, 1940) suggested (guessed!) that planetary volume was proportional to orbital period (i.e. surface area proportional to distance). Both of these propositions were rejected by Jeremiah Horrocks in around 1640. Pierre Gassendi had measured the apparent diameter of Mercury during the 1631 transit and Horrocks had found that Venus, in transit, subtended an angle of 1′ 16″ at Earth, in comparison with the 31′ 30″ subtended by the solar disc. Horrocks concluded (assumed?), like Wendelin, that the planetary diameters were proportional to their distances from the Sun, and that all planets subtended an angle of 28″ at the Sun (see Whatton, 1859). If this were so, the solar parallax would be half this value, that is 14″.

In the context of the Earth-Sun distance, Christiaan Huygens (1659) wrote that "... no tolerable method for measuring that distance has yet been found. For whether they try to discover it by means of eclipses or of the dichotomies of the Moon, it can easily be demonstrated that these efforts have been in vain." He therefore decided to follow the Keplerian approach. No direct observations were used, and the whole argument depended upon a 'feeling' for how big the Earth should be. Two things were known. From Kepler's harmonic law one had the relative ratios of the semi-major axes of the planetary orbits, but no direct measurements of any specific orbit size. And from contemporary telescopic observation one had, with the exception of Earth, fairly reasonable estimates of the angles subtended by the discs of each known planet at Earth. Huygens then simply assumed that Earth, being a typical terrestrial planet, would have a size that was half way between the sizes of Mars and Venus, its two neighbours. Huygens had made extensive measurements of planetary sizes. In *Systema Saturnium* he wrote: "We have said that the diameter of Mars is $^1/_{166}$ the diameter of the Sun, and that the diameter of Venus is $^1/_{84}$. Taking, then, for the earth's diameter the mean of these two diameters, we find that it is $^1/_{111}$ of that of the Sun.... I grant that the calculations rest on a slippery basis..." (Huygens, 1659). Well, slippery, indeed, but Huygens' $^1/_{111}$ is fortuitously very close to today's accepted value of $^1/_{109.1}$. Taking the mean angular diameter of the Sun as 0.507°, Huygens then estimated that the Earth-Sun distance was 25086 $r_e$.

The first sound steps towards a reasonable estimation of the astronomical unit were via Mars. If the distance between Earth and Mars could be measured accurately, then the astronomical unit could be calculated. The Earth-Mars distance could be most accurately measured when Mars was at opposition, at its closest to Earth, this happening about every 780 days.

The first serious measurements were made in 1672 by the French team of Giovanni Domenico Cassini, Ole Römer, Jean Richer, and Jean Picard (see Olmsted, 1942. They observed Mars at opposition (when it is about 0.38 au from Earth) from both Paris and Cayenne (in French Guiana, South America). Cassini analysed these observations and obtained a value of 9″.5 for the solar parallax, and an Earth-Sun distance of 21600 $r_e$. John Flamsteed, however got $\pi = 10''$, Jean Picard $\pi = 29''$ and Lahire $\pi < 6''$, so uncertainty continued on a gigantic scale (see Cassini, 1772). Flamsteed (1672) also observed Mars, in 1672 October, and obtained a Martian parallax of 25″ and a solar parallax of 10″.

Isaac Newton, for one, was very reluctant to accept the French values. He was not too happy with Flamsteed's either. The effect of atmospheric refraction was severe, and most astronomers regarded the similarity of the two solar parallax results as somewhat fortuitous (see Hufbauer, 1991). In fact the period between about 1626 and about 1704 was a Period of Confusion. During this interval a host of different figures for the Sun-Earth distance was used, more or less at random. Anything between 3000 $r_e$ and 25000 $r_e$ seemed to do! Young (1895:375) concluded that "Until nearly 1700 no even reasonably accurate knowledge of the sun's distance had been obtained."

Newton was somewhat confused about the whole problem and changed his mind several times. He used a value of $\pi = 20''$ in the 1687 first edition of the *Principia* (see Van Helden, 1985:147), but in the second edition (published in 1713) gave Cassini's and Flamsteed's value of 10". [This halving of the parallax led, among other things, to an increase in the accepted mass of the Sun by a factor of eight, no mean jump in 26 years!]. In the third edition of *Principia* (1726), Newton used $\pi = 11''$, 12″, and 13″ in different places; he also used 10″.5, this being justified as the midpoint of the not less than 9″ and not more than 12″ found by James Pond and James Bradley in 1719 (see Rigaud, 1832). Meanwhile, in the second edition of *Optik's* (1717), he used 12″.

In passing it is worth noting that the solar system, as discussed by Newton in the *Principia*, differed greatly from the one proposed by Nicolaus Copernicus in 1543. Four Jovian and five Saturnian moons had been added, and the scale of the system had been expanded by an order of magnitude. This had magnified the Sun so that it now truly deserved its central role. Knowledge of the angular diameters of the planets had lead to a tremendous enlargement of the accepted physical dimensions of the outer planets. Coupled with these effects, attempts by Galileo, Newton, and Huygens to estimate the distance between the Sun and the bright stars had helped make the Universe itself almost inconceivably large. The new cosmic dimensions, learned by all educated men and women, were one of the wonders of the age.

Around 1550 the cosmos was very Ptolemaic, the *Almagest* being the supreme authority. The Sun was thought to be 1200 $r_e$ away from Earth, 10.5 times bigger than Earth, and the naked-eye stars were at a distance of about 20,000 $r_e$. By 1760 the Sun had become 110 times the size of Earth, and the bright star Sirius was estimated to be at a distance of around $7 \times 10^8$ $r_e$.

## 4 THE COMMONLY-ACCEPTED VALUE, AND THE ERA OF GROWING DISINTEREST

More careful work on the parallax of Mars, and especially Maraladi's measurement in 1704 seemed to convince contemporary astronomers that they were reasonably close to the correct value. Only the archaistic accepted the Greek value for the solar parallax after that date. Astronomers agreed that the true Earth-Sun distance was somewhere in the range 16,000 to 24000 $r_e$. What was needed to further improve the situation was hard work, careful observations, accurate results and a reduction of systematic and experimental errors. Three completely new techniques helped greatly. These were: (i) the timing of the transits of Venus across the solar disc from many known positions on Earth, (ii) the use of ground-based measurements of the velocity of light together with the accurate assessment of the constant of aberration to measure the mean orbital velocity of Earth, and (iii) the quantification of the dynamic perturbation of the Martian, Cytherean and Earth-Moon systems by their neighbours.

The accuracy of the Mars parallax method was also improved greatly by doing it 'diurnally', that is looking at the way in which the position of Mars changed against the celestial background as a specific observatory was moved in space as Earth spun during the night (see later). By the late nineteenth century the Mars opposition parallax technique had also been applied to certain asteroids. The increased sensitivity and accuracy of astronomical spectrometers enabled the annual variation of Earth's orbital velocity to be measured directly, thus providing a fourth approach to the problem, with this bearing fruit in the early twentieth century. More recently, in the last few decades, pulse radar techniques have taken over and the distance has been measured directly.

Needless to say the two-station parallax method could also be applied to Venus as well as Mars, but the brightness of Venus, and the fact that the Cytherean atmosphere produced a somewhat indistinct limb, did not help. Gillis used a Santiago (Chile) to Washington (USA) arc in 1849/1852, but with limited success. The parallax method could also be applied directly to the Sun. The accuracy of the measurements of the solar apparent declination (as seen from observatories at different latitudes) were, however, greatly reduced due to the imprecise nature of the 'boiling' solar limb and the effects of solar heating on the adjustments of the viewing instruments.

Edmond Halley was a great advocate of the Venus transit method, this keenness being spawned by his observation of a transit of Mercury on 1677 October 28 from the South Atlantic island of St. Helena. Halley was extremely unimpressed by the plethora of values being used for the astronomical unit at the time, and published his famous 'advert' for the Venus transit method in 1716. Unfortunately the next transit, in 1761, was a long way off. In the interim, Halley (1716) suggested that the value of $\pi = 12''.5$ should be used, simply because this made Earth (which had a moon) bigger than Venus (which did not), and made Mercury bigger than the Moon.

In 1737 William Whiston was using 10″ as the solar parallax but wrote "... the Sun's parallax ... is not yet accurately determined by astronomers; so that no exact number can be certainly pitch'd upon, till farther observations put an end to our doubts." (Whiston, 1737:34-35).

By 1760, on the eve of the Venus transit observations, Halley's Comet had already been seen to retreat 4 times further from the Sun than the Saturnian aphelion, and the 12″ typical value accepted for the solar parallax made the Sun 17,000 $r_e$ away from Earth and 150 times bigger (today it is given as being 109 times bigger).

An Earth-orbit baseline of some 40,000 $r_e$ gave the more optimistic searchers of stellar parallax some hope. In 1669 Robert Hooke thought that he had succeeded with the star Gamma Draconis (see Hooke, 1674). Flamsteed tried unsuccessfully with the star Polaris (see Wallis, 1699).

Unfortunately the great effort expended on the observations of the transit of Venus did not produce a very satisfactory result (see, for example, Woolf, 1959 and Meadows, 1974). The 1761 June 6 transit was observed by 120 scientists at 62 different locations, and the resulting solar parallax values ranged from 8″.28 to 10″.60 (i.e. Earth-Sun distance from 24900 to 19500 $r_e$). Two factors were to blame for the magnitude of this parallax range. One was the inaccuracy of the timing of the Cytherean ingress and egress, due to the black-drop effect. Errors in estimating when Venus had left the solar limb, or reached it, could easily be between

10 and 15 seconds of time. The other was due to poor knowledge of the latitudes and longitudes of some of the observing sights (see Hufbauer, 1991).

The subsequent observations of the 1769 June 3 transit narrowed the parallax range to 8.43-8″.80. These transit observations were at least providing genuine measurements of the solar parallax, which might not have been the case with either the early observations of the parallax of Mars (where the instrumental, systematic, and refraction-induced errors were thought by many to be of the same magnitude as the required angle) or with the lunar dichotomy estimations. Apart from the black-drop effect, and the need for the observers to travel to distant, and often unpleasant parts of the globe, there was a third serious defect in the transit approach and that was the rareness of its occurrence. Venus transited and transits the Sun on 1761 June 5; 1769 June 3; 1874 December 8; 1882 December 6; 2004 June 7; 2012 June 5; 2117 December 10 and 2125 December 8. All scientists like to repeat their observations in order to check the veracity of their results and improve their experimental performance. In this context, the period of 105.5 years between, say, the 1769 and 1874 transits was clearly impractical in both respects, and this alone favoured other approaches to the parallax problem.

There were three ways of collecting data for the transit method. Halley's method required observations from Earth's polar regions, plus a knowledge of the latitude of the observing sight, and measurements of the total time it took the planet to cross the disc (i.e. it had to be seen at both ingress and egress). Delisle's method utilized stations near the equator, plus accurate knowledge of their longitude and latitude, and measurements of either the time of ingress or the time of egress. The third method required continuous measurements of the exact position of the planet on the solar disc, as a function of time. Both photographic (mainly in 1874) and visual heliometric approaches were tried for the latter.

After all the rigours of the Venus transit observations of 1874, Harkness, (1879) still gave the parallax error as ± 1.7 %. As a result, "Astronomers, accordingly, looked round for fresh means or more refined expedients for applying those already known. A new phase of exertion was entered upon.... [to solve] the *quæstio vexata* of the sun's distance." (Clerke, 1885: 282).

Gravitational methods can be used to measure the Earth-Sun distance and one of these, the lunar parallactic inequality approach, was first suggested by Matthew Stewart in 1763. This method relies on the fact that the Sun is not an infinite number of times further away from Earth than the Moon, but only about 400. The Sun affects the Moon's orbit around Earth, the solar force accelerating the Moon when it is moving towards it and decelerating the Moon when it is moving away from it. As the Sun is not an infinite distance away, its disturbing force on the half of the Moon's orbit that is on the sunward side of Earth differs from the disturbing force on the opposite half, and the amount of difference is a function of the Earth-Sun distance. So, for example, the solar retarding force exerted on the Moon as it moves from being new to first quarter differs from the force exerted on the Moon as it moves from first quarter to full. Due to this, the Moon's position varies by as much as minus two minutes of arc and plus two minutes of arc with respect to the position it would have had if the Sun were an infinite distance away. Now two minutes of arc corresponds to four minutes of time, a quantity that can easily be accurately measured.

Unfortunately this accuracy is somewhat diminished by the fact that the lunar surface is uneven and that different lunar limbs have to be used for measuring the lunar position during the different quarters of the orbit. Laplace published a value for the solar parallax using this method (see Wallis, 1699).

A second gravitational approach depends upon measuring the perturbing forces that Earth exerts on the orbits of its neighbouring planets Mars and Venus. Using Kepler's Harmonic Law and Newton's second law we can write

$$M_O + M_E = 4\,\pi^2\, D^3 / G\, T^2, \text{ and}$$

$$g = G\, M_E / r_e^2,$$

where $M_O$ and $M_E$ are the masses of the Sun and Earth respectively, D is the Earth-Sun distance, G is Newton's constant of gravitation, T is the length of the year, g is the average acceleration of gravity at Earth's surface and $r_e$ is the average radius of Earth. Combining these equations gives

$$D^3 = [(M_O / M_E) + 1]\, T^2\, r_e^2\, g / 4\pi^2$$

The quantity ($M_O$ / $M_E$) can be obtained (with effort) from measurements of the rate at which the longitudes of the nodes and the position of the line of apsides of the orbits of Venus and Mars change with time. Leverrier developed this approach, and was so impressed with its potential that he would have nothing to do with the Venus transit observations of 1874. Spencer Jones (1924:109) also dismissed the transit approach, and wrote disparagingly: "Although this method is not capable of giving results of a high order of accuracy, it is of considerable interest historically ...".

A third gravitational approach assumed that the mass of the Moon was known, and had been calculated from, say, its tidal effects on Earth. As it is the centre of gravity of the Earth-Moon system, and not the centre of the Earth that moves on an elliptical orbit around the Sun, the Sun is displaced from where it is expected to be, throughout the month, by a maximum of 6″.3 (a quantity known as the lunar equation). It can easily be shown that the solar parallax is given by

$$\pi = 6.3'' \, (r_e / EM) \, (M_E + M_C) / M_C,$$

where EM is the Earth-Moon distance and $M_E$ and $M_C$ are the masses of the Earth and Moon respectively. Unfortunately the lunar equation is a very small angle, which is extremely difficult to measure with accuracy.

A completely new approach to the quantification of the astronomical unit came via the combination of a knowledge of the velocity of light and the constant of aberration of starlight. The fact that light travelled at a finite velocity was discovered by Ole Römer in 1675, when he analysed the variations in the intervals between the eclipses of the Jovian satellites as a function of the Earth-Jupiter distance. For many years the velocity of light was given in terms of the "light-equation" (i.e. the time it takes light to travel an astronomical unit). Delambre obtained a value of 493.2 seconds in 1792, and Glasenapp, two years later, found 500.84 seconds. Both these produced velocities of light that depended upon contemporary knowledge of the Earth-Sun distance, so were useless in the present endeavour.

The first successful 'ground-based' experimental measurements of the velocity of light were made by Fizeau, who obtained 308 x $10^8$ m $s^{-1}$ and Foucault who obtained 298 x $10^8$ m $s^{-1}$. These velocities, when combined with the 'light-equation', immediately lead to a value for the astronomical unit (see Foucault, 1862).

James Bradley discovered the aberration of starlight in 1726. Here the velocity of light combines vectorially with Earth's orbital velocity to displace the position a star against the celestial background. A star that would be at the pole of the ecliptic if Earth were stationary is displaced by an angle of about 20″.5. [Here we have an example of a parameter of the first type, i.e. one where the initial measurement produced a value that was very close to the 'correct' value that is obtained later on. Bradley first measured the constant of aberration to be 20″.25, but "Upon further consideration he was induced to fix it at 20″." (Grant, 1852: 340). Delambre found 20″.255. By 1844 Baily was quoting 20″.4192, and later on M Struve made it 20″.445. Nyrén suggested 20″.492 in 1882. Notice that constants of aberration of 20.46, 20.48, 20.50, 20.52 and 20″.54 correspond to solar parallaxes of 8.808, 8.799, 8.790, 8.782 and 8″.773 respectively.]

It is clear that, by the early 1860s, when Fizeau and Foucault had measured the velocity of light accurately, the constant of aberration was known with an even greater precision, so the calculation of an accurate value for the solar parallax was a simple matter. Struve, for example found $\pi$ = 8″.86 (see Table 1).

The original Mars parallax method, first attempted by the French (see above), was seriously disadvantaged by the fact that it required two observers, working at a considerable latitudinal distance from each other. These observers were out of touch and were using different instruments. The method gave too large a value for the parallax and Young (1895) thought this might be due to the red colour of the planet affecting the astronomical refraction. A more satisfactory approach was to measure the diurnal parallax, a quantity that maximized for an observer near Earth's equator. A single observer using a single instrument (usually a heliometer) could measure the change in right ascension and declination of Mars produced by the approximate 2 $r_e$ shift of the observer in the time interval between Mars' rising and setting on a single night (Mars, being near opposition at the time, transits the meridian at midnight, and is thus in the sky throughout the night). This method was used by Gill during the 1877 opposition of Mars. Observing from the Ascension Islands, he obtained $\pi = 8''.783 \pm 0.015$.

The parallax method was also applied to asteroids, and especially those that pass close to Earth. The fact that asteroids appeared as star-like points, and not planetary discs, improved the accuracy. This approach was first suggested by Johanne G. Galle in 1873, and he used 8 Flora to obtain $\pi = 8''.87$. In 1886, Robert Ball wrote: "Let us hope that ere long the next transit of Venus approaches, the problem of the sun's distance will have been satisfactorily solved by the minor planets." (Ball, 1886:210). In 1888-1889 Gill observed asteroids 7 Iris, 12 Victoria and 80 Sappho, all of which came within 0.85 au of Earth, and found $\pi = 8''.802$.

The asteroid 433 Eros was discovered in 1897 and its close passage (0.27 au) in 1900-01 produced more precise values of $\pi = 8''.806 \pm 0.004$ (from visual observations using telescopes of long focal length) and $\pi = 8''.807 \pm 0.003$ (from photographic observations). At its opposition of 1930-01 Eros was only 0.17 au distant, and Spencer Jones (1941) obtained $\pi = 8''.790 \pm 0.001$ (cf. Atkinson, 1982). Thirty telescopes in fourteen different countries were used to produce 2847 photographic plates of this nearby object.

In 1912 Hough, using a high dispersion spectrograph at the Cape of Good Hope, measured the annual variation in the Doppler shift of the light from a set of seven bright near-ecliptic stars to measure the mean orbital velocity of Earth. The resulting value of the solar parallax was $8''.802 \pm 0.004$. Working with Arcturus alone gave $8''.805 \pm 0.007$.

Between about 1705 and 1880 the quoted standard deviation of the astronomical unit went from about 11 % to 1 % of its value. By 1931 it had been reduced further to around 0.01 %. The period between 1705 to 1931 saw *The Diminution of The Standard Deviation about a Commonly-Accepted Value* (i.e. Stage 5 in the evolution of this parameter). By 1931 the job of quantifying the astronomical unit had almost been done, and then followed the final stage, *The Era of Growing Disinterest*.

Rabe (1950:) obtained the solar parallax using a dynamic method in which he studied the perturbation of Earth on near-by objects, deriving $\pi = 8''.79835 \pm 0.00039$. (i.e. 1 au = 149,532,000 ± 7,000 km). The post-WWII period was somewhat enlivened by the completely new technique of pulsed radar ranging. This has been applied to the Sun, nearby planets and asteroids since 1961 (see, for example, Ash *et al.*, 1967; Hey, 1973:122; and Shapiro, 1968). At the nearest approach of Venus in 1961 it took a radar pulse about 5 minutes to travel from Earth to Venus and back. The American, British and Soviet results led the International Astronomical Union to adopt a solar parallax of $8''.794$ and an astronomical unit of 149,600,000 km at its 1964 General Assembly. Further detailed work led the 1976 meeting of the IAU to adopt a solar parallax of $\pi = 8''.794148 \pm 0.000007$, corresponding to an astronomical unit of 149,597,870 ± 120 km. Stix (1989:3) gives the astronomical unit as 149,597,870 ± 2 km, while the Jet Propulsion Laboratory Planetary Ephemeris uses 149,597,870.61 km (see Lang, 1991:11).

## 5 CONCLUDING REMARKS

The measurement of the distance between Earth and the Sun has been approached in about seven sensible ways. Young (1895) went so far as to give each contemporary and historic method a mark out of a hundred, and his grading is listed in Table 2, in order of decreasing efficacy. More recent methods have been added to the top of this table.

In this paper we have suggested that the fundamental parameters of astronomy can be divided into two types, this division being dependent upon the history of their development and the accuracy of their quantification in any historical epoch.

Type one parameters have a very simple two-stage history. In the first stage, the parameter is not known. In the second it is known with reasonable accuracy. Here the first attempts to measure the parameter achieve very nearly the correct answer. Typical examples of 'type one' parameters, are the radius of the Earth, the distance to the Moon, the radius of the Moon, the length of the month and the year, the velocity of the Sun around the galactic centre, the Eddington Limit, Oort's constants, the Schönberg-Chandrasekhar limit, etc..

The history of a quantity of the 'type two' is more interesting. Instead of there being only two stages in their development, there are six. In this brief paper we have considered the history of the astronomical unit, but there are many other similar quantities in the field of astronomy. A few examples are the age of the Earth, the temperature of the solar photosphere, the mass of the Sun, the solar luminosity, the cosmic hydrogen to helium ratio, the distance to nearby stars, the number of stars in the sky, the size of the Milky Way Galaxy, the number of galaxies in the Universe, the mass of the Universe, the Hubble Constant, the density parameter of the Universe, the Cosmological constant, and so on.

Table 2. Methods of measuring the astronomical unit, ordered according to the marking scheme suggested by Young (1895). Here 0 is bad and 100 is good.

| Mark | Method |
|---|---|
| 100 | Modern Radar |
| 100 | Annual variation in radial velocity of ecliptic stars |
| 100 | Near Earth Asteroid diurnal parallax |
| 95 | Perturbation of the orbits of Mars and Venus |
| 90 | Mars diurnal Parallax (from a single observatory near the equator) |
| 90 | Constant of aberration + knowledge of c |
| 80 | Light equation + knowledge of c |
| 75 | Venus transit (measuring planet's position on solar disc, using a heliometer) |
| 75 | Venus transit (noting planet's position on solar disc, using photography) |
| 75 | Asteroid diurnal parallax (from a single near-equatorial observatory) |
| 70 | Lunar parallactic inequality |
| 50 | Venus transit (Delisle Method) |
| 40 | Venus transit (Halley Method) |
| 40 | Monthly perturbation of solar position (i.e. measuring the lunar equation) |
| 25 | Mars parallax (from two observatories, north and south) |
| 20 | Venus declination from many observatories |
| 20 | Asteroid parallax (north + south observatories) |
| 0 | Lunar dichotomy (Aristarchus) |
| 0 | Hipparchus method (timing lunar eclipses and assuming $^1/_{19}$ for ratio of Earth-Moon and Earth-Sun distances) |
| 0 | Parallax of Sun directly |

We have divided the history of these 'type two' parameters into six different stages. The temporal start- and end-points of these stages provide interesting mileposts on any journey through astronomical history. Often they are associated with major advances in instrumentation or with key paradigmatic shifts in our approach to the subject. When it comes to the astronomical unit the Stage 1, *Age of Ignorance*, ended around 300 BC with the flourishing of Greek astronomy under the likes of Eratosthenes and Hipparchus. They introduced Stage 2, and *The First Measurements and their Acceptance* lasted until the work of Kepler around 1604. Nineteen centuries is a long time without change. Not only was there no specific reason during that time period to doubt the original value of the astronomical unit, but, more importantly, the one inadequate lunar dichotomy method of measurement was not superseded by something better.

Kepler's desire to quantify the sizes of planets and the scale of the heliocentric solar system led to the brief two decades or so (1604-1630) of Stage 3, *The Realization of Error*. By the 1630s quite a few astronomers harboured strong suspicions that the Greeks were wrong, and that in fact the Solar System and its central Sun were much larger than had been previously thought. From 1630 to around 1705 the situation was very confused, and many different values were produced for the solar parallax. Much of this confusion arose because the lunar dichotomy method was shown to be not only completely inadequate, but also not open to improvement. Results obtained by this method during the early seventeenth century therefore differed widely. The new, 1672, approach to the astronomical unit, via the measurement of the Earth-Mars distance at opposition, was also very imprecise when first attempted. Here astronomers were trying to measure the angular distance between Mars and nearby stars at specific times (i.e. the measurements made in Cayenne and Paris had to be co-temporal). Successful observations depended upon the use of the newly-invented eye-piece micrometers, as well as on the accuracy of the contemporary clocks. A further weakness of this new method was that the required angle (i.e. the parallax of Mars) was obtained by calculating the difference between two numbers, both of which were known imperfectly.

The telescopic eyepiece micrometer was invented around 1640 by the Yorkshireman William Gascoigne, and the next sixty years saw huge improvements in this device (see, for example, King, 1955). This fact, coupled with important developments in telescope design and mountings during the late seventeenth and early eighteenth centuries, revolutionized the accuracy with which astronomical positions could be measured. This accuracy was vital when it came to measuring the position of Mars. Pledge (1939:291) plots a graph (which he attributes to H Mineur) showing the way in which the angular accuracy, $\beta$ seconds of arc, of an astronomical positional measurement varied as a function of the epoch of the measurement. Typical values given in this graph are listed in Table 3. These follow a relationship of the form

$$\log \beta'' = (1.45 \pm 0.14) - (14 \pm 1) \times 10^{-3} (T - 1700), \quad [\text{AD } 1600 < T < \text{AD } 1900]$$

where T is the date of the observation. It can be seen from this relationship, and from Table 3, that the errors of the Cassini measurements in 1672 were probably comparable with the magnitude of the parallax (~ 28″) that was being measured. It is also noticeable just how speedily the accuracy of astronomical angular measurement improved at this time.

The accuracy of time measurement has been reviewed by Howse (1980). Again the interval between AD 1650 and 1900 saw great improvements. If the best clocks of the period were gaining or losing about $\Delta t$ seconds per day, the data in Howse (ibid.) indicate that

$$\log \Delta t = 0.76 - 0.0114 (T - 1700). \quad [\text{AD } 1650 < T < \text{AD } 1900]$$

Table 3. The accuracy of astronomical angular measurements (± $\beta$), taken from Pledge (1939).

| Astronomer | Date (AD) | Accuracy (±$\beta$) (seconds of arc) |
|---|---|---|
| Tycho Brahe | 1585 | 240 |
| Flamsteed | 1700 | 10 |
| Piazzi | 1800 | 1.5 |
| Bessel | 1845 | 0.3 |
| Auwers | 1880 | 0.1 |
| Newcomb | 1900 | 0.03 |

The *Period of Confusion* about the value of the astronomical unit was followed, round about AD 1705, by a general realization that the Sun was of the order of 150,000,000 km away from Earth (i.e. at a distance of about 24000 $r_e$). This was the start of Stage 5, the period of *The Diminution of the Standard Deviation about a Commonly-Accepted Value*. This stage in the history of the astronomical unit lasted from about 1705 to about 1935 and was enlivened by a host of new experimental approaches to the distance measurement, and a strengthening of the realization that the astronomical unit lay at the heart of astrophysics, in as much as all the important quantities such as stellar distance, size, mass, temperature, and luminosity depended upon it. Stage 5 saw considerable competition between the advocates of the different measurement techniques, many of which had similar precision. This competition added greatly to the excitement of the endeavour and to the pace of improvement.

At the start of Stage 5 much was expected of the method that relied upon the timing of the transits of Venus across the solar disc. Unfortunately many inherent problems and inconveniences led to disappointing results and a waning of interest by the time of the most recent transit, in 1882 (see Dick, Orchiston and Love, 1998). The Mars opposition parallax method was transformed both by the increase in measuring accuracy and by taking the measurements from a single site close to the equator during a single night, as opposed to using two sites in different hemispheres. In the late nineteenth century the diurnal parallax approach was extended to asteroids and then revolutionized by the discovery of certain asteroids that come much closer to the Earth than Mars does. The ground-based measurement of the velocity of light by laboratory physicists meant that this quantity could be combined with the constant of aberration to give an accurate value for the Earth-Sun distance. Also, the ever-increasing accuracy of astronomical telescope scales and time pieces meant that the varying solar gravitational effects on the motion of the Moon around Earth, and the effect of the Sun/Earth mass ratio and the Sun/Earth distance on the perturbation of the orbits of Venus and Mars, could also be measured, leading again to accurate estimates of the astronomical unit. Both of these methods were working well in the 1870s, and this furthered the growth of the disillusionment with the transit of Venus method. Work on the annual variation in the radial velocity of certain stars, and observations taken during the close flyby of Earth by asteroid Eros meant that, by 1935, the problem of quantifying the astronomical unit was essentially solved.

Returning to the topic of the competition between different techniques, Russell, Dugan, Stewart (1926:188) wrote: "The mutual agreement of the values arrived at by very different methods (none differing from the mean by more than 1 in 1500) is very striking." Again referring to the competition, van Helden (1995) reported on the worrying way in which the raw data of the 1761 and 1769 Venus transits were analysed time and again in the hope that they would yield results that were more in agreement with other and more recent results.

The Sixth Stage, *The Era of Growing Disinterest*, has lasted from about 1935 to the present day. The disinterest is illustrated beautifully by the change in prominence given to the measurement of the astronomical unit in the university textbooks of the period. Young (1888, first edition, and 1895, second edition) devoted the whole of his chapter 16, some 18.3 pages, to the subject; Russell, Dugan, Stewart (1926) assign a mere 2.2 pages; Payne-Gaposchkin (1955) also 2 pages; Motz and Duveen (1966) just the one page; and the recent 1400-page classic by Carroll and Ostlie (1996) has only two lines. What is more, this Sixth Stage saw the introduction of a completely new technique, one relying upon the timing of reflected radar pulses. This technique did not 'compete' with previous techniques, it completely overwhelmed them. The 1926 accuracies of 1 part in 1500 were replaced, in the 1980s, by accuracies of better than 1 in 70,000,000. The job had been done.

## 6 ACKNOWLEDGEMENTS

In writing this paper I have referred extensively to van Helden (1985) and would like to thank him for all the encouragement he has (inadvertently) given.

## 7 REFERENCES

Ash, M.E., Shapiro, I.I. and Smith, W.B., 1967. Astronomical constants and planetary ephemerides deduced from radar and optical observations. *Astronomical Journal*, **72**:338-350.

Atkinson, R.d'E., 1982. The Eros parallax, 1930-31. *Journal for the History of Astronomy*, **13**:77-83.

Ball, R.S., 1886. *The Story of the Heavens*. Cassell & Co., London.

Carroll, B.W. and Ostlie, D.A., 1996. *An Introduction to Modern Astrophysics*. Addison-Wesley, Reading.

Caspar, M. (ed.), 1940. *Johannes Kepler. Gesammelte Werke. Volume 6.* Beck, Munich.

Cassini, J.D., 1772. *Histoire Abrégée de la Parallax du Soleil*. Paris.

Clerke, Agnes, 1885. *A Popular History of Astronomy during the Nineteenth Century*. Black, Edinburgh.

Dick, S.J., Orchiston, W. and Love, T., 1998. Simon Newcomb, William Harkness and the nineteenth century American transit of Venus expeditions. *Journal for the History of Astronomy*, **29**:221-255.

Dreyer, J.L.E., 1906. *History of the Planetary Systems from Thales to Kepler*. Cambridge University Press, Cambridge.

Encke, J. F., 1824. *Der Venusdurchgang von 1769*. Becker, Gotha.

Flamsteed, J., 1672. An extract of Mr. Flamstead's Letter ... concerning the Appulses ... of the Moon, and other Planets to Fixed Stars; together with an Observation of the Planet Mars, made by the same. *Philosophical Transactions of the Royal Society of London*, **7**:5118-5119.

Foucault, J.B.L., 1862. A value for the solar parallax obtained using the velocity of light and the constant of aberation. *Comptes Rendus de l'Academie des Sciences*, **55**:502.

Grant, Robert, 1852. *History of Physical Astronomy from the Earliest Ages to the Middle of the Nineteenth Century*. Baldwin, London.

Halley, E, 1716. Methodus singularis qua Solis Parallaxis sive distantia a Terra, ope Veneris intra Solem conspiciendæ tuto determinari poterit. *Philosophical Transactions of the Royal Society of London*, **29**:454-464.

Harkness, W., 1879. *American Journal of Science*, **20**:393.

Hey, J.S., 1973. *The Evolution of Radio Astronomy*. Elek Science, London.

Hooke, R., 1674. *An Attempt to Prove the Motion of the Earth by Observation*. London (see, also, Gunther, R.T., 1931. *Early Science in Oxford, Volume 8 (Cutler Lectures of Robert Hooke)*. Oxford University Press, Oxford, pp. 1-28, where it is reprinted).

Howse, D., 1980. *Greenwich Time and the Discovery of Longitude*. Oxford University Press, Oxford.

Hufbauer, K., 1991. *Exploring the Sun: Solar Science since Galileo*. Johns Hopkins University Press, Baltimore.

Huygens, C., 1659. *Systemma Saturnium*. See *Oeuvres Complètes de Christiaan Huygens, Volume 15*. The Hague, Martinus Nijhoff, pp. 342-345.
Jones, H.S, 1924. *General Astronomy*. Edward Arnold, London.
Jones, H.S, 1941. The solar parallax and the mass of the Moon from observations of Eros at the opposition of 1931. *Memoirs of the Royal Astronomical Society*, **66**:11-66.
Kepler, J., 1617. *Epitome Astronomiae Copernicanae*. Plank, Linz.
King, H.C., 1955. *The History of the Telescope*. Charles Griffin & Co., London.
Lang, K.R., 1991. *Astrophysical Data: Planets and Stars*. Springer-Verlag, New York.
Meadows, A.J., 1974. The transits of Venus in 1874. *Nature*, **250**:749-752.
Motz, L., & Duveen, A., 1966. *Essentials of Astronomy*. Columbia University Press, New York.
Newcomb, S., 1867. Investigations of the distance of the Sun and the elements which depend on it. *Washington Observations for 1865*, Appendix II:1-29.
Olmsted, J.W., 1942. The Scientific expedition of Jean Richer to Cayenne, (1672-1673). *Isis*, **34**:117-128.
Paris, 1896. *Conference International des Etoiles Fondamentales*. Process-Vebaux, Paris.
Payne-Gaposchkin, C., 1956. *Introduction to Astronomy*. Eyre & Spottiswoode, London.
Pledge, H.T., 1939. *Science Since 1500: A Short History of Mathematics, Physics, Chemistry, Biology*. His Majesty's Stationery Office, London.
Rabe, E., 1950. Derivation of fundamental astronomical constants from the observations of Eros during 1926-1945. *Astronomical Journal*, **55**:112-126.
Riccioli, G.B., 1651. *Almagestum Novum, Volume 1*. Benati, Bologna.
Rigaud, S.P. (ed.), 1832. *Miscellaneous Works and Correspondence of James Bradley*. Oxford University Press, Oxford.
Rogers, E.M., 1960. *Physics for the Inquiring Mind; The Methods, Nature and Philosophy of Physical Sciences*. Princeton University Press, New Jersey.
Russell, H.N., Dugan, R.S., & Stewart, J.Q., 1926. *Astronomy: A Revision of Young's Manual of Astronomy. Volume 1: The Solar System*. Ginn & Co., Boston.
Shapiro. I.I., 1968. Spin and orbital motions of the planets. In J.V. Evans, & T. Hagfors (eds.). *Radar Astronomy*. McGraw-Hill, New York, pp.143-185.
Stewart, M., 1763. *The Distance from the Sun to the Earth Determined by the Theory of Gravity*. Sands, Kincaid & Bell, Edinburgh.
Stix, M., 1989. *The Sun: An Introduction*. Springer-Verlag, Berlin.
van Helden, A., 1985. *Measuring the Universe: Cosmic Dimensions from Aristarchus to Halley*. University of Chicago Press, Chicago.
van Helden, A, 1995. Measuring the solar parallax: the Venus transits of 1761 and 1769 and their nineteenth-century sequels. In R. Taton, R., & C. Wilson (eds.). *The General History of Astronomy. Volume 2. Planetary Astronomy from the Renaissance to the Rise of Astrophysics Part B: The Eighteenth and Nineteenth Centuries*. Cambridge University Press, Cambridge, pp. 153-168.
Wallis, J., 1699. *Wallis Opera Mathematica. Volume 3*. Lichfield, Oxford.
Wendelin, G., 1626. *Loxias, seu de Obliquitate Solis Diatriba. In qua Zodiaci ab aequatore ... tandem eruitur et in canonem suum refertur ...* Antwerp.
Wendelin, G., 1635. Letter to Gassendi, dated May 1. See *Petri Gassendi Opera Omnia. Volume 6*. Lyons, pp.427-429 (1658).
Whatton, A.B. (ed.), 1859. *Memoire of the Life and Labours of the Rev. Jeremiah Horrox ... To which is Appended a Translation of his Celebrated Discourse upon the Transit of Venus across the Sun*. London.
Whiston, W., 1737. *A New Theory of the Earth. Second Edition*. Macintosh, London.
Woolf, H., 1959. *The Transits of Venus: A Study of Eighteenth-Century Science*. Princeton, New Jersey.
Young, C. A., 1895. *A Textbook of General Astronomy*. Ginn & Co, Boston.

David W Hughes is a Reader in Astronomy at the University of Sheffield, UK. He has spent his academic life researching the minor bodies of the solar system and specifically into cometary decay, asteroidal evolution, cosmogony and the relationships between comets and meteoroid streams. He is author of *The Star of Bethlehem: An Astronomer's Confirmation* (Walker & Co., 1979).