

## THE SURFACE DENSITY PROFILES AND LENSING CHARACTERISTICS OF HICKSON COMPACT GROUPS OF GALAXIES

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### ABSTRACT

A statistical method is developed to infer the typical density profiles of poor galaxy systems without resort to binning of data or assuming a given center to each system. The method is applied to the accordant redshift quartets in Hickson compact groups (HCGs). The distribution of separations in these groups is consistent with a unique generalized and modified Hubble surface density profile, with best-fit asymptotic slope  $\beta = 1.4$  and core radius  $R_c = 18 h^{-1}$  kpc, although a King profile ( $\beta = 1$ ) is also consistent with the data (with  $R_c = 6 h^{-1}$  kpc). These distributions are more concentrated than what has been determined previously for these groups. HCGs are unlikely to act as strong gravitational lenses, but analogous systems 5–10 times more distant should produce a nonnegligible fraction of giant arcs.

*Subject heading:* galaxies: statistics — gravitational lensing

### 1. INTRODUCTION

The 100 compact groups (CGs) cataloged by Hickson (1982) have been among the most systematically observed galaxy systems, and yet there is little consensus on the nature of these systems, between those who believe that most of the Hickson compact groups (HCGs) are roughly as dense in three dimensions as they appear to be in projection (e.g., Hickson & Rood 1988) and those who argue that CGs are mostly chance alignments of galaxies along the line of sight, within larger “loose” groups (Mamon 1986) or cosmological filaments (Hernquist, Katz, & Weinberg 1995).

The nature of CGs may be clarified through the characterization of their galaxy surface number density profiles,  $\Sigma(R)$ . Indeed, if CGs are caused by chance alignments within much larger systems, then the galaxy positions should be relatively random, and the corresponding density profiles should be roughly homogeneous. The galaxy number density profile in CGs is also interesting by itself, as compared with the density profiles of the gaseous and dark matter components, in connection with the theory of the formation and evolution of galaxies within groups.

The small number of galaxies (typically four) per HCG makes it very difficult to estimate the density profile. Hickson et al. (1984) have shown that all 100 HCGs, rescaled to the same size, show a concentrated density profile. More recently, Mendes de Oliveira & Giraud (1994, hereafter MG94) have argued that HCGs, once rescaled to a common size, are consistent with a King (1962) surface density profile, with typical core radius of  $15 h^{-1}$  kpc. Unfortunately, such studies suffer from the uncertainty introduced by the definition of a center for a system of typically four galaxies, and from the assumption that this center is at the galaxy barycenter. Never-

theless, there are statistical methods to determine the density profiles of discrete systems that do not make use of the definition of the system center, relying instead upon the distribution of projected particle (galaxy) separations (see the mathematical foundations in Salvador-Solé, Sanromà, & González-Casado 1993, hereafter SSG, and Salvador-Solé, González-Casado, & Solanes 1993, hereafter SGS).

Alternatively, one can obtain good estimates of the surface density profiles of the X-ray-emitting diffuse intergalactic gas, which, so far, has been firmly detected with the *ROSAT* Position Sensitive Proportional Counter in seven HCGs out of 17 pointings (see Mulchaey et al. 1996, hereafter MDMB, and references therein), suggesting that those HCGs are physically bound entities. These *ROSAT* observations reveal projected gas density profiles consistent with  $\Sigma_{\text{gas}}(R) = \Sigma_{\text{gas}}^0/[1 + (R/R_c)^2]^{(3\beta_{\text{gas}}-1)/2}$ , and with core radius  $4 \leq R_c \leq 30 h^{-1}$  kpc and  $0.38 \leq \beta_{\text{gas}} \leq 0.92$  (see MDMB and references therein). In comparison, Abell clusters have X-ray core radii typically 15 times larger (Jones & Forman 1984; Mushotzky 1994).

The efficiency of a structure as a gravitational lens is determined by its projected mass density profile,  $\Sigma_{\text{mass}}(R)$ , and the relative lens-observer and source-observer distances (Bourassa & Kantowski 1975). Extrapolating the small intergalactic gas core radii to the distribution of total binding mass suggests that CGs may be concentrated enough to be candidates for gravitational lenses (MG94). If the binding mass in groups is more concentrated than the intergalactic gas, as is the case in clusters (Henriksen 1985; Hughes 1989; Durret et al. 1994), the lensing efficiencies of groups are enhanced.

Hence, various signatures of gravitational lensing (giant arcs, arclets, shear, apparent magnitude enhancement) in CGs of existing catalogs (Hickson et al. 1992; Prandoni, Iovino, & MacGillivray 1994) or of future deeper surveys, if observed, would be of major interest because they would allow a quantification of the dark matter distribution in CGs through a method independent of X-ray analyses. This quantification has important consequences for the theories of formation and evolution of galaxies and their groupings in the framework of the large-scale structure of the universe.

In this Letter, we implement a statistical method to determine average galaxy density profiles in the case of poor

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statistics, with no assumed group center. The mean galaxy surface number density profile of compact groups is computed from these statistical methods, and the implications for the lensing properties of compact groups are studied.

## 2. THE GALAXY SURFACE DENSITY PROFILE

For a catalog of groups (whose centers are uncorrelated), the galaxy two-point correlation function,  $\xi(s)$ , is (Peebles 1980)

$$\xi(s) = \frac{\langle N_i^2 (\Sigma_i * \Sigma_i)(s) \rangle n_G}{n^2}, \quad (1)$$

where  $\Sigma_i(s)$  is the projected galaxy density profile of the  $i$ th group normalized to unity,  $s$  is the projected intergalactic separation,  $N_i$  is the number of galaxies in the  $i$ th group,  $n_G$  and  $n$  are the mean number density of groups and galaxies, respectively, the angle brackets are the average taken over the groups of the catalog, and  $(\Sigma_i * \Sigma_i)(s)$  is the convolution of the projected density profile of the  $i$ th group, which can be calculated by means of the deconvolution method (SSG; SGS):

$$(\Sigma * \Sigma)(s) = \mathcal{F}_1 \circ \mathcal{A} \{ \mathcal{A} \circ \mathcal{F}_1^{-1} [2P(\geq s)] \}, \quad (2)$$

where  $\mathcal{F}_1$  and  $\mathcal{A}$  are the one-dimensional Fourier and Abel transforms, respectively, and  $P(\geq s)$  is the fraction of galaxy pairs with intergalactic separation  $\geq s$ .

The average galaxy surface density profile for groups in a given catalog can be determined through a fitting procedure by means of equations (1) and (2) above. Let us assume that the groups in the catalog are discrete realizations of an analytical profile,  $\Sigma(R)$ . We use here the *generalized modified Hubble profiles* (GMHPs; see Jones & Forman 1984), whose shape is characterized by the core radius,  $R_c$ , and the slope,  $\beta$ :

$$\Sigma(R; R_c, \beta) = \frac{\Sigma_0}{[1 + (R/R_c)^2]^{(3\beta-1)/2}}, \quad (3)$$

where  $\Sigma_0$  is the central projected density. Our free parameters  $R_c$  and  $\beta$  are obtained by calculating  $\xi^{\text{data}}(s)$  for the data catalog and  $\xi^T(s; R_c, \beta)$  for the analytical profile, using equations (1) and (2), and by minimizing

$$X^2(R_c, \beta) = \frac{1}{N_{\text{sep}}} \sum_{k=1}^{N_{\text{sep}}} \frac{[\xi^{\text{data}}(s_k) - \xi^T(s_k; R_c, \beta)]^2}{\sigma_k^2}, \quad (4)$$

where  $N_{\text{sep}}$  is the number of bins of intergalactic separations,  $s_k$ . The dispersions  $\sigma_k^2 = \langle (\xi_k - \langle \xi \rangle)^2 \rangle$ , required in evaluating equation (4), are evaluated from Monte Carlo simulations of artificial group catalogs, generated with GMHPs, with the same number of galaxies per group and of groups per catalog as the real group catalog. Since the different  $\xi_k$  are not independent random variables, the number of degrees of freedom of  $X^2$  is not known, and we have to resort to Monte Carlo simulations to evaluate empirically its distribution (instead of the usual  $\chi^2$  statistics), for estimating confidence levels. Note that only information on the projected intergalactic separations is required in the deconvolution method. This circumvents the delicate problem of determining the center of the groups. Moreover, noise suppression by filtering is also implicit in this method.

In catalogs of groups, one has a low number of galaxies per group and of groups per catalog. In this case of *poor statistics*, the reliability of the method has been tested with simulations

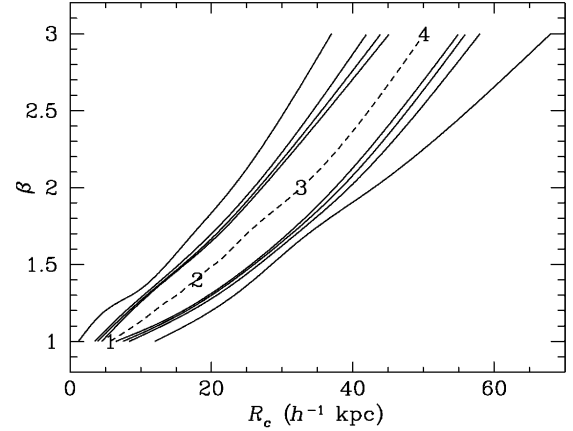


FIG. 1.—Best-fit surface density profile (eq. [3]) parameters of HCGs, for confidence levels of 100%, 95%, 90%, and 85% (solid curves) and curve of optimum fits (dashed curve). The numbers refer to our adopted models (with “2” our best solution).

of artificial catalogs whose groups are realizations of GMHPs with fixed  $\beta$ . Our results show that the method is able to recover the input value of  $R_c$  to a typical 90% confidence precision of a factor 1.6, and to distinguish between different values of  $\beta$  in the case of poor statistics (Montoya et al. 1995).

The method explained above has been applied to the HCG sample of 42 accordant redshift quartets (Hickson et al. 1992). We have tested the null hypothesis  $\mathcal{H}_0$  that the groups of this sample are realizations of a unique GMHP with  $1 \leq \beta \leq 3$ . For each value of  $\beta$ , we have calculated  $X^2$  (eq. [4]) for the sample, and the cumulative probability  $F(X^2)$  from Monte Carlo simulations. The resulting confidence levels in the  $(R_c, \beta)$ -plane are plotted in Figure 1. The optimum fit is obtained for  $\beta = 1.4$  and  $R_c = 18 h^{-1}$  kpc (hereafter, model 2), corresponding to  $F(X^2) = 0.16$ . For  $\beta = 1, 2$ , and 3, the best fit is for  $R_c = 5.7, 32.7$ , and  $50.1 h^{-1}$  kpc (models 1, 3, and 4, respectively). Our fitted surface density profiles have been obtained without any rescaling of the sizes of the groups. Such rescalings are not necessary because we get good fits without them. To show this, we have compared the distributions of the minimum, median, and maximum intergalactic separations for each group in the catalog with those for artificial catalogs of 42 quartets, each of them being a realization of a unique GMHP with  $\beta = 1.4$  and  $R_c = 18 h^{-1}$  kpc. Table 1 shows the median Kolmogorov-Smirnov probabilities of not being able to reject the hypothesis that the distribution of separations of the catalog are consistent with that of the Monte Carlo catalogs with  $\beta = 1.4$  and  $R_c = 18 h^{-1}$  kpc. We conclude that any rescaling (using the minimum, median, and maximum intergalactic separations as typical lengths) leads, indeed, to much poorer fits, whatever values of both  $\beta$  and  $R_c$ .

MG94 have also computed the galaxy surface number density profile for a sample of HCGs. Their fitting procedure requires a group center (for which they adopt the barycenter), and, moreover, they scale all groups in their sample to a

TABLE 1  
KOLMOGOROV-SMIRNOV TEST PROBABILITIES

	$S_{\text{min}}$	$S_{\text{med}}$	$S_{\text{max}}$
Simulation-simulation .....	0.987	0.909	0.752
Simulation-catalog .....	0.752	0.256	0.909

common size by normalizing each group to the largest galaxy distance to the group barycenter,  $R_{\max}$ . Both of these operations introduce important uncertainties in their results. On one hand, the center of a system of few (typically four) galaxies is not well defined. Beers & Tonry (1986) have shown that any possible inner galaxy concentration is washed out when the barycenter is used as the center of galaxy clusters. So, given  $\beta = 1$ , MG94's core radius should be much larger than ours, and indeed it is 2.5 times larger (Fig. 1). On the other hand, it is not clear that groups should have a normalized common size and that  $R_{\max}$  should be a useful measure of that size. Instead, our results show that our sample of HCGs can be considered as a realization of a unique profile, and that the dispersion in  $R_{\max}$  arises from statistical fluctuations.

The galaxy density profiles found here are steeper than the gas density profiles obtained from X-ray surface brightness data (see § 1). Qualitatively, this can be expected in the framework of the  $\beta$ -model (Cavaliere 1974), which predicts  $\beta_{\text{spec}} = \sigma_v^2 / (kT / \mu m_p) = \beta_{\text{gas}} / \beta_{\text{gal}}$ . Indeed, the galaxy system is cooler than the X-ray-emitting gas, since HCGs have  $\beta_{\text{spec}} \simeq 0.7$  (Mamon & Henriksen 1996). The typical values of  $\beta_{\text{gas}} = 0.6$  and our best fit of  $\beta_{\text{gal}} = 1.4$  are consistent with  $\beta_{\text{spec}}$  in the context of the  $\beta$ -model, given the uncertainties in these three parameters. Hydrodynamical simulations of X-ray cluster formation, including galaxy formation (e.g., Frenk et al. 1996), also predict a velocity bias and spatial gas/galaxy segregation.

### 3. LENSING BY COMPACT GROUPS

Due to their peaked density distributions (§ 2), HCGs could constitute efficient gravitational lenses. An HCG at angular distance  $D_l$  with central projected mass density  $\Sigma_{\text{mass}}(0)$  may produce an elongated arc image of background sources at a distance  $D_s$  if (Bourassa & Kantowski 1975)  $\Sigma_{\text{mass}}(0) \geq (c^2 / 4\pi G)(D_s / [D_l D_{ls}])$  (strong lensing condition, hereafter SLC). For  $\Omega = 1$  and a smooth universe, the angular distance is  $D_i = 2(c/H_0)(G_i - 1)/G_i^3$ , where  $G_i = (1 + z_i)^{1/2}$ ,  $i = l, s$ , and one can write (Blandford & Kochanek 1987)  $D_{ls} = 2(c/H_0)(G_s - G_l)/(G_s^3 G_l)$ . The SLC imposes, for a given lens, a minimum source redshift:

$$z_s^{\min} = \left[ \frac{AG_l(G_l - 1) - G_l^4}{A(G_l - 1) - G_l^4} \right]^2 - 1, \quad (5)$$

where  $A = 8\pi G \Sigma_{\text{mass}}(0) / (cH_0)$ . A physical solution  $z_s^{\min} > z_l$  is obtained, after some algebra, directly from the SLC when  $\Sigma_{\text{mass}}(0) > (1 + z_l)^{1/2} [c^2 / (4\pi G D_l)]$ . Assuming that the galaxy system has isothermal and isotropic kinematics, one can solve for the total mass density in the Jeans equation, and integrating over the on-axis line of sight, one finds  $\Sigma_{\text{mass}}(0) = 3\beta\sigma_v^2 / (2GR_c)$ , where  $\sigma_v$  is the group one-dimensional velocity dispersion. Dropping the factor  $(1 + z_l)^{1/2} \simeq 1 + z_l/2$ , the condition for  $z_s^{\min} > z_l$  can now be written as

$$D_l \left( \frac{\sigma_v}{c} \right)^2 > \frac{R_c}{6\pi\beta}. \quad (6)$$

In Figure 2, we plot (*thick histograms*) the cumulative fraction of HCGs<sup>5</sup> from our sample for which the product  $D_l(\sigma_v/c)^2$  exceeds different thresholds (eq. [6]), marked by arrows. For model 1, three HCGs (8, 60, and 82) in our sample verify equation (6), implying (eq. [5])  $z_s > 0.19, 0.20$ , and 0.08,

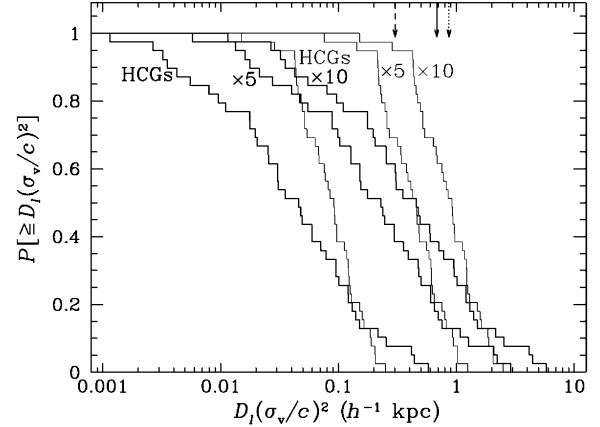


FIG. 2.—Cumulative fraction of HCGs with values of  $D_l(\sigma_v/c)^2$  greater than the given thresholds (eq. [6]), for our sample of HCGs and for the same samples placed at 5 and 10 times their original distances. The arrows show the thresholds for strong lensing satisfied for models 1 (*dashed arrow*), 2 (*solid arrow*), and 3 (*dotted arrow*). The threshold for model 4 is similar to that for model 3. The thick histograms use measured velocity dispersions, while the thin histograms use  $\sigma_v = 315 \text{ km s}^{-1}$ .

respectively. For our other models, no HCG in our sample fulfills the strong lensing condition. The three HCGs that are potential strong gravitational lenses have much higher line-of-sight velocity dispersion than the average (517, 472, and 714  $\text{km s}^{-1}$  compared with a median of 237  $\text{km s}^{-1}$  for our HCG sample), and this could easily be due to sampling and projection effects, as is suggested by the lack of HCGs detected in X-rays with gas hotter than 1 keV (MDMB). Put another way, since most HCGs detected in X-rays have temperatures close to 0.9 keV (see MDMB), with Mamon & Henriksen's (1996) estimate of  $\beta_{\text{spec}}$ , we infer *true* one-dimensional velocity dispersions of  $\sigma_v = 315 \text{ km s}^{-1}$ . The thin histograms in Figure 2 illustrate the situation if *all* HCGs have  $\sigma_v = 315 \text{ km s}^{-1}$ , and we infer that no HCGs induce strong gravitational lensing.

HCGs are relatively nearby ( $z_{\text{median}} = 0.03$ ), and future catalogs should provide more distant CGs. In light of this, we have considered a sample of CGs with the same properties as before, but with  $D_l\sigma_v^2$  multiplied by a factor of 5 and 10, and we have plotted in Figure 2 the corresponding cumulative fractions of CGs whose  $D_l(\sigma_v/c)^2$  values exceed the same thresholds as before. For model 1, the strong lensing criterion is fulfilled for 38% and 62% of equivalent CG samples 5 and 10 times more distant, respectively, using the measured velocity dispersions for  $\sigma_v$ , and 69% and 95%, respectively, using the canonical value  $\sigma_v = 315 \text{ km s}^{-1}$ . Thus, we obtain more efficient lensing in comparison with MG94, who, with the same profile, predict that 37% of a sample of HCGs 10 times more distant will produce strong lensing. For our best-fit model 2, we expect strong lensing for 18% and 38% of our sample, respectively (see Fig. 2), using the measured velocity dispersions, and 21% and 64% with  $\sigma_v = 315 \text{ km s}^{-1}$ . These results confirm MG94's suggestion that CGs in deeper surveys (and/or with larger velocity dispersion than for HCGs) could turn out to be efficient strong lenses.

Lens systems cause an enhancement of the apparent magnitude of a source placed behind them. Assuming a perfect alignment and  $D_s = 2D_{ls}$ , the fraction of HCGs that amplify background sources by  $\geq 0.5 \text{ mag}$  is 21% for model 1 and 8% for models 2 and 3 using measured  $\sigma_v$ 's, and 26% for model

<sup>5</sup> Correcting Hickson et al.'s (1992) velocity dispersions by  $[n/(n-1)]^{1/2}$ .



1 and negligible for the others using  $\sigma_v = 315 \text{ km s}^{-1}$ . In comparison, MG94 find 16%.

#### 4. DISCUSSION

The uniqueness of the absolute surface density profiles of HCGs may seem puzzling when considered within the hierarchy of galaxy systems. It arises from the characteristic mass density and velocity dispersion required for virialized groups, from a combination of sharply decreasing cosmological distributions of these properties and minimum cutoffs for virialization (Mamon 1994). Consequently, virialized groups have a characteristic radius and, by extension, a characteristic density profile.

Because our method for determining the density profiles of groups does not make use of a choice for a group center, relying only on the projected separations, our fitted surface density profiles are more concentrated than that obtained by MG94. Our profiles are even more concentrated than for most HCG diffuse intergalactic gas distributions, as determined from X-rays. Part of this inferred concentration may be due to small-scale correlations, i.e., binarity, in the galaxy distribution (see Walke & Mamon 1989). Note that, whereas Hickson

(1982) selected only the bright galaxies, the spectroscopically confirmed faint members of HCGs have somewhat larger projected separations (S. Zepf 1996, private communication).

Although the lensing efficiency of HCGs is highly model dependent, we confirm MG94's result that HCGs should not constitute strong lenses, as confirmed by the absence of giant arcs in CCD images of HCGs (see Hickson 1993). CGs in 5–10 times deeper surveys should produce a few giant arcs. Our results do not exclude that a fraction of HCGs are due to projection effects from loose groups or long filaments. In this case, real groups in Hickson's catalog should be even more concentrated, and our conclusions on lensing would be reinforced.

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