THE MASS OF THE ASTEROID 15 EUNOMIA FROM OBSERVATIONS OF 1313 BERNA AND 1284 LATVIA

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ABSTRACT

The mass of the asteroid 15 Eunomia was determined from its perturbations of 1313 Berna and 1284 Latvia. The perturbation of Berna gives a mass for Eunomia of $(4.2\pm1.1)\times10^{-12}$ $M_\odot$. The perturbation of Latvia gives an upper bound on the mass for Eunomia of 8×10^{-12} $M_\odot$. The mass determined using Berna gives a density of only 0.79 ± 0.21 g cm^{-3} assuming that Eunomia is a homogeneous sphere with a diameter of 272 km. However, Eunomia is not spherical; hence, its volume is very poorly known. Since a sphere has the highest volume to mean projected area ratio of any body the density of Eunomia may be much higher than indicated. Radar range observations will determine the position of the perturbed asteroid to a higher precision than the uncertainty in the current ephemerides. The estimated reduction in the uncertainty for the mass of Eunomia from a single ideal radar time delay measurement of Berna is a factor of 2. This reduction in the uncertainty makes radar ranges the best way to reduce the uncertainty in the mass of Eunomia. Both Latvia and Berna will be in position to be observed by the Arecibo radio telescope at opposition during 1998 and 2000. [S0004-6256(97)03307-4]

1. INTRODUCTION

15 Eunomia, mean diameter 272 ± 6 km (Tedesco 1989), is the eighth largest asteroid and the largest of the S-type asteroids. Eunomia is a prime target for mass determination since there has been no previous mass determination for an S-type asteroid. Hilton *et al.* (1996), henceforth HSM96, presented a list of 460 encounters between asteroids that should be useful in determining the masses of up to 34 asteroids. Twenty-six encounters were found involving Eunomia. The best encounters involved 1313 Berna and 1284 Latvia. All three of these asteroids are in similar orbits (Table 1), so Eunomia has encountered both Berna and Latvia on numerous occasions (HSM96; Scholl *et al.* 1987).

Section 2 presents a determination of the mass of Eunomia from current observations of Berna and Latvia, Sec. 3 discusses the problems in these two mass evaluations and what can be done to improve the determination of the mass of Eunomia, and Sec. 4 presents the conclusions.

2. THE MASS OF EUNOMIA

Mass determinations of Eunomia were made using the ephemeris-generating software known as the Planetary Ephemeris Program or PEP (Ash 1965). PEP is a high precision ephemeris program capable of simultaneously integrating up to 10 bodies and include perturbations from an additional 20 bodies, and fitting the ephemerides to several different data types including, but not limited to, transit telescope, photographic astrometry, and radar range data. PEP is capable of solving for many model parameters including orbital elements and the masses of perturbing bodies, if desired.

For the PEP integrations, the ephemerides of the Moon and the perturbing planets were provided by the JPL ephemeris DE200 (Standish 1990), and the masses of the planets used were the masses incorporated in the production of DE200. DE200 was chosen for the ephemerides of the perturbing planets because it is the standard planetary ephemeris used, for example in the *Astronomical Almanac*. There are more recent ephemerides such as DE403 (Standish *et al.* 1995); however, the differences in asteroid orbits caused by using these ephemerides rather than DE200 are orders of magnitude smaller than the uncertainties in the orbits of the asteroids themselves.

Perturbations due to the asteroids 1 Ceres, 2 Pallas, 4 Vesta and Eunomia were included in the force model. The ephemerides for Ceres, Pallas, and Vesta were taken from previous integrations, fit to transit circle observations made by the Royal Greenwich Observatory (1897–1940), the Carlsberg Meridian Catalog (1984–1995) (CAMC), the U.S. Naval Observatory at Washington and El Leoncito, Argentina (USNO 1949, 1964, 1968, 1982, 1992), and the Bordeaux University in Floriac, France taken from the Minor Planet Center. The masses used for Pallas and Vesta were the same as used in DE200. However, the Viateau & Rapaport (1995) mass for Ceres, 5.0×10^{-10} $M_\odot$, was used because modern determinations of its mass are significantly different from the one used to generate DE200. The ephemeris for Eunomia was integrated and fit to 954 observations acquired from the Minor Planet Center and the CAMC. These observations cover the period 1864 July 21 through 1995 March 7. The initial mass estimate for Eunomia, 1.6×10^{-11} $M_\odot$, was based on a spherical body with a radius of 272 km (Tedesco 1989) and an assumed density of 3 gm cm^{-3}.

TABLE 1. Orbital elements for 15 Eunomia, 1284 Latvia, and 1313 Berna in the ecliptic coordinate system.

Asteroid	a^a (AU)	e^a	i^a (°)	Ω^b (°)	ω^b (°)
Eunomia	2.64367	0.1469	13.168	293.55685	97.21384
Latvia	2.64511	0.1507	12.011	303.26905	114.38917
Berna	2.65669	0.1688	14.093	298.76446	98.77112

[a]Mean distance, eccentricity, and inclination are proper elements from Knezević & Milani (1989).
[b]The longitude of the ascending node and argument of perihelion are osculating elements on the mean equator and equinox of J2000.0 taken from Ephemerides of the Minor Planets (1996).

2.1 *Mass Determined by Perturbations of 1313 Berna*

Berna underwent a series of close encounters with Eunomia in the recent past. Figure 1 shows the log acceleration (dotted line) and change in speed (solid line) of Berna, per unit solar mass of Eunomia as a function of time from the discovery of Berna in 1933 through 2000. Based on a mass of 1.6×10^{-11} $M_\odot$ for Eunomia, the acceleration of Berna between 1954 to 1970 is from 10^{-13} to 10^{-12} AU day^{-2}. This acceleration is sufficient that if all of the change in velocity goes into the mean motion in the same direction, the change in the position of Berna will be about 12″ after 40 years from that of the unperturbed orbit.

The Minor Planet Center catalogs 47 observations of Berna made between 1911 July 19 and 1991 January 17. Forty of these observations between 1933 August 24 and 1991 January 17 are of high enough precision to be used in making a mass determination of Eunomia based on the perturbation of Berna. Stone (1996) made eleven additional observations of Berna during the most recent opposition in 1996 March. The mass obtained for Eunomia from the 51 remaining observations of Berna was 4.4×10^{-12} $M_\odot$ with a formal uncertainty of 1.1×10^{-12} $M_\odot$.

There is a strong correlation between the mass of Eunomia and the catalog corrections for the observations from Uccle. Why? Only 19 astrometric observations of Berna were made before the close encounter of 1961 to 1969 between Berna and Eunomia. Thirteen of these pre-encounter observations were made at Uccle. The Uccle observations, which make up all observations prior to 1954 were all made during the first two oppositions (1933 and 1935) in which Berna was observed. The mean motion of an asteroid must be inferred from its change in position on the sky so the determination of the mean motion of Berna prior to its encounter with Eunomia, and the resulting mass of Eunomia, are both dependent on the determination of systematic errors in the data from Uccle. Four different solutions were made for the mass of Eunomia to determine if there were any significant systematic errors in the Uccle observations. These solutions were:

(1) The observations of Berna and Eunomia from Uccle are treated as part of the same set in making a determination for the catalog corrections for Uccle.

(2) The catalog corrections for Berna from Uccle are treated as independent from the Uccle observations of Eunomia because the Eunomia observations were made nearly 40 years after the Berna observations.

(3) Three observations of Berna made at Uccle producing outlying residuals, evident in Fig. 2, were removed. The observations of Berna and Eunomia were treated as a single set for making catalog corrections.

(4) The outlying observations were removed and the catalog corrections of Berna from Uccle were treated independently from the Eunomia observations.

The results of these four solutions are shown in Table 2.

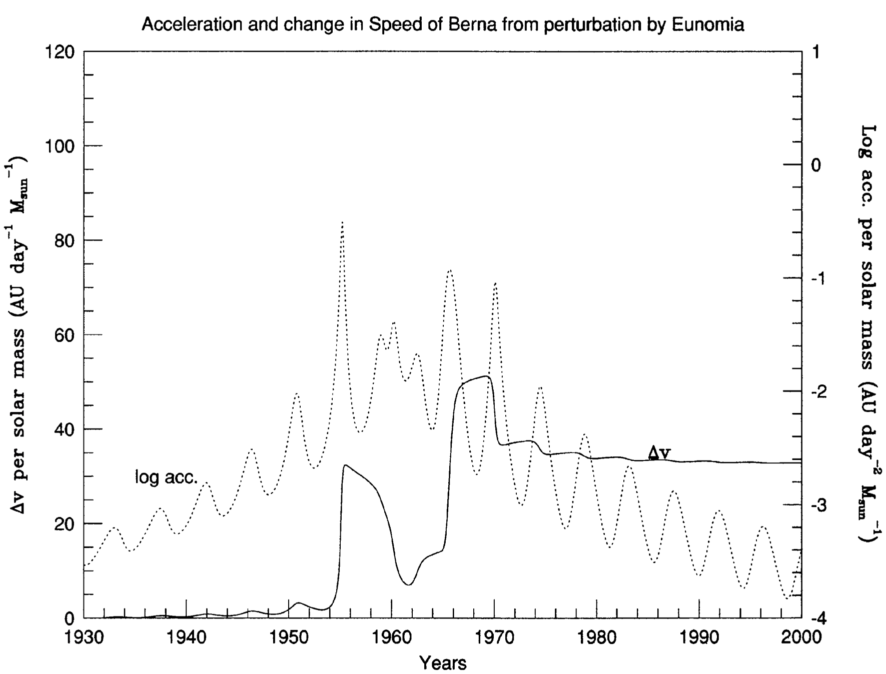

FIG. 1. The log acceleration (dotted line) and change in velocity (solid line) per unit solar mass caused by 15 Eunomia on 1313 Berna.

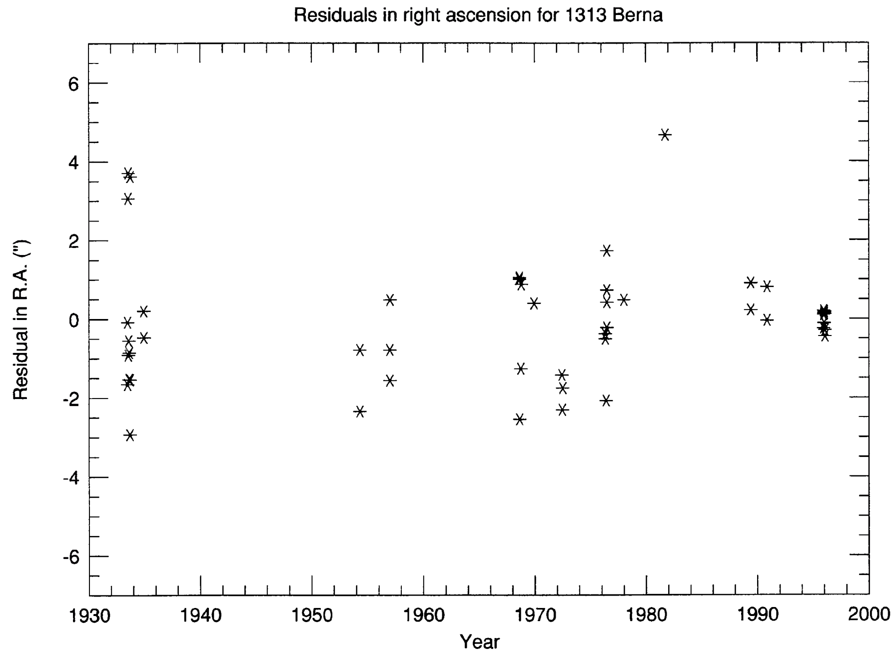

FIG. 2. Residuals in right ascension for all observations of 1313 Berna.

The models show that the initial solution was biased by the treatment of early and late observations from Uccle as having the same catalog corrections. The outlying observations had no significant effect on the solution. The rms value of the residuals was 1.″2 including the outlying observations and 1.″0 without them. If perturbations from Eunomia are not included, the rms value for the residuals in right ascension is 1.″6 including the outlying observations and 1.″4 without. Calculating the Uccle catalog corrections for Berna independent of those for Eunomia changed the mean residuals less than 0.″02 in all cases.

The mass adopted here for Eunomia is $(4.2\pm1.1)\times10^{-12}$ $M_\odot$; however, the difference between this mass and the other derived masses is insignificant.

The density of Eunomia, based on the Tedesco 1989 mean diameter and assuming a spherical body, is only 0.79 ± 0.23 g cm^{-3}. This density is very low and would imply that Eunomia consists of a rubble pile with a fill factor of about 0.3. This scenario is unlikely. There are two problems in determining the density of Eunomia.

First, the density of Eunomia is based on the assumption that it is a spherical body. Fig. 3, taken from Ostro & Connely (1984), shows the shape of Eunomia based on visual and infrared variations in its light curve. This shape is highly non-spherical, so a determination of its volume based on a spherical shape is going to be subject to large errors. For example, a tetrahedron with the same mean projected area as a sphere has only 49% of the sphere's volume. Thus, if Eunomia were a tetrahedron it would have a density of 1.6 g cm^{-3}.

Second, there may be some residual systematic error in the observations of Berna that has caused the mass of Eunomia determined from its perturbation of Berna to be low. It takes a change of only 5×10^{-4}″day^{-1} in the determination of the mean motion of Berna, resulting in a change of 7″ in the mean longitude after 40 years, to change the mass determined for Eunomia by 10^{-11} $M_\odot$. The easiest way to assure that the mass is not subject to systematic errors in the observations of the perturbed asteroid is to confirm the mass by making a determination using a different perturbed asteroid. Formal uncertainties, such as those used here, underestimate the actual uncertainties by a factor of two or three, so a second mass determination is very desirable. For Eunomia another highly perturbed asteroid exists: 1284 Latvia.

TABLE 2. The mass of Eunomia determined for a fixed value of the mean distance of 1284 Latvia.

Model	Mass of Eunomia 10^{-11} $M_\odot$	Uccle Equinox Correction (″)	Uccle Equator Correction (″)
1	1.7 ± 0.9	0.5 ± 0.7	0.4 ± 0.4
2	4.4 ± 1.1	1.6 ± 1.0	1.0 ± 0.6
3	3.6 ± 0.9	0.1 ± 0.6	0.6 ± 0.4
4	4.2 ± 1.1	0.6 ± 1.0	1.3 ± 0.6

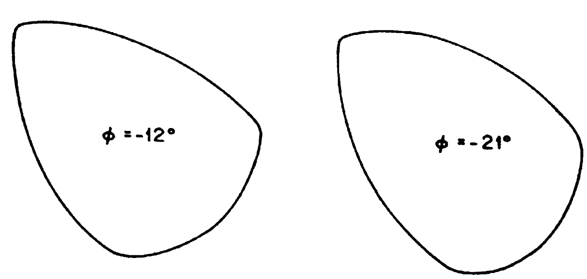

FIG. 3. The shape of 15 Eunomia derived from its rotation light curve at two solar phase angles. Taken from Fig. 7 of Ostro & Connely (1984).

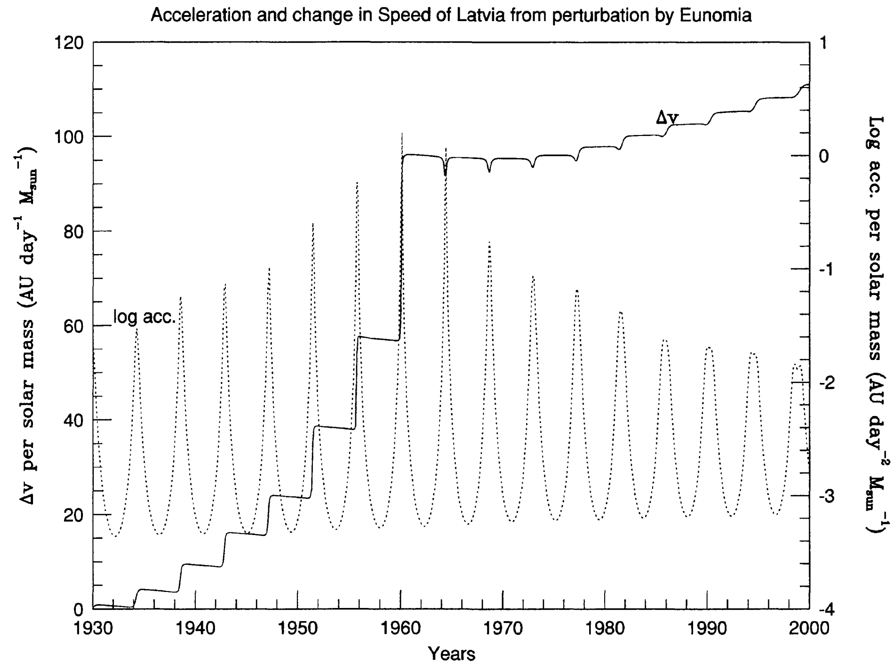

FIG. 4. The log acceleration (dotted line) and change in velocity (solid line) per unit solar mass caused by Eunomia on 1284 Latvia.

2.2 Mass Determined by Perturbations of 1284 Latvia

Latvia, discovered shortly before Berna, is also in an orbit very similar to that of Eunomia (Table 1). Figure 4 shows the log acceleration and change in speed of Latvia caused per unit solar mass of Eunomia. If the change in the velocity vector all goes into changing the mean motion in the same direction, the change in longitude of Latvia, based on the mass of Eunomia found from perturbation of Berna, should be about 9″ at 40 years after the closest encounter, some three times greater than the change for Berna.

The Minor Planet Center provided a total of 91 observations of Latvia covering the period from 1925 through 1991. Fifty-three observations from 1933 July 27 through 1991 February 12 were of sufficient astrometric quality to be useful in making a mass determination for Eunomia. Eleven additional observations from the opposition in 1996 April

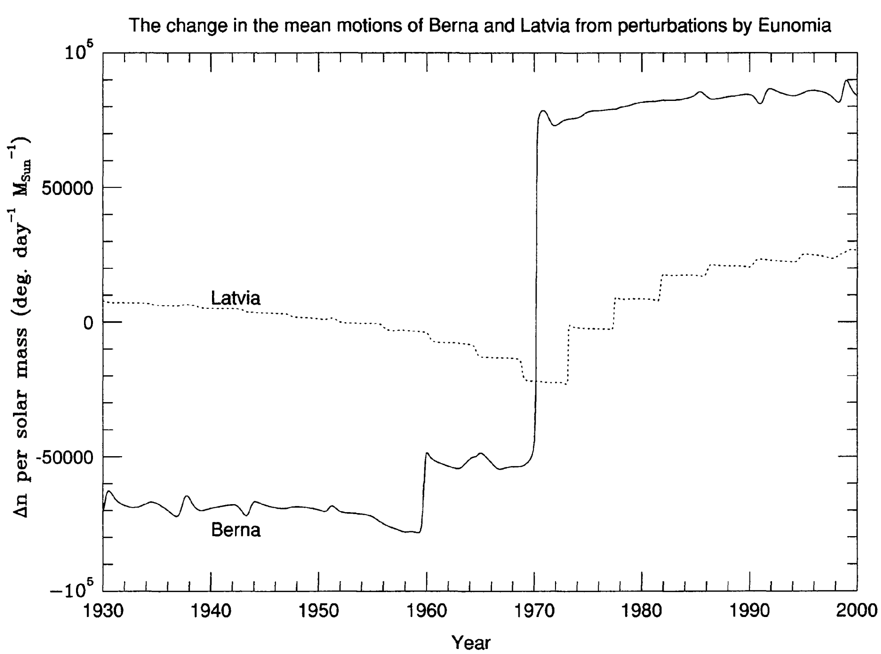

FIG. 5. Change in the semi-major axes for Latvia (dotted line) and Berna (solid line) from encounters with Berna. The amplitude of the changes depend linearly on the mass of Eunomia, hence arbitrary units are used.

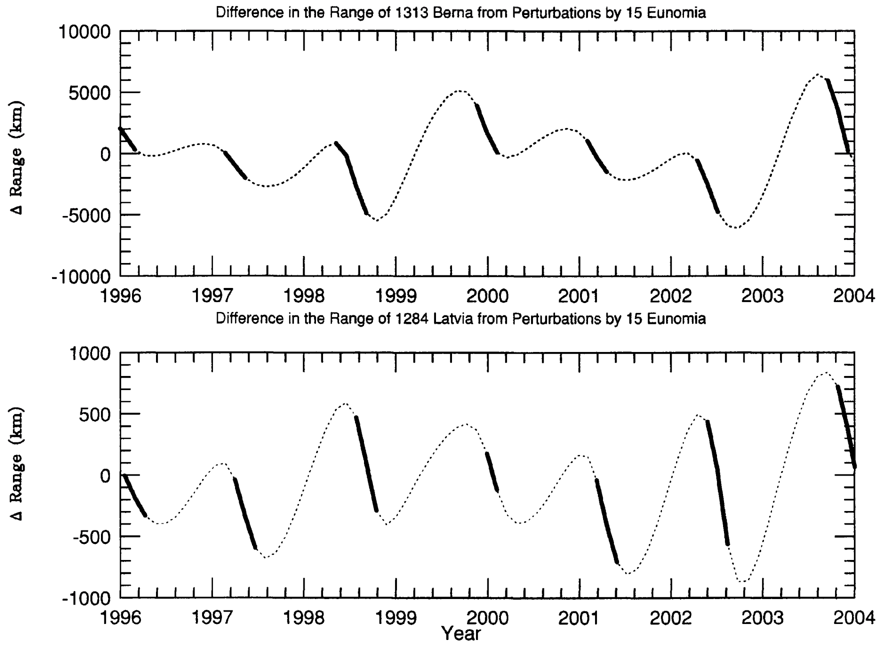

FIG. 6. Change in the ranges of 1313 Berna and 1284 Latvia from perturbations by 15 Eunomia. The heavy lines indicate where Latvia is within 30° of opposition.

and May were provided by Stone (1996) for a total of 64 observations.

Unlike Berna, no single observatory plays a disproportionate role in the determination of the mass of Eunomia.

The mass found for Eunomia was $(1.5\pm6.9)\times10^{-12}$ $M_\odot$. Because of the size of the uncertainty, this estimate can only act as an upper bound on the mass of Eunomia. It does show, however, that the mass of Eunomia is significantly less than initially estimated. The mass determined from Berna is well within the upper bound set by the perturbation of Latvia.

The rms value for the residuals of the observations of Latvia was 0".9.

3. DISCUSSION

At this point two questions need to be asked. First, why is the mass determination using observations of Latvia so much poorer than that produced using observations of Berna? Second, what can be done to reduce the uncertainties and produce a more reliable mass for Eunomia?

There are two possible reasons for the difference in the uncertainties produced by the two different solutions for the mass of Eunomia. Either the observations of Latvia are generally of a poorer quality or there is a major difference in the encounters that makes the determination of the mass of Eunomia from its perturbation of Latvia less sensitive than that from the perturbation of Berna.

The problem does not lie in the observations. First, there were 28% more observations of Latvia than Berna used in determining the mass of Eunomia. Second, the residuals for Berna (after including Eunomia as a perturbing body) are 10%–33% larger than those for Latvia.

3.1 Changes in the Mean Motion

Figure 5 shows the change in the mean motions of Latvia and Berna as a result of perturbations by Eunomia. The change in the mean motion scales linearly with the mass of Eunomia, leaving the shapes of both curves unchanged. For the mass of Eunomia determined by Berna, height of the overall change for Berna is 0".3 day^{-1}. The evolution of the mean motions of Latvia and Berna is quite different. Almost all of the effect on Berna occurs in two large jumps in the same direction. For Latvia, however, the jumps are much smaller, continue over at least twelve encounters, and are somewhat symmetric about the closest approach near 1970. The long term effect of Eunomia on Berna is about five times greater than on Latvia because of the smaller long term change in the mean motion of Latvia. A series of test

TABLE 3. Uncertainty in the mass of Eunomia when including simulated radar delay measurements.

Berna Number of Oppositions	Total Number of Observations	$\Delta M_{Eunomia}$ (10^{-13} $M_\odot$)
1	1	6.6
1	4	5.3
2	2	5.4
2	8	5.2
Latvia		
Number of Oppositions	Total Number of Observations	$\Delta M_{Eunomia}$ (10^{-12} $M_\odot$)
1	1	3.9
1	4	3.8
2	2	3.9
2	8	3.4
3	12	1.3

TABLE 4. Osculating[a] equatorial orbital elements for Eunomia, Latvia, and Berna.

15 Eunomia Element	Value	Uncertainty	Units
Mean distance	2.642245210	4×10^{-9}	AU
Eccentricity	0.18726113	3×10^{-8}	
Inclination	30.022063	3×10^{-6}	degrees
Ascending Node	338.077359	6×10^{-6}	degrees
Argument of perihelion	50.43045	1×10^{-5}	degrees
Mean anomaly	204.441564	9×10^{-6}	degrees
1313 Berna Element	**Value**	**Uncertainty**	**Units**
Mean distance	2.65526446	7×10^{-8}	AU
Eccentricity	0.2076096	3×10^{-7}	
Inclination	31.35195	4×10^{-5}	degrees
Ascending Node	338.53004	4×10^{-5}	degrees
Argument of perihelion	56.6900	1×10^{-4}	degrees
Mean anomaly	175.9654	2×10^{-4}	degrees
1284 Latvia Element	**Value**	**Uncertainty**	**Units**
Mean distance	2.6438975	1×10^{-7}	AU
Eccentricity	0.1714928	2×10^{-7}	
Inclination	30.72072	3×10^{-5}	degrees
Ascending Node	341.97128	4×10^{-4}	degrees
Argument of perihelion	73.7694	1×10^{-4}	degrees
Mean anomaly	183.7839	1×10^{-4}	degrees

[a]The epoch for the osculating elements is 1996 November 11.

solutions confirmed that the change in Berna's mean motion as a function of Eunomia's mass was a factor of five greater than for Latvia.

3.2 Improving the Mass of Eunomia

The uncertainty in the mass of Eunomia in absolute value is smaller than that for any mass determination for Ceres, Pallas, or Vesta, but as a percentage of the mass of Eunomia, it is still quite high. Therefore, it is worthwhile to ask what can be done to improve the mass.

A reduction in the uncertainty of the mean motion of the perturbed asteroid would significantly reduce the uncertainty in the mass of Eunomia. There are three ways to reduce the uncertainty of the mean motion: (1) radar delay observations of the perturbed asteroid, (2) re-examination and discovery of old observations, and (3) future optical observations.

Both Latvia and Berna have mean radii of about 40 km (Tedesco 1989) and a distance from the Earth at mean opposition of 1.65 AU. Hence they both should be within the detection threshold of the improved Arecibo radar system. Both asteroids will be within the declination range of Arecibo during the years 1998 and 2000 at the same time they are near opposition. The greatest source of uncertainty in radar time delay measurements of these asteroids is the shape of the target. It is assumed that the uncertainty in the center of reflection with respect to the center of mass of the perturbed asteroid is 1/4 the mean diameter of the asteroid or 10 km. Figure 6 shows the change in the expected radar range of Berna caused by perturbations due to Eunomia. The portion with the heavy line indicates where Berna is within 30° of opposition.

A test was conducted to determine the amount of improvement that can be expected from radar delay measurements. Simulated radar delay observations were generated by computing a set of radar ranges from an ephemeris for the perturbed asteroids, adding random noise, and then generating a new ephemeris including the simulated radar observations. The times chosen for the radar ranges were while Berna and Latvia were within 30° of opposition and within the declination band of Arecibo. The results from the use of simulated data in Table 3 are quite encouraging. Including a single ideal radar delay measurement chosen at random within the available window reduced the uncertainty in the mass of Eunomia by nearly a factor of 2 for either perturbed asteroid. Additional radar ranges would be necessary to remove biases not included in the idealized observations as well as further improve the asteroid ephemerides.

The numbers of astrometric observations of Berna and Latvia are so small that discovery of additional observations of the perturbed asteroids would be useful in reducing the uncertainties in their orbits which, in turn, would refine the mass of Eunomia. However, to produce the same reduction in uncertainty that a few radar range measurements would provide requires about 150 additional observations to be found for either Berna or Latvia. For Berna there is the possibility of improvement if the observations of Berna from Uccle can be remeasured and reduced using a modern astrometric catalog for the reference star positions. However, these observations can only improve the pre-encounter orbit of Berna.

Future optical observations would also reduce the uncertainty in their post-encounter orbits. To reduce the uncertainty in the position of a perturbed asteroid by a factor of 2 using optical observations, however, would require observations made regularly over the next 30 years. Thus radar range observations of the perturbed asteroids are by far the best means of improving the mass of Eunomia.

4. CONCLUSIONS

The mass of the S-type asteroid 15 Eunomia was determined to be $(4.2\pm1.1)\times10^{-12}\ M_\odot$ from observations of the perturbed asteroid 1313 Berna. The uncertainty given is the formal value determined from the least-squares process and may underestimate the true uncertainty by a factor of 2 or 3. From observations of the asteroid 1284 Latvia the upper bound for the mass of Eunomia was determined to be $8\times10^{-12}\ M_\odot$. These both indicate a mass much smaller than the mass expected based on a spherical body with the *IRAS* value for the mean radius of Eunomia and a density of 3 g cm^{-3}. However, Ostro & Connely (1984) indicate that Eunomia is far from spherical and Reed *et al.* (1997) indicate that it is probably not homogeneous. Hence its volume and mean density remain highly uncertain.

Analysis of how the mean motions of Latvia and Berna were changed by the perturbation of Eunomia shows two different reactions. For Berna most of the change occurred in two large jumps in the mean motion both in the same direction. For Latvia the change in the mean motion occurred in a series of small jumps at each of several close encounters. The size and direction of the jumps were somewhat symmetrical over time, so the final mean motion of Latvia was nearly the same as the initial one. Hence the change in the longitude caused by the perturbation of Eunomia is greater for Berna than for Latvia.

Reducing the uncertainty in the mean motion of Latvia and Berna will reduce the uncertainty in the mass of Eunomia. This can best be done using radar delay measurements. Numerical experiments were made to determine how much improvement can realistically be expected from a series of such radar observations. A single ideal radar delay measurement of Berna can reduce the uncertainty in the mass of Eunomia by half. Both Berna and Latvia will be within the declination band observable by Arecibo at oppositions in 1998 and 2000.

Additional reduction in the uncertainty should come from either the discovery of additional pre-encounter observations, remeasurement of plates of Berna made at Uccle, or making future optical observations. However, radar range measurements offer the best method to reduce uncertainty in the mass of Eunomia.

The best-fit osculating elements for Eunomia, Latvia, and Berna for 1996 November 13 are given in Table 4.

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