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Auroral Observations on AD 1770 September 16: the Earliest Known Conjugate Sightings

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SUMMARY

Observations of the *aurora australis* on the night of AD 1770 September 16 have been recorded in the journals of Joseph Banks and Sydney Parkinson, written on board *HMS Endeavour* during the first voyage of Captain James Cook to Australia. Both descriptions of the *aurora australis* refer to a red light or glow in the southern sky, accompanied by rays, or stripes, of a brighter coloured light extending directly upwards. Observations of the *aurora borealis* on the same night have been recorded in Chinese provincial histories, which refer to auroral displays in the northern sky. The Chinese provincial histories also indicate that the *aurora borealis* was observed each night during the interval 1770 September 16–18, and include several references to a red light in the north or north-west. Japanese histories indicate that red auroral displays were also observed from several places in Japan on the night of 1770 September 17. Assuming that the red light seen in both hemispheres was predominantly 630-nm ('red line') emission from excited atomic oxygen, the magnetic field model of Bloxham & Jackson, valid for the interval 1690–1840, is used to show that these early contemporaneous auroral descriptions are consistent with conjugate auroral observations during an intense geomagnetic storm. These observations provide the earliest example yet known of conjugate auroral sightings.

1 OBSERVATIONS OF THE *AURORA AUSTRALIS*

Important early scientific observations of the *aurora australis* occurred during the first voyage of Captain James Cook to Australia on board *HMS Endeavour*. Observations were recorded both by (Sir) Joseph Banks, botanist and zoologist on the expedition, and by Sydney Parkinson, botanical or natural history draughtsman. The *Endeavour* Journal of Joseph Banks 1768–1771 contains the following entry on AD 1770 September 16, which is presented as a verbatim rendering of the relevant text in the holograph manuscript journal (1):

> About 10 O'Clock a Phænomenon appeard in the heavens in many things resembling the Aurora Borealis but differing materialy in others: it consisted of a dull reddish light reaching in hight about 20 degrees above the Horizon: its extent was very different at different times but never less than 8 or 10 points of the compass. Through and out of this passd rays of a brighter colourd light tending directly upwards; these appeard and vanishd nearly in the same time as those of the Aurora Borealis, but were entirely

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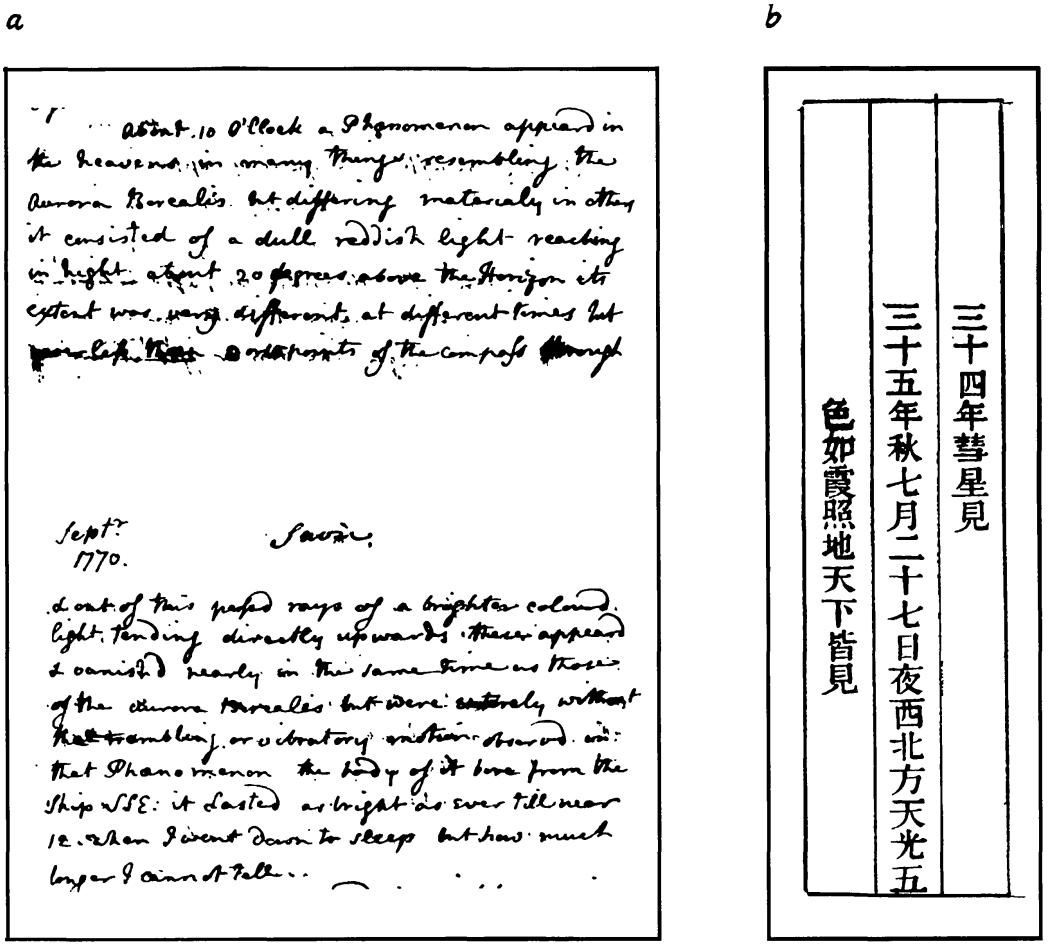


FIG. 1. Conjugate auroral observations on AD 1770 September 16: part of the 'raw data' recorded in contemporary histories and journals. (a) Description of the *aurora australis*, as recorded in the holograph manuscript journal of (Sir) Joseph Banks, according to visual observations made by Banks himself on board *HMS Endeavour* (approximate geographic location: 10.°5 S, 122.°8 E). (b) Description of the *aurora borealis* (second and third columns from the right), as recorded in the history of Ji-zhou county in Hebei province, China, according to visual observations actually made at Ji-zhou (40.°1 N, 117.°4 E).

> without that trembling or vibratory motion observd in that Phænomenon. The body of it bore from the Ship SSE: it lasted as bright as ever till near 12 when I went down to sleep but how much longer I cannot tell.

The pertinent parts of pages 326 (last paragraph) and 327 (first paragraph) in the original holograph manuscript journal of Joseph Banks are reproduced in Fig. 1(a). The posthumous printed version of the journal written by Sydney Parkinson contains the following complementary description for the same night (**2**):

> In the night, between ten and eleven o'clock, before the moon was up, we saw a remarkable phænomenon, which appeared in the south quarter, extending one point west, and two east, and was about twenty degrees high, like a glow of red rising from fire, striped with white, which shot up from the horizon in a perpendicular direction, alternately appearing and disappearing.

These two descriptions of the auroral display are largely consistent. Both refer to a (dull) red light or glow in the southern sky, which extended from

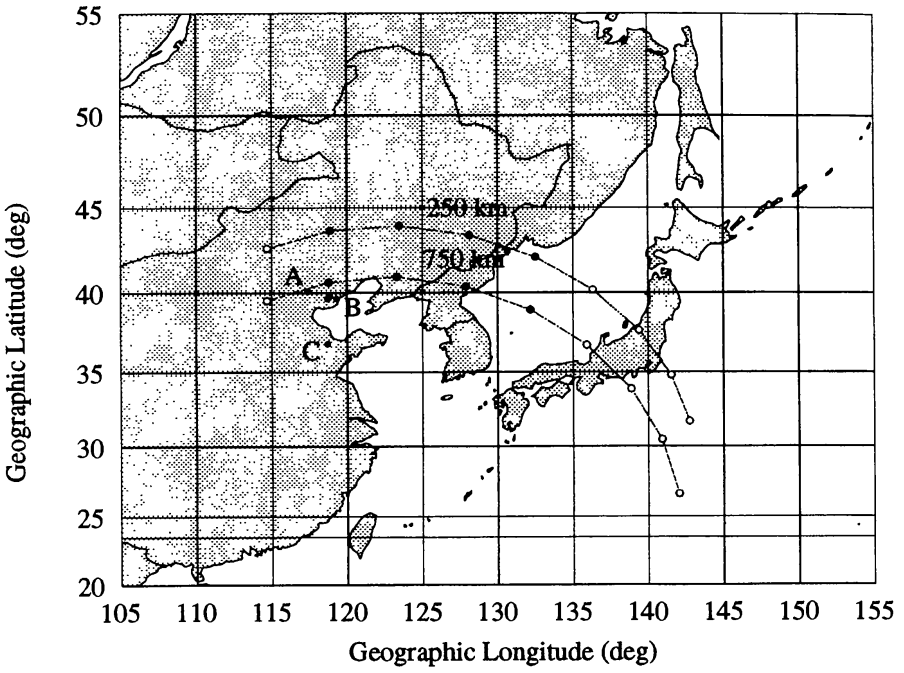
the horizon to a height (elevation) of about 20°. Banks describes "rays of a brighter colourd light tending directly upwards", which passed through and out of the red light. Parkinson says that the glow of the red light was "striped with white" and extended upwards from the horizon in a perpendicular direction. Banks states that the auroral display first appeared at about 22:00 LT and lasted at least until near 24:00 LT; Parkinson notes that the display appeared between 22:00 LT and 23:00 LT. On the basis of these two accounts, it can be claimed with confidence that the aurora definitely existed at 23:00 LT (as it happens, the Moon would not have risen until 03:00 LT on September 17). The time 23:00 LT is used to determine the approximate position of the *Endeavour* during the auroral display, although it should be noted that the ship carried no chronometer (**3**).

There is only one notable discrepancy between the two accounts. Banks indicates that the (azimuthal) extent of the auroral display varied with time but was "never less than 8 or 10 points of the compass", whereas Parkinson claims that the display extended "one point west, and two east". The term *point* refers to the division of the circumference of the magnetic compass into 32 equally spaced points, separated by an azimuthal angle (ψ) of 11°.25 (11°15′). Banks noted that the body of the auroral display "bore from the Ship SSE", and hence his assertion that its extent was "never less than 8 or 10 points of the compass" implies that the minimum range of the azimuthal angle ψ (measured clockwise from north) was approximately $110° \leqslant \psi \leqslant 200°$ (taking an average of 9 points of the compass). Conversely, Parkinson's statement that the aurora "appeared in the south quarter, extending one point west, and two east," implies that ψ was restricted to the approximate range $160° \leqslant \psi \leqslant 190°$.

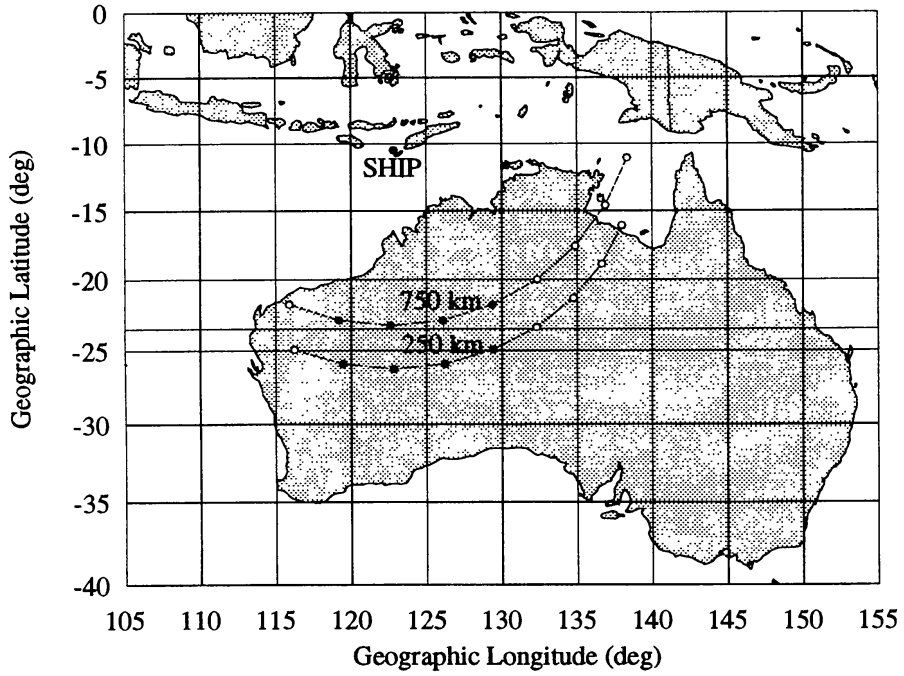
2 APPROXIMATE POSITION OF *HMS ENDEAVOUR*

The journals of Captain Cook (**3**) indicate that, on the evening of 1770 September 16, the *Endeavour* had already passed between the islands of *Rotte* (Rote) and *Anaboa* (Semau). By 18:00 LT the ship was "clear of all the Islands" and at that time "the South part of *Anaboa*, which lies in the Latitude of 10°15′S, bore NE distant 4 Leagues". Once clear of the islands, the ship was steered on "a west Course all night untill 6 o'Clock in the Morning" when "an Island (Savu) bearing WSW" was unexpectedly in sight. The ship was then steered directly towards Savu and "by 10 o'Clock" was "close in with the North side". According to the *Endeavour* Journal of Joseph Banks 1768–1771 (**1**), by 10:00 LT on September 17 the ship was "very near the East end" of the island of Savu. Although the *Endeavour* carried no chronometer (**3**), the approximate times quoted in the various journals and logs are used here solely to determine the proportional distance between two known geographical positions.

Assuming that the *Endeavour* sailed due west in a 'straight line' with constant speed between 18:00 LT on September 16 and 10:00 LT on September 17, its geographic coordinates at 23:00 LT on 1770 September 16 can be estimated to be $10°.45 \pm 0°.1$ S and $122°.82 \pm 0°.2$ E. The estimated position of the *Endeavour* is represented in Fig. 2(b) by a small solid circle



(a) Northern Hemisphere



(b) Southern Hemisphere

FIG. 2. Conjugate auroral observations on AD 1770 September 16: magnetic conjugacy defined by a magnetic field model. (a) The two arcs of small open and solid circles over China and Japan are determined by stepwise numerical integration along magnetic field lines using the magnetic field model of Bloxham & Jackson (see Section 4 for a detailed explanation). (b) The approximate position of *HMS Endeavour* at the time of observation of the *aurora australis* (23:00 LT) is shown by a small solid circle to the

to the south-west of Timor; the radius of this circle provides an overestimate of the uncertainty ($\pm 0.^{\circ}25$) in the position of the ship. An estimated latitude of $10.^{\circ}45 \pm 0.^{\circ}1$ S is consistent with the entry in the journal of Sydney Parkinson, who recorded on 1770 September 16 that the "latitude, by observation, was 10°24′, about four or five leagues from the southernmost part of Timor" (**2**). The estimated uncertainties in latitude and longitude are determined partly by the fact that the speed of the ship may not have remained constant for 16 hours and partly by the fact that it was probably not steered exactly due west after the island of Savu was sighted at 06:00 LT on September 17.

Using the magnetic field model developed by Bloxham & Jackson (**4**), valid for the interval 1690–1840, the magnetic declination (D) at the estimated position of the *Endeavour* at 23:00 LT on 1770 September 16 was $-1.^{\circ}37 \pm 0.^{\circ}05$ E. This numerical estimate for D is corroborated by the maps of declination presented in fig. 8(b) and (c) of the paper by Bloxham & Jackson (**4**). Since D is small, bearings from the *Endeavour*, which were presumably measured with the aid of a compass, can be referred directly to a geographic frame of reference without significant loss of accuracy.

3 OBSERVATIONS OF THE *AURORA BOREALIS*

Chinese, Japanese and Korean historical records have been investigated for references to auroral displays seen around the time of the observations from the *Endeavour*. Chinese and Japanese records prove extremely promising, although bad weather apparently dogged potential Korean observers.

Many local histories (*fang-zhi*) of China record auroral displays on dates corresponding to AD 1770 September 16, 17 and 18 (i.e. the 35th year of the Qian-long reign period, 7th lunar month, 27th to 29th day). These various accounts are conveniently copied in the extensive compilation of early Chinese astronomical records by Beijing Observatory (**5**).

Of most direct interest in the present context are five records on the night of AD 1770 September 16 (the 27th day of the lunar month). Four of these are from counties in Hebei province (fairly close to Beijing). The fifth is from Shandong province. There are similarities between the four accounts from Hebei, but sufficient differences to suggest a degree of independence. The descriptions may be translated as follows:

(i) "At night in the sky due north the five colours like 'rosy clouds' (*xia*) shone on the Earth." (*Yong-ping Fu-zhi*: History of Yong-ping county in Hebei province.)

(ii) "At night in a due north direction, the sky was bright with the five colours. As if rosy clouds shone on the Earth." (*Ruan-zhou-zhi*: History of Ruan-zhou county in Hebei province.)

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south-west of Timor. The poleward circular arc of small open and solid circles over Australia indicates the geographic position of the red light seen on the horizon by both Banks and Parkinson, and the equatorward arc indicates the geographic position of the red light seen at an elevation of about 20°.

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(iii) "At night in the north-west direction, the sky was bright with the five colours. As if rosy clouds shone on the Earth. They were seen everywhere." (*Ji-zhou-zhi*: History of Ji-zhou county in Hebei province.) This description is a translation of the printed Chinese text in the second and third columns (from the right) of Fig. 1(b) (the first column describes the sighting of a comet).

(iv) "At night in a due north direction, the sky was bright with the five colours like rosy clouds. This light shone on the Earth." (*Chang-li Xian-zhi*: History of Chang-li county in Hebei province.)

(v) "At night a red light stretched across the sky." (*Shou-guang Xian-zhi*: History of Shou-guang county in Shandong province.)

The expression 'rosy clouds' (*xia*) used in the first four entries above is very archaic. In combination with other terms it can imply 'red sky in the morning' or a 'rosy sunset'. Here it seems to indicate that, although the auroral display was multicoloured, there was a prominent reddish glow. The counties from which the accounts (i) to (iv) originated are only a few tens of kilometres apart. They have mean geographic coordinates as follows: Yong-ping (39.°8 N, 118.°8 E); Ruan-zhou (39.°7 N, 118.°7 E), Ji-zhou (40.°1 N, 117.°4 E), and Chang-li (39.°7 N, 119.°3 E). It appears that the various observers or scribes compared notes to some extent. The account from Shandong province is obviously independent. The mean coordinates of Shou-guang are 36.°8 N, 118.°7 E.

The various Chinese records from September 17 (eight in number) and September 18 (three) may be summarized by quoting the descriptions in the *Qing-shigao* ('Draft History of the Qing Dynasty'). Translations from chapter 41 of this work by Yau (**6**) – with slight emendations – are as follows:

AD 1770 September 17. Qian-long reign period, 35th year, 7th month, 28th day. "A red light rose in the northern direction; at midnight it gradually receded. To the north-west of the Chang-shan mountain a red vapour was seen filling the sky. Within it there was a white vapour like separated silken threads. After the fourth watch, then they dispersed."

AD 1770 September 18. Qian-long reign period, 35th year, 7th month, 29th day. "At night there was a vapour like fire extending across and covering the north-west. It stretched several tens of *zhang* (many tens of degrees) and contained a red light. It rose in ranks like a forest of swords and spears pointing upwards."

Matsushita has summarized historical information on ancient aurorae seen in Japan since the seventh century (AD 620–1909) (**7**). The chief reference book that he consulted was the *Nippon Kisho-Shiryo* ('Historical Data on Meteorology in Japan'). According to Matsushita, there are 15 independent Japanese reports of an auroral display on the night of 1770 September 17, but there is no mention of a display on the night of September 16. An independent search of the historical records has revealed 14 references to Japanese auroral observations on the night of September 17. Two illustrative descriptions may be translated as follows:

(i) "On the 28th night of the 7th month at 21:00–23:00 in the north there was a red vapour covering the sky, its colour was like fire. In it there were also some strips of greyish (blue/white) vapour of differing lengths, the long ones were four or five *zhang*, in the middle of the sky the short ones were over one *zhang*; they lasted until it became light. This was a heavenly phenomenon which since ancient times has never existed before." (*Zoku Kō Nendai Ryakkei*: 'Notes on the Era of the Emperors: A Continuation').

(ii) "At 19:00–21:00 on the night of the 28th of the 7th month there was red vapour in the north. After a short time it shone east and west, mixed with it were white vapours like the ribs of a fan; it disappeared at 1:00–3:00. Both the public and the astronomers were on the look out for it and in Kyoto there were 7 days ritual fasting." (*Hannichi Kanwa*: 'Brief Discussions').

The most likely source of Korean records of these auroral displays is the *Sunjongwon Ilgi* ('Diaries of the Court Secretariat'), an extensive day-to-day chronicle. The date corresponding to AD 1770 September 18 is the day *xin-wei* in the 7th lunar month of the 46th year of King Yongjo, but no aurora is recorded on this or an adjacent date. The *Sunjongwon Ilgi* regularly reports astronomical observations of all kinds, but is silent on such matters around this time. However, the chronicle notes the fall of heavy rain on September 18. Apparently, overcast skies prevented auroral observations.

4 ALTITUDE AND CONJUGACY OF THE AURORAL DISPLAYS

The descriptions of the *aurora australis* recorded by Banks (**1**) and Parkinson (**2**) both indicate that the red light extended from the horizon to an elevation of about 20°. Presumably, this red light was predominantly 630-nm ('red line') emission from excited atomic oxygen in the ^1D state. Assuming further that the lowest altitude of the visible red aurora was 250 km, the minimum distance from the ship of the red light seen on the horizon was about 1800 km. The poleward circular arc of small open and solid circles over Australia, labelled 250 km (0° elevation) in Fig. 2(b), indicates the geographic position and extent of the red light seen on the horizon, as projected on the Earth's surface using spherical geometry, and the actual geocentric radius (6377·4 km) of the Earth at the estimated position of *HMS Endeavour* (10°.45 S, 122°.82 E). The small open and solid circles essentially represent points of the compass: solid circles define the azimuthal extent of the *aurora australis* according to Parkinson (**2**); open circles indicate the approximate additional azimuthal extent according to Banks (**1**). (The term *arc* is used here in a purely geometrical context and does not imply an auroral arc in the conventional sense.)

The poleward arc of small open and solid circles over China and Japan, labelled 250 km (0° elevation) in Fig. 2(a), defines the points that are magnetically conjugate to the corresponding (compass) points shown in Fig. 2(b). The locations of the conjugate points in the northern hemisphere are determined by stepwise numerical integration along magnetic field lines using the magnetic field model of Bloxham & Jackson (**4**). Each integration is

started at an altitude of 250 km in the southern hemisphere and terminated at the same altitude in the northern hemisphere. Small solid circles labelled A, B and C in Fig. 2(a) indicate the locations of the Chinese visual sightings of the *aurora borealis* on AD 1770 September 16. The circle A signifies the site at Ji-zhou in Hebei province; the cluster of circles B signifies the close sites at Yong-ping, Ruan-zhou and Chang-li, also in Hebei province; and the circle C signifies the site at Shou-guang in Shandong province. All five sites are significantly southward of the arc of conjugate open and solid circles, which specifies the geographic position (as projected on the Earth's surface) of conjugate 630-nm emission from O(^{1}D) at an altitude of 250 km in the northern hemisphere.

The elevation of the conjugate arc of red-line emission at an altitude of 250 km in the northern hemisphere (Fig. 2a) is less than 35° at all five sites in China, which are at least a few degrees to the south of the arc of 630-nm emission. This upper limit to the elevation is quite consistent with the detailed descriptions in the Chinese provincial histories, which state explicitly that the red aurora was seen in the northerly or north-westerly direction. However, if the lowest altitude of the red light seen on the horizon in the southern hemisphere were actually as low as 150 km, corresponding to a minimum distance of about 1390 km from the *Endeavour*, the conjugate arc of red-line emission in the northern hemisphere would have been observed near the zenith from the three close sites in Hebei province and to the south of the zenith from Ji-zhou. Such elevations are inconsistent with the detailed auroral descriptions in the Chinese provincial histories and thus the red light observed on the horizon in the southern hemisphere must have been at an altitude significantly above 150 km. Similarly, if the lowest altitude of the red light seen on the horizon in the southern hemisphere were 200 km, corresponding to a minimum distance of about 1610 km from the *Endeavour*, the conjugate arc of red-line emission in the northern hemisphere would have been observed at an elevation of about 56° from Ji-zhou. This elevation is inconsistent with the textual description presented in Fig. 1(b). Therefore the red light seen on the horizon by both Banks (**1**) and Parkinson (**2**) could not have been at an altitude much below 250 km.

Both Banks (**1**) and Parkinson (**2**) state that the *aurora australis* extended to a height (elevation) of about 20°, which defines the highest altitude of the visible red light in the southern hemisphere. Stepwise numerical integration along magnetic field lines can be used to determine the altitude corresponding to an elevation of about 20°, as seen from *HMS Endeavour*. The equatorward arc of small open and solid circles over Australia, labelled 750 km (≅ 20° elevation) in Fig. 2(b), defines the projected geographic position of points at an altitude of 750 km on magnetic field lines that pass through the corresponding (compass) points on the poleward arc at an altitude of 250 km (= 0° elevation). Actually, the elevation of the points on the equatorward arc (750 km) varies between about 14°.9 and 20°.1, although seven of the nine points lie between 18°.0 and 20°.1. This variation is in acceptable agreement with the approximate elevation of 20° cited by both Banks and Parkinson. Therefore the *aurora australis* could not have extended much above 750 km on AD 1770 September 16, if the corresponding lowest altitude was about 250 km. Since this early observation of the *aurora australis* occurred almost

exactly one year after sunspot maximum (AD 1769·7) (**8**), the conclusion that the 630-nm emission originated from the approximate altitude range 250–750 km is in good agreement with modern observations (**9**, **10**).

The equatorward arc of small open and solid circles over China and Japan, labelled 750 km in Fig. 2(a), defines the projected geographic position in the northern hemisphere of points that are magnetically conjugate to the corresponding points at an altitude of 750 km in the southern hemisphere. The highest elevation of this arc from Ji-zhou is 86°, and it would also appear to pass reasonably close to the zenith if viewed from Yong-ping, Ruan-zhou or Chang-li: the highest elevation from Shou-guang is 57°. Thus the coincident observations of the *aurora australis* and *borealis* on 1770 September 16 are entirely compatible, *provided* that it was customary practice for Chinese observers to describe an auroral display extending from an elevation of less than 35° to the north (or NW) up to almost the zenith as being to the north (or NW) or in the northerly (or NW) direction. Such a convention would be completely consistent with that adopted by modern observers, who specify the angular position in the sky of the *lower borders* of auroral forms (**11**).

5 CONCLUSIONS

Since the conjugate auroral displays described above were observed at low magnetic latitudes in both hemispheres on 1770 September 16, and the *aurora borealis* was seen in China each night throughout the interval September 16–18 (and possibly on September 15 and 19), these remarkable auroral observations clearly coincided with a major geomagnetic storm. This conclusion is corroborated by the fact that auroral displays were observed from several places in both Japan (**7**) and Europe (**12**, **13**) on September 17. To allow for the fact that the magnetosphere can be greatly distorted during a major geomagnetic storm, the stepwise numerical integrations along magnetic field lines, used to derive the arcs shown in Fig. 2(a), have been repeated, utilizing the magnetospheric magnetic field model published by Tsyganenko (**14**). Even for the case of the highest level of geomagnetic disturbance ($K_p \geqslant 5^-$), the equatorward and poleward arcs in Fig. 2(a) are essentially unchanged if the external sources of magnetic field in the model of Tsyganenko are added to the internal magnetic field model of Bloxham & Jackson for epoch AD 1770·7. Therefore the observations on 1770 September 16 of the *aurora australis* made by Banks (**1**) and Parkinson (**2**) on board *HMS Endeavour*, and of the *aurora borealis* made by Chinese observers from sites in Hebei and Shandong provinces (**5**), clearly constitute conjugate auroral observations. These observations, which were made two-and-a-quarter centuries ago during a major geomagnetic storm, constitute the earliest example yet known of conjugate auroral sightings.

ACKNOWLEDGEMENTS

We thank Dr A.Jackson for supplying the computer version of the magnetic field model developed by Bloxham & Jackson (**4**). The original holograph version of the *Endeavour* Journal of Joseph Banks 1768–1771 is held in the Mitchell Library, State Library of New South Wales, Australia,

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