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Definition of the Celestial Ephemeris Origin and of UT1 in the International Celestial Reference Frame*

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Abstract. The adoption of the International Celestial Reference System ICRS, and of the corresponding Frame, ICRF, by the 23rd General Assembly of the International Astronomical Union, calls for a redefinition of the departure point on the true equator. Several possibilities have been suggested. This paper considers the use of the non-rotating origin (Guinot 1979). The "Celestial Ephemeris Origin" (CEO) is defined here as the non-rotating origin on the equator of the Celestial Ephemeris Pole (CEP). Developments valid at the microarcsecond, based on the best model for precession, nutation and pole offset at J2000.0 with respect to the pole of ICRF, are provided for computing the CEP coordinates and the position of the CEO. It is shown that an operational definition of UT1 based on the CEO leads to values which are insensitive at the microarcsecond level to future improvements of this model.

Key words: astrometry – ephemerides – reference systems – time

1. Introduction

At its 23rd General Assembly in 1997, the International Astronomical Union (IAU) adopted an International Celestial Reference Frame (ICRF) that realizes the International Celestial Reference System (ICRS), as specified by IAU Resolution A4, 1991.

According to its 1991 definition, the ICRS is such that the barycentric directions of distant extragalactic objects show no global rotation with respect to these objects. For the study of Earth rotation, a geocentric coordinate system is required. It was specified in 1991 that the orientation of the geocentric system follows the kinematical condition of absence of global rotation of geocentric directions of the objects that realize the ICRS. To avoid ambiguities, we shall designate by CRS this geocentric system.

Until the adoption of the ICRF, the conventional celestial reference frame, the FK5 based on positions and proper motions of bright stars, was oriented so that at a given date, the *epoch*, the positions are referred to the best estimate of the location of the mean pole and mean equinox. The proper motions were evaluated so that, for a given model of precession, they provide the best access to the mean pole and mean equinox of epoch, at any other date. The ICRF was aligned with the FK5 at J2000.0. However, a conceptually fundamental change is that no attempt is made to refer the positions of the sources to the mean pole and mean equinox at J2000.0, although a significant offset has been observed. In addition, it was decided in 1991 that further improvements of the ICRF will be accomplished without introducing any global rotation. Thus *the connection between the peculiarities of the Earth's kinematics will have to be severed* (Note 4 of Recommendation VII of IAU Resolution A4, 1991). In this context, one may argue that the simplest way to study and describe the orientation of the Earth in the CRS is to use three parameters (Eulerian angles), instead of the usual five parameters required by the use of an intermediate axis (axis of the CEP, instantaneous rotation axis or others). Although this can be an interesting development for the future, the usual five parameters will still be needed, for the theory of Earth rotation and for geophysical interpetations. This paper makes use of them.

The historical abandonment of the link of the CRS with the motion of the Earth requires that an offset at epoch be introduced in the description of the precession/nutation of the Earth's rotation pole, this offset being experimentally determined and being revisable in conventionally adopted models of precession/nutation. On the other hand, the current conventional model is known for more than ten years to be inadequate for highly accurate observations. Corrections to this model, the so-called "celestial pole offsets", are derived from Very Long Baseline Interferometry (VLBI) observations.

In the spirit of this new definition of the CRS, and as a consequence of observational progress, it is desirable to adopt a definition of the equatorial system of date and of the Earth rotation parameters (ERP) based on the location and motion of the rotation pole only, i.e., without any relation to the orbital motion of the Earth. Several possibilities for selecting a departure point on the moving equator, replacing the equinox, and depending

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* Table 1 is only available in electronic form at the CDS via anonymous ftp to cdsarc.u-strasbg.fr (130.79.128.5) or via http://cdsweb.u-strasbg.fr/Abstract.html

Correspondence to: capitain@danof.obspm.fr on the pole of the CRS and the pole of rotation have been suggested (Kovalevsky & McCarthy 1998; Mathews 1999). One possibility is the use of a kinematically defined point that has been called the *non-rotating origin* (NRO) (Guinot 1979). The properties of the NRO have already been studied (Capitaine et al. 1986). However, the progress of the observations requires a more accurate realization of the NRO, based on the best precession/nutation model, while keeping the coherence with previous developments. It is also necessary to evaluate more accurately the consequences of a change of precession/nutation model in the realization of the NRO and in the determination of Universal Time UT1. When the NRO is associated with the Instantaneous Rotation Pole (IRP), it provides the modulus of the instantaneous vector of rotation of the International Terrestrial Reference System (here denoted TRS) with respect to the CRS. However, the concept of the NRO can be applied to other definitions of the rotation pole, such as the Celestial Ephemeris Pole (CEP), in its present definition or in other definitions which may be required by newly discovered sub-diurnal terms of the Earth's rotation. In the following the designation NRO is used for the concept. The application to the CEP will be designated as the *Celestial Ephemeris Origin* (CEO). The corresponding origin of longitude in the terrestrial system will be designated as the *Terrestrial Ephemeris Origin* (TEO).

After reviewing the concept of the NRO, basic definitions and formulae, this paper reports on the influence on the determination of UT1 of errors in the precession/nutation model and of changes of the definition of the rotation pole, when the CEO and TEO are used. It also provides numerical developments positioning the CEO on the moving equator with an uncertainty of a few microarseconds between years 1900 and 2100, consistent with the requirements of current observations, to be used in the coordinate transformation between the CRS and the TRS. It is a contribution to the Working Group on the New International Celestial Reference System (ICRS), and the Hipparcos catalogue, formed by the 23rd IAU General Assembly in 1997 (Appenzeller 1998).

2. Basic definitions and formulae

2.1. Space and time references

The study is developed in the framework of classical kinematics. The relativistic effects are important in processing the observations for measuring the ERP, but they are still negligible in their kinematical description, with the exception of the geodesic precession and geodesic nutation (Fukushima 1991).

The International Earth Rotation Service (IERS) refers the ERP to International Atomic Time TAI (and to Coordinated Universal Time UTC). Their time derivatives are referred to the scale unit of TAI, i.e. the second of the International System of Units as realized on the rotating geoid. We therefore assume that the continuous time scale t we need is TAI. It must be noted that TAI is a realized time scale whose theoretical counterpart is $TT - 32.184$ s, TT being Terrestrial Time (Guinot 1995). By definition of TT, $TAI - (TT - 32.184 \text{ s}) = 0$ on 1977 January 1^{st}; but at later dates, it may reach 20 μs, which is of the order of the uncertainty of measurements of UT1. However, the uncertainties of TAI and of its rate are, by far, negligible when dealing with the geophysical interpretation of UT1.

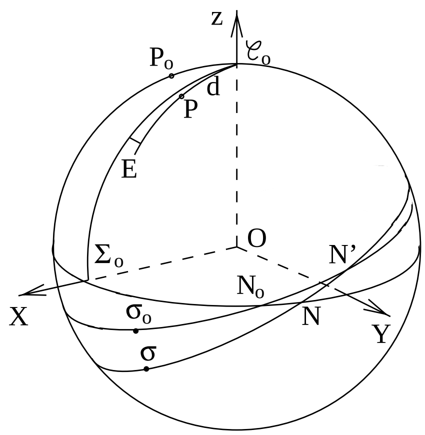

Fig. 1. Definition of the NRO, sigma, with respect to the ICRF corresponding to a finite displacement of the pole P between dates t_0 and t.

2.2. Coordinates of the rotation pole

In the CRS, the Z-axis is oriented in the direction of the pole, designated by $\mathcal{C}_0$, and the X-axis in the direction of the origin of right ascensions, designated by Σ_0; the Y-axis completes the direct trirectangular coordinate system. The direction of the pole of rotation (IRP or CEP) is expressed by direction cosines $X(t), Y(t), Z(t)$ (thus, these quantities describe the whole motion of the pole in the CRS including precession, nutation and offset at epoch). The polar coordinates d and E of the pole (see Fig. 1) can equivalently be used, such: $X = \sin d \cos E, Y = \sin d \sin E, Z = \cos d$.

The TRS, appears in this study only through the orientation of its axes. The TRS is geocentric, the centre of mass being defined for the whole Earth, including oceans and atmosphere, and the time evolution in orientation should create no residual global rotation with respect to the crust (Boucher 1990). Details on its realization, including the adopted motions of the sites in its realization, Terrestrial Reference Frame (TRF), can be found in the IERS Conventions (McCarthy 1996). The w-axis is oriented towards the (geographic) pole, the u-axis towards the origin of longitudes Π_0, the v-axis completing the direct trirectangular system is oriented towards the East. The direction cosines of the pole of rotation are $u(t), v(t), w(t)$. Note that the usual *coordinates of the pole* $x(t), y(t)$, as in IERS publications, are $x = u, y = -v$.

2.3. Rotation of the Earth

The reference systems CRS and TRS are rigid systems of axes to which the residual motion of the source (if any) and of the particles of the Earth are respectively referred. By definition, the *rotation of the Earth* is the relative rotation of these two systems.

The usual representation of the rotation of the Earth by five time series involves an intermediate axis, which can be selected at will in one of the systems, provided that its motion be continuous. The choice is guided by criteria such as convenience, theoretical and/or physical significance. In a similar manner, the departure points on the equator at date t of the intermediate axis in TRS and CRS can be selected at will, provided that the motions of these points in the TRS and CRS be known, so that the arc $\theta(t)$ between these points on the equator leads to the true orientation. The criteria for choosing these points are the same as for the intermediate axis.

2.4. The concept of non-rotating origin

Consider a rigid body. As only directions matter, this body is represented by a celestial sphere, with reference cartesian axes OXYZ. Consider a point P(t) in continuous motion on the sphere and a system of cartesian axes $\mathrm{O}\xi\eta\zeta$, with $\mathrm{O}\zeta$ along OP and let be σ the point of the equator of P where $\mathrm{O}\xi$ pierces the sphere. The orientation of $\mathrm{O}\xi\eta\zeta$ is determined by the condition that in any infinitesimal displacement of P there is no instantaneous rotation around $\mathrm{O}\zeta$. To stress this property, σ was designated as the non-rotating origin on the moving equator of P. In a finite displacement of P between dates t_0 and t, the motion of the point $\sigma(=\sigma(t))$ is provided by evaluating the quantity s (=$s(t)$) defined by (Capitaine et al. 1986):

$$s = \sigma\mathrm{N} - \Sigma_0\mathrm{N} - (\sigma_0\mathrm{N}_0 - \Sigma_0\mathrm{N}_0), \tag{1}$$

where $\sigma_0 = \sigma(t_0)$, N_0 and N are the ascending nodes of the equators at t_0 and t in the equator of CRS (Fig. 1). Then s is given by

$$s = -\int_{t_0}^{t} \frac{X\dot{Y} - Y\dot{X}}{1+Z}\,dt - (\sigma_0\mathrm{N}_0 - \Sigma_0\mathrm{N}_0), \tag{2}$$

the dot denoting the time derivative. A simpler form in polar coordinates is

$$s = \int_{t_0}^{t} (\cos d - 1)\dot{E}\,dt - (\sigma_0\mathrm{N}_0 - \Sigma_0\mathrm{N}_0). \tag{3}$$

To refer the development of $s(t)$ at t_0, it is convenient to use by convention:

$$\sigma_0\mathrm{N}_0 = \Sigma_0\mathrm{N}_0, \tag{4}$$

where σ_0 is on the true equator of epoch, whatever be the model in use. This convention is adopted in the following.

This being done, it remains the constant $s(t_0)$ introduced by the integral. To avoid ambiguities, this constant is kept, its numerical value being given by the development of s in Table 2a. In particular, it is taken into account in the relation (22).

2.5. Earth rotation parameters using the non-rotating origin

For a description of the Earth rotation, the concept of the NRO is applied both in the CRS and in the TRS, for a common rotation axis. Which axis? The axis of the IRP is first considered on account of its simple theoretical definition.

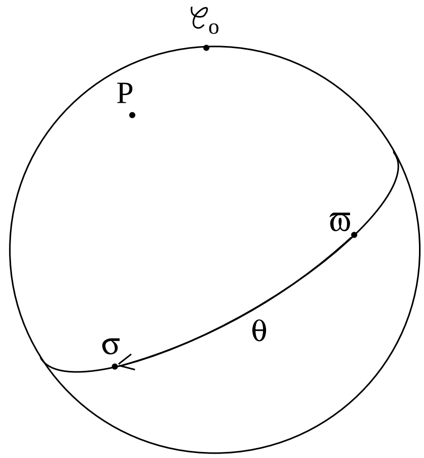

Fig. 2. Definition of the stellar angle

In the CRS, the NRO is provided by the formulae of Sect. 2.4. In the computation of s, the total motion of the true pole has to be used. It is not possible to separate entirely the effects of precession, nutation and also of offset at epoch, because the development of s includes mixed terms which reach a few $0.01''$ after a century. Some of these mixed terms also appear in the equinox formulation. The value of s reaches about $0.04''$ in a century.

In the TRS, to avoid confusion, different notation is used. The NRO is ϖ; it is derived from the position of the origin of longitudes by evaluation of s', with a convention similar to that for s:

$$\begin{aligned} s' &= \varpi\mathrm{M} - \Pi_0\mathrm{M} \\ &= -\int_{t_0}^{t} \frac{u\dot{v} - v\dot{u}}{2}\,dt = \int_{t_0}^{t} (\cos g - 1)\dot{F}\,dt, \end{aligned} \tag{5}$$

M being the ascending node of the equator at t on the equator of TRS, and F, g the polar coordinates of the rotation pole (the third direction cosine w has been taken equal to 1, with an error largely below 1 microarcsecond in several centuries). Some components of s' have to be evaluated, in principle, from the measurements of polar motion. After a century s' reaches about $0.0004''$. The constant $s'(t_0)$ is smaller than 0.5 microarcsecond.

Let us consider the arc θ on the common equators at date *t*, reckoned positively in the retrograde direction from the terrestrial NRO, ϖ, to the celestial NRO, σ (see Fig. 2). The definition of the NRO's ensures that the derivative $\dot{\theta}$ is strictly equal to the instantaneous angular velocity ω of the Earth around the selected polar axis. Thus θ represents rigorously the sidereal rotation of the Earth around this axis. To avoid some ambiguities arising from the adjective *sidereal*, θ has been called the *stellar angle* (Guinot 1979), a denomination kept here. The definition of UT1 by its relationship with mean sidereal time (Aoki et al. 1982) is based on the condition that it be proportional to θ, taking account of the precession only. Although it has not been said explicitly, the concept of NRO has been applied and thus UT1 is, by definition, based only on the pole of rotation. The reference to sidereal time, which involves the motion of the ecliptic, is misleading.

More generally, it must be stressed that, whichever be the choice of the departure points on the moving equators, the concept and use of the NRO is unavoidable for the evaluation of the angular velocity of the Earth, and also for the definition of UT1 if one wishes that the time derivative of UT1 be proportional to ω. Thus the definition of UT1 takes the simple form:

$$UT1 - UT1_0 = k\theta, \tag{6}$$

where k is a constant selected so that, on the average, the Sun crosses the prime meridian at 12 hours UT1.

The use of any other departure point on the equator introduces a spurious instantaneous rotation. This rotation is:

- equal to $\dot{s}$ for the point Σ defined by $\Sigma \mathrm{N} = \Sigma_0 \mathrm{N}$,
- equal to $\dot{E} \cos d$ for the node N,
- of the order of $\dot{s}$ for the intersection, K, of the meridian $\mathcal{C}_0\Sigma_0$ with the moving equator as well as for the intersection, H, of the meridian $\mathrm{P}\Sigma_0$ with the moving equator.

Any of these points can be used as an origin of the orientation angle of the TRS w.r.t. the CRS around the polar axis. If the angular velocity of the Earth and its integral form UT1 is needed, the spurious rotation of this point has to be considered (i.e. the concept of the NRO).

2.6. *Transformation between the celestial and terrestrial systems*

The coordinate transformation to be used from the TRS to the CRS at date t, using the basic quantities referred to the NRO as defined in previous sections is:

$$[CRS] = PN(t).R(t).W(t)\,[TRS]\,, \tag{7}$$

where the fundamental components $PN(t)$, $R(t)$ and $W(t)$ are the transformation matrices arising from the motion of the pole axis in the CRS, from the rotation of the Earth around the pole axis, and from polar motion respectively. They are given below in a similar way as in the IERS Conventions 1996 (McCarthy 1996) and using the notations of 2.2, 2.4 and 2.5:

$$\begin{aligned} W(t) &= R_3(-s').R_1(-v).R_2(u), \\ R(t) &= R_3(-\theta), \\ PN(t) &= R_3(-E).R_2(-d).R_3(E).R_3(s), \end{aligned}$$

$PN(t)$ can be given in an equivalent form involving directly X and Y as:

$$PN(t) = \begin{pmatrix} 1 - aX^2 & -aXY & X \\ -aXY & 1 - aY^2 & Y \\ -X & -Y & 1 - a(X^2 + Y^2) \end{pmatrix} .R_3(s), \tag{8}$$

with

$$a = \frac{1}{(1 + \cos d)}, \tag{9}$$

which can also be written, with a sufficient accuracy as

$$a = \frac{1}{2} + \frac{1}{8}(X^2 + Y^2). \tag{10}$$

3. Instantaneous rotation pole *versus* Celestial Ephemeris Pole

The CEP has been theoretically defined so that it should have no diurnal motions both in CRS and TRS (Seidelmann 1982); its use therefore cancels the effects of diurnal terms in s and s'. Then, the stellar angle θ_c derived from the computation of the NRO's based on the motion of the CEP differs slightly from θ associated with the IRP, this latter representing strictly the sidereal rotation of the Earth. However, it is found that the difference $\Delta\theta = \theta_c - \theta$ is is quite small and purely proportional to t at the microarsecond level:

$$\Delta\theta = -0.000073''(t - t_0), \tag{11}$$

with t in centuries (see 5.3). It can be neglected in geophysical interpretations of the variation of θ (or of UT1 proportional to θ). The linearity in time of $\Delta\theta$ would be kept with a new definition of the CEP to take account of short periods in space and within the Earth.

In view of the advantages of the CEP, it is used henceforward with the associated NROs, CEO and TEO. The arc θ refers to these origins.

4. Computation at a microarsecond accuracy of the basic quantities in the ICRF

4.1. *The celestial pole coordinates, X, Y*

Developments as function of time of the celestial coordinates X and Y of the CEP to be used for consistency with the IAU 1980 nutation series have been given (Capitaine 1990). These developments have been computed from the previous standard expressions for precession and nutation with a consistency of $5 \times 10^{-5''}$ after a century. For consistency with the 1996 IERS nutation series and improved numerical values for the precession rate of the equator in longitude and obliquity, similar developments of X and Y can be derived. They can obtained from the following expressions for X and Y (Capitaine 1990):

$$\begin{aligned} X &= \bar{X} + \xi_0 - d\alpha_0 \bar{Y}, \\ Y &= \bar{Y} + \eta_0 + d\alpha_0 \bar{X}, \end{aligned} \tag{12}$$

where ξ_0 and η_0 are the celestial pole offsets at epoch and $d\alpha_0$ the right ascension of the mean equinox at epoch in the CRS. Quantities $\bar{X}$ and $\bar{Y}$ are given by:

$$\begin{aligned} \bar{X} &= \sin\omega \sin\psi, \\ \bar{Y} &= -\sin\epsilon_0 \cos\omega + \cos\epsilon_0 \sin\omega \cos\psi, \end{aligned} \tag{13}$$

where ϵ_0 is the obliquity of the ecliptic at J2000, ω is the inclination of the true equator of date on the fixed ecliptic of epoch and ψ is the longitude, on the ecliptic of epoch of the node of the true equator of date on the fixed ecliptic of epoch; these quantities are such that:

$$\omega = \omega_A + \Delta\epsilon_1, \qquad \psi = \psi_A + \Delta\psi_1, \tag{14}$$

where ψ_A and ω_A are the precession quantities in longitude and obliquity (Lieske et al. 1977) referred to the ecliptic of epoch and $\Delta\psi_1$, $\Delta\epsilon_1$ are the nutation angles in longitude and obliquity referred to the ecliptic of epoch. $\Delta\psi_1$, $\Delta\epsilon_1$ can be obtained with an accuracy better than one microarcsecond after one century from the nutation angles $\Delta\psi$, $\Delta\epsilon$ in longitude and obliquity referred to the ecliptic of date, following Aoki & Kinoshita (1983) by:

$$\begin{aligned}\Delta\psi_1 &= \frac{(\Delta\psi \sin\epsilon_A \cos\chi_A - \Delta\epsilon \sin\chi_A)}{\sin\omega_A},\\ \Delta\epsilon_1 &= \Delta\psi \sin\epsilon_A \sin\chi_A + \Delta\epsilon \cos\chi_A,\end{aligned} \tag{15}$$

ϵ_A being the precession quantity in obliquity referred to ecliptic of date and χ_A the precession quantity for planetary precession along the equator (Lieske et al. 1977).

In the following the direction cosines X and Y are multiplied by the factor $1296000''/2\pi$, in order to provide in arcseconds the approximate value of the corresponding angles with respect to the polar axis of CRS.

The development as functions of time of X and Y has been performed, retaining all the terms larger than 1 μas after one century in order that the procedure using X and Y be consistent at this level of accuracy with the classical procedure using precession and nutation quantities. The current numerical development of the precession quantities have thus been completed with the needed number of zeros. This numerical development is based on the IERS 1996 series of nutation including planetary nutations, on the current developments of precession with the following corrections to the precession quantities (McCarthy 1996):

$$\delta\psi_A = -0.2957''/\mathrm{c} \qquad \text{and} \qquad \delta\omega_A = -0.0227''/\mathrm{c} \tag{16}$$

and on the following numerical values for the offsets at J2000.0 (IERS Annual Report for 1997):

$$\begin{aligned}\xi_0 &= (-0.01713 \pm 0.00001)'',\\ \eta_0 &= (-0.00507 \pm 0.00001)'',\\ d\alpha_0 &= (0.078 \pm 0.010)''.\end{aligned} \tag{17}$$

The development as functions of time of X and Y has the following form:

$$\begin{aligned}X =&\\ &-0.017130'' + 2004.''193319t - 0.''4271605t^2\\ &-0.''1986210t^3 - 0.''0000461t^4 + 0.''0000058t^5\\ &+\sum_i [(a_{\mathrm{s},0})_i \sin(\text{ARGUMENT}) + (a_{\mathrm{c},0})_i \cos(\text{ARGUMENT})]\\ &+\sum_i [(a_{\mathrm{s},1})_i t \sin(\text{ARGUMENT}) + (a_{\mathrm{c},1})_i t \cos(\text{ARGUMENT})]\\ &+\sum_i [(a_{\mathrm{s},2})_i t^2 \sin(\text{ARGUMENT}) + (a_{\mathrm{c},2})_i t^2 \cos(\text{ARGUMENT})],\end{aligned} \tag{18}$$

$$\begin{aligned}Y =&\\ &-0.''005202 - 0.''0219421t - 22.''4072863t^2\\ &+0.''0018416t^3 - 0.''0000037t^4 + 0.''0000019t^5\\ &+\sum_i [(b_{\mathrm{c},0})_i \cos(\text{ARGUMENT}) + (b_{\mathrm{s},0})_i \sin(\text{ARGUMENT})]\\ &+\sum_i [(b_{\mathrm{c},1})_i t \cos(\text{ARGUMENT}) + (b_{\mathrm{s},1})_i t \sin(\text{ARGUMENT})]\\ &+\sum_i [(b_{\mathrm{c},2})_i t^2 \cos(\text{ARGUMENT}) + (b_{\mathrm{s},2})_i t^2 \sin(\text{ARGUMENT})].\end{aligned} \tag{19}$$

The numerical values of $(a_{s,j})_i$, $(a_{c,j})_i$, $(b_{c,j})_i$,$(b_{s,j})_i$ for $j = 0, 1, 2$ are given by Tables 1a and 1b (only published in electronic form). The amplitudes $(a_{s,0})_i$, $(b_{c,0})_i$ are equal to the amplitudes $A_i \times \sin\epsilon_0$ and B_i of the IERS 1996 series for nutation in longitude $\times \sin\epsilon_0$ and obliquity, except for 20 terms in each coordinate X and Y in which appears a contribution from crossed-nutation effect. The six last terms of Table 1a (from $i = 264$) are complementary terms coming from such crossed-nutation effects. The coordinate X and Y also contains crossed terms between precession and nutation: in each coordinate, the number of terms in $t\sin$ or $t\cos$ is of the order of 120 and the number of terms in $t^2\sin$ or $t^2\cos$ of the order of 25.

It can be noticed that the numerical series provided by Tables 1a and 1b can be easily modified to be consistent with a new series of nutation as soon as it is available.

Developments (18) and (19) of X and Y limited to the terms larger than 0.1 mas are sufficient for an evaluation of s with uncertainty smaller than 1 μas after a century and for the study of the sensitivity of s to the uncertainty of precession quantities and of the positioning of the pole and the equinox at epoch.

4.2. The quantities s, s' positioning the CEO and the TEO on the equator

For pratical computations of s and s', the rigorous formulae (2) and (3) may be replaced by:

$$s(t) = -\frac{1}{2}[X(t)Y(t) - X(t_0)Y(t_0)] + \int_{t_0}^{t} \dot{X}(t)Y(t)dt, \tag{20}$$

with the approximation $Z = 1$. This approximation requires the addition of a small correction of the order of 1 μas after a century:

$$\delta s = -\int_{t_0}^{t} (2a-1)\frac{X\dot{Y} - Y\dot{X}}{2}\, dt, \tag{21}$$

a being expressed by (9). This correction has been applied in the following.

The numerical development of s compatible with the IAU 1980 Theory of Nutation and the Lieske et al. (1977) precession has been given with an accuracy of 0.05 mas after a century (Capitaine 1990).

The numerical development of s compatible with the IERS Conventions 1996 is provided by (20) using the development of

Table 2a. Development of $s(t)$ with all terms exceeding 0.5 μas during the interval 1975-2025 (unit μas)

$$\begin{aligned} s(t) \ = \ & -XY/2 + 2184 + 3810t - 106t^2 - 72573t^3 \\ & + \sum_k C_k \sin\alpha_k (\text{see below}) \\ & + 2t\sin\Omega + 4t\cos 2\Omega \\ & + 744t^2\sin\Omega + 57t^2\sin 2\odot + 10t^2\sin 2☾ - 9t^2\sin 2\Omega \end{aligned}$$

α_k	C_k
Ω	-2641
2Ω	-63
$\Omega+2\odot$	-12
$\Omega-2\odot$	$+11$
$2\odot$	$+4$
$\Omega+2☾$	-2
$\Omega-2☾$	$+2$
3Ω	$+2$
$-l_\odot+\Omega$	-1
$-l_\odot-\Omega$	$+1$
$l_☾+\Omega$	$+1$
$-l_☾+\Omega$	-1
$l_\odot+2\odot$	-1

Table 2b. Complementary terms exceeding 0.5 μas for the intervals 1900-1975 and 2025-2100 (unit μas)

$$\begin{aligned} \Delta s(t) \ = \ & +28t^4 + 13t^5 - 23t^3\cos\Omega - 1t^3\cos 2\odot \\ & + \sum_i D_i t^2 \sin\alpha_i (\text{ see below}) \end{aligned}$$

α_i	D_i
$-l_\odot$	$+6$
$l_\odot+2\odot$	$+2$
$l_☾$	-3
$2☾+\Omega$	$+2$
$l_☾+2☾$	$+1$
$-l_\odot+2\odot$	-1
$2\odot+\Omega$	-1
$-l_☾+2☾$	-1
$-l_☾+2☾-2\odot$	-1

X and Y as functions of time, given by (18) and (19). The constant value $s(t_0)$ has been evaluated to be - 2184 μas. Retaining the terms larger than 0.5 μas, this development is as follows. It is separated into two parts, Table 2a for terms larger than 0.5 μas over 25 years and Table 2b for additional terms extending the development over one century.

In the polynomial development of s, the term in t is mainly due to cross terms nutation $\times$ nutation and the others are precession terms. The periodic and Poisson terms are mainly due to cross terms precession $\times$ nutation except for a few periodic terms due to cross terms nutation $\times$ nutation.

Concerning s', terms arise from the geophysical excitation of the polar motion. Their effect on θ is estimated assuming that the Chandlerian nutation remains in the range of amplitude observed since 1900. Terms with annual and Chandlerian periods are generated by the offset of the mean position of the pole at J2000.0 and by its westwards drift. Their amplitudes should remain smaller than 1 μas until 2100. A secular variation of θ appears, with a rate of about - 50 μas/century.

4.3. The stellar angle, θ

The numerical relationship between the stellar angle θ and UT1 as given by Capitaine et al. (1986), has been derived to be consistent with the conventional relationship between GMST and UT1 (Aoki et al. 1982). It can be chosen as the conventional relationship providing a primary conventional definition of UT1 attached to the ICRF. Completing the numerical coefficients to provide an accuracy of 1 μas gives:

$$\begin{aligned} \theta(T_u) \ = \ & 2\pi(0.7790572732640 \\ & + \ 1.00273781191135448 T_u \times 36525), \end{aligned} \quad (22)$$

where

$$T_u = (\text{Julian UT1 date} - 2451545.0)/36525. \quad (23)$$

5. Sensitivity of UT1 to change in the model of the pole trajectory when using the CEO

5.1. Background

The precession/nutation model used in processing obervations is improved from time to time. The effects on the values of the ERP of these corrections are well known when using the traditional equinox: reduction of the spurious diurnal terms in the terrestrial pole coordinates u, v (when the celestial pole offsets are not determined simultaneously); steps in UT1 and UT1 rate $dUT1/dt$. The step of UT1 can be cancelled by a change of the UT1/GMST relationship, as done in 1984 (Aoki et al. 1982).

We consider here the resulting effects on the terrestrial pole coordinates, and on UT1, when the CEO is used instead of the equinox, when a new model of the celestial pole trajectory is adopted for processing observations. We remark first that the effects on u, v are identical, whether one uses the CEO or the equinox. Only the effects on the stellar angle θ need to be considered.

Assume that the change of model occurs at date t, reckoned in centuries since J2000.0 ($t_0 = 0$). The corrections to the coordinates of the pole are functions of time $\Delta X(t), \Delta Y(t)$. In this application, the position of the pole is sufficiently close to $\mathcal{C}_0$ so that terms in d^4 can be omitted. The correction to θ at date t is then (Capitaine et al. 1986):

$$\begin{aligned} \Delta\theta(t) \ = \ & \Delta s + \frac{1}{2}[X(t)\Delta Y(t) - Y(t)\Delta X(t)] \\ & - \frac{1}{2}[X(t_0)\Delta Y(t_0) - Y(t_0)\Delta X(t_0)], \end{aligned} \quad (24)$$

where Δs is obtained by integration over (t_0, t).

Three applications are considered.

(a) First, in 5.2, $\Delta\theta$ is evaluated for the change from the IAU 1980 Theory of Nutation (Seidelmann 1982; Wahr 1981) and IAU 1976 Precession to the model of the IERS Conventions (McCarthy 1996), for the introduction of the mean pole offset and right ascension of γ at J2000.0, as provided by the 1997 IERS Annual Report. This evaluation is not truly representative of the properties of θ, since the IAU model is known to be largely in error.

(b) Second, in 5.3, $\Delta\theta$ is evaluated for the change from the CEP to the IRP.

(c) Then, in 5.4, an evaluation of the magnitude of $\Delta\theta$ is based on the uncertainties of modern models.

The values of UT1 depend also, in principle, on the errors in the terrestrial pole coordinates u, v through the evaluation of the TEO. This effect will be estimated.

5.2. Change from the IAU model to the IERS model

Retaining the terms in $\Delta\theta$ which exceed $10^{-7''}(0.1\ \mu\mathrm{as})$ in the application, using the development as function of time of the celestial pole coordinates X and Y, with

$$\begin{aligned} X_1 &= 2004,193319''/\mathrm{c}, \\ Y_1 &= -0.022194''/\mathrm{c}, \\ Y_2 &= -22,407286''/\mathrm{c}^2, \end{aligned}$$

$$\begin{aligned} \Delta\theta(t) \;=\; & -[Y_2 t^2 + \sum_i b_i \cos(\omega_i t + \alpha_i)]\xi_0 \\ & + [X_1 t + \sum_i a_i \sin(\omega_i t + \alpha_i)]\eta_0 \\ & + \frac{1}{2} X_1^2 t^2 d\alpha_0 + \frac{1}{2} X_1 \Delta Y_1 t^2 \\ & - t \sum_i b_i \cos(\omega_i t + \alpha_i)\Delta X_1 \\ & + \frac{1}{2} t \sum_i \omega_i (a_i \Delta b_i + b_i \Delta a_i) \\ & + X_1 \sum_i \frac{1}{\omega_i} \sin(\omega_i t + \alpha_i)\Delta b_i - \Delta\theta(t_0). \end{aligned} \tag{25}$$

In Eq. (25), $a_i, b_i, \omega_i, \alpha_i$ are the amplitudes, angular velocities and phases of the nutations. One can use the values $a_i = (a_{s,0})_i, b_i = (b_{c,0})_i$ provided by Table 1a (only published in electronic form). Symbol Δq represents the difference of values of q in the sense IERS model minus IAU model.

On the other hand, the IERS Conventions (1996) provide correction to precession quantities which leads to:

$$\begin{aligned} \Delta X_1 &= 0.1176''/\mathrm{c}, \\ \Delta Y_1 &= -0.0227''/\mathrm{c}, \\ \Delta Y_2 &= 0.0026''/\mathrm{c}^2. \end{aligned}$$

Using these values, with t in centuries as well as the values (17) of the offsets at J2000.0 and the values of Δa_i and Δb_i, $\Delta\theta$ is as follows, with indication of the origin of the components. The unit is the microarcsecond.

$$\begin{aligned} \Delta\theta \;=\; & \\ -50t + 2t^2 \quad & : \ \text{precession} \times \text{mean pole offset at } t_0 \\ +4t^2 \quad & : \ \text{precession} \times \text{equinox offset} \\ -119t^2 \quad & : \ \text{precession in longitude} \\ & \quad \times \text{precession in obliquity} \\ +5t \quad & : \ \text{nutation errors} \\ +1\cos(\omega_1 t + \alpha_1) \quad & : \ \text{nutation} \times \text{offset} \\ -1\sin(\omega_1 t + \alpha_1) \quad & : \ \text{nutation} \times \text{precession} \\ +5t\cos(\omega_1 t + \alpha_1) \quad & : \ \text{nutation} \times \text{precession} \\ -1t\cos(\omega_3 t + \alpha_3) \quad & : \ \text{nutation} \times \text{precession} \\ +1 \quad & : \ -\Delta\theta(t_0) \end{aligned} \tag{26}$$

In this expression, subscripts 1 and 3 designate the nutations with periods 18.6 y and 13.67 d. The largest term originates from the cross term between precession in longitude and the secular term in obliquity which was not considered in the IAU model. After a century, it changes the rate $dUT1/dt$ by $5.0\ 10^{-15}$ when using a fixed relationship $UT1/\theta$. The second largest term is linear in t and the corresponding change of $dUT1/dt$ is $1.0\ 10^{-15}$. This is to be compared to the relative inaccuracy in frequency of cesium time standards in 1999: $2.0\ 10^{-15}$.

5.3. Change from the CEP to the IRP

The difference between the stellar angle computed for the IRP instead of the CEP can be provided by (25), using this formula both in the CRS and the TRS. It is indeed necessary to take into account the change of the pole coordinates in the CRS as well as the corresponding "diurnal nutation" in the TRS, the amplitudes of the periodic terms in the CRS and the TRS being linked as a consequence of the kinematical properties of the IRP.

In the application of (25) in the CRS, the differences $\Delta a_i, \Delta b_i$ are for the change of the amplitudes of the nutation i in the celestial pole coordinates from the CEP to the IRP (i.e. the opposite of the so-called "Oppolzer terms"). In the TRS, a_i, b_i are for the amplitudes of the diurnal terms of the polar motion arising from the use of the IRP. It leads, in $\mu\mathrm{as}$, to:

$$\Delta\theta = +73t. \tag{27}$$

5.4. Application to modern models

The uncertainty of quantity q is, in this subsection, denoted by $\sigma(q)$; the IAU Working Group on Nutation gives in its report (1998) the following values of the uncertainties of the 1996 IERS model for precession-nutation:

- on the precession in longitude, $\pm 0.020''/\mathrm{c}$, to which correspond the uncertainties on X_1 and Y_2:
  $\sigma(X_1) = 0.008''/\mathrm{c}$, $\sigma(Y_2) = 0.0002''/\mathrm{c}^2$,
- on the amplitudes of nutations, $0.0004''$ for the 18.6 y-term, $0.00001''$ for the terms of periods 1 y, 0.5 y and 13.67 d, and less for other terms.

The uncertainties on the celestial pole and equinox offsets at epoch t_0 are given by (17). The uncertainty of the precession in obliquity can be estimated to be lower than $0.001''/\mathrm{c}$, to which correspond $\sigma(Y_1) \leq 0.001''/\mathrm{c}$. With these values, the uncertainties on the terms of (25) are as follows, the unit being the microarsecond as previously:

$$\begin{aligned}
Y_2 t^2 \sigma(\xi_0) &: \; 0.001 t^2 \\
X_1 t \sigma(\eta_0) &: \; 0.10 t \\
\frac{1}{2} X_1^2 t^2 \sigma(d\alpha_0) &: \; 0.5 t^2 \\
\frac{1}{2} X_1 \sigma(Y_1) t^2 &: \; \leq 5 t^2 \\
b_i \sigma(\xi_0) \text{ and } a_i \sigma(\eta_0) &: \; \leq 0.0004 \\
t b_i \sigma(X_1) &: \; \text{ maximum } 0.36 t, \\
&\quad\; \text{for the 18.6 y term} \\
\frac{1}{2} t \omega_i (a_i \sigma(b_i) + b_i \sigma(a_i)) &: \; 0.52 t, \text{ for the 18.6 y term} \\
&\quad\; 0.30 t, \text{ for the 0.5 y term} \\
&\quad\; 0.78 t, \text{ for the 13.67 d term} \\
&\quad\; \leq 0.1 t, \text{ for other terms} \\
(X_1/\omega_i) \sigma(b_i) &: \; \text{ maximum } 0.11 t, \\
&\quad\; \text{for the 18.6 y term.}
\end{aligned} \tag{28}$$

The largest contribution to the uncertainty in θ comes from the uncertainty of the secular term in obliquity; it may reach 5 microarseconds (0.3 μs in UT1) in 2100. The corresponding uncertainty on the relative frequency of UT1 is at the level of $2.0\ 10^{-16}$.

Concerning polar motion in the TRS, if the amplitude of Chandlerian nutation shows larger variations than observed in the past, its contribution may have to be evaluated. This might be a task of the IERS. Anyway, the extrapolation over a few years should not lead to errors larger than 1 μas.

6. Conclusion

The concept of non-rotating origin applied to the motion of the Celestial Ephemeris Pole (CEP) leads to the Celestial Ephemeris Origin (CEO) in the celestial reference system (CRS) and to the Terrestrial Ephemeris Origin (TEO) in the terrestrial reference system.

The CEO depends on the whole history of the motion of the CEP in the CRS. This may appear as a disadvantage with respect to a purely geometrical definition of a departure point on the moving equator based only on the position of the pole at the considered date t. However, it has been shown that, even at the level of accuracy of a few microarcseconds over one century, developments in terms of t leading to the CEO remain simple. Moreover, these developments are practically insensitive to possible improvements of modern models for precession and nutation and of evaluation of mean pole and mean equinox offsets at epoch. Developments valid at the microarcsecond, based on the best model for precession, nutation and pole offset at J2000.0 with respect to the pole of ICRF, are provided for computing the CEP coordinates and the position of the CEO.

The TEO provides a strictly defined origin in the TRS. Its position depends in principle on the measurements of the polar motion, but it can be extrapolated over many years with an uncertainty smaller than 1 microarcsecond.

The concept of non-rotating origin on which is based the CEO and the TEO is unavoidable to derive the angular velocity ω of the Earth from observations and for defining UT1, if one wishes that the time derivative of UT1 be proportional to ω. If other types of departure point on the moving equator are used, their relationship with the CEO and TEO have to be provided. In addition, the definition of UT1 based on the CEO does not require modifications in order to maintain the continuity of UT1 when the models for precession, nutation, pole offset and equinox offset are improved. Other definitions, especially that based on the equinox, are sensitive to these modifications.

In the case where the orientation of the Earth in space is described by Eulerian angles, if the angular velocity of the Earth and its integral form UT1 is still needed, the concept of the CEO is necessary to derive these quantities from the new Earth orientation parameters.

It can be stressed that, due to the kinematical definition of the NRO, all the developments relative to the NRO can be easily extended to the GR framework with respect to a kinematically non-rotating celestial frame.

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